Logic for CS

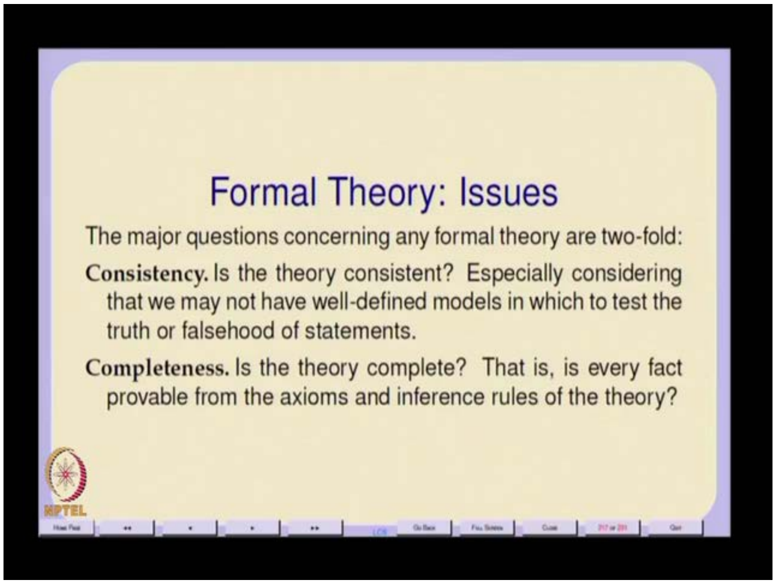
Prof. Dr. S. Arun Kumar
Department of Computer Science
Indian Institute of Technology, Delhi

Lecture - 15
The Hilbert System: Soundness

So, we were doing Hilbert system and natural reduction system. So, what will one of the things that we need to consider is the consistency and completeness of the system itself. So, this so word consistency that I, am using here is different from that set of formulae being consistent. It is that of a, whole formal theory being consistent here so that is something we have to worry about. The other thing is that we have to worry about, completeness that is that and complete actually means every logical consequence. Whether, every logical consequence can be proven in the system I mean. So, that is and we will consider only the Hilbert system for a specific reason so, which I will come to actually.

So, the major issues you know formal theory really is a theory consistent. And, that is the consistency of the theory so. And, this is especially and it is a, good idea to start with propositional logic and so formal theory. Because, there at least we have a finitary semantics which we can vary measure against. But, normally with any mathematical theory you have only a collection of abstract axioms. You do not have any formal semantics because you are trying to capture this semantics of the entire formal theory actually through the notion of axiom and proofs and theorems.

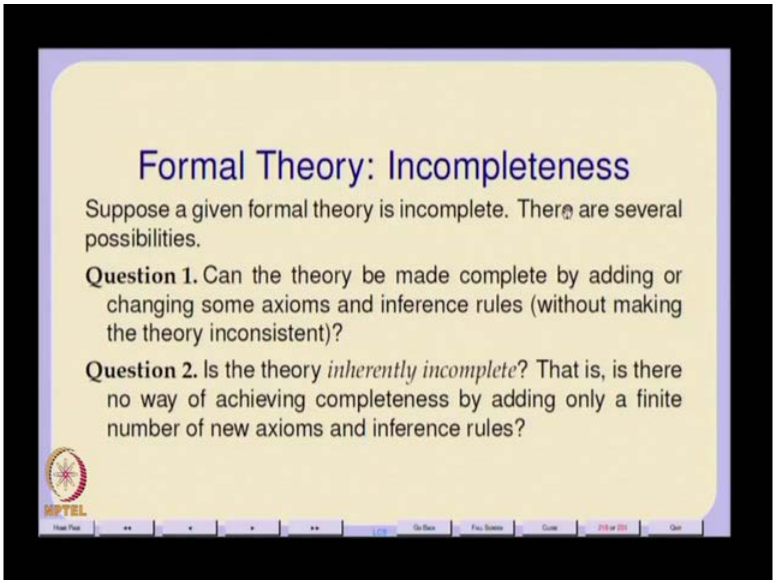
(Refer Slide Time: 02:12)



So, in most cases you do not have a semantics really available to you only have a theory available. And, then there is question of is it consistent firstly. And, is it in some sense complete and you need to do this. The development of the theory is independent of is therefore needs to delinked from a semantics. But, as beginners in formal theory we will tie it down to a, semantics initially for propositional logic. For the moment we will come to something like for sort of logic you will see that you know it is not possible to tie it down to a semantics. Because, very often semantics may not be finitary. And so whereas, it is a good excise first tie down to propositional logic because that finitary.

And then proceed towards this look at actually what should be reasonable definitions of consistency and completeness. So, consistency is the main thing is that in the case of so since we may not have. So, we want to define this consistencies and consistency and completeness also in terms of the of purely syntactic notions. Because, very often as I said when you look at mathematical theories you may not have a, semantics and you are so and we are trying to capture the semantics through finitary proof theoretic means. So, in fact what you what we normally expect is that everything that you want should be provable from the axioms. And, that is and in that sense the axioms and all the formal theorems of the theory actually capture the semantics of the theory.

(Refer Slide Time: 02:12)



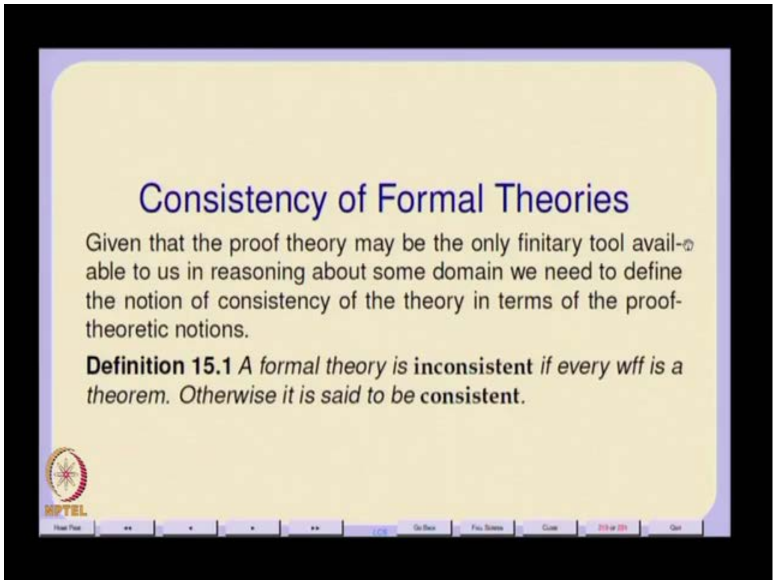
So, supposing we have given a formal theory which is incomplete. Then there are some possibilities one possibility is so when I, say incomplete and when I say formal theory is complete am essentially saying that every fact about the formal theory is either an axiom or is provable form the axioms. Now, if a theory is incomplete it means that there are some facts which cannot be provable. Which cannot axioms and which cannot be provable from the axioms system. Then you essentially have two options one possibility is that a incompleteness might be simply because you did not have sufficient number of axioms. For example if, you take the Hilbert's proof system and just removed axiom schema n then the system becomes obviously incomplete. Because, a most of the formulae that involved negation will no longer be provable. In fact for instance naught phi arrow phi for example will not be provable or phi arrow naught phi will not be provable. And so, that therefore the theory becomes incompleteness incomplete just because of a lack of an axioms.

So, one possibility in more in the more complicated cases is can I complete the theory by adding some more axioms or some more rules of inference without of course making theory inconsistent either. The question of what exactly makes a keeps a theory consistent or inconsistent is something that we will look at we consider it means to be carefully defined. How do we know that a theory is consistent or inconsistent? That is one thing the other possibility is that the theory may be inherently incomplete. Now, here I am using the word the phrase inherently incomplete in some informal sense. And, all I mean by that is even by adding a finite number of extra axioms. Which, keep the theory consistent you may not be able to complete the theory. So, in fact if you look at a the.

So, this a adding of extra axioms is what takes you for example you take this whole hierarchy of algebraic systems you start from semi groups. A, semi group is just something which has an associative binary operator. So, when you add an extra axiom about, the existence of an identity element for that semi group. Then, you get a richer structure called a monoid if, you postulate the existence of inverses unique inverses for that. Which, give you the identity element of such that each element and its inverse on the product operation gives you the identity element. Then, you actually have groups and you can take groups you can have groups. Which, then you can add an extra operator for example and you can add distributer laws you can come up with rings of even before that semi rings. And, you can get more and more you can get richer and richer structures by adding some things. So, but with just a single binary operation the question of adding an extra axiom to check to gave to get a complete theory is something that is always open. In, fact intuitively that is a most natural intuitive thing to do. But, the question still is can, You still prove is it still possible that some of your theories are incomplete? And, they might be inherently incomplete. And, here again by you might have to add an infinite number of axioms sometimes in order to get in order to complete a theory.

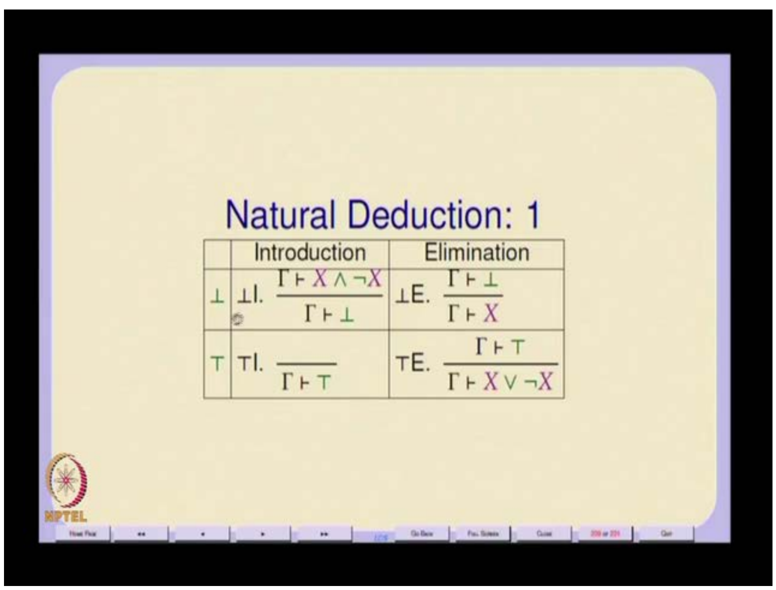
But, the moment you add an infinite number of axioms and the notion of the formal theory and finitary and this all those things go out of the window. And, so in that sense the theory might be inherently incomplete. The other possibility of course is that, instead of adding an infinite number of axioms. If I, can add a single axiom schema or a finite number of axiom schemas which capture all the infinite number of axioms. Then, the theory is no longer inherently incomplete it is still finitary. Because, of the fact that I can add those axioms in a in well defined patterns which, are at number of different patterns. And, so then the theory becomes decidable provable and so on and so forth. I, mean these are the actually the issues concerning completeness. But, we will take a farmer simpler attitude at the moment I just putting it against propositional logic which, am first proving certain completeness results.

(Refer Slide Time: 09:30)



So, will say that a theory is consistent if it is not possible to prove all the well formed formulas of the theory. This is a simple I, mean this definition relies on the fact that if you can prove a contradiction then from that contradiction you can prove any formula.

(Refer Slide Time: 10:10)

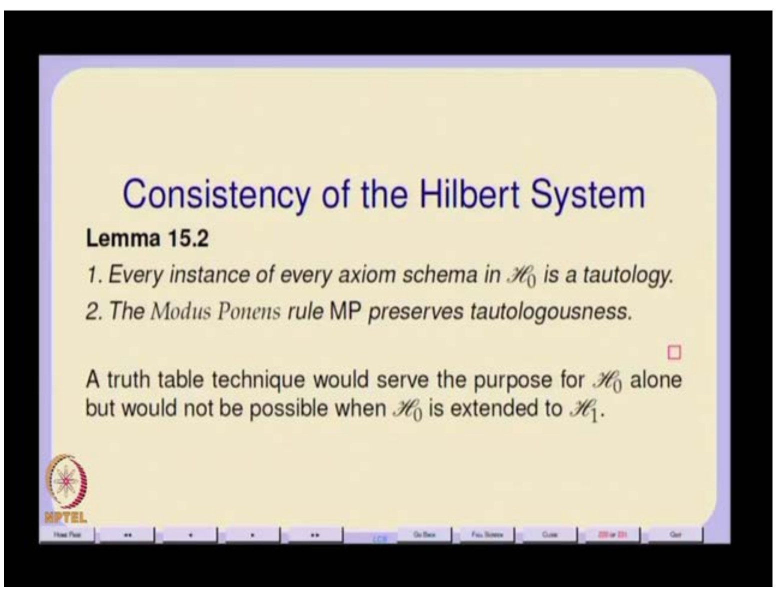


In fact if you look at, your natural reduction prove system is just quickly go back look at this, bottom introduction and elimination rules. So, the introduction rule just says that if I somehow manage to prove a contradiction of the form X and naught X then I can infer bottom. But, having inferred bottom I, can eliminate bottom by inferring any formula. So, if my axioms of my formal theory can provide me a contradiction. Then, from that contradiction I can infer any formula of the theory. So, our notion of consistency therefore is just this if all formulas of the theory approvable then here theory is inconsistent. Which i mean the theory is really desperately in need of some changes. And, it is actually uninteresting and that is another way of looking at it and inconsistent theory is logically uninteresting.

So, that is that is going to be a notion of that is going to be a notion of inconsistency of theories. So, otherwise so, what we are saying is now that issue can show that there is at least one well formed formula. Which, is not provable then, the theory is consistent that is actually the definition. So, but this is slightly different technically slightly different from our normal notion of consistency of the theory. We, normal notion of consistency of a theory is just that it should not be able to prove any contradictions. But, an immediate consequence of that is that if you can prove a contradiction then you can prove anything in the theory every formula in the theory. And, it is actually closely philosophically tied to the notion of material implication. I mean actually it is tied to the notion of logical consequence and material implication. And, the fact that we have a certain truths semantics so material implication. And, we have a notion of definition of logical consequences actually comes from that definition of the truth of material implication. Because if, you look at material implication as a causal relation then the kind of truth semantics that we have given for the arrow no longer holds. So, philosophically causality is different from implication. Even linguistically causality something causes something is different from if something then something else. And, all that we are saying is that in the case of mathematical theories I, suppose to say physical theories. In the case of mathematical theories there is no causality there is a clear what is known as platonic existence. I, mean facts exists and your logical theory only presents the facts in a certain order. That order in which the facts is not a causal relationship between a theorem and axiom though, informally we actually treat it like causal relationship it is not. And our prime example of that we have already seen before. We, have seen these two systems right h naught and h naught prime. So, they both have slightly different the n axiom is slightly different in both cases. In each case the axiom of the other system can be proven from each system. So, clearly there is no causal relationship between these systems. And, these axioms all that we are saying in a formal theory is that there is a huge set of facts we happen to chose some of those facts as axioms. And, we are deriving other facts from these axioms using the rules of interests.

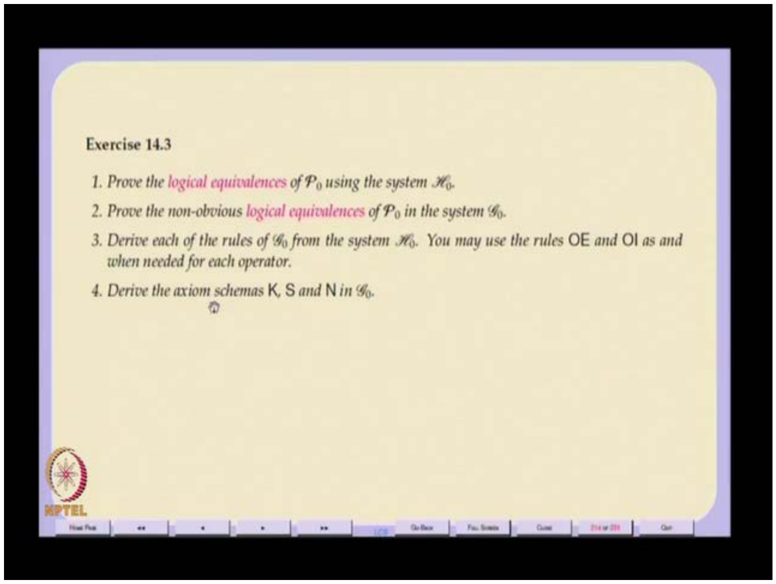
And, therefore there is no causal relationship between of between one fact and another there is no causal relationship between one fact chosen as an axiom. And, another fact which happens to be a theorem in your presentation. All, that your formal theory is all that you are doing is you are making a systematic presentation of the facts rather than actually defining a causal relationship. And, in that sense mathematics is very different from let us say the physical sciences. Where, in the physical sciences in physics for example a fundamental notion is that there is such a thing is cause. And, there is such a thing is effect I mean if a certain particle accelerates at a certain non 0 rate. Then you can infer that that has a cause namely a, force acting on it. So, the notion of cause and effect and actually in the physical sciences the fact that effect always follows the cause. And there is a response time difference between the two all that is absent from arithmetical theories I mean. So, you have to look at these issues of consistency completeness and so on in the larger context of what exactly does it mean to have. And, to have a certain kinds of definitions of these theories. So, we are just talking our presentations of theories basically what, might be called a platonic presentation of these theories.

(Refer Slide Time: 16:43)



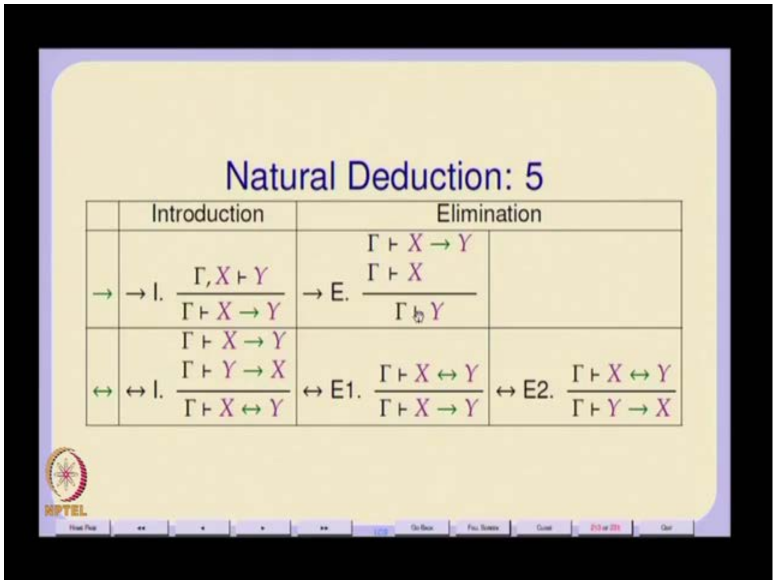
So, let us come to the consistency of the Hilbert system. So, one thing of course is that the reason we are just considering only the Hilbert system is because of this excise. You take the natural reduction system.

(Refer Slide Time: 16:57)



And, every rule of the natural reduction system can for example we derived from the Hilbert system. So, you can look upon every rule in g naught the genson natural reduction system. As essentially I, derived rule of h naught if you like. On the other hand it is also possible to take natural reduction as the as your axiom system. And, there is already form of modus opponents in natural reduction system in natural reduction. Because, we had this arrow elimination rule we have this arrow elimination is modus ponens in sequent form.

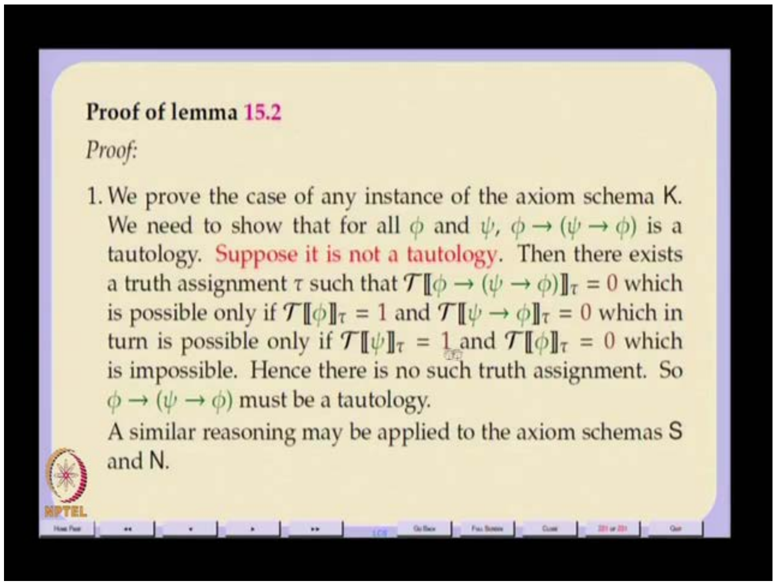
(Refer Slide Time: 17:43)



So, modus ponens is already there all that you need to do is to derive the axiom schemas K S and N in the natural reduction system. In which case what it means is one thing is that having just 3 axiom schemas is very convenient technically convenient. Because, it is the TDM of dealing with 100 of rules of inferences is absent. So, will take the view that firstly the axiom schemas of the Hilbert system can be proven from the natural reduction system. And, modus ponens is anywhere is there. And, if you can prove that the Hilbert system is complete then, it automatically shows that the natural reduction system is also complete. So, it us says is that lot of bother that is one thing. Of course there is a notion of consistency but, we can show that from by using the reduction theorem and so on so forth. We, can show that there is a natural reduction system does not create does not actually yield in consistent theorems collection of inconsistent theorems.

But, I will not go into that really. So, but in so, I will just concentrate on the Hilbert system and because it is much smaller its minimal it is technically therefore shorter to deal with. So, one thing is this every instance of every axioms schema h naught is tautology. I mean this is something that you can just sit and prove I, mean you take any I mean in fact you can just. So, but when we are talking about, consistent completeness. We, are now just talking about it is interaction with the semantics. And, so that is one thing the other thing is modus ponens rule actually preserves tautologousness. I mean these are the with these two facts I, mean if you can prove these two which, is fairly easy to prove.

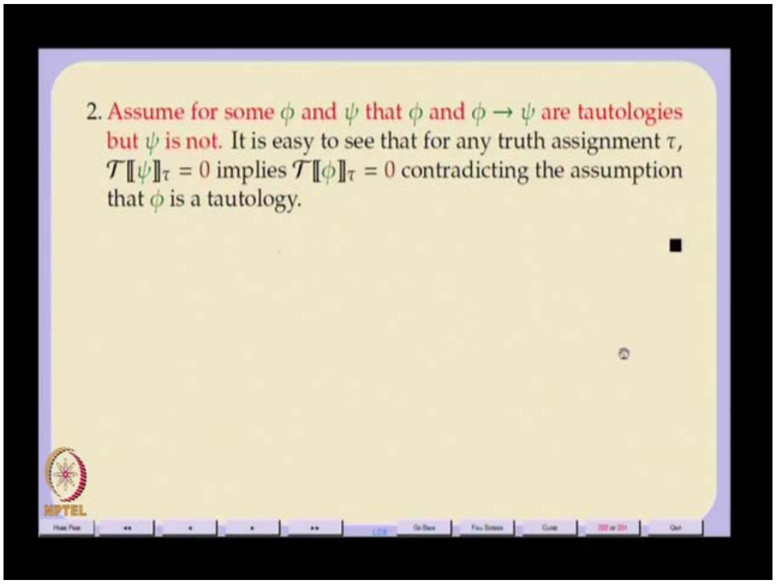
(Refer Slide Time: 19:47)



So and it is really easy to prove using the semantics of propositional logic. Because, we have truth tables based semantics which is all finitary. But, supposing we did not have something that was finitary then actually we would have to prove in some indirect fashion. And, so this proof that I am giving even though it is of an obvious and trivial nature. Is, essentially that of something which allows you to deal even with the case when semantics might be infinitary.

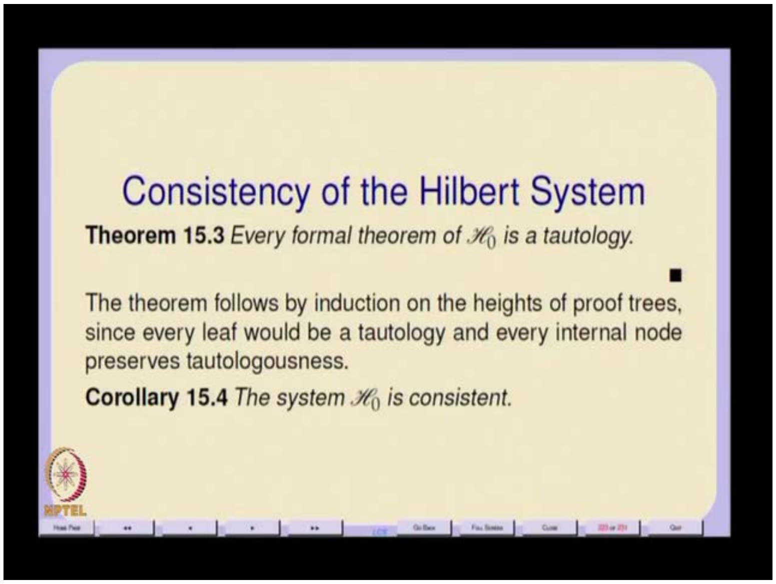
And, therefore you would not have finitary semantics to deal with. So, actually so you just let us as I simple example let's just take the axiom schema K. Which, essentially means that for all phi for all formulae phi and psi we need to show that phi arrow psi arrow phi is a tautology. And, so instead of 1 obvious thing is to do is to take the truth table semantics of this and show that it is always all the in the last column of the truth table you always get a 1. But, as I said we need to consider the case when the semantics may not be finitary. So, in that case this you need a somewhat more general kind of proof. And, as usual since we are we are dealing with logic also as branch of mathematics we use mathematical reasoning in our normal way. So, we prove it by contradiction suppose it is not a tautology. Then, there exist some truth assignment tau such that this is true which, is possible only if phi is true under that truth assignment and psi arrow of phi is false under that truth assignment. Which, in turn is possible only if psi is true and phi is false. But, then we also require that phi is true and since phi cannot be true and false the same truth assignment. Therefore, we have got a contradiction and therefore this must be a. So, it means that there does not exist any truth assignment which, false is phi arrow psi arrow phi and, therefore it must be a tautology. So, you can use a similar reasoning for S and N also. And, therefore you essentially you have essentially proven that every instance of every axiom schema and h naught is a tautology. And, now what you have what you have to show is that the modus ponens rule preserves tautologousness. I mean the modus ponens rule preserves lots of things it preserves at it might preserve satisfiability, it might preserve truth, it might preserve validity and it proves lot of. But, in this particular instance we are interested in tautologousness.

(Refer Slide Time: 22:44)



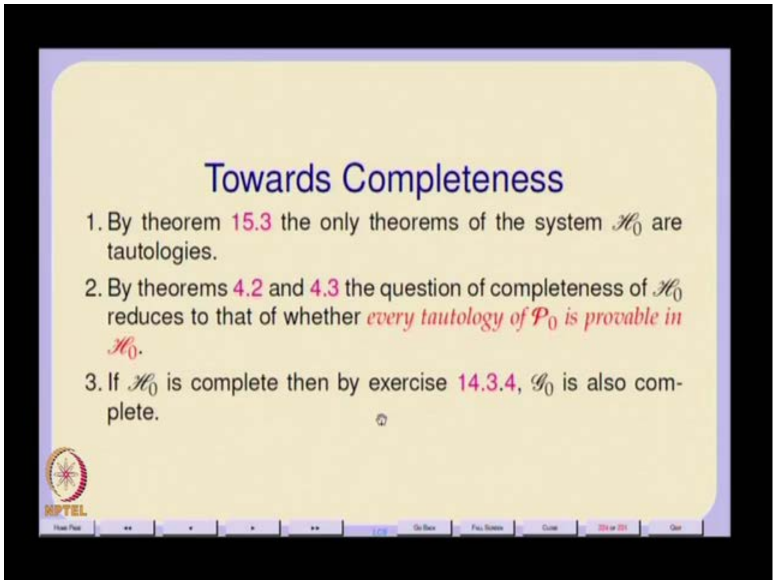
Which, means that if phi and phi arrow psi are both tautologies then, psi is a tautology. So, here again you can go through the proof in a pretty straight forward manner just assume that phi and phi arrow psi arrow tautologies. But, psi is not a tautology in which case the any truth assignment will imply that something it will contradict some assumption here. So, here we have preserving tautologousness. And, that is because essentially what we are saying is that the Hilbert's system if, you look at our formal theorem. You, might use a deduction theorem and create assumptions and so on so forth. But, our formal theorem something that, is proven without any assumptions. If you look at, our formal theorem and all the formal theorems are actually tautologies. And, so that is actually the so the Hilbert system.

(Refer Slide Time: 23:49)



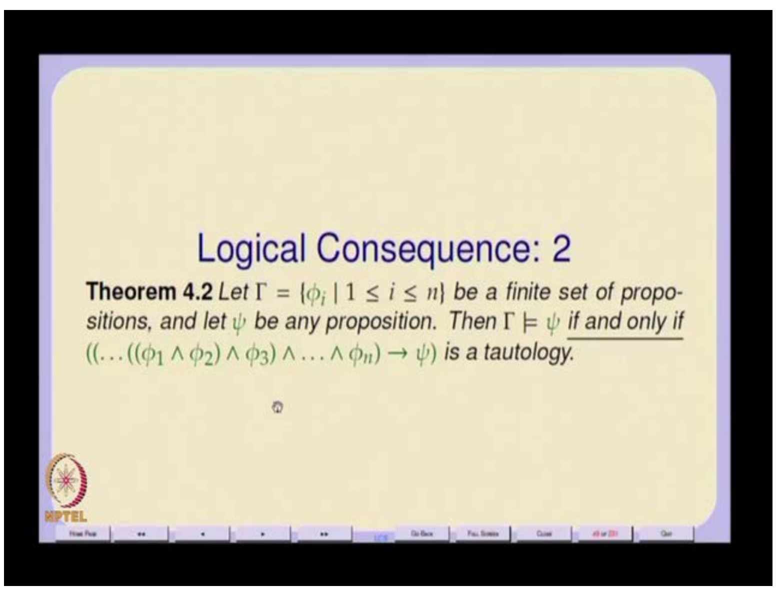
So, one thing is so everything that is provable in the Hilbert system is a tautology and we will do no other exists non tautologous formulae in the language L naught. And, therefore the Hilbert system is consistent I mean it is. So, we are not doing a usual thing like you know approving that you cannot prove any contradiction it is of too complicated. Technically it is simpler to show that you can know that you can prove only tautologies. And, there are prove there are formulae for example like naught of phi arrow phi which, are not tautologies. And therefore, the Hilbert system is consistent it is a very clean and neat way of handling the issue of consistency. But, it also has a, but if there is also this. So, now if every formal theorem of h naught is a tautology the of course the notion of completeness reduces to a simpler notion. Can every tautology be proven that is it, that is really all that there is to it. And, actually so this so the fact that every formal theorem of H naught is a tautology you can prove by induction on the height of the proof trees. So, therefore the system H naught is consistent.

(Refer Slide Time: 25:21)



But, now the Completeness is also is also just related to tautologousness. But, actually that is that is perfectly fine because we had this.

(Refer Slide Time: 25:38)



So, we had these theorems of logical consequence which essentially said that all logical consequences can be expressed in terms of tautologies. So, the notion of completeness also reduces to just that of tautologies. And, the fact that we have got a deduction theorem means that if you want you can move some of their sub-formulae on the to the left hand side of the turn side. So, provability which is a more general notion than theorem hood theorem hood is proving a formula without any assumptions. Provability is given a set of assumptions gamma you prove some formula. So, because of their deduction theorem we can move backend forth and either side between provability and theorem hood. And, therefore it is sufficient to just consider the case of whether every tautology is proven. And, if every tautology is provable in H naught then, H naught is complete. And, therefore the G naught is also complete since everything in H naught can be proven from G naught so, G naught is also complete. The question of whether G naught remains consistent is something you have to think about. I, mean you have to show that each application of the rules of influence will not lead you to will not allow you to prove any formulae. But, we leave for that for a moment will live with the simplest system of h naught.

Student: (Refer Time: 27:16)

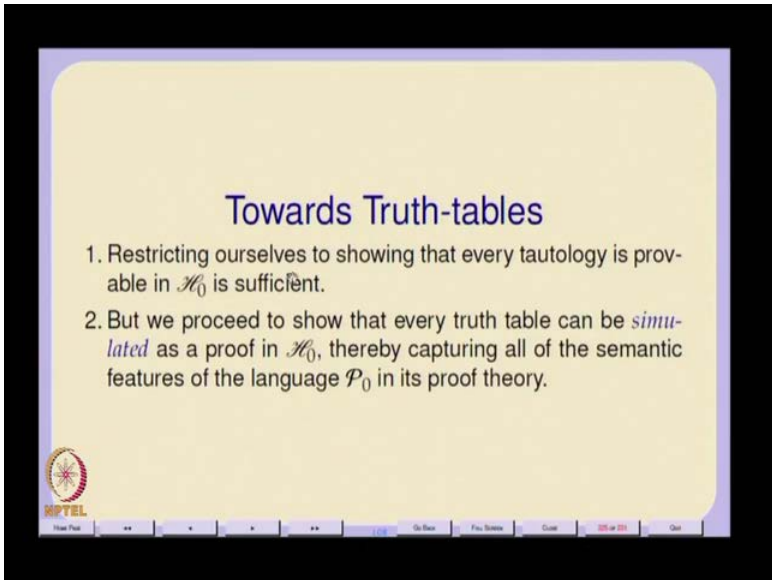
I, am just saying that a definition of consistency of H naught definition of consistency is that not all formulae should be provable. And, I am saying that a every theorem that you can prove in H naught is a, tautology. And, they will do exist formulae which are not tautologies. And, therefore H naught is consistent. So, that so but more importantly actually so technically speaking therefore it is sufficient to show that every tautology is provable in H naught to show that it is complete. But, there is something about a formal theory as I said, you are essentially trying to capture semantics in purely syntactic fashion that is what you are doing in a formal theory. You, start with a, new theory with a set of axioms. Which, are essentially which some suppose to capture the semantics all the models which are true for that theory. You take the whole idea is that you if, you were to take I mean. So, these kind of notions again come from traditional Euclidean and Non-Euclidean geometries. I, mean you take lets go back to that famous problem of the parallel postulate. If I, take all of Euclid’s axioms with the notion of point line and plane are defined.

And, I change the parallel postulate I say that through a given point where the notion of point is undefined not on a given line where the notion of a line is also undefined on a plane. Where, the notion of plane is undefined there does not exist any line parallel to the given line. So, I take this is a formal statement of a new axiom which replaces Euclid’s parallel postulate. So, now you got all these postulates and some more these, postulates are supposed to capture the intuitive notion of a point line or plane without actually defining them because, they are undefined terms. Now, what is a theory give you what kinds of models does this theory with this Euclid's parallel postulate replaced by one. Which, says that there does not exist any line parallel to a given line through a point external to the given line. And, the hunt for and what you are trying to say so what you do you are trying to prove theorems about this and you try to come up with some models. Which, will satisfied and in fact there is a model which satisfaction for example if you are to take if, you were to interpret a point as a point on the surface of a sphere. A line as a great circle of that sphere and, the plane as the surface of the sphere itself then suddenly this geometry make sense. So, the points are all points let us say markbell attitudes and longitudes maybe in a polar coordinate system.

And, so you what you got are models so a model of Euclidean plane geometry is really the cartesian coordinates X Y which and what we learn is analytic geometry in school and later in college and so on. So, that is a model for Euclidean plane geometry and for this Non-Euclidean geometry where the where the postulate changes I, have a different geometry based on the surface of a sphere I mean. And, in fact this parallel postulate this new postulate which says that if, the lines are only all the great circles on this sphere. Then, any two great circles always intersect in two points for example. So, therefore that is another thing I mean so you will be any two lines any two non-parallel lines intersect in a unique point goes away. Now you will have in two points so, you get a complete different it is I mean.

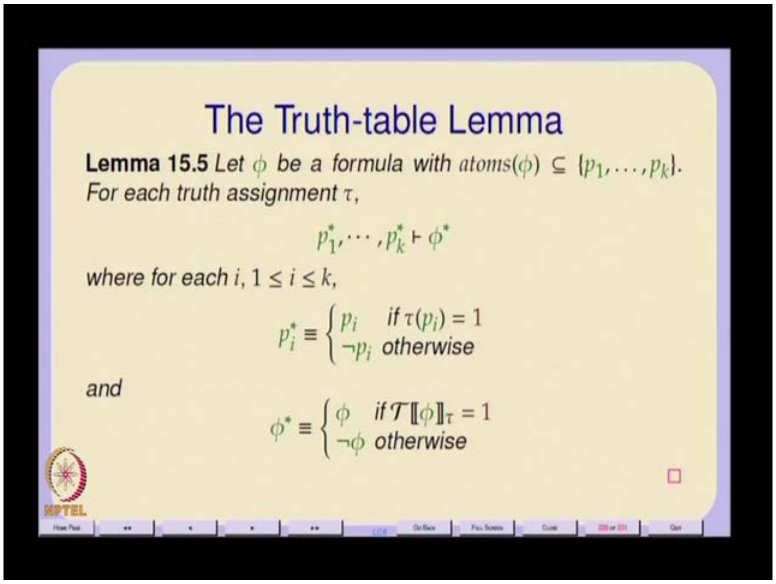
So, now the fact that you could come up with a model is a different matter the theory could exist even if you dint have any models. In which case what you are trying to do is you are trying to capture all the properties of the theory only through those axioms. And, that is that is an interesting aspect of this. So, this so one question is in the case of propositional logic of course the question of capturing semantics still remains. And, one obvious question is, Can we capture truth tables semantics? Throughout when and there so what will do this. We will show that our the Hilbert system is actually expressive enough and powerful enough to capture truth table semantics. From which it, will follow that, any tautology can be proven. I mean I just have to simulate truth table for that tautology and i have proven that tautology. What coupled with the fact that I know that the only things I can prove tautologies. And so I am consistent and at the same time I have captured all the semantics that is what I am that is we will work to it.

(Refer Slide Time: 33:46)



So, essentially even though we need to restrict ourselves to showing only that showing that every tautology is provable in the system will we proceed to show that we can actually simulate every Truth table within the system H naught. And, if you can if you can essentially if you can essentially capture every truth table you capture all the semantics I mean basically. And, therefore there is nothing there is no gap between your formal theory and your semantics I mean it is the two are perfect. So, anything that can be expressed in the semantical domain can be expressed also and theoretical domain and vice versa. And, so I mean so just like logical consequence corresponds to provability and tautologous corresponds to theorem hood. We, can also capture truth table and that is what we will proceed it to. So, what so this is I named the truth table lemma. It is not an official name anywhere so what I am saying is.

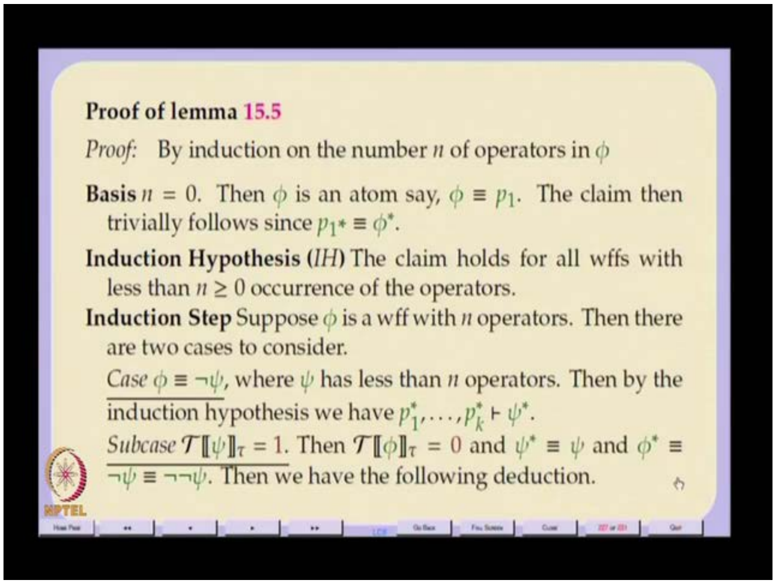
(Refer Slide Time: 35:19)



So, let take any formula phi of course strictly speaking this truth table is only an unformal notion I mean. What, we actually had inner semantics truth assignments truth assignments for an infinite collection of variables. And, therefore we should we should work with that so informally I am calling it the truth table lemma. But it is we are actually going to deal with truth assignments. What however, we are guaranteed is that every formula phi has only a fine is made up of only a finite number of atoms. And, since we are talking about the Hilbert's system the only operators are not end arrow. So, let us assume that phi has atoms only drawn from this finite set P1 to Pk for some K greater than or equal to 1. Now, take any truth assignments tau of course so and what I am, claiming is that I can prove so this is a formal theorem of the Hilbert system.

Where, also this P1 star to Pk star are also each Pi star is Pi if the truth assignment if under tau Pi is been assigned 1 and, if it is 0 then I take naught Pi. Similarly this phi star is phi if under the truth assignment tau phi has got the truth value 1 otherwise I said naught phi. So, all I am claiming is that this from the assumption P1 star to Pk star I, can prove phi star. In that sense all that is relevant for the phi is captured which, is essentially capturing the truth table of phi given a truth assignment.

(Refer Slide Time: 37:21)



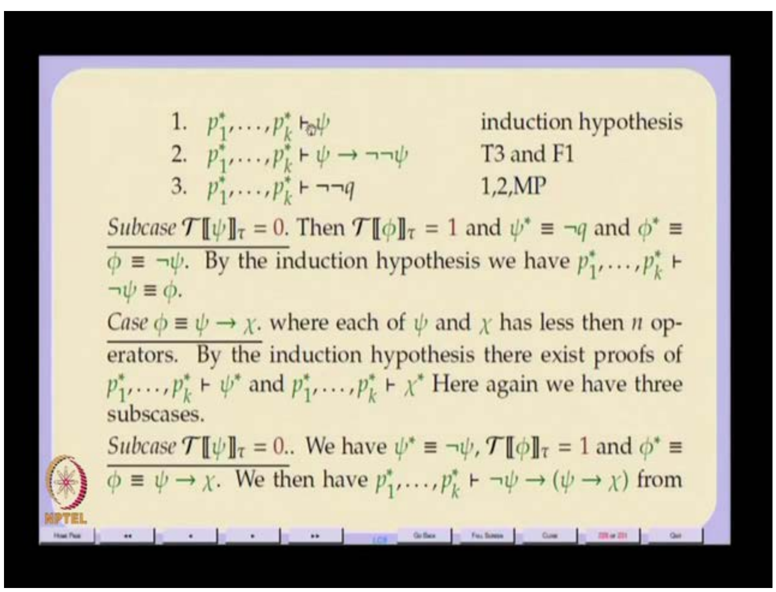
So, let us look at the proof of this and how we are going to actually go about it. So, one thing is so what will do is we are saying that this is can be done for every formula phi given some super set of all the atoms which make a phi some finite super set of all the atoms which make phi. So, what will do is do this by induction will prove this by induction on the number of operators in phi. So, when n is equal to 0 when the number of operators is equal to 0 then, phi must be clearly an atom. So, without loss of generality let us assume P1then clearly then P1 star and, phi star are the same thing. So, if the truth assignment assigned true to P1 then phi star must be also P1 so all you are saying P1 proves P1. And, even if you add p two to p k as assumptions it is not mind so that's so the basis is trivial because then P1 star is the same as phi star. The, induction hypothesis is that the claim holds for all with less than n occurrences of the operators.

And, now suppose in phi is well-formed formula with n operators. Then, we have two cases to consider basically we have only two operators the language. Because, we are dealing with the language L naught which contains just arrow and negation. So, we have two cases corresponding to those operators. So, the first case is when phi is of the form naught psi and of course psi has less than n operators. So, further I can assume the induction hypothesis that so this is this look at in the terms of truth table all that you are saying is phi is of the form naught psi is less complex than phi by 1operator. And, you are in your in your truth table you would have these columns for P1 to Pk. And, then you would have the formation with the operators and the n minus 1th column following will have the truth values of psi. The, n'th column would have the truth value of phi in the truth table. So, all that you are saying is so now with this psi has less than n operators.

And, so the truth table corresponding to psi is essentially a sub-truth table of the truth table corresponding to phi. And, essentially by the induction hypothesis you can prove P1 star to Pk star you can prove psi star from P1 star to Pk star. So, you take so your truth assignment any the truth assignment tau corresponds to some row of this truth table. It gives you a unique row in that truth table based on what are the assignments to the individual atoms P1 to Pk so, that is one unique row. And, you are looking at in that row what is the what is the truth value of psi psi cloud be true or false. So, psi star is appropriately chosen and you and. So, therefore you assume that you already proven that this already this proof is already this.

Now, of course there are two possibilities psi might be 1 or it might be 0. In that in that particular row if it is 1then of course phi is not psi. So the truth value of phi would be 0. And, so therefore phi star would be the same as psi and phi star would be not psi. Which, is naught since this psi star should be so psi star would be psi star would be naught psi. But, this is there is some because phi has got the truth value 0 phi star would be naught of naught of psi. By our definition here, it should have been naught phi, phi star should be naught phi and phi is naught psi so, phi star is actually naught psi. Then, you have the following deduction it is very simple.

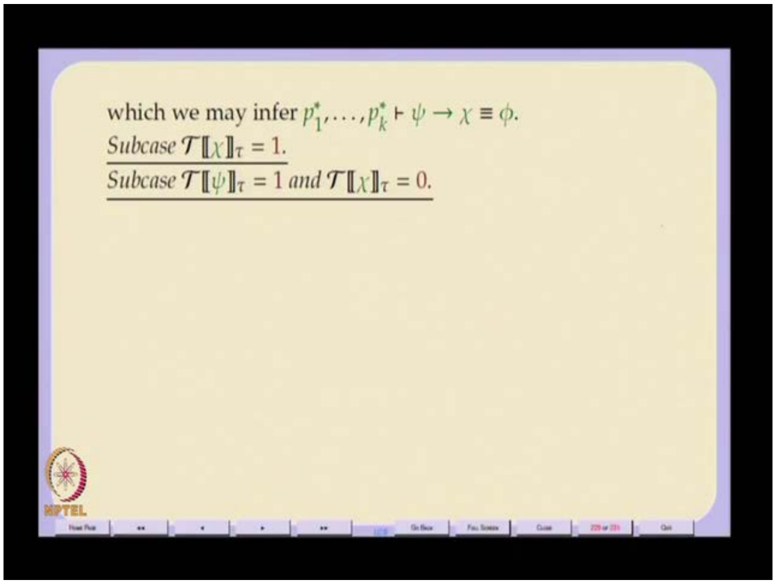
(Refer Slide Time: 41:53)



So, one thing is that you already have proven psi from P1 star to Pk star. And we know somewhere that we have proved this double negation psi arrow naught psi.

This, should be this should be a naught psi here. And, by modus ponens. And, therefore you have essentially proven phi.

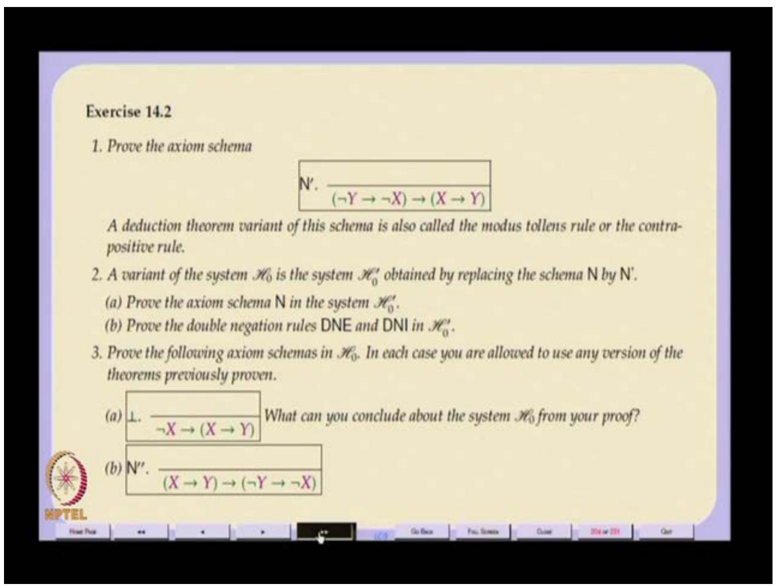
(Refer Slide Time: 42:24)



So, you take the case when so this was the sub-case when psi was false we should take the sub-case when psi is true. This should have been 1 here, I made some mistake. And, then what you can assume by you then what will happen is your psi star would be some naught phi star would be naught psi and by induction hypothesis you have this.

So, this is the case when psi was assigned to 0 in which case T of phi is 1 psi star should be naught psi and phi star would be also phi star would be phi therefore it will be naught psi. And, by the induction hypothesis we already have P1 star to Pk star proves naught psi which is the same as phi. This is the case of negation you have two sub-case. Next the natural thing is to do consider the case when arrow. And, in the case of arrow actually it is there should be four sub-cases. But it is not necessary to consider four it’s enough to consider three sub-cases so one sub-case. So, two of sub-cases are when psi is been assigned false then, it does not matter what Kai is been assigned. So, regardless of what Kai has been assigned you can just consider the sub-case when psi has been assigned false. Similarly if Kai has been assigned true then it is not necessary to consider what psi has been assigned. That, leaves only the case when psi is assigned true and Kai assigned false. So, out of the four cases three of the cases are taken care of by the two sub-cases by the independence of either Kai or psi. And, you need to consider these three sub-cases and in like manner you just use induction hypothesis and some of the theorems that you proven before. So, actually it is important for you to proven all this gone through all this exercises. I, hope you all diligently been solving all the exercises you know. And look at all these fantastic exercises and all there are whole lot of other theorems also which you need to prove. But, essentially you do this case analysis and you can prove that that truth tables can be captured so every row of every truth table therefore can be captured by your by the Hilbert style system.

(Refer Slide Time: 45:23)



And, from that what you actually get therefore is that a basically from that actually what you get is this that. Therefore, in particular for all tautologies all I am saying is that I have to capture the truth table of that tautology that is it. And, I can capture all truth tables therefore I can capture all the truth tables so all the tautologies. And, from a previous consistency result I know that I cannot prove anything other than tautologies. So, actually what we are what we have done here is you taken a row of that truth table of each any truth table.

And, you are saying that there is a tautology formula exactly corresponding to that row. Where, you take negation of the atom you also you take that row to be essentially a big kind of truth values. So, if the truth assignment gives a atom truth value of 1 then you take the atom itself otherwise you take it is negation. And, you are essentially taking that naught, taking that and of all that. And, you are showing that that implies a last truth value which is your phi star. Which, is exactly what logical consequences of what. So, essentially what so the completeness of the system therefore just relies on this. Therefore, the Hilbert style system is complete. Because, first I can prove only tautologies and I can express every row of every truth table as a, tautologous formula. And, I can claim that I can prove every one of those tautologies. So, I have a general method of proving it from the Hilbert style system I have a method of proving every row of a truth table. And, hence the Hilbert system is complete is actually an quite interesting proof the but what makes that even more interesting is that it is not applicable beyond propositional logic.

The moment you comment to something like for sort a logic you require completely different techniques. And, therefore it is an education and itself to studied for propositional logic first before we get on to first tautology. So, with that I think we have essentially done a fairly thorough study of propositional logic. For many of you I hope who thought propositional logic versus trivial I hope it is opened your eyes to the possibilities of complexities even propositional logic. And, to the fact that you have learnt that least of you know techniques which you dint know could for there already there in propositional logic. So, there is a lot propositional logic is a very rich subject like number theory you know. So, this is it is always possible to come out with some new and interesting patterns or ways of doing things in propositional logic. But, as you become more and more general you will get restricted in your techniques. Because, many of these techniques will not be applicable there. So, what will do is we will start first tautologic from the next from the next time.