Logic for CS
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Lecture - 09
Consistency and Completeness

Hey, we were looking at the tableau method. And now, we should start moving towards a doing something is that are less implementation and more formal oriented. So, one of the, I means first thing that I will do is in terms of. For, any logic a there are certain soundness and completeness requirements that you usually want to prove. And, so this so first thing we will do the tableau method is to prove a consistency and completeness. And, then will get on to formal proof theory after this. And, then try to prove the soundness and completeness proof theory and may, be we will also look at the certain other properties like compactness and so on. So, we lets west establish some of the notation.

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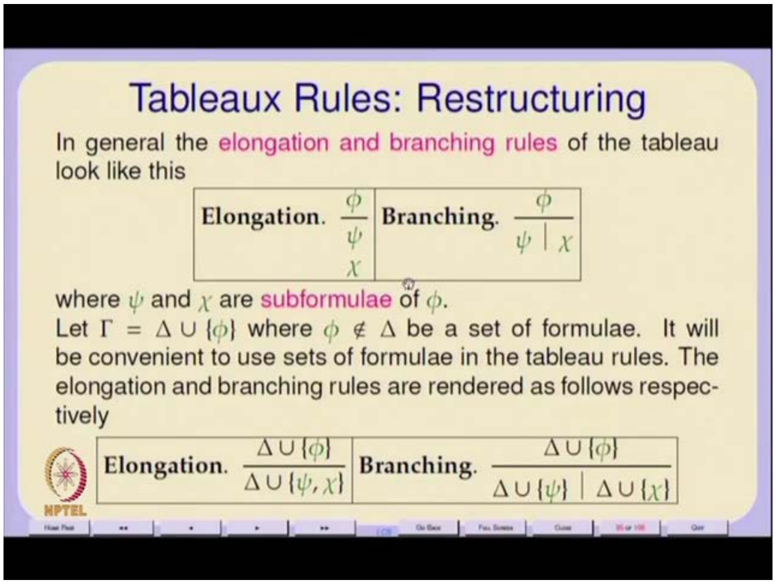

So, one thing is that first thing I would like to do is to restructure the Tableau Rules. So, in general we had these tableau rules for proposition logic.

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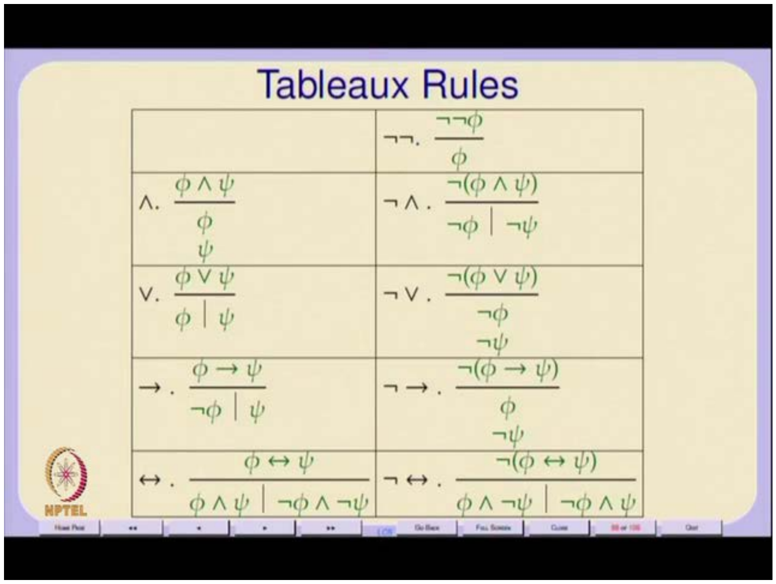

And, essentially what we said was that you creator a tree a tableau is a tree in which it is called and I mean it the tableau actually comes from a notion of the semantic tableau.

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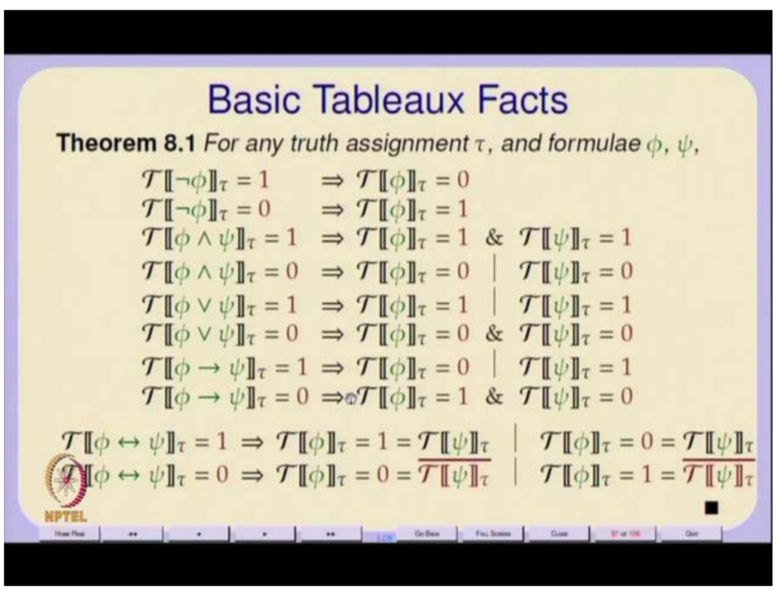

Because, if you remember what it was inspired essentially by this identities which directly related to the notion of the truth or false. So, however this tableau rules go beyond a notion of a user's semantic tableau in the sense that they satisfy an extra property. And, that is that in general the rules are such that there is a numerator and denominator or one or more denominators. And, the denominator except for the case of this last by conditioner you can think of this denominator as an essentially being sub formulae of the numerators. To extend that, sometime occurs the notion or negation is used instead of assigning truth and false rule which is what happens in a semantic tableau. So, in an analytic tableau sometime there are negation symbols which needs to be introduce a like for example this naught psi here in this denominator is clearly not a sub formula of the numerator.

However, modulo the appearance of negation symbols they essentially satisfy what is known as sub formula property. And, that is the denominators are the usually sub formulae of the numerator and that is what. So, what this gives is and the negation rule ensures that you do not have to many negations appearing. And, what it all say ensures therefore is that, your tableau will always is always guaranty to be finite tree and I mean in this you cannot go beyond finite tree. And, that finite tree is determined by the some of the sizes of the formulae that your that was there in the root of the tableau that originally there in the tableau.

So, and tableau rules also have two kinds of rules when Smallian originally defined them he called them Alfa and beta rules. I preferred to call them elongation and branching because that is a way I that is that helps me not to get confused as for as an nobody calls it an elongation and branching. So, but he called them Alfa and beta rules and basically Alfa rules of a conjunctive kind of an and kind and beta rules where, of a disjunctive. So, essentially the elongation rules for would be called Alfa rules and the branching rules would be beta rules. And, since you are dealing binary operators which remains me also. This, you think of all this as single operators it is not necessary to necessarily consirable them as two different operators.

But, since we normally write them like, this express them in terms of two different operators but they could be consider to be a single operator. So, it has nice structure that basically for all operators and their negations there are rules which are either of an elongation type or of branching type. And, in a case like this there are both rules are branching in the case of double negation you might think of it either as an elongation rule or as branching rule it does not matter its truly in either case. But, very often it is I mean whenever its conveniently I will be think of it does not an elongation root at as time some at think of it as branching root.

So, essentially in that sense so all these elongation rules of this kind and the branching rules are this kind. And, what we can do is in order to what we can do is we can restructure the tableau it is. So, essentially what we think of we can think of tableau whereas, the rules as rules something a very similar to the sets of literals of literals sets of sets of literals which we use for resolution. So, one way of restructuring the tableau rules in terms of sets of formulae which is what I essentially what we are saying is when you have an argument that argument is converted into a set of formulae. And, you want to prove either the unsatisfiability of that argument or you want to prove there is a false kind of assign assignment for the data.

So, in either case we can think of elongation rules as essentially being of these kind. So, if there is an elongation rule in the original tableau calculus of this kind. Then, essentially we can think of an elongation rule in which it start with a set of formulae. Of, which in which a set of formulae gamma in which we choose 1 formula phi to which to apply either an elongation rule or branching rule depending upon the what the operator what the root operator of that formula is. Now, depending on that root operator of that formula if it is ppsi and if it is an elongation rule then, we can we can just think of gamma as not having phi anymore but, instead having the two denominators ppsi and phi. In, the case of Branching what happens is that you can think of it as creating two separate sets remove phi and one set of possibilities is delta union psi. And, the other set of possibilities delta union phi.

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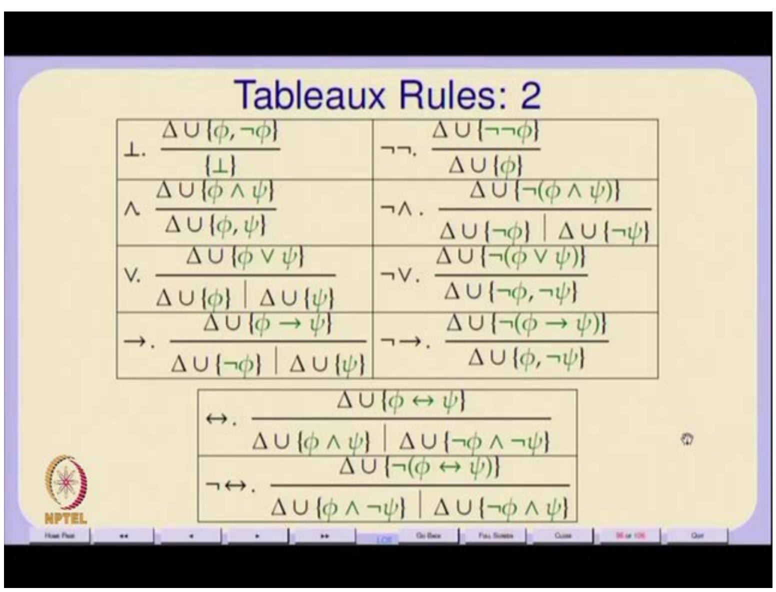

And, we could actually this kind of restructuring will help us in a certain way which it is convey mix a certain proofs low so it makes the tableau rule look ugly it actually is obvious. It also allows to put in an extra rule which I just stated verbally you know that of that of closure of a tableau closure of a path in that tableau and, that is made clear by this. So, essentially what we have saying is usually this rule expressed as a complementary pair of literals. But, I see absolutely no reason why we cannot just take. Since, we are interested in the forms in the shapes of formulae if, it is obvious the there are two formulas in the original set gamma. Where, one formula can be is in syntactically identical to the negation of the other. There, is absolutely no reason why we cannot considered to be complement pairs.

So, what I have done instead of using atoms and atoms p and naught p I am saying that you just look at the patterns of the formulae. If, you find one pattern which is a negation of the other in the set of formulae then that set of formulae is essentially inconsistent. And, therefore essentially that entire branch of the tableau can be closed with this part. Now, one of the things that so there is so now all the rules can be expressed in terms of these sets of formulae. And, it allows us there is an interesting thing which is that. Now, I do not need to look at the entire trees that actually generated I need to look at only they will lease of that tree. Because, I am anyway carrying all the formulae in that sense you can think of that the tree construction as an updation of set of formulae which from an implementation of view actually is quite efficient. So, firstly we do not need to consider we do not need to actually construct trees.

Because, you can think of these this horizontal line and this vertical line. As, essentially boundaries which cannot be crossed. Which, was not so in the case of this in the case of rules expressed in this form. In, the case rules expressed in this form you always cross this horizontal line boundary. Because, you wanted to look for complementary pairs along the entire history of the tableau. But, you are not allowed to cross this vertical line I mean you are not allowed. In, the two parts on either side of the vertical line were not allowed to interact. Because, no influence between them once branching was done essentially they went their independent way. They probably branch for that but, this was like a barrier between two parts of the tableau. In, this case what we are saying is by carrying all these a formulas in both numerator and denominator.

Essentially, we are saying that this is like just updation. And, I can regard this horizontal line also has barrier I should not go above this horizontal line ever I, have to only look at all the formulas at the leaf of the tableau. So, everything else in the tableau is just history and it is not important. So that, is what these rules give us so they allow us to actually create this barrier. And, that the entire tableau essentially historical entity of which what is corrects are only the leaves. Where, each leaf each node of the tableau is decorated by is labeled by a set of formulae. And, each step of the tableau can be regarded as an updation of state of that of the set of formulae. So, otherwise these rules I exactly like the original rules.

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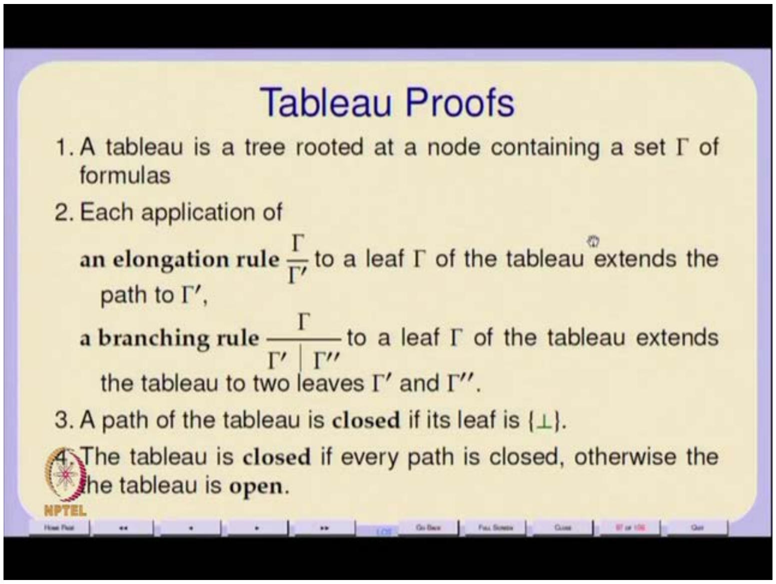

So, now with these with these new set of rules a tableau is tree rooted at a node containing a set gamma of formulas each application of an elongation rule of the form gamma down gamma prime to leaf. Notice that now, there is a gamma prime because elongation rules instead of two formulas became the set of formulae set of two formulae to a leaf gamma of an elongation rule to the leaf gamma of the tableau extends the path to gamma prime. And, a branching rule to in actually there should be a to include gamma prime may be. But, no it is extents to path to gamma prime that is perfect. A, branching rule of this form to a leaf gamma of the tableau extents the tableau two leaves gamma prime and gamma double prime.

A, path of the tableau is closed if its leaf is bottom this is something that was not there in the original set of rules I only stated it. But, now we actually do have rule for it and therefore that is basically what we are saying is you cannot extent that path anymore there is absolutely no rule to which can be applied to the symbol. So, the tableau is closed if every path is closed so essentially we should have got bottom a set of bottom of leaves all of which are bottom otherwise, the tableau is open which means that there is at least on open path such that to that leaf of that open path you cannot apply any rule.

So, a tableau which is I am not but it will come later so we will say a completed tableau is 1 is a tableau in which no rule is applicable on any of the leaves. So, a completed tableau could have either closed paths or even and some open paths.

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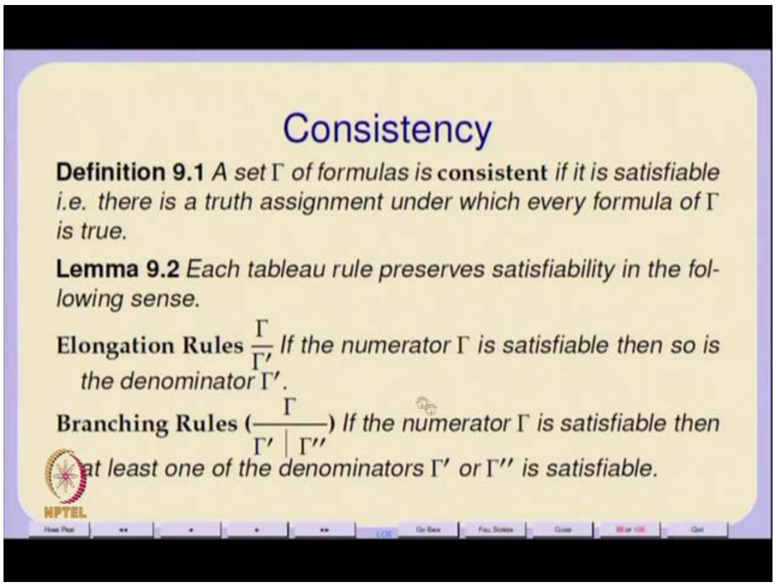

So, now as I said the problem or validity of

Student: (Refer Time: 15:43) being complete but having ones.

We, will come to that, is has to do that satisfiability. So, a set gamma of formulas since the notion of validity of an argument and logical consequences so one has been reduced to that has satisfiability of a set of formulae. We, will say a set of formulas just consistent if it is satisfiable that means there is a truth assignment under which every formula of the set is true for the same truth assignment. So, one thing is Each tableau rule preserves satisfiability in the following sense. Now, that we are dealing with sets of formulas we have to look at the notion of satisfiability in a more precise fashion for set of formulas. In, the case of an Elongation Rules if the numerator gamma is satisfiable then so is the denominator gamma prime. In the case of Branching Rules if the numerator gamma is satisfiable then, at least one of the denominators is also satisfiable. So, there is so the construction of the tableau in effect gives you something which is disjunctive in nature. So, you are talking about essentially or of ands of formulae's before actually going into negation without actually going into normal forms. So it is actually a disjunction of conjunctions and so it is satisfiable if at least one of the denominators is satisfied.

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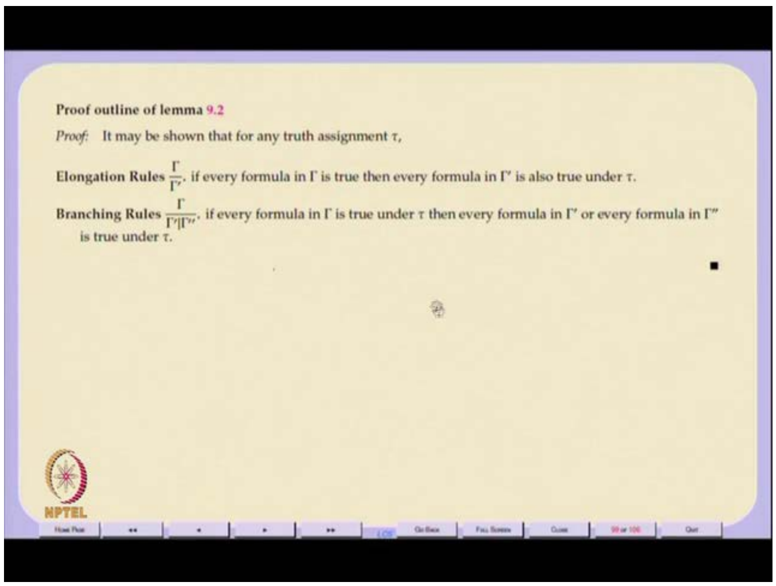

Now, so one thing is so you can that the proof is very simple you just take an arbitrary as truth assignment which for which gamma is true. And, show that therefore if I chosen phi formula phi in that gamma is true. And, an elongation rule has been applied then, by case analysis you know the denominators you can show that each of them would also be true if that original phi was true. Similarly, in the case of branching rules again you can do a case analysis on the operators which are actually branching which actually give you branching rules. And, show that if one of them is true then, if the original formula phi is true then the two derived formulae of the two derived formulae psi and chi at least one of them would be true.

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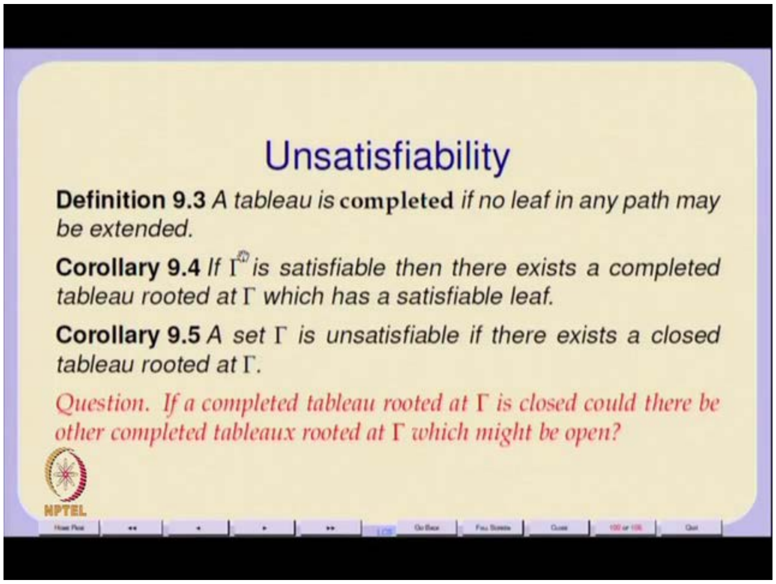

So, we want go into too much so we would say a tableau is completed, if no leaf in any path may be extended which, means there is absolutely no rule that is applicable on any of the leaves. And, if gamma is satisfiable then, there exists a completed tableau rooted at gamma which has satisfiable leaf. So, complete that should answer your question a completed tableau in which you are not derived. In, which there is at least one path that is not have bottom minute as leaf in it is a satisfiable leaf. And, if there is roof if there is no rules applicable. And, a set gamma is unsatisfiable if there exists a closed tableau rooted at gamma. Which, means every leaf every path ends in a bottom then the tableau is then the original set of formulae is gamma that you started with unsatisfiable.

So, the mean question in this, particular cases is forgiven set gamma there could be many different tableau. Because, it all depends on what formula phi in the set of formulas I choose as a candidate for on which to apply a rule. So, that is very highly nondeterministic choice which, can allow for multiple tableaus. And, what we have set here is that essentially if I can find one close tableau then we will say that the set gamma is unsatisfiable that is what says. But, then there is question of is it true that there exists other tableaus for the same set gamma. Which, have open a, completed tableau which have open paths when that is some. So, issue completeness is really this question in the case of tableau consider all the tableaus that I have.

And, the second issue the more important completeness actually is the question of take any tautology of proposition logic can I prove that it is tautology using tableau method. The issue of completeness there is can I proof for every tautology that there is a tableau which will establish that it is a tautology. In, order to establish that has formula is tautology it is necessary for me to be able to take the negation of the formula. And, use the tableau method to prove that it is unsatisfiable. If I, can proof supposing for every tautology there does exist a tableau which proves it. Which, means which prove that negation is unsatisfiable is a, guaranteed that every tableau for that formula is also guaranteed to unsatisfiable. If, you can prove this then essentially then, the question of completeness for logical arguments is automatically answering. If, every tautology can be proven using the tableau method then, every valid argument the validity of every argument can be established.

So, essentially these are the questions which we require some basic machinery to be established at proposition logic level itself. So, it becomes essay for us to handle this question at when we come with more complicated logics. So, it will seen obvious to you at proposition logic level. But, we will bring in some machinery which will use later that will also for other logics. So, the main question now is the so the main questions are essentially, Can I establish the tautology tautologussnes of every tautology for an give tautology?

Is, there a tableau which does not established it is tautologussnes. And, if so then what do I do with it? And the second important question which is hidden. But, is as an important as we did for the case of tautology checking is. Supposing, I got formula which is not tautology, Then, can I prove that it is not tautology by getting a completed tableau and truth assignment its negation? Supposing I have formula phi which I want to be prove be a tautology which means an attempt a tableau the formula naught phi. Now, if phi is not tautology then not phi is not contradiction if not phi is not contradiction then, it has satisfying truth assignment.

And, therefore is possible to satisfying the truth assignment this is equivalent to establishing a counter example when you claim that something that not hold. So, hidden in all this notion of completeness and are these two facts not just whether, I can prove all tautologies are complete. But, also for all formulae which are not tautologies can I establish a satisfying truth assignment. So, these are important questions and then, the next question is given that a tautology even if every tautology is provable by tableau those tableau is for those tautologies which are completed which are not closed. So, this for corollary 9.5 point just says that. A set is unsatisfiable if, there exist a closed tableau rooted at gamma. Are there other tableaus rooted at gamma which are not closed I mean that is so, these are the kinds of questions for which we have to establish machinery to answers these questions. So, the issues of completeness and so are not complete are not totally obvious there are deeper things. One is that a finding counter example another is that of checking whether, I mean is this corollary only says it is like looking needle an infinity hairstyle. I mean how do I know that there are in other tableaus which, are completed tableaus which are not closed? And, I have to establish those things.

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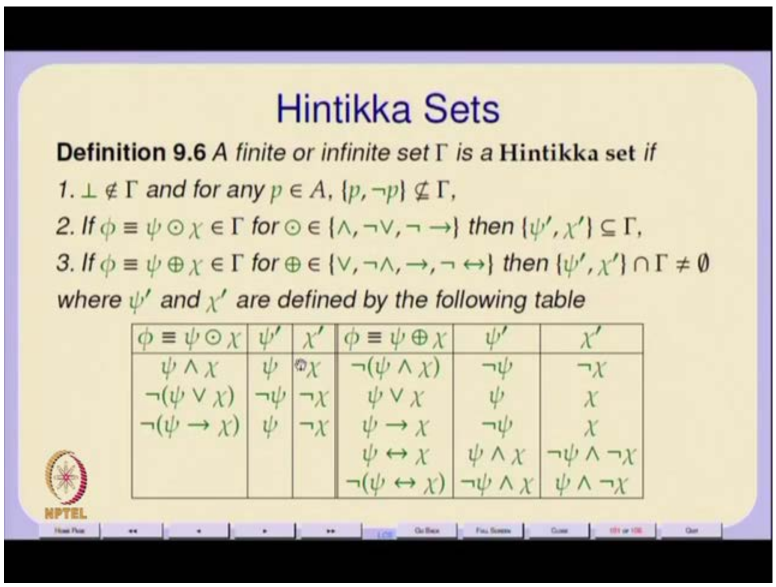

So, the machinery will start with is something known as Hintikka Sets. So, finite or infinite set by the way now it does not when we looking at this, when we looking at tableaus of course we are looking at finite sets. But, in general you can have an infinite set of formulae. For, example there is enough mathematics out there where, certain theorems do not have finite proofs where you require what are known as infinity proof rules in order to establish a proof for those theorems. So, there are such things that happen in mathematics and so therefore it is perfectly reasonable to also consider infinity sets of formulae. So, finite or infinite set gamma is a Hintikka set named after logician Hintikka who studied these things. So, one is that it should not a bottom minute and it should not have any complementary pair.

So, here I have gone back to complementary pair of literals. But, essentially it means that let us go back to this. The point is if phi is not an atom it some complicated formula. And, you all you have both phi and naught phi in this set. Then, what actually would happen is? Even, if I replace this rule by having only atoms p and naught p here. Then, phi and naught phi by application of all these rules actually eventually will lead complementary pair of literals. So, this rule is just like a short circuiting that process if you can see write for the start. That, one formula one complicated formula is obviously a negation of another complicated formula. You might as stop the process write there it is a short circuiting process rather than being regress. All formulations are tableau rules will actually have complementary pairs of literals here rather than general formulae.

So, the set gamma should not have complementary pair and if phi is essentially a multiplicand kind of formula there is the ways of looking at you look at the operators a basically ten operators we can think of them as either additive or multiplicative. In fact, the once which have elongation rule or multiplicative operators and, the once which have a branching rules or additive operators. So, I am looking at these of operators naught of or naught of arrow so we are essentially saying that the formula will be of the form naught of psi arrow phi. But, I am looking at this is single operator not of arrow similarly, in the case of naught of or similarly in the case of naught and so on. So, this if phi is a multiple is actually the root if you look at phi as an abstract syntax and its root operator is multiplicative that means it is one of these.

So, which means that you have to go one step down through negation also in order to determine whether it is multiplicative or additive. Then, there is a psi prime chi prime which also belongs to gamma if it is a multiplicative operator then, there are two formulae of the form psi o dot chi. Then, there is a psi prime chi prime also belonging to gamma. And, if it as an additive operator psi chi where the additive operators are these. Then, one of this, intersections says that one of psi prime or chi prime should be in gamma thus what we are saying is that it cannot be the case that neither be the psi prime or chi prime is in gamma. So, the psi prime and chi prime in each case for the multiplicative operators and the additive operators are defined actually here. If you look at this table this table exactly corresponds to the tableau rules for the multiplicative operators correspond exactly to the elongation rules. And, the additive operators correspond exactly to the branching rules. And, the two components in the denominator are the psi prime and chi prime. So, this is how will define a Hintikka Sets.

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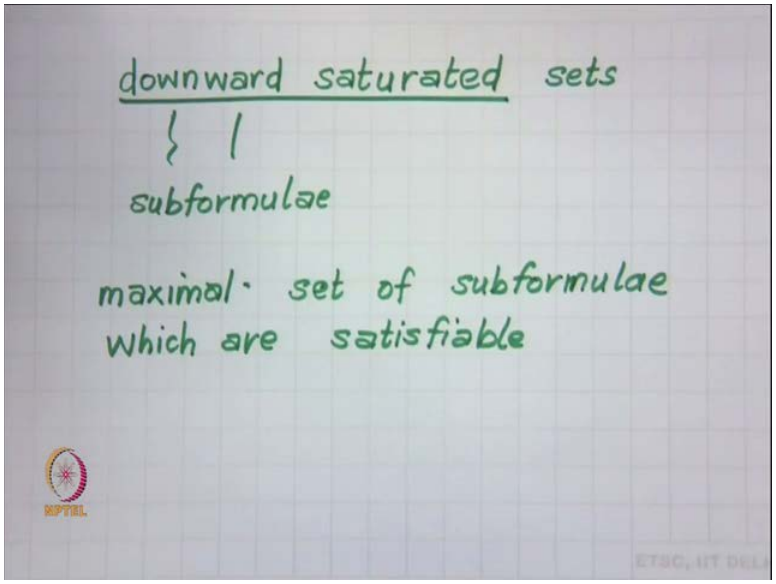

So, essentially a Hintikka is a set which is what is known as downward saturated. So, the downward saturated in the sense downward is because you are looking at essentially sub formulae. And, what the motivation for constructing Hintikka sets is actually to look at some maximal set of sub formulae. Which, are satisfiable so, if a set gamma is satisfiable what I can do let me start with some delta. If, the if all the formulas in that set delta are satisfiable then what I can do is I can saturate the set by closing it under these conditions these, are the Hintikka conditions. So, first if it is satisfiable then it cannot have bottom minute and it cannot have complementary pair in it. And, therefore and actually cannot even have any pair of formula which are complement of each other. But, more importantly what I saturated essentially on formulas of smaller size which preserve certain satisfiable properties.

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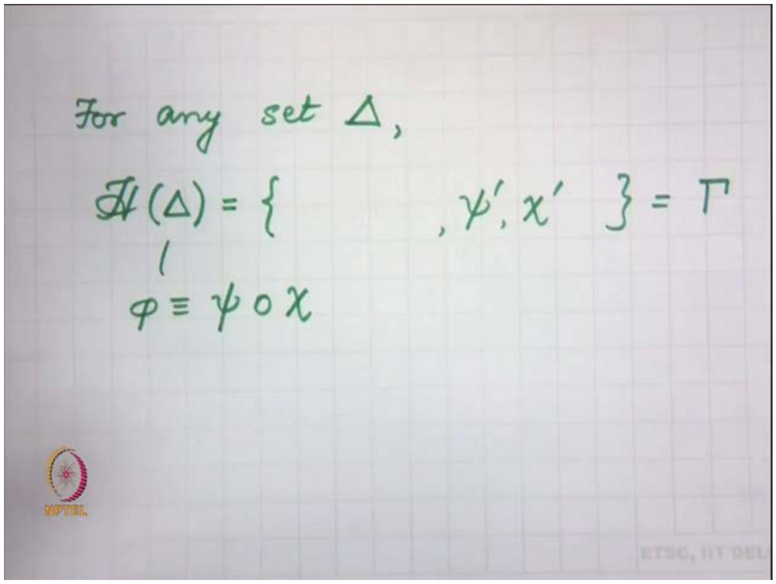

So, essentially if the original so I can actually define for any set delta I can define the Hintikka operator H of delta. Such, that essentially these thee conditions hold so, it includes. So, this is some set gamma such that if delta has a formula of the form phi is some psi some operator chi. Then, depending on whether this operator is additive or multiplicative the corresponding psi prime and phi prime are also added into these sets. So, I can think of I can think of a Hintikka operator h which closes any set of formulae on such that these conditions hold that is an alternative definition of a Hintikka set.

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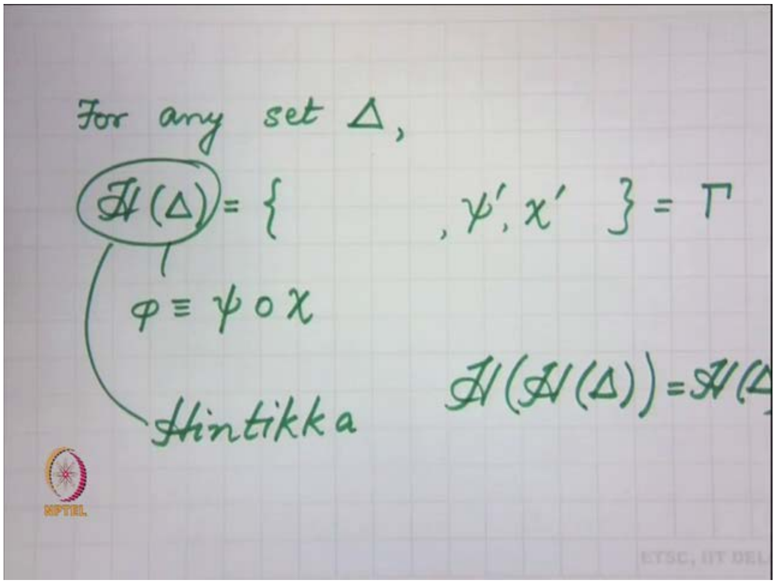

So, I can so essentially this H of delta even delta is not originally Hintikka set. This, set H of delta will become Hintikka set and it is clear that the Hintikka this operator H is a closure operation. Because, you take you apply this operator twice you do not get anything new you only get H of delta. So, it is a closure operator a closure operations are as saturate or also called as saturation operations. And, not only is it closure it is a closure it is a downward closure so in that sense your saturating set downward by taking essentially formulas of smaller degree than the original formula. So, your populating the set with formula of smaller degree so it is downward saturated in that sense.

So, will so this is the way the saturation proceeds. If, there is a formula of the form psi and chi then, include both psi and phi in the formula that is what that is from step two. If, a formula is of the form psi or phi then, choose one of them and put that in the set. So, actually this Hintikka operator will give me set f sets of formulae. Because, for all for all the additive operations there are two possible sets which, can be obtain. So, but each of them will be a Hintikka set so this actually so, this formulation is not exactly correct it has to be that this H gives me a set of sets of formulae. Where each set each element of that set of sets of formulae is a Hintikka set by itself. Therefore, satisfies is downward closure property downward saturation property.

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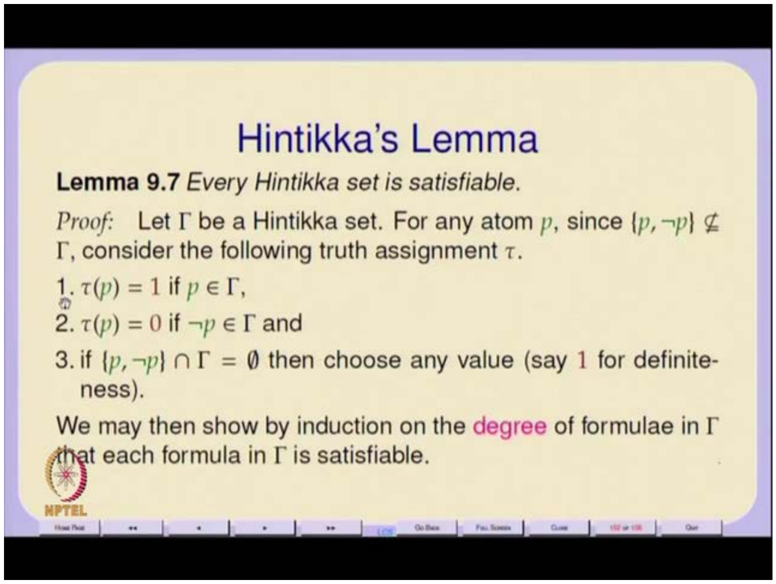

So, then you have Hintikka's Lemma which essentially is has to do its satisfiability. So, let gamma be a Hintikka set that means it satisfies those three properties. For any atom p since the complementary pairs are not included in that gamma. If, its Hintikka set they cannot be they cannot be any complementary pairs. We, take a truth assignment where such that if p is positive then we give it we assign it 1 and, if it is a p occurs negatively then assign it 0. For, all other atoms we just choose any value any truth value we choose either 1 or 0 randomly it does not matter.

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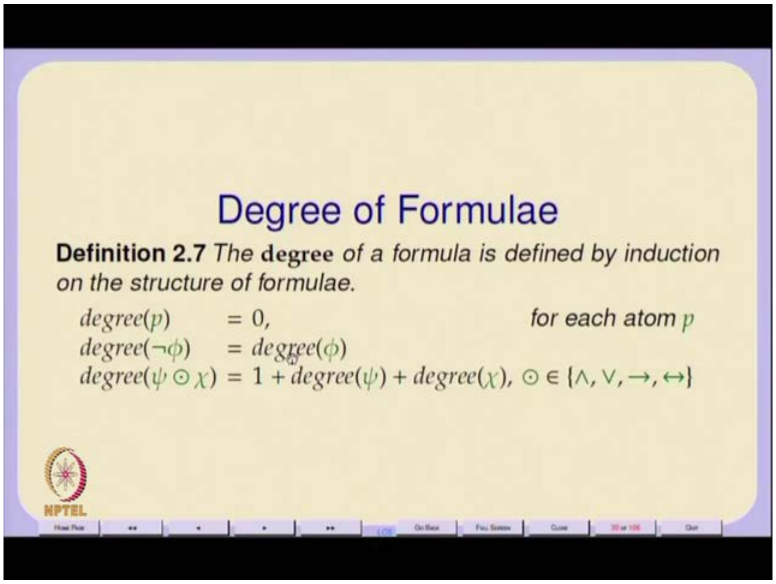

So, then you can show by induction the degree of a formula and this is formal definition of a degree basically I am not it is like size except that I am not including negation. So, the degree of each atom is 0 the degree of negation of formula is just the degree of the formula itself. And, for any other binary operator the degree increases by 1. You, take the sum of the degrees of the components and increased by 1. So, it is not exactly the size of the formula because for negation I have because I have firstly for atoms I am saying the degree is 0. And, for negation not increasing the degree so it is not the size of the formula. But, the notion of the size can be defined similarly. So, what you can show by a induction on the degree of the formula formulae in gamma is that each formula in gamma in satisfiable. If, on this truth on this truth assignment I am not going to formally prove it by induction because this it is sort of clear. So, essentially Hintikka sets are satisfiable sets.

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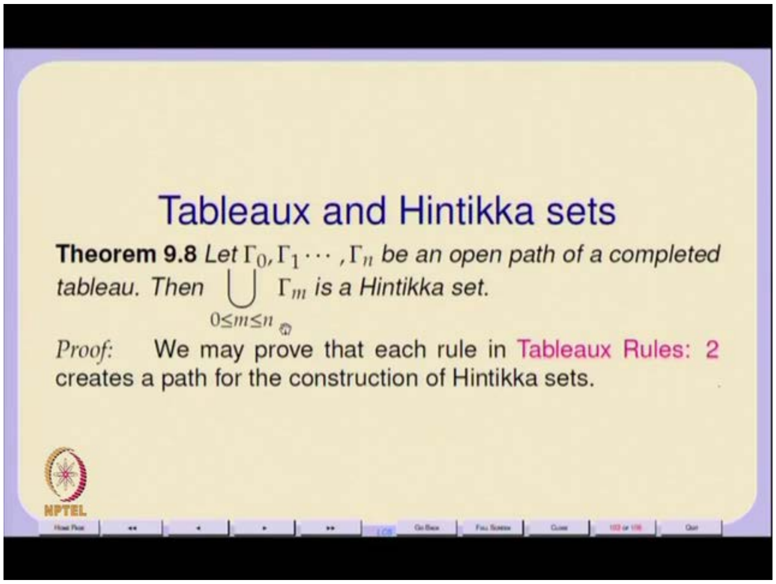

And, so let us take so here I am going to consider the new formulation of the tableau rules as sets of formulae. So, take a completed tableau and take an open path so supposing there is a open path in the completed tableau take the open path. And, look at all the formulae in that open path the union of all those formulae is a Hintikka set. And, that is because the table for the construction of downward saturated sets exactly is mimicd by the tableau rules. For the set of rules that is why I am taking a union over the entire path to restore back the original formulae. Which, you got split by the sub formula property.

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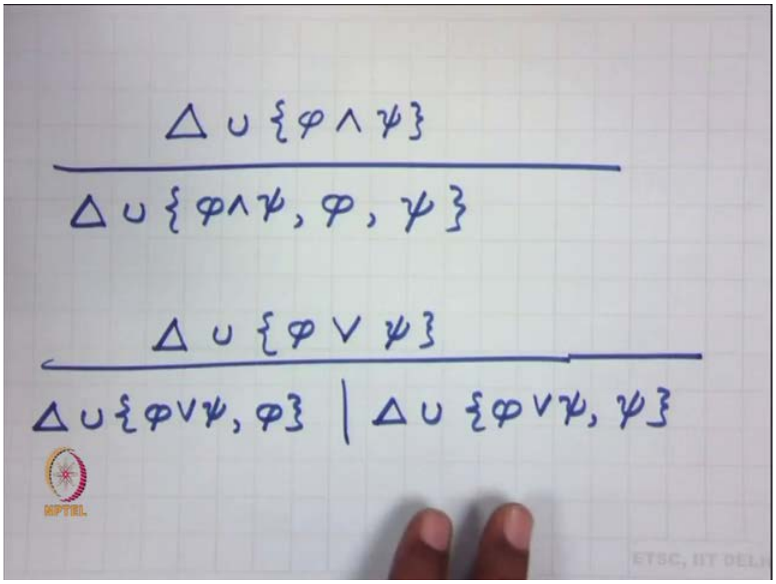

So, if I take all the original formulae and so, basically so another way have looking at it. If, I look at all these rules there is the third formulation. Which, we can do if I look at all these rules as of this kind delta union lets a phi and psi. Let us take this and I actually write as delta union phi and psi and include phi and include psi for typical elongation rule. Then, what I will be then I do not need to consider the entire path. Similarly, for this is in the case of elongation rule in the case of branching rule like this if I had phi or psi. If I think of this as delta union phi or psi and I include psi delta union phi or psi and I include psi I could do this in which case I would have reformulate the theorem as saying essentially every leaf in the tableau if this is my this is how my tableau rules go. Then, the theorem would be restated as essentially I do not need to consider the entire path I just need to consider a leaf. That is like this is like a downward closure operation.

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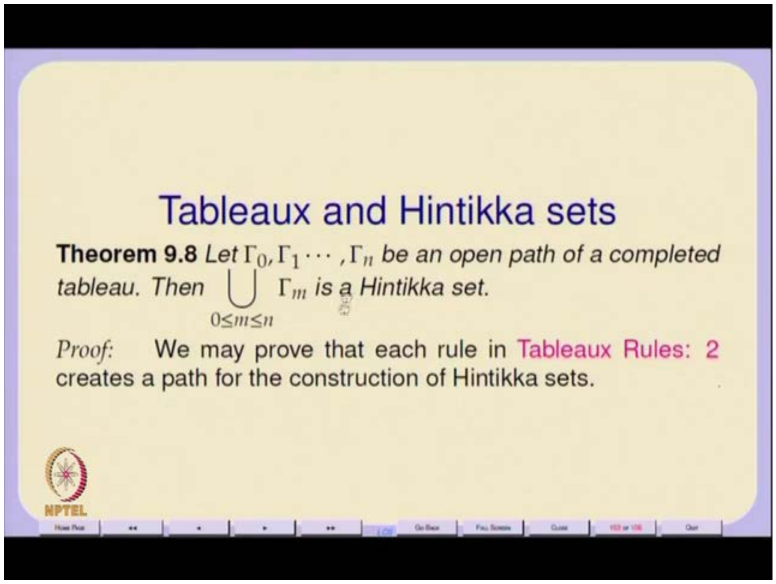

So, this is I am considering the tableau rules 2 I this is a I have to take the union over an entire path and that is a Hintikka set. So, you can prove that each rule in the tableau rules two creates a path for the construction of Hintikka sets. So, that is the relationship between tableaus and Hintikka set and now we are the completeness.

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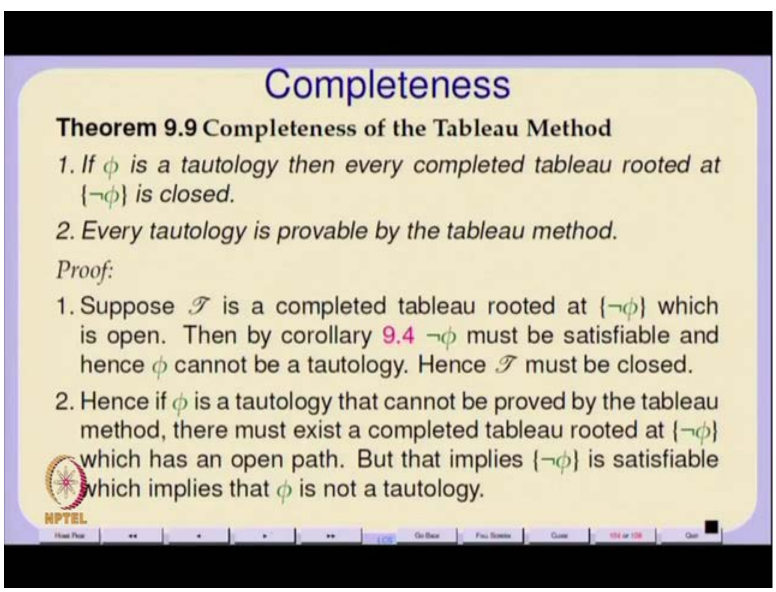

So, first thing if phi is the tautology then every completed tableau rooted at not phi is closed that is very simple let T be a completed tableau rooted at not phi which is open so, we are proving by contradiction. If, it is open then by this corollary 9.4 this must be satisfy it has satisfiable leaf. So, which means a, which means that phi cannot be a tautology and therefore it means if phi is a tautology then this tableau must be closed. Secondly every tautology is provable by the tableau method and that is if phi is tautology that cannot be proved by the tableau method there must exist a completed tableau rooted at not phi which has an open path but if it has an open path. Then that employs not phi is satisfiable and this which will in turn employ that phi is not tautology.

So, this Completeness so essentially what it therefore what it shows is that a, it gives us it answers this original question that we raised. That, you take any unsatisfiable set you take any tableau rooted at that unsatisfiable set it does not matter. You, will the tableau will eventually close every path will get closed. So, this is and essentially saying that you know there is a certain sense in which proofs by contradiction or actually easier to perform. Because, they are somewhat more deterministic there actually gold directory. So, in general when you take when you take any mathematical proof there is a possibility of going directly. One is you take the hypothesis and start trying to derive the conclusion.

But, the problem with that could be that you might actually go off on to some tangents and never lead which will never lead to the conclusion I mean you may prove a lot of other interesting Lemma ’s and theorems in the process. But, you may not ever prove the original theorem. Whereas, if you take if you try to prove a contradiction there is a good chance that your proof is goal directive. Because, what you are going to do is you are going to take the hypothesis and the negation of the conclusion and try to prove a contradiction. So, it is much more goal directed that is the reason why these methods like a resolution and tableau actually work so well. And, that is because of this theorem which, essentially tells you that every tableau for an, unsatisfiable formula will close. And, therefore the tableau method is complete.

So, by essentially by Hintikka sets what you are saying is that if the tableau is open then you take any open path that is Hintikka set. And, what the previous Lemma tells you is I can find satisfying assignment for that Hintikka set. So, I can find satisfying assignment for that Hintikka set then I can definitely find a satisfying assignment set for original set of formulae right. And, that satisfying assignment is very easily determined from the Hintikka set by just looking at all the atoms all the literals that occur in the Hintikka set through a way all the complicated formulae. Finally, it mean order to satisfying truth assignment just look at the literals in the Hintikka set and that is the satisfying assignment just defined by this.

So, your tableau method is complete and in similar fashion actually resolution the resolution method is also complete the tableau method is complete in the sense that for every tautology. So, firstly every, logic logical argument can be converted into set of formulae. Such, that the argument is valid if and only if this set of formulae is unsatisfiable. Secondly, for every set of unsatisfiable formulae every tableau will close. And, for every set of consistent formulae every open path in every tableau will actually give you a truth assignment which satisfies a original set that is. So, that is what this completeness that is and many proofs of completeness in model logics and forested logic will actually use the notion of the Hintikka set and downward saturation.

And, the important think about that downward saturation is that finally you have to look at only the literals. And, in order to find a satisfying assignment you just require this, Other any questions? So, the first assignment I am not been able to put it up because some more the network is not working you have to implement tableau method for propositional logic as soon. As I can I will put up the formal data type. The data type is same is that in the tautology checker I have not had the time to correct the link that you mentioned. But, I will correct it today so that you can the tautology checker source code is available to you.

So, you use the same data type and you use the same signature that you use for the tautology checker you also use it to implement tableaus. And, you have choice you can implement the tableau either by the first set of rules or by the second set of rules or even by this third set of rules that is not matter to me. But, these third set of rules will be carrying baggage you know this the second set of rules at least it is just like state updation. And, that might be there can if you have the choice you can also be use the first set of rules but implement one tableau rule line in as efficient as possible. So, that we have a tableau method for proposition logic. And, submission formalities so and so for will come through model on the in the department I think the institute model is not working. So, the model in the department running on Jaijaivanti which means that you have to be in the institute when you submit because the Jaijaivanti will not be visible outside the institute. So, you have to be in the institute when you submit. So, that is that I hope will set that so then we will we will do compactness next time and then will start first order logic and proof theory for we start with proof theory and then first order logic.