Logic for CS
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Lecture - 24
Existential Quantification

So, we are looking at certain first order proofs and so on. Today so, will look at existential quantification so will go through existential quantification today. There, is something special about existential quantification even though it is a derived operator of the helved style systems. So, in fact I am going to the devote the entire lecturers to just existential quantification. So, what it is what is special about, it and this is something about ignored by all mathematicians and computer scientists in everybody they just take it for granted. But, there are certain deep and certain issues in existential quantification. Which, need to be taken care of in any minimal axiomatization of predicate calculus. And, therefore by extension for any first order theory that existential quantification place a fairly important role but, before that let us look at. So, I have a couple of exercises here to which have also add some more exercises and i will put up this slides only after added those exercises.

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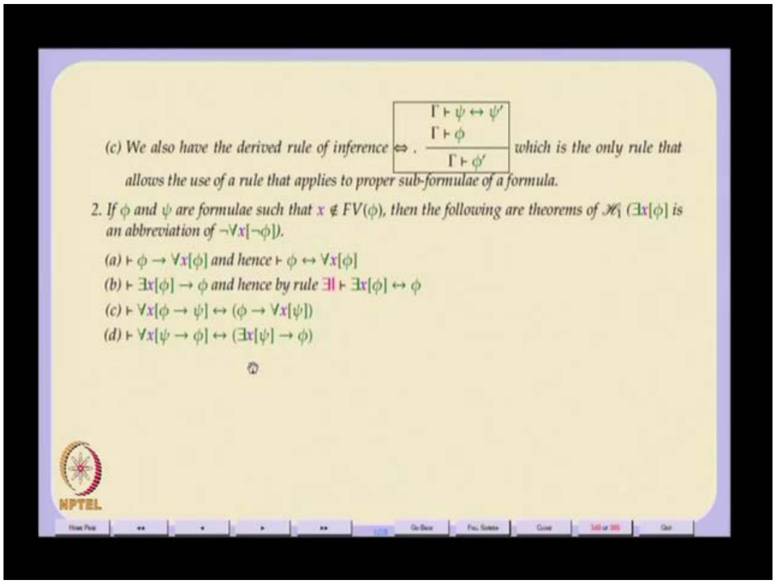

So, but lets you look at this exercises initially it has to do so all are quantified formulae are some out deeply concerned with the existence or nonexistence of free variables that is. So, there are certain things that you can prove if, you then there is free variable does not exist in a formula. And, so there are some certain surprising things that happen. So, let us assume phi and psi are formulae I know you cannot be able to see it. But, in particular what happens is will take a, the existential quantifier as I, mean there exists phi as an abbreviation for not of for all x not phi by the standard Demorgan extension. So, now let us assume that there is variable x which is not free variable of phi. So, and this quantification for all x is deliberate it is essentially got to do that variable x. So, x does not occur anywhere inside the body of phi as a free variable. So, effectively quantification does nothing to the truth or the validity of the formula phi. So, for all x phi and phi will have the same truth values therefore same validity whenever under any interpretation in particular what happens in the proof system is that you can take so for. So, therefore phi actually implies logically implies for all x phi and by the universal instantiation by for all elimination. You, also get that for all x phi will logically imply phi if x is not free variable of phi. So, which means that phi and for all x phi logically equivalent. And, this exercise is to just is to show that you have to sot of prove this things within the Hilbert system h1 the other thing is so this happens in this case. So, in fact the same thing is happens to the existential quantifier also there exists x phi implies phi and we are going to do this rule there exists introduction which will show that there exists x phi is logically is equivalent to phi. And the interesting thing is now psi could have excess of free variable so now when you look at phi or psi which look at its universal quantification that turns out since x does nothing at all to phi. So, therefore this is the logically equivalent to phi arrow for all x psi so the for all quantify can be moved in to and and localize to psi. But the more interesting thing is supposing x is a free variable of psi but x is not free variable of phi. Then when you have for all x psi arrow phi then that is logically equivalent to saying that if they does exist an x for which size true then phi must also be true. So, there is an inversion of quantifier which happens in this last portion. So, that is an important thing so that inversion does not takes place here this is important also for other reasons which will become clear. When we do normal forms so and so on but for the present for the purpose of this today's lecture actually it is important to this notice this deportion.

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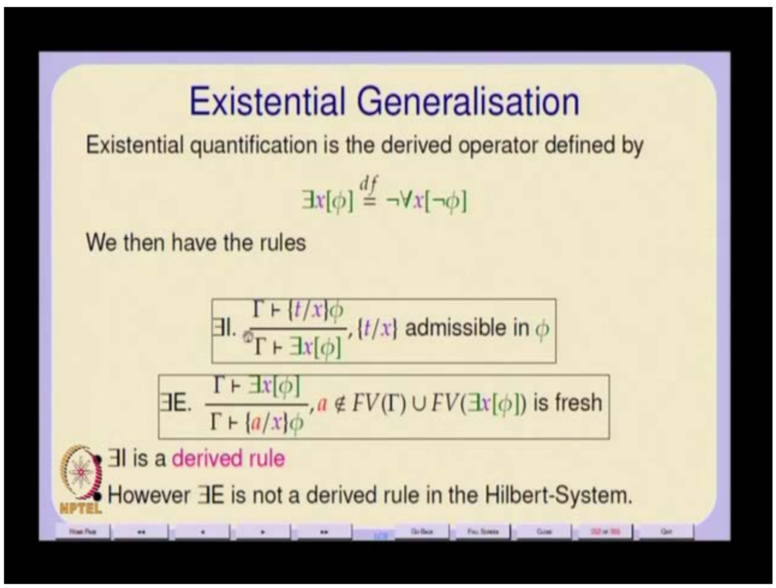

So, let us start today's lecture. So, the point is that the standard thing in any computer science or logic text book is to use. What you in all derive rules for Hilbert systems when we introduce new operators basically had introduction and elimination rules for the operators. And that is the trend you can continue for this derived operator there exists x. So, there exists x phi is just defined as not of for all x not phi. And then we can have these two rules this is there exists introduction which says that if phi is of the form such that it looks like t has been substitute for free variable x in phi. If that is the form it adopts then i can take phi in its original form and existential quantified. There is a slit difference between for all rule for all introduction and there exists introduction there they was an implicit notion of an arbitrary variable y. And arbitrary is what you spoke we did not actually implemented in anyway. Here if t were variable then basically you can think of it as particular variable rather than arbitrary variable. Our interpretations of logic are always over nonempty domains. So, if there does exist any valuation at all which makes t for x phi true. Then you can confidently assert that they does exist an x which makes an phi true.

And that is so if there is even one so not all t's nor all terms t nor all variables may be substituted for x. But if there is even one that can be substituted and for x in phi and phi can be made true according or interpretation then we can confidently assert there does exist x which will be phi true. So, there is it looks suspiciously similar to the for all introduction but is different. And that difference has to be some of capture and we are going to spend essentially the rest of this lecture is capturing that difference. But more importantly you take the elimination rule there exists elimination and here your essentially making the distinction between arbitrary and particular. So, i am going to make this distinction by essentially referring to it as what we what we do in normal mathematics. Between the distinction between variable and constant a variable can basically take any value a constant is a particular value its therefore fixed in a certain sense. So this red a is suppose to signify a constant and but the thing is that our language did not have any entity called a constant. And it does not have I mean we are talking about purely syntactical formalization so which means we just have a collection of variables but somehow we have to capture the idea that this a is the particular constant. And so if a just happens to be some element in the set of variables it should satisfy this condition of being fresh. These this is what makes it particular there is an issue of freshness also which comes in which we have to somehow capture.

And of course by fresh essentially what I am saying will may will be made clearer a little later. But the basic minimum condition that you require of this a is that it should not be a free variable of any of the assumption of gamma. And it should not be a free variable of there exist x phi this is the basic minimum condition the other extra condition is that it should be fresh we will look at that. So, the third important thing about these two introduction and elimination rules is that where as there exists i is a derived rule there exists e cannot be derived the Hilbert system. Whereas, there exists E is something that you always naturally in any mathematical proof you actually whenever you have an existential statement. And you have to start proving from that you immediately put in a symbol for a constant a, and you proceed with a proof. Which means the first question is if this, there exist e cannot be derived then in the Hilbert system. Then is a Hilbert system incomplete for one thing secondly a how can we justify proofs in mathematics where we automatically put in a constant symbol a, and say let this b the thing with this property. So, will look at this but first just look at this there exists introduction is a derived rule.

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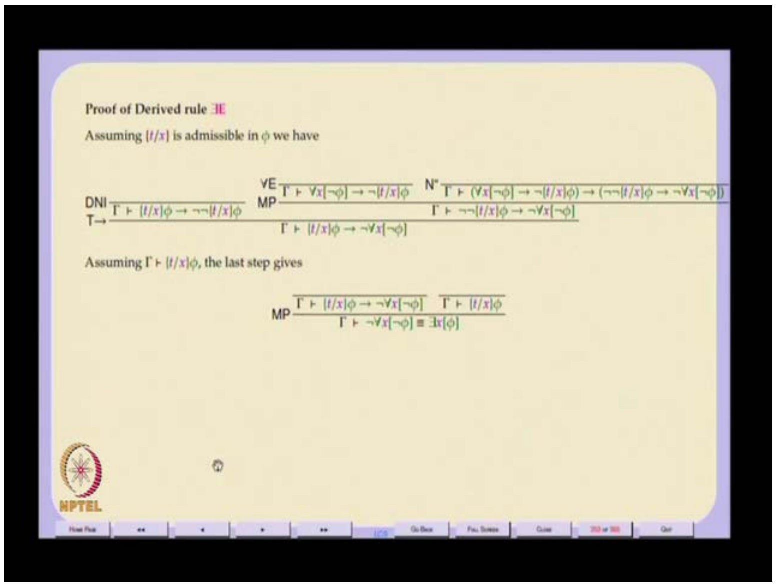

And here again because the our trees are like I do not know my trees are like bunion trees I think so there to vast to wide and the go behind that a text width of these screen. But for those of you who have binoculars and so and so forth may be you can use them. So, essentially what we are saying is that so the proof actually requires the fact that t for x is admissible in phi the. So, assume t for x is admissible and phi then I can take a, for all x not phi arrow not of t for x phi. And then, I can also use this N double prime rule which is familiarly to all of you contra positive rule which you all use in your examination. So, which essentially says that so in this particular instant it would says that for all x not phi arrow not of Tx for x phi in essentially implies. That i can put a double negation on the right hand side of the arrow make that the left hand side and put a negation on the left hand side of the arrow and make it the right hand side so, that is what this is. So, I have not of t for x phi here. So, that becomes naught t for x phi and I have for all x naught phi here and that becomes not for all x phi. So, this is the contra positive rule a derived rule which was there in one of the exercise. So, this is N double prime so and of course from this and this by no dissonance I can essentially remove this so I get double negation naught of t for x phi arrow not of all x not phi. And there is this double negation introduction it says that t for x phi arrow naught t for x phi. And between these two I can use transitivity to get the t for x phi arrow not of for all x not phi not is just there exists x phi. So, you get this so this is a derived rule.

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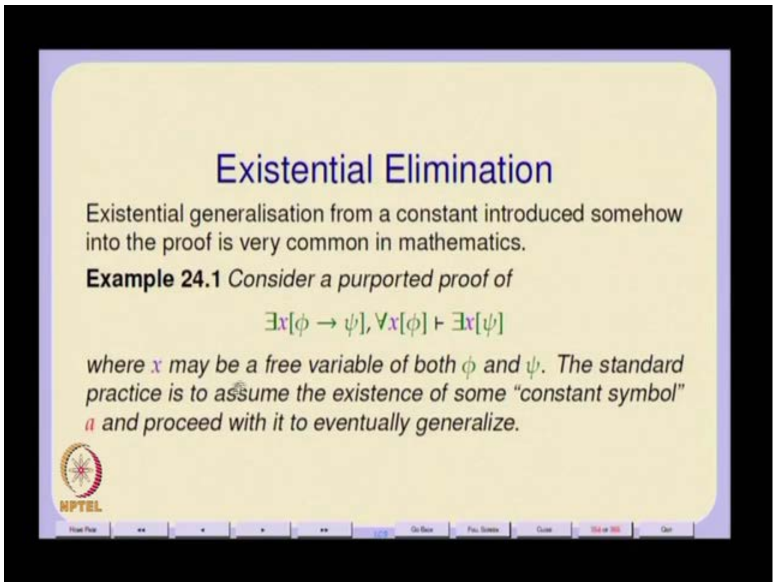

Now let us go further so now let us look at existential elimination because that is the problem it is not something that can be derived in the Hilbert style system. But the interesting thing about proof systems is that it is quite possible as i said to get a different set of proof rules which are not necessarily derivable from another sets. So, you can have two independent sets of proof rules which have both sound and complete. And in fact that is attitude most logicians were take when it comes to the natural deduction of system versus a Hilbert style system. They would not derived natural deduction system from the Hilbert style system they will look them is regard them is to independent proof system. In the case of proposition logic what we did was we actually derived every rule of the natural deduction systems from the Hilbert style system. And we have been doing that except for this there exists elimination. So, now what will do is let us look at standard proof of such statement this is something that should be true in any predicate calculus. If there exists an x such that phi arrow psi and for all x phi from that should be possible to prove that there exists x psi. And this proof should be possible precisely an all these proofs relay on the fact that you are in any semantics of first order logic you have nonempty domain. So, that this statements like for all x phi do not become vacuously true. So, it is possible to in instantiate for all x phi and find a term such that t for x phi would true. So, now the standard practice in mathematics. And in fact in any natural deduction rule system would be to assume the existence of some constant symbol a. And then proceed with it to sot of generalized so what so as in the case of universal quantifier you would do. The same thing in the case of existential quantifier two any proof in the case of universal quantifier which first instantiated by using for all elimination get some term proceed with proof. And then generalized again back to a for all in the case of exist existential quantifier also we do essential the same thing.

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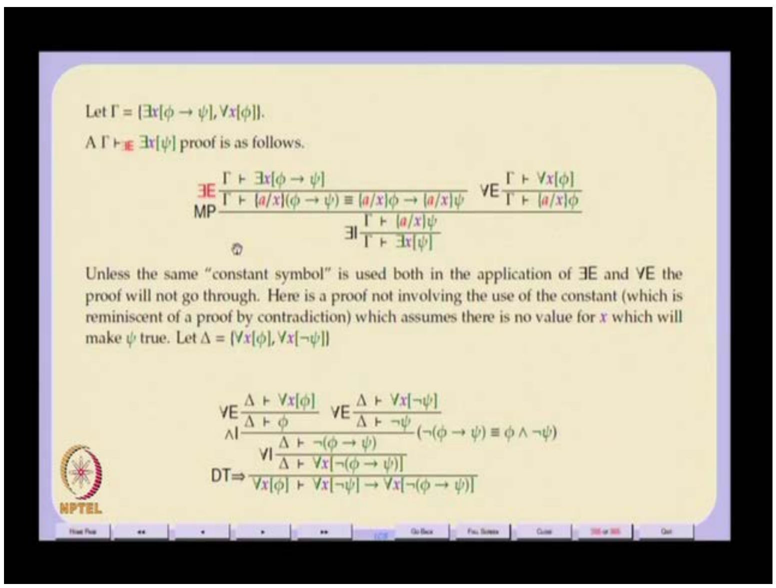

So, here is a proof where I am assuming there exists elimination so even if you cannot read it the red color should guide you to what I am saying. So, the red color bright enough actually I can read this from here i do not know whether you can read from there if it is. So, essentially what we are saying is i take this there exists x phi arrow of psi and i eliminate the existential quantifier. By having new constant a, for x phi arrow of psi and of course this a for x phi arrow of psi is purely syntactic. I mean its substitution is a syntactic operation and so it is syntactically equal to just this one a, for x phi arrow a for x psi. So, that I mean that does not require proof rule because it is a syntax. And this is metasyntactic equal equivalent I mean. So, essentially think of it as a for x phi arrow a for x psi that is what you are getting. And this for all x phi I am actually going to instantiated with the same a if I do not instantiated with the same a then I cannot apply modispondance. Because now the propositions are different this pattern is would be different from this pattern only way modispondance be applied this pattern is identical to this pattern. So, then with modisponance I essentially get a for x psi which i now introduce existential quantifier and therefore I get there exists x psi. The most interesting thing about this is this is how this is essentially a proof tree of standard mathematical proof of an given some existential assumption. And some universal assumption you prove an existential statement and in the process of proving that you actually instantiate the existential statement use that same constant to instantiate universal statement.

Proceed with the proof till you get the right form and then existentially generalized that is the standard prime. So, but it tells out to need to go through this if I stuck to my Hilbert system. And only whatever rules can be derived from it then I do not need to go through this. Because what I am going to do is I am going to do is something that looks subspecialty like the proof by contradiction but actually is not proof by contradiction in that proposition sets. So, what I am going to do is I am going to assume essentially the negation of this. And the negation of this there exists x psi is actually is really for all x not psi I am going to take this assumption and then I am going to discharge it somewhere. So, now what I have in my assumptions are all universally quantified statements. And, so now I do not need to any existential quantifier. And the proof was its follows let I am taking my assumption delta to be for all x phi and for all x naught psi. So, for all x phi of course is can be instantiated to phi because in particular I can instantiate I do not need to actually substitute term for x for the free occurrences of x and phi I can leave it as it is. And for all x naught psi can also be instantiated to naught psi. So, now x might be of free variable in both phi and naught psi.

The other thing is that we had a derived rule since all are propositional rules were derived rules from the Hilbert system I can apply one of them for example I can apply and introduction. If I apply and introduction essentially I can take phi and not psi but of course my language i am restricting it and is a derived operator and it is essentially defined as a naught of phi arrow of psi. So, from these two essentially by and introduction and using the fact that and is defined this way as phi and naught psi is defined as not of phi arrow of psi I get naught of phi arrow of psi. And now this x here throughout was arbitrary I mean this no particular reason to worry about x. So, which means I can apply my universal generalization. So, I can introduce the universal quantifier and essentially from delta I can prove for all x not of phi arrow of psi. And then these are all close formulae. So, the deduction theorem is applicable. So, I can move this assumption for all x not psi to the hand right side and i get for all x naught psi arrow for all x naught of phi arrow of psi.

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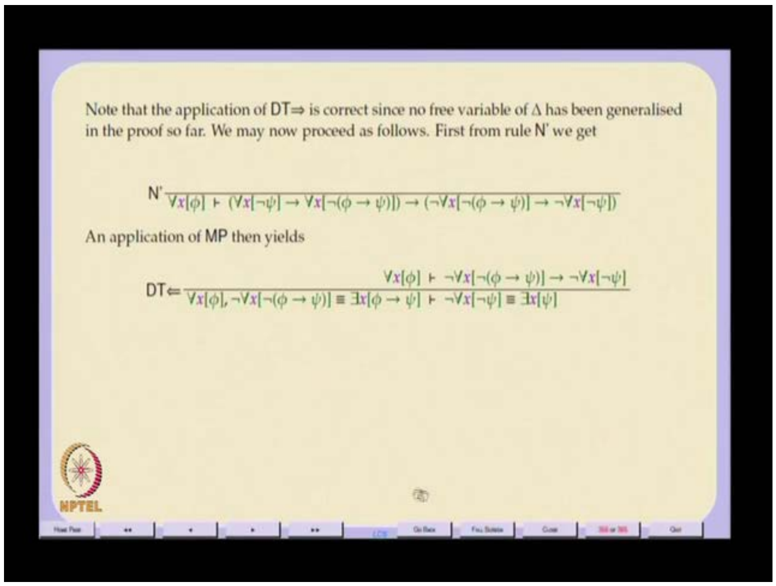

And then I apply n prime n prime is very much like n double prime but when. So, n prime essentially says for all x naught psi arrow of for all x not of phi arrow of psi arrow negation of this arrow negation of this. And that is what it is and applying this an again here again I had the split the tree cut it into bits that it fits into this screen. So, essentially applying n prime and then modisponance on this I get for naught of for all x naught of phi arrow of psi not of for all x naught psi. This now I can move this to the left side they are close formulas there essentially like propositions all the conditions of and this is inverse of the deduction theorem. And that has no conditions whether its propositional or predicate logic it does not matter I can always apply. So, I can move this formula to the left. So, what I get now for all x phi and this formula not of for all x not of phi arrow of psi is actually there exist x phi arrow of psi. Which was original assumptions which were suppose to prove and this right hand side not of for all x not of psi is just there exists psi. So, essentially without using existential elimination i have you can prove this you prove any statement actually. But the problem is that it is not clear how to translate proofs using existential quantifiers existential how to translate proofs using existential elimination to proofs which do not using existential elimination. And there is no direct translation so which means that you have to do some more work to proof that your Hilbert's of firstly that your Hilbert system is complete secondly that for. If there is a proof using existential elimination then there exists a proof which does not using existential elimination. So, I have to proof both of them so that is what we are going to do.

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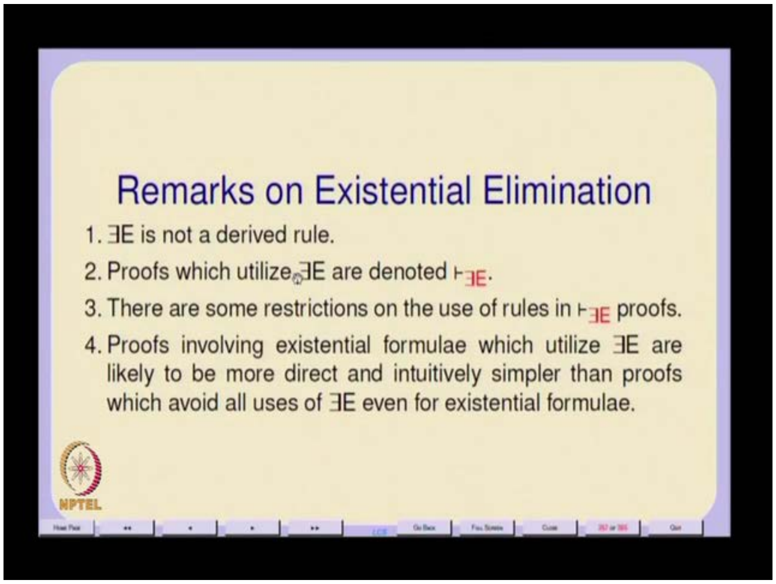

So, let us look at existential elimination. So, firstly of course a, it is not derived rule and what we are going to do is proofs which use existential elimination I will subscript them with the. If anywhere in the proof there is an occurrences of existential elimination then I will say that I proved it using this. So, the whole proof gets bloodied with this with this red mark on the provability's. So, and then further this was actually simple proof there are some restrictions on the use of existential elimination proofs. And actually those restrictions are intuitively clear when most of us do or proofs are using existential statements but how were we should actually formalize then that is one thing. But the interesting thing is that as you can see that this proof using existential elimination is very short very intuitive compare to this proof which does not existential elimination. So, that is why it is easy to use existential elimination and we should and therefore you get if, you can proof that for every proof which uses existential elimination. There is also corresponding proof which does not use it then we are essentially safe. And this we need to do this especially because this is not a derived rule there is no derivation of the rule. And there is no direct translation of the proof. I mean for example a translation of proof is something that we have used i do not know whether you are done it in one of your courses. But if you look at the principle of mathematical induction and the principle of complete induction. You, take proofs by principle of mathematical induction in proofs way complete induction. So, what you do there is actually a direct verbal translation of any.

So, you can take that the actual language sentences in the proof of by the principle mathematical induction and by just changing. The property you can get an exact syntactically identical proof except for the fact that it will be using the principle of complete induction. The induction hypothesis naturally changes you can do direct translation of proofs. So, if you take similarly if you take proofs by structural induction one of the things that you might want to do for example the principle of structural induction gives us a convenience of case analysis. And using the structure but really it is no more powerful than a proof by mathematical induction. So, it is possible the proof that it is no more powerful than the principle of mathematical induction rather. The proof the meta proof that for every proof by structural induction there is also a proof by mathematical induction. Take that meta proof can be translated from structural induction directly into mathematical induction by just making certain syntactic transformation. In, these cases however there are no such syntactic transformation. So, many of this proofs so we have a twofold problem really firstly the rule is not a derived rule secondly the proofs are not translatable. As this shows I mean for the presently will assume this the only proof that is available.

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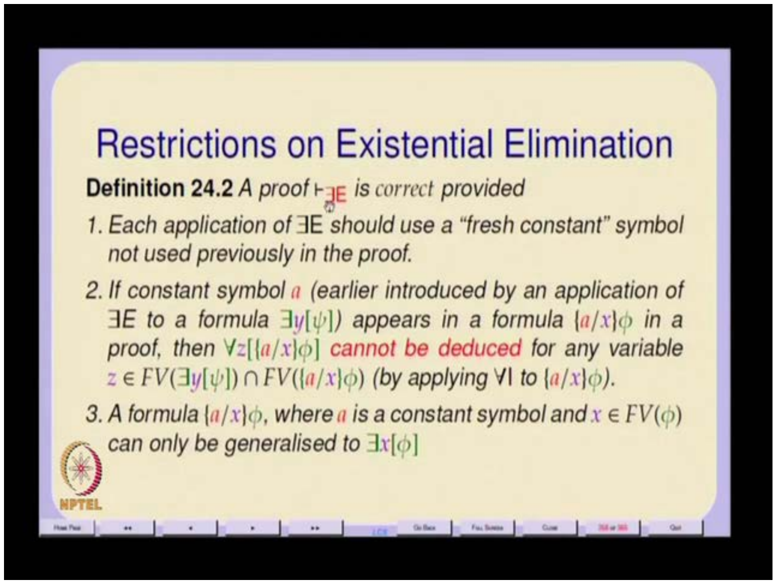

Further there are certain restrictions which we implicitly follow so I will call this informally I call a proof by using there exists elimination to be correct provided. And these are discipline that we follow and in fact we implicitly follow in any mathematical proof. Firstly each application of existential elimination should use the fresh constants numbers.

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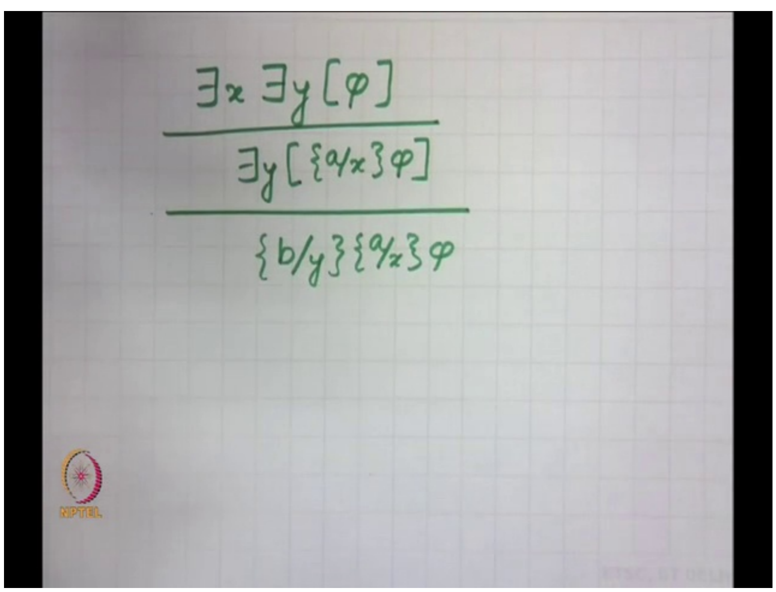

So, for example if I had formulae with let us see two existential quantifiers then I would have to use u constants for each instantiation. This because the semantics says that whatever constant I use to instantiate x in phi may not be the same constant for y. So, I have use two different constants so I might actually get there exists y a for x phi. And then if I want instantiated once more then Hilbert have to assume that there exists some other b for y a for x phi. So, I have each time I use existential elimination I have to use a new symbol. And more over this symbol these two symbols a and b actually should not have occurred anywhere in the proof let us assume that this is some intermediate statements in the proof. Then what we are saying is there is there is some proof from which you concluded this any of the symbols that were there in the proof should be different from this a and b. Otherwise you are likely to have this is a discipline we actually implicitly follow when we do these proofs. So, that is so the meaning of fresh is just that you have to pull out a completely new symbol which does not exists anywhere in the proof. So, that is why I am saying that this any of this is a link none of these is a link. So, that is why I am saying that this has not belong to free variables of gamma union free variables of there exists x phi is not sufficient. What you want to do is given gamma and a whole proof you have to look at the entire proof to make sure a does not occur anywhere there. When you are using a as a constant of existential elimination. So, it is not the question of being syntactic about the assumption and the formula. Here you have to look at the whatever, preceded it in every step there you should make sure that the you are choosing an a which is really fresh. Which does not occur anywhere there. So, that is of course one restriction and there is another thing one thing is that a, and b are essentially terms. So, there is a if a and b are just a terms of variables then there is a tendency to actually universally generalize. One thing is you cannot universally generalize on this a, and b that sense they are special. That is why I am calling them constants that is one thing the second thing is that.

Supposing there is some free variable here suppose in y has supposing in phi has some z which is a free variable. The truth of z and with a and b may not allow you to directly do a universal quantification over z for example. So, universal quantifications may have to be postponed till you have done in an existential quantification and there is still that free variable. Otherwise they are have you might end up with proof but not just that z that occurs in this any free variable z which occurs in. So, here is the exact thing so firstly for all z a for x phi cannot be deduced for any variable z which belongs to the free for variables there exists y psi intersection. So, you so supposing so essentially what we are saying is supposing you have a proof you have some large proof.

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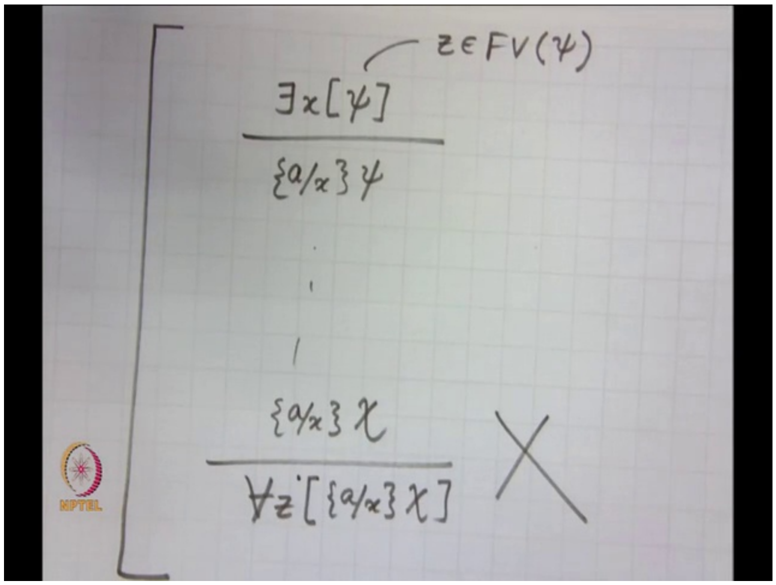

At some place there was there exists psi and z was a free variable of psi and you applied existential elimination. And you got let us say a for x psi you go down some steps you know that you cannot universally generalize on z here. But, may be you go down some few steps something happens and you are left with some a, for x some chi which is derived from this. And then you decide to generalize on z this is also not allowed. So, what you are saying is you go through the entire proof where ever there was where ever you use. So, what is the idea of proof system is to preserve truth. The fact that this is true in your proof might depend entirely on the existence of a. It may not be independent of the existence of a. So, which means the therefore for different constants a, and a prime the corresponding x which makes this formula chi true might be different. So, for different a, and a prime you might require different values b and b prime for x to make it true. In which case you cannot universally generalize on this z. I should reword it so the z which makes this chi true here might vary with the constant a. So, for different constants a you might have different values of z. So, therefore you cannot universally quantify on z so which means that you have to go through the entire proof. Look at all those steps which you involve constants look at all those free variables. Make sure you do not generalize on those you do not know universal generalization on those free variables.

Till, you had eliminated the constant you might eliminate the constant in one of two ways one is that the constant might disappear as part of something. For example if you use and elimination the constant might be one term and may not be on the other. And you may have eliminated in which case then after that you do not have that constant occurring anywhere and you could get the generalized. The other way is that you might take the fact that the constant is there you might existentially generalize. Therefore eliminate the constants and then you universally generalize and that we preserve truth. So, these are the certain issues which make any mathematical proof which make the debugging of the mathematical proof very hard. How what is the structure of the proof are they using existential elimination in some way are they generalizing before eliminating the constant. So, this restriction is actually very important if a constant symbol a earlier introduced by an application of an existential elimination to a formula. There exists y psi appears in a formulae a for x phi in a proof then for all z a for x phi cannot be deduced for any variables z which is free in there exists y psi. Intersection free variables of a, for x phi by applying the universal instantiation. So, buggy proofs come because this step two is somehow violet a large number of buggy proofs.

Actually happens because of this they also may many bugs in mathematical proofs. And which are very hard to detect also happen because sometimes of the fact that the universal generalization precedes the existential generalization.

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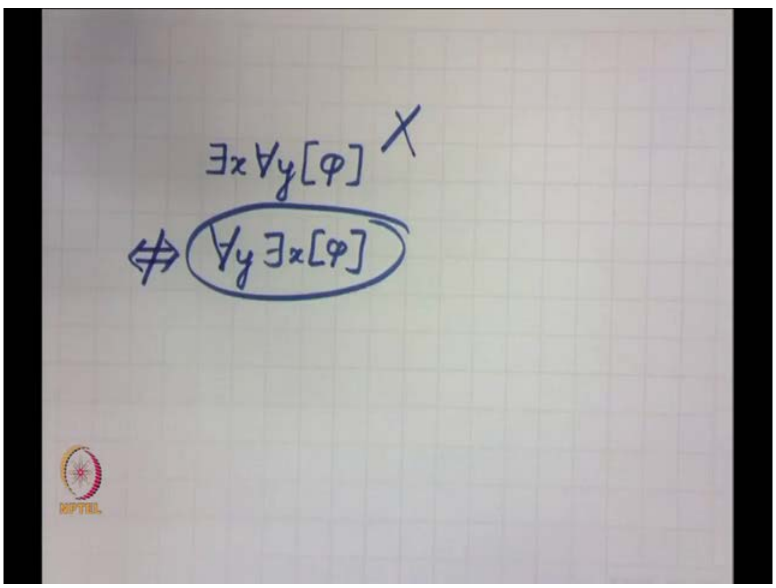

So, what I mean is there exists x for all y phi is not logically equivalent to for all y there exists x phi. Because semantically they have different interpretations so before if your proof did have instance of existential elimination. Then you cannot do the universal quantification before you have eliminated that constant. In which case in most of the time you are looking at this and not this. This is typical this is something that these two formulae are like a typical example is that of in any algebraic systems the existence of identity and inverse. These two formulae actually are like weak versions of that. In the first case you are saying that essentially there exists an identity element. Means I can write phi in terms of x and y such that x is essentially an identity element for some operation. In this case I am really talking about inverses for every y there is different x for every y there is an x. But for every y there is a different x for different y there might be different x. And so the symantical interpretation of these two formulae are different and in a proof you have to ensure that you preserve the truth of that. So, this actually affects the entire proof otherwise you will confuse with inverses with identities. A formula a, for x phi where a is constant and x belongs to free variables of phi. That is another thing you should only generalize it to when you eliminate. That constant by generalization you have to generalize only existentially you cannot generalize universally. That is but the most stretchers thing is that point two.

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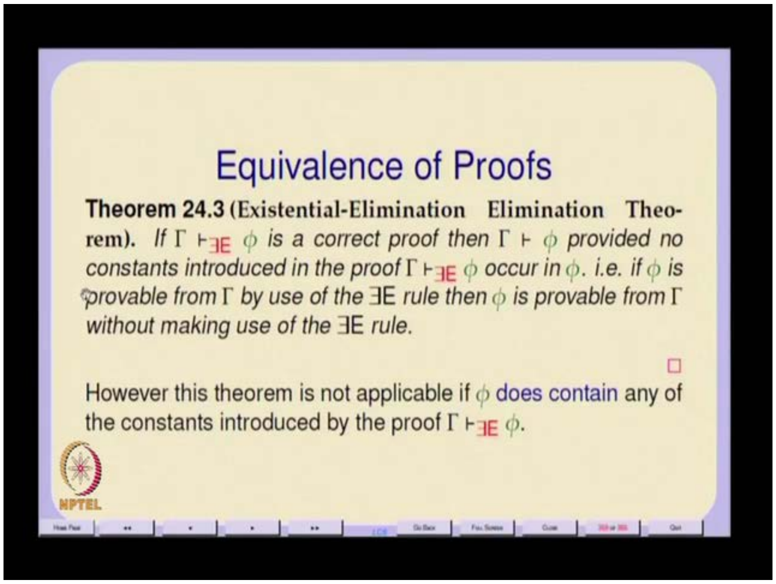

So, now will say correct we'll say a proof with existential elimination is correct if it actually satisfies this these three conditions. Now here is the theorem which have called existential elimination elimination theorem. But I think its self explanatory all I am saying now is that if there is a correct proof of phi from assumptions gamma which uses existential elimination. Then there is a proof which does not use existential elimination. And such that phi can be proved from gamma. So, the one condition that I need here is that this phi should not use any of the constants. That were introduced in existential elimination otherwise I will never be able to prove this. So, typically what we are saying is that phi should be either be a conclusion. Which is already existentially in which the existential quantifier has been introduced for each of the constants. Are those constants are eliminated by some other means but, phi should not contain any of those constants that we are introducing in this proof. So, you take this is the proof that is given to you what you are and phi has been given such that it does not contain any of these constants. Notice that phi does contains any of those constants. Then it is not a statement that is provable without the introduction of the constant. That is obvious so we will look at only those statements phi which do not have any of the constants that were introduced in this proof. So, that is last portion this theorem is not applicable if phi does not contain any of the constants introduced by the proof of this phi does contain.

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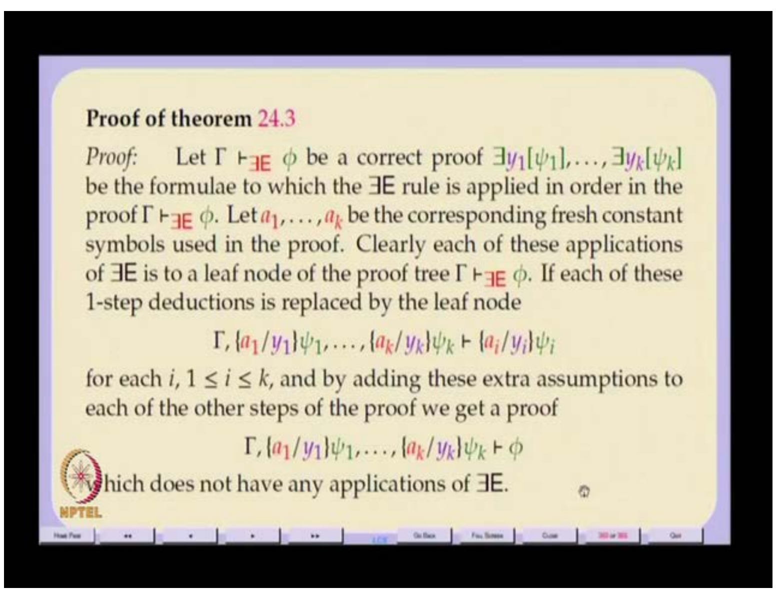

So, let us look at these proofs and it is so supposing there is a correct proof of phi from gamma. And let us say that correct proof involves there is a small error here which I have to correct which thought of only later. But when we come to that I will talk about it so if you introduced existential elimination. Then somewhere in the proof not necessarily in the assumption gamma but somewhere in the proof you had this existential formulae which were instantiated. Now what I am saying is so if these were existentially quantifiers are eliminated. Then of course the freshness condition says that for each of these formulae. You should have chosen a fresh new constant. So, let us assume which was corresponding constants a one to ak so wherever there is something here which. This statement clearly each of these applications of there exists a is to a leaf node of the proof tree but this statement is wrong. All I am saying is now what we do is I look at a fresh proof I have already introduced. This constants a one to ak I put this corresponding instantiated versions of these psi one to psi k as extra assumptions in addition to. So, take a proof tree so this proof tree unfortunately I do not have red pen here.

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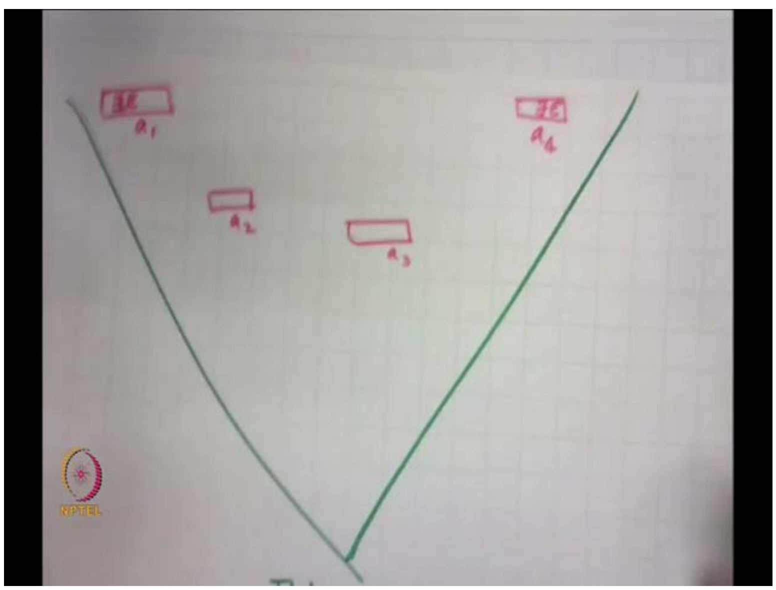

So, we have this proof tree which somehow we have some huge proof trees in which is rooted essentially at gamma proves phi. And at various stages either from gamma itself or at various intermediate stages there are applications of existential elimination. And which gave us this constants a one to ak now what I am saying is take this proof tree.

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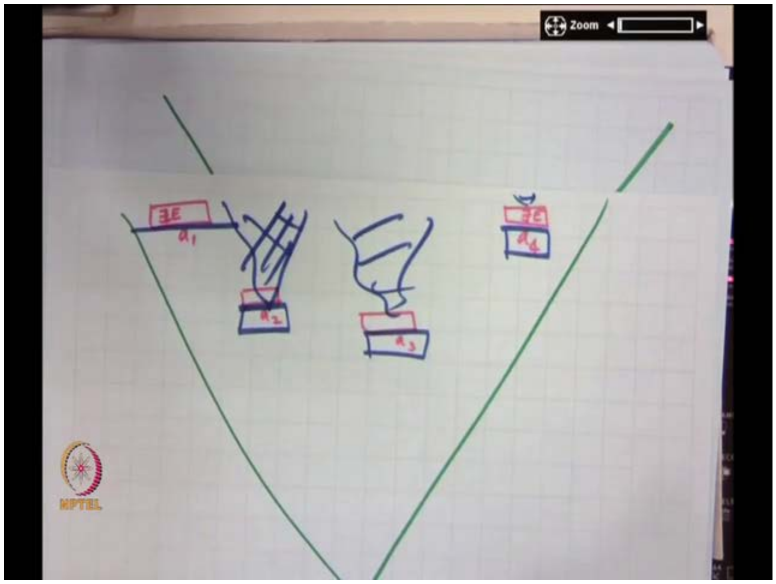

And I am going to construct another proof tree where the assumptions are gamma i am going to write this simply in red to say. That I use this a one for y one and a two for y two and so on. And essentially from this i have proved let us say phi. And of course none of these constants a one to ak occurs in phi. Now this proof tree is such that I take this proof tree and retain only this step there was a sub tree here may be but it ended in this step. So, I throwed this sub tree entirely because now I retain only this. So, this is become part of my assumptions so I throughout all these sub trees which gave me this constants. And retain only the steps with the constants in each of these steps here there was gamma. But now what I am going to have sought of pruned tree with leaves like this. Where there are these first steps are some occurrences of this a one a three a four let us say. And what is going to happen is in any intermediate step in this proof tree the gamma is replaced by gamma with all these psi one to psi k. So, the monotonicity of proofs tells me that all these steps do not get affected by doing this I just added extra assumptions. And essentially from those extra assumptions so in all these cases let me call this gamma prime. So, essentially I have gamma prime everywhere here throughout the proof. And so gamma prime is so, I get this proof tree and this proof tree is valid. But the problem with this proof tree is this proof tree proves only with the assumption psi 1 to psi k. So, now all this is for you to read I wanted to skip all these because I have explained all this through the proof tree.

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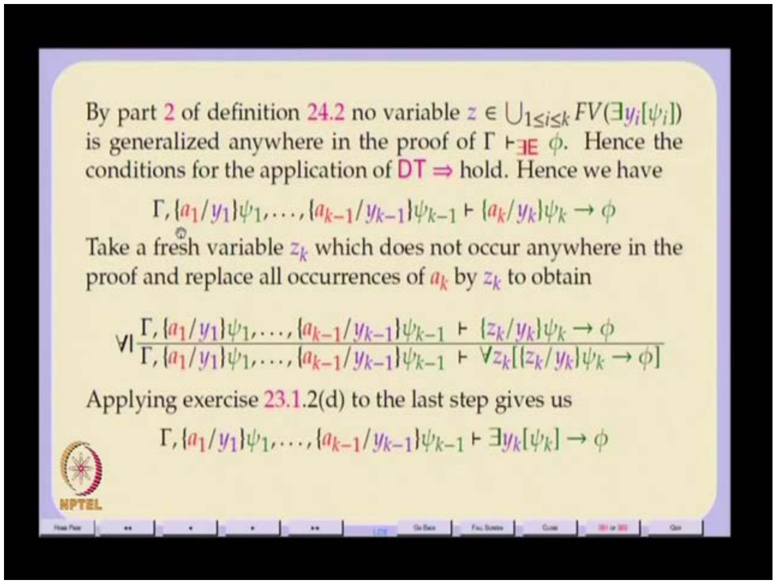

So, now what I am going to do is I take this proof tree. And I choose one of them I choose let us say psi k in this proof tree I can replace ak by a brand new variable zk. So, zk replaces ak so I am going to replaces ak now zk is actually a variable. So, what do we have now we have essentially a proof tree.

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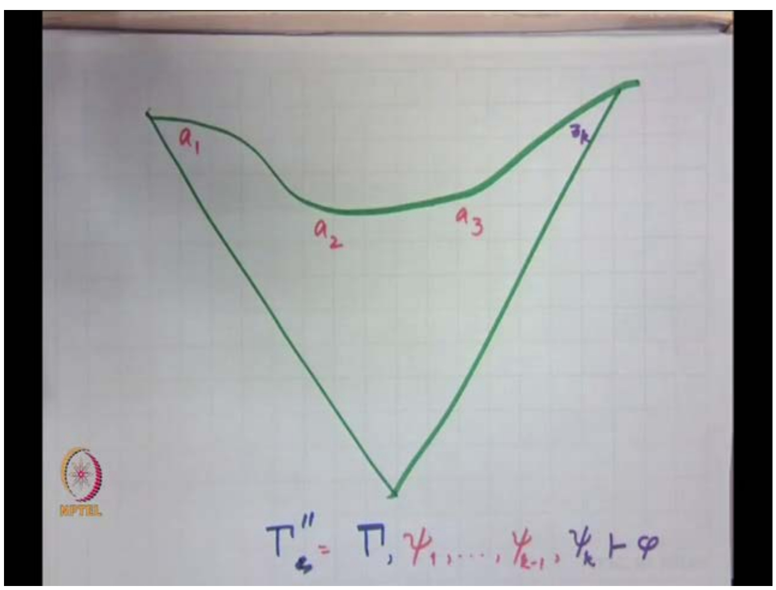

Like this I am trying to make it look as much as this is possible except that now we have gamma double prime. And what is gamma double prime gamma double prime is essentially that I still have all these is gamma along with these red psi one two psi k minus one and then now I get essentially a purple psi k proves phi. So, what i did was in this proof tree I just replaced all this all occurrences of ak by this variable zk. By brand new variable zk which does not occur anywhere in the proof and I get essentially this proof tree. And what do I have? I still have my red constants a one two ak minus one let us say. But the last one is zk now what I can do is you can see that the deduction theorem is directly applicable. Why because I have not done any quantification on any of the free variables on the left hand side zk is brand new. So, I have not done any quantification on zk. So, which means now this proof tree can be replaced by essentially moving psi k to phi. So, the deduction theorem is applicable and I can move psi k out. So, when once I have moved psi k out that is this step so now since zk is absolutely new here.

Student: (Refer Time: 55:13) Is it, then read the rest of the proofs your self's those who have a class please go I think I want to finish this just takes a few steps.

The point is this now since zk is a brand new variable now i can actually do a universal generalization and universally quantify on zk. So, I get this whole thing gamma double prime proves for all zk this psi k arrow phi. And since zk does not occur free in phi therefore I can use that exercise on free variables and this becomes there exists zk psi k arrow phi. So, this is so actually of course I replaced that zk for yk. So, I can quantify on yk essentially I get this step. There exists yk psi k arrow phi and then i can do this to each of the case.

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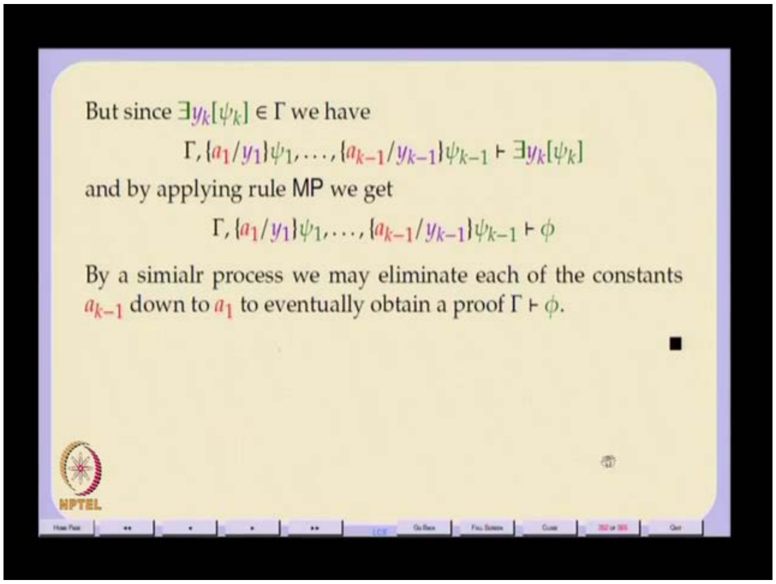

And finally I will actually get a proof of gamma proves phi without the use of any of this. So, this is not a translation is a transformation of the proof tree. So, essentially now you are free to use the existential elimination axioms rule.