Logic for CS
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Lecture - 11
Maximally Consistent Sets

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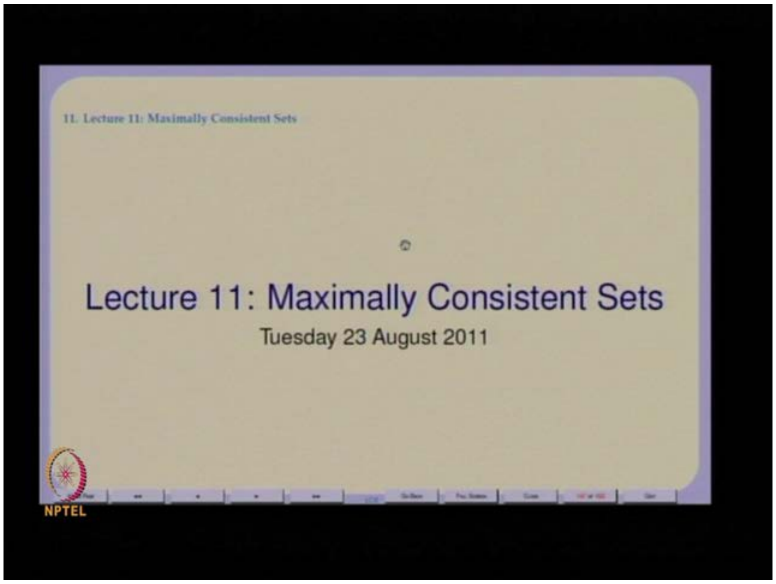

So, let us start so last time we did the compactness theorem today will do something known as maximal consistency but, just briefly go through the compactness theorem. So, essentially what we are seeing is that if you take so you the Compactness theorem is essentially says that. You take any countably infinite set of propositions it is satisfiable if all its non-empty finite subsets are satisfiable.

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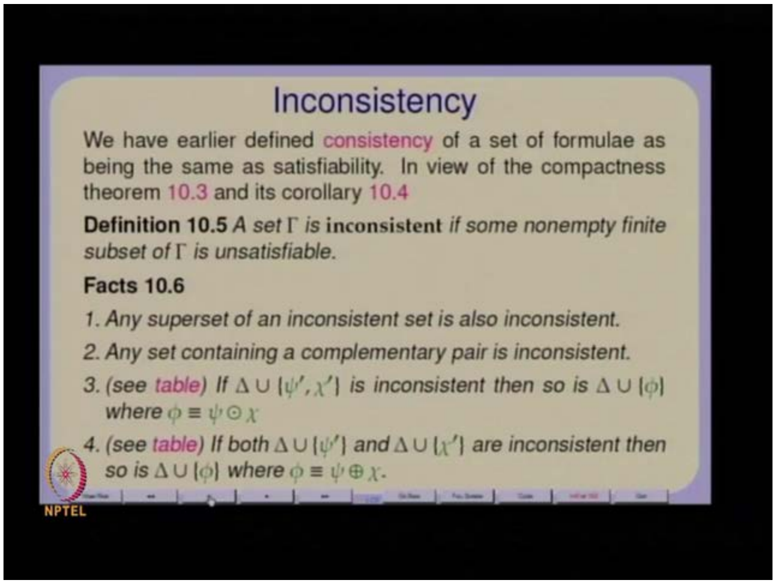

And we went through this proof and a simple corollary is that any finite or infinite set of formulae is satisfiable if and only if all its non-empty finite subsets are satisfiable.

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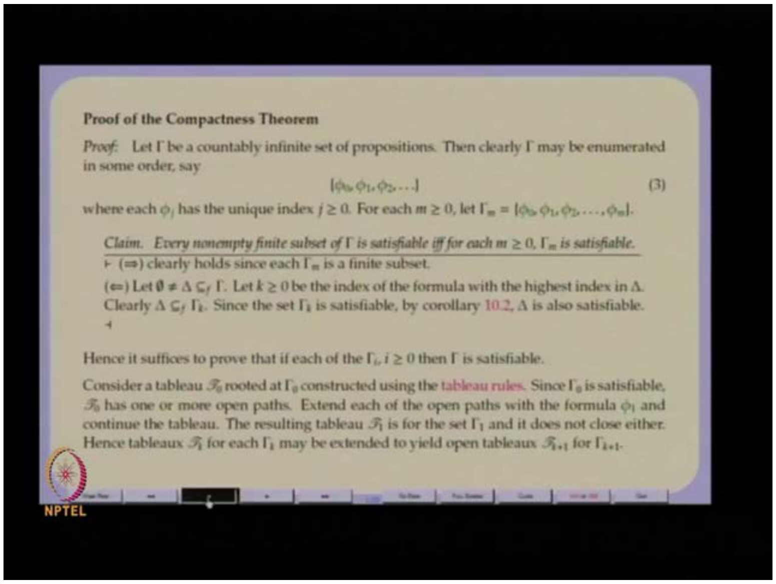

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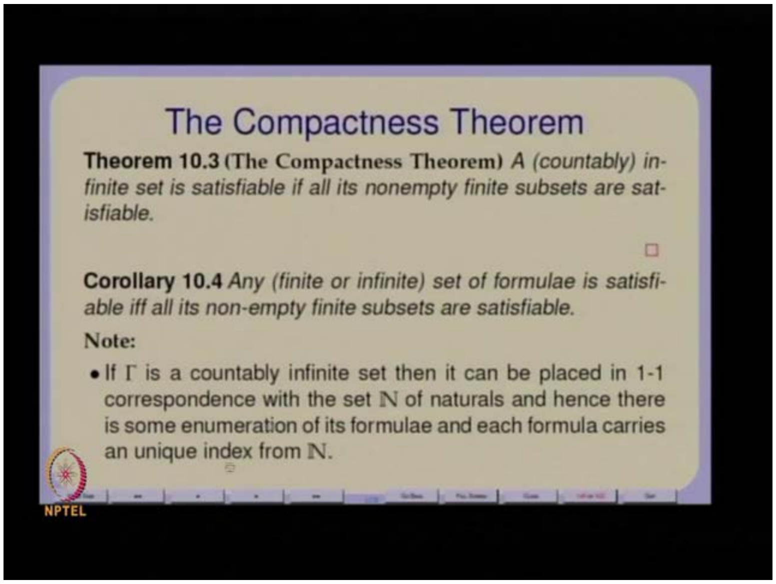

So, the proof of the compactness theorem is something that we actually proved it using the tableau itself. The essentially created Hintika sets and we use Chemi's Lemma there are other proofs of the compactness theorem which do not require the use of tableau rules which can be which can be proved from other things. There are also proofs of the compactness theorem which do not require the use of Chemis Lemma and Hintika sets. But let us for the moment let us stay with this.

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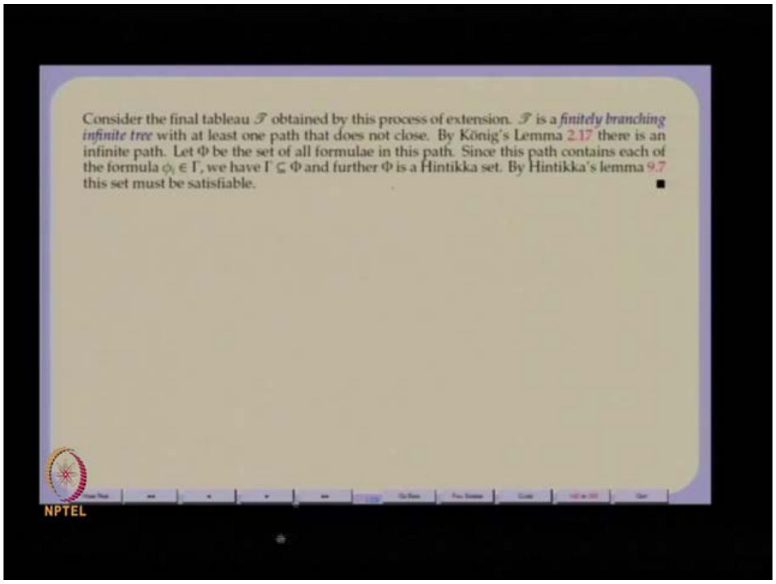

So, the compactness property is so important that is always good to have many different kinds of proofs. As sort of validation that validation that you have statement is actually correct. So, 1 consequence of the compactness theorem is that of inconsistency. And that is that we define consistency of the set of formulae as essentially that all the formulae should be simultaneously satisfiable. And so, in since it is a same as satisfiability of a set of formulae. Now, in view of the compactness theorem and its corollary actually more importantly its corollary. Is that I can negate both the hypothesis and conclusion of this corollary and essentially state that a set is inconsistent a set gamma is inconsistent if and only if at least 1 non-empty finite subset of it is inconsistent. Or unsatisfiable so inconsistency is a same as unsatisfiability or rather it is a same as simultaneous the negation of simultaneous satisfiability.

So, the other thing occurs is that therefore just like a subset of a consistent set is always consistent a superset of an inconsistent set is of course always inconsistent. And in particular if, you go through the tableau method and so on so forth any set containing a complementary pair is inconsistent. And through the tableau methods if for any formula phi if you have both the of phi of the form psi multiplicative operator Kai. Then, if you have both's psi prime and Kai prime in your set of formulae. Then, the set is inconsistent and in the case of an additive operator if you have if both the sets two sets which are identical except them 1 of them contain psi prime and other contains Kai prime.

If, both of them are inconsistent then the original set containing is phi is also inconsistent. So, this is what we did about inconsistency.

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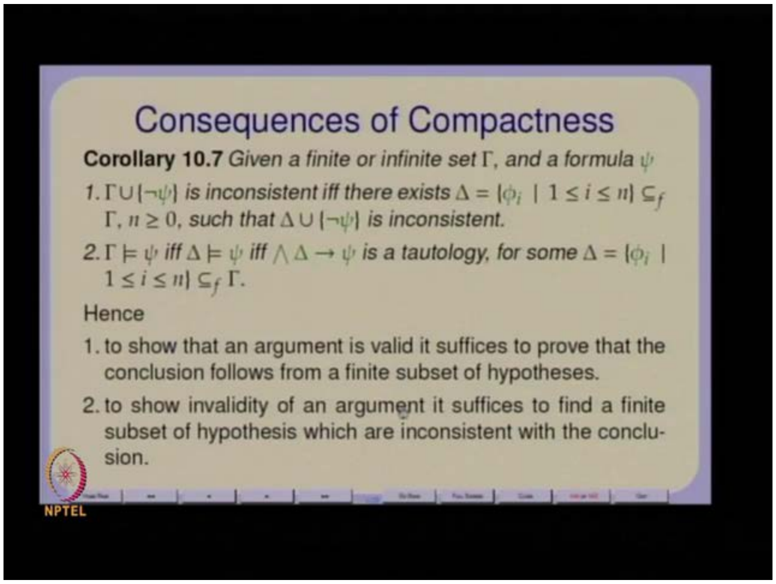

And other so, what it means now is, if you want to given any finite or infinite subset of formulae. And some formula psi you gamma union not psi is inconsistent if and only if there exist a finites subset of gamma such that it is called the delta such that delta union naught psi is inconsistent. And similarly all these things so what it means is that to show that an argument is valid. It suffices to prove that if I negate the conclusion i. It suffices to prove the some subset of the hypothesis along with the negation of the conclusion is unsatisfiable.

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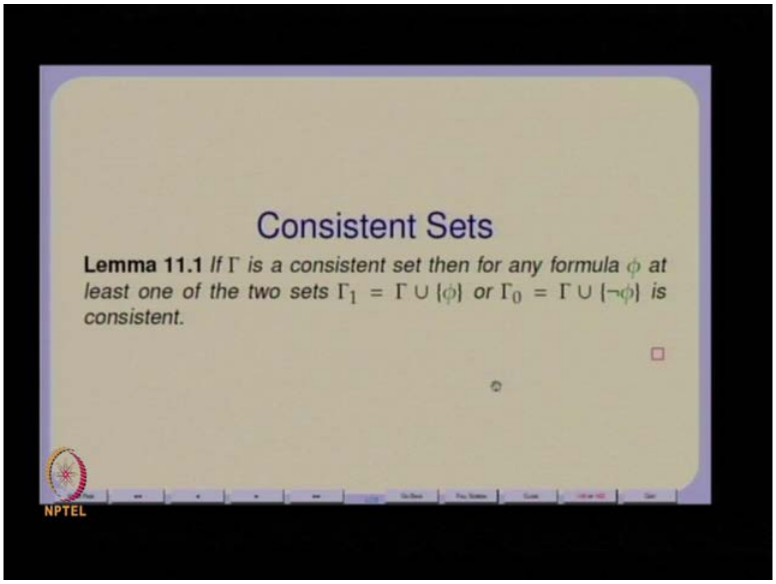

So, now the next the next question 1 can ask or sets of formulae. Is, that of consistency we can extend consistency the other way instead of going downwards to finite subsets. We can think of a take a set of formulae gamma and see how, What elements from can add to it? So, 1 thing is that if, gamma is a consistent set. Then, you can add for any formula of phi you can add either its either phi it is a or its negation and still maintain consistency.

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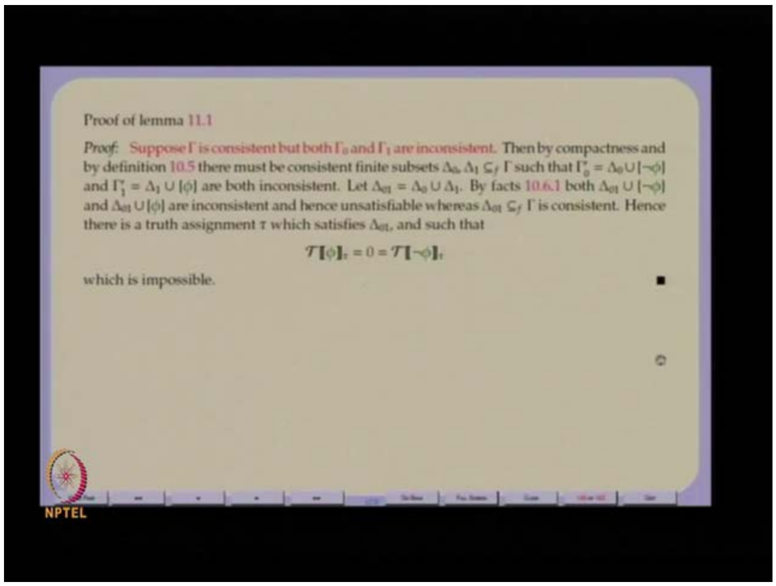

And that is actually fairly easy to see using proof by contradiction if gamma is consistent. But, let us say the addition of the formula phi gives you. So, gamma 1 contains gamma union phi and gamma naught contains gamma union naught phi. If both of them are inconsistent then it is clear that there must be some finite subset subsets of gamma 1 and gamma naught. So, let us call them delta 1 and delta naught respectively. Where, delta 1 contains phi delta naught contains naught phi. And if you remove phi and naught phi then, I, mean if you can take since gamma is inconsistent you can take finite subsets gamma naught and gamma 1. Add naught phi and phi respectively and there if gamma naught and gamma 1 are both inconsistent. Then, this corresponding gamma naught prime. Which is delta 1naught union naught phi and gamma 1 prime which is delta 1 union phi will both being inconsistent.

Whereas, delta naught and delta 1 themselves which are just finite subsets of gamma will be consistent because, gamma is same to be consistent. So, if gamma naught and gamma 1 are inconsistent then there exist finite subsets delta naught and delta 1 of gamma such that delta naught union naught phi and delta 1 union phi are both inconsistent. Let us call them gamma naught prime and gamma 1 prime. So, now consider a delta naught 1 to be the union of delta naught and delta 1. Since, delta naught and delta 1 are both consistent subsets of gamma and they both are finite delta naught 1 which is the union of these two sets is also I should be find it easy to check that.

Then delta 0 1 is also consistent because it is a finite it is a union of two finite sets two finite consistent sets. And then what happens is that both delta 0 1 union naught phi and delta 0 1 union phi are inconsistent. And hence on satisfiable if that is so then, there is a truth assignment tau which satisfies every formula and delta 0 1. And such that for both phi and naught phi it gives me 0 which is important. So, there is a simple proof by contradiction to show that given any consistent set. I can add for any formula phi either the phi itself or the formula naught phi and still maintain consistency. Now, we can extend this further to how many such different formulae can we add. So, before that let us look at a something known as a property of finite character.

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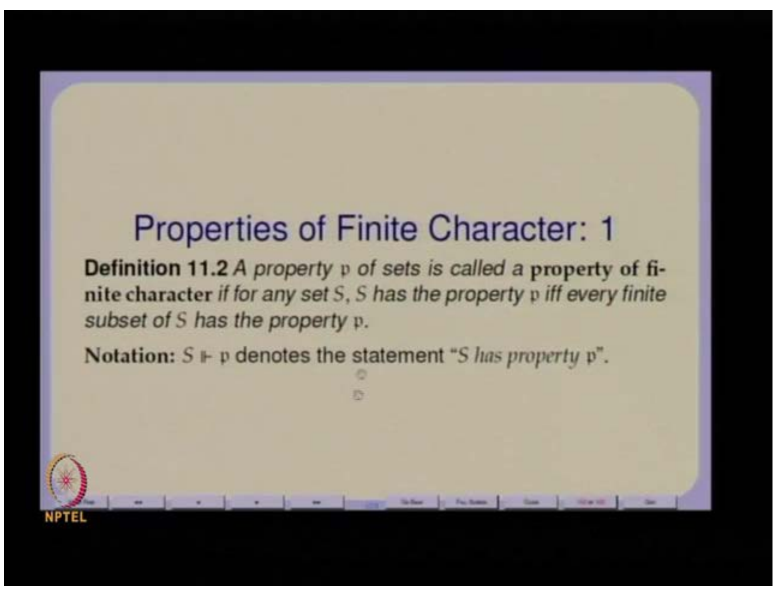

So, and in particular this is important because, its compactness is actually one of this example of a property of finite character. So, property p of sets this by the way this is called nothing to do with relay logic its general set theoretic mathematics. So, you take any property p of sets is called a property of finite character if for any set S. S has the property p if and only if every finite subset of S also has a property p. So, compactness or the corollary to the compactness theorem essentially tells you that compactness is a property of finite character. There are so will use this notation to denote the S has property p. But, there are actually properties of finite character are obvious useful. In fact they are always useful to characterize various kinds of infinitary properties.

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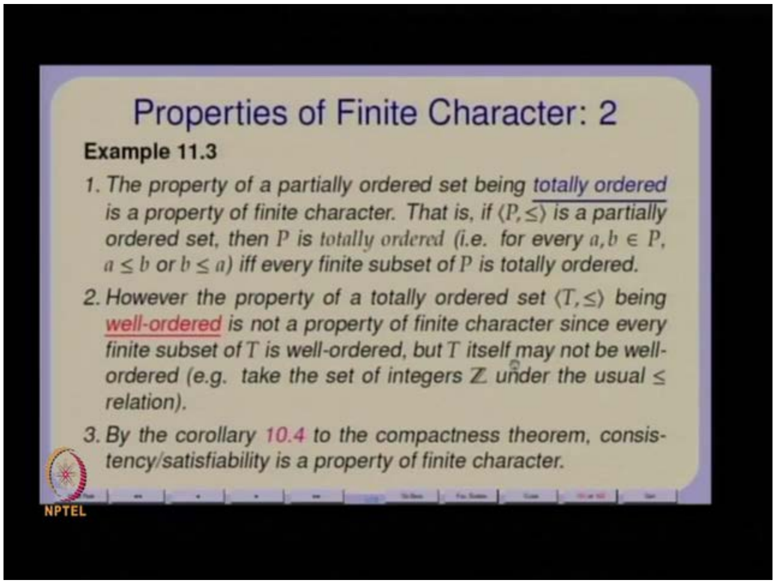

So, for example you take so here, is some examples you take the property of being of a partially ordered set being totally ordered. So, essentially a set p under some less than or equal to relation is partially ordered. Then it is totally ordered if and only if every finite subset of p is always totally ordered. And so, that is the property of being totally ordered for partial orders for the universe of partially ordered sets is a property of finite character. Here, the on the other hand there are properties like being well-ordered. What is well-ordering? I mean that the set will not have infinite descending chain. So, you take any totally ordered set every finite subset of it is well-ordered. In the sense that they can given any finite subset a totally you cannot find any an infinite descending chain.

Since, is you are taken up some subset but the whole set itself may not be well-ordered under that less than or equal to relation. So, well-ordering of total orders so in the universe of totally ordered sets, the property of being well-ordered is not a property of finite character. So, whereas compactness is a property of finite character there are actually other properties of finite character. Which you may have encountered in other parts of some mathematics like for example the question of k color ability whether a graph is k colorable. So, in particular we take the question of infinite graphs being k colorable. That means you have a let us say an undirected an infinite undirected graph.

So, its vertex set is infinite and the edge also is infinite. And they are all undirected edges and what you are saying is you have set of k colors. And know to vertices which are connected by an edge should bear the same color is it possible to color the graph I mean this is the map coloring problem or the graph coloring problem take n to infinite sets. So, it is possible to show that an infinite graph is k colorable if, and only if, every finite sub-graph is also k colorable. And so k color ability for example is a property of finite character. So, and other finite the question of if you lo at I mean an almost exact analogy to k color ability is of tiling.

I mean you have set of tiles some sizes let us say rectangular tile or some such thing or hexagonal tiles whatever. And you want to tile the entire plane so the tiles go to overlap edges are maintained. And this question of whether that and infinite plane can be tiled with given a set of tiles is equivalent to the question that is possible. If, and only if, every finite sub-plane is also tillable with those tiles with only those tiles. So, that tile ability and actually the problem of contilability can be mapped on to the problem of k colorability also. So, properties of finite character are important in mathematics in general. And compactness is one example of a property of finite character.

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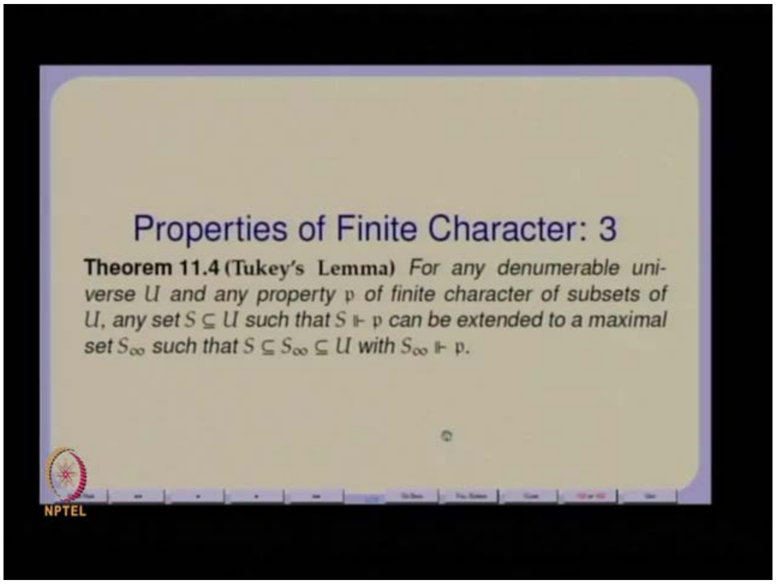

And we look at this property of finite character we look at this to. So, if you were to take properties of finite character in their most general form then there is an interesting lemma by Tukey. Which says that given any set you take a denumerable universe and any property of finite character. Then any set which satisfies property which has the property p can be extended to a maximal set which continuous to have this property. So, what we saw previously was that of consistency. If gamma is a consistent set then, can you add more and more elements. So, one question that you can ask is, How many different elements from the set of all propositions p naught? So, it universe now is a set p naught of all possible propositions can I add to gamma to keep it consistent. Which, means if is maximal if, I have a maximally consistent set then adding any more formulae into it will make it inconsistent. So, that is what I mean by a maximally consistent set. So, let us first look at this proof of Tukey's lemma so is essentially says that any given a property p of finite character for sets subsets of an denumerable universe u. Any set S which has the property p can be extended to a maximal set which has this property.

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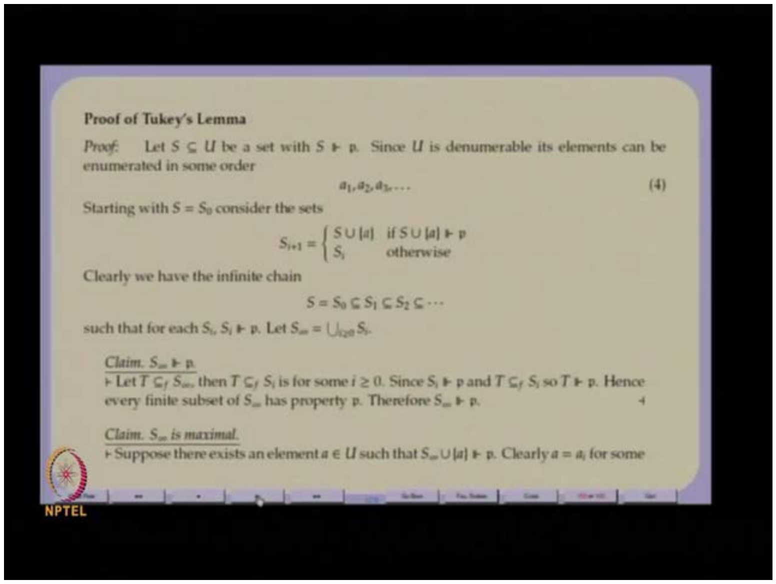

So, the proof is important because of the fact that many things in computer science also follow similar in the semantics of programming languages follows similar approach. So, what I can do is I can since, so I have a denumerable universe U. And so, which means that its elements so all the elements of U can be enumerated in some module let us say a1, a2, a3. So I am using only the positive integers I am not using 0. So, assume that there is a set S subset of this universe which has the property p. And I call that set S 0 and then what do I do is I extend that set gradually. So, my Si plus 1 for any i is, consist of so this a here should i actually be ai plus 1 for i greater than or equal to 0. So, all I am saying is so this s this S should also have the subscript i. So I look at so for in order to construct the i Si plus 1 I look at ai plus 1. If the addition of the ai plus 1 still preserves the property. Then I call that added I add that ai plus 1 and call that set Si plus 1 otherwise Si plus 1 is the same as Si. So, now what happens is since we are only adding elements what you get is an infinite chain starting from S. And this infinite chain is a monotonically increasing chain is at least monotonic.

So, no set is Si is guaranteed to be subset of Si plus 1. And what I can do is I can take the union of this infinite chain and call that s infinity. So this union S infinity I claim is a maximally is a maximal extension of S. Which will satisfy the property p given that S satisfies the property p. So, first S infinity should satisfies the property p well this is because you take any so if S infinity is an infinite is a possibly infinite set. So, in order since p is a, property of finite character it is enough to show that an arbitrary finite subset of S infinity also has the property.

So, take let us take any finite subset T of S infinity then 1 thing is because, this is a an increasing chain and this is this is S infinity is a union this p must be contained in some Si. And since Si satisfies property p any subset of Si also satisfies the property p because p is a property of finite character. So, therefore T must also satisfies the property p so therefore then since p is a property of finite character s infinity also satisfies the property p. So, S infinity therefore satisfies the property p the next thing to show is that it is maximal. So, supposing there is an element a, that you can add it to S infinity. Then but this element a would be some a i in this enumeration a to a1 a2 a3 etcetera which, means if this S infinity union a if S infinity union a satisfies this property and a is actually the ith element in this enumeration. Then, what it means is that Si plus 1 would just be Si union ai.

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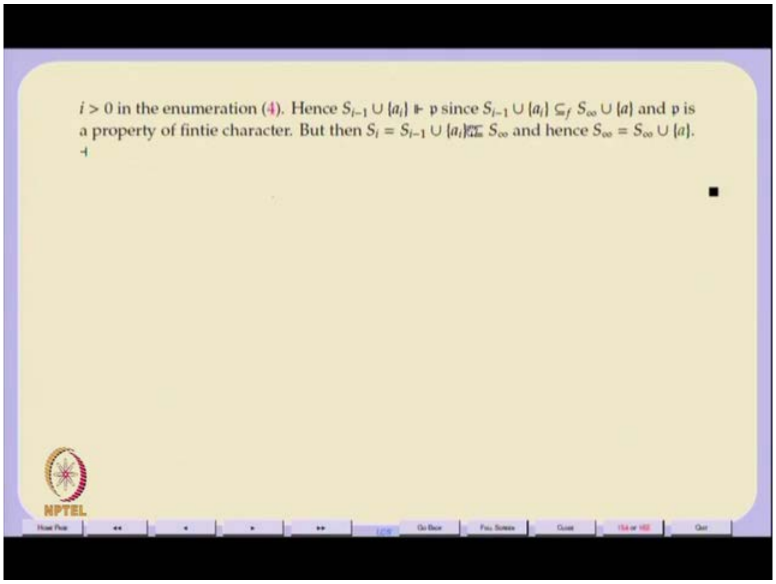

And Si plus 1 we know that satisfies this property p. And this Si plus 1 this, Si union ai plus 1 would actually be a subset of S infinity union a. And p is a finite p is a property of finite character and if Si contains ai it, means S infinity is also contains a before s infinity does not get extended.

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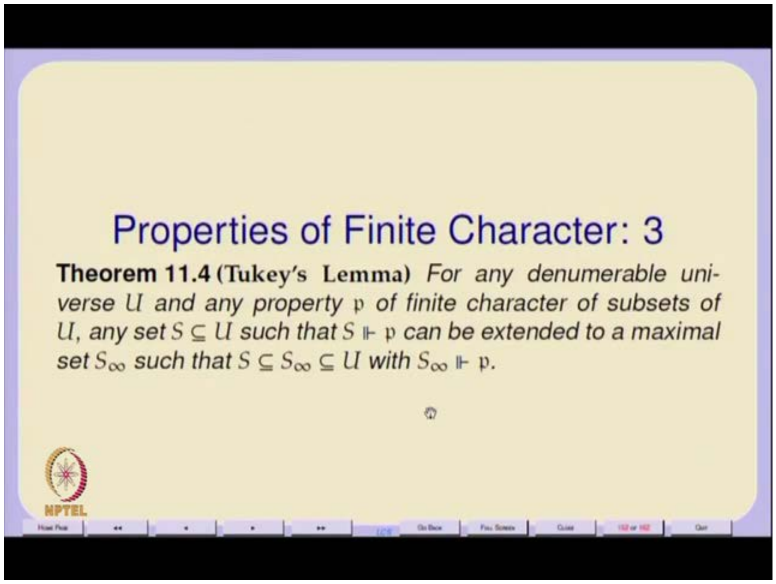

So, all that we are saying is so therefore you take this. So, any property of finite character will actually can be extended to some maximal set. Such that the addition of any new element so, the for any set s that satisfies the property p it can be extended to set s infinity. Such that the addition of any new element actually a negates the property.

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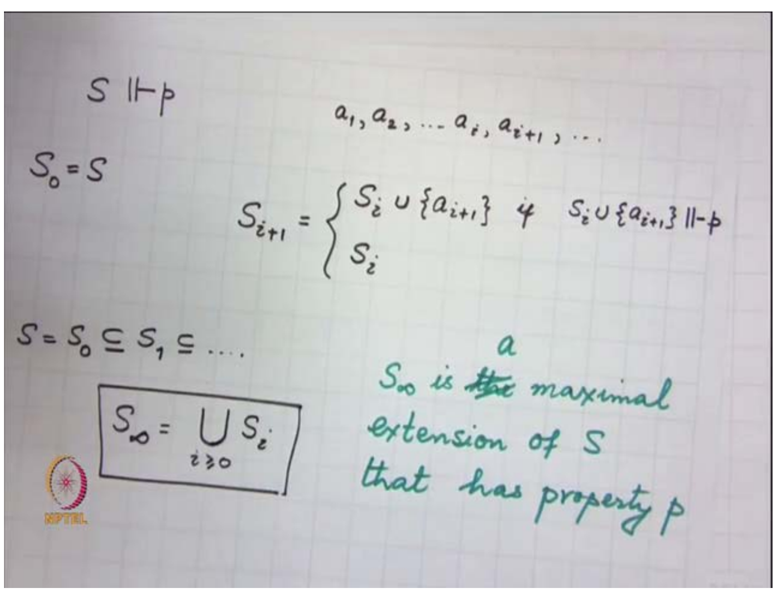

So, what I am saying is so assume that S has the property p assume S is a set which has the property p. You have enumeration of this universe is a denumerable universe you have this enumeration. Now, what I do is I start with S naught equal to S and what I do is I construct my Si plus 1. So, Si plus 1 equals Si union ai plus 1. If Si union ai plus 1 satisfies the property p. If Si union ai plus 1 does not satisfies the property p. Then, my Si plus 1 is the same as Si fine. So, I go through this construction through this entire enumeration. So, what do I have a S equals s naught and S naught is a subset of S1 it might S1 might contain a1 or it might not contain a1. But, it contains all the elements of S naught and so on and so forth I have this infinity. And now, my the S infinity that I am constructing is simply the union of all these Si Where i is greater than or equal to 0. So, that is so this is my construction. And, now what I am claiming are two things that this is a maximal extension of S so essentially what my claim is that. This S infinity is a maximal extension of S that satisfies that has property p.

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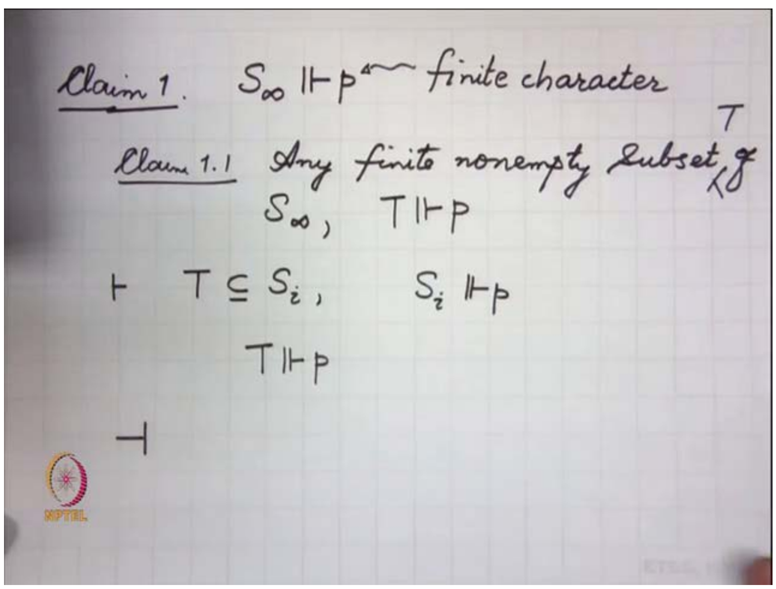

This is my claim and this is the claim, So I am going to prove it in two ways firstly that first thing of course is that. I, have to show that so, the first part of the claim. Claim 1 is that S infinity has the property p. Keeping in mind that this p is a property of finite character what I am what I need to show is to show that so the it supposes to show this sub claim. That, any finite subset non empty subset of this infinity any non empty subset T of S infinity has the property P.

So, this is sufficient so the proof of this claim is all that is required. So, I assume T is some finite subset. Now, what I, am claiming is that T is actually since T is a finite subset. It consists of the elements a1, a2, a3, a4 and so on so forth assume. So therefore, it has and it is a finite subset. So, it has an element with a maximum index for example. So, which means that T is subset of some Si. where Si belongs to this chain. So if t is a subset of this finite Si and we know that Si satisfies the property p. therefore any p and p is a property of finite character. Therefore, any subset of Si satisfies this T. Therefore T also satisfies this p fine so that is there ends this proof the next claim the X infinity is of finite character.

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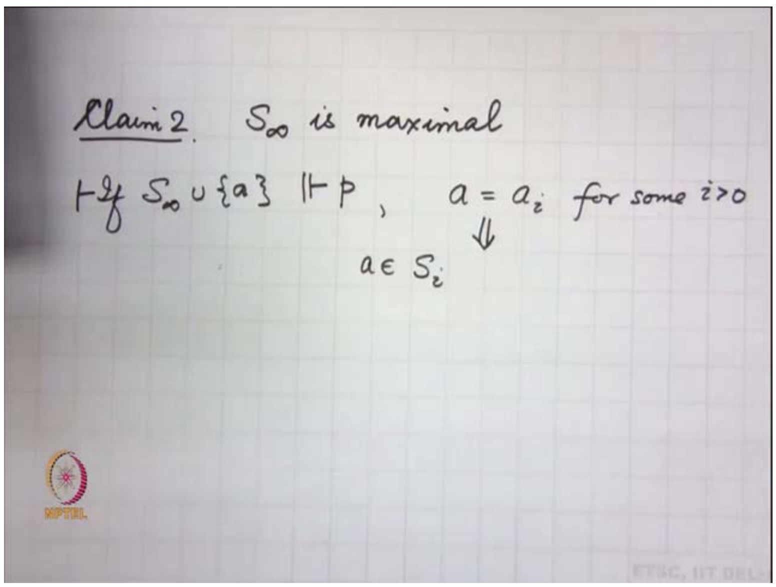

So, the next claim which is claim 2 is simply that S infinity is maximum and is maximal in the sense. That it cannot be you cannot add any more elements and still get a larger set which satisfies property p. So the obviously the so the question is, Suppose S infinity union a satisfies property p? So, if S infinity union a satisfies property p then, this a must be equal to some ai in the enumeration for some i greater than 0. So which means that since this is a property of finite character firstly this means that this implies that this a belongs to Si. Because, in the Si was constructed from Si minus 1 by adding this element ai if Si minus 1 union ai satisfies the property.

Student: sir there is (Refer Time: 29:16) How can we ensure that if s infinity union ai satisfies property p then Si minus 1 union ai will also satisfy property p there have been no such claims? No p is a property of finite character so any finite subset will also satisfies the property p. So, there is no problem with that. So, Si so, then so what we are saying is that s infinity satisfies the property p. So, Si minus 1 also satisfies the property p and S infinity Si minus 1 union ai would also satisfied the property. Which means that a, was already in s infinity and therefore you have not extended S infinity any further.

Student: Sir, in this proof hardly assuming the thing that (Refer Time: 30:17)

No, we have we have assuming that p is a property of finite character. And, that means that it does not matter whether you take a finite set which has this property or an infinite set which has this property. Every finite subset of it also has the property but, I will go through a different form of this proof may be if you have the time.

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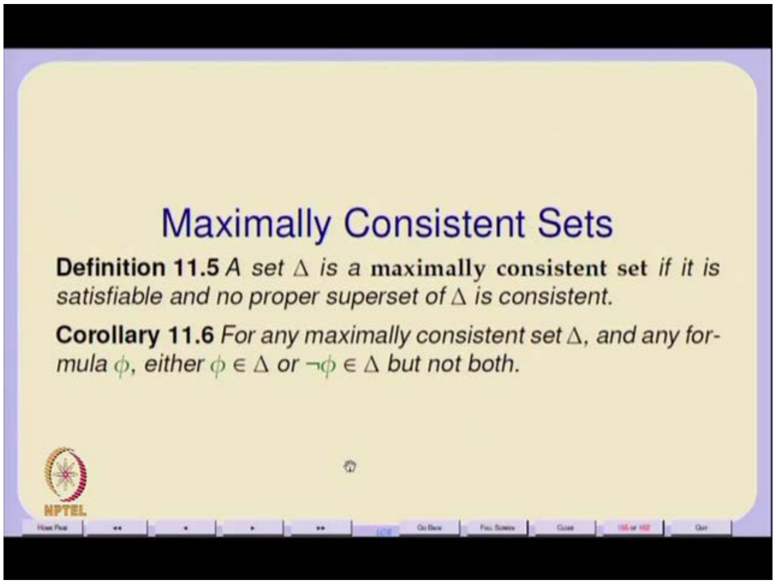

So, now so you have a maximal set max a maximal extension there could be many such maximal extensions. So, in particular if a if, I were to take a if I were to take a set delta I will call it a set of propositions. And will call it maximally consistent if it is satisfiable and no proper superset of it is satisfiable. That means if I had any more element to it then, it should not be possible for me to then, what I get is unsatisfiable I get an inconsistency by adding any new element to it. So, the if, I were to do that it follows from the semantic so propositional logic that for if I were to have a any maximally consistent set delta. For any formula phi either phi or naught phi would be a member of this delta. Because for any truth assignment which satisfies all the elements of delta only 1 of phi or naught phi can be false both cannot be false. So, therefore it should be possible to add 1 of them. And, still maintain consistency still maintain satisfiability. So, there is a truth so for every truth assignment in which all the elements of delta are true either phi is true or naught phi is true. So, I can add 1 of them and still maintain consistency for that truth assignment.

So, the fact that such a, truth assignment exist is enough. So, this is 1 corollary of maximal consistency and Linden Baum's theorem is the next important theorem. Which is essentially like a consequence of the fact that compactness is the property of finite character.

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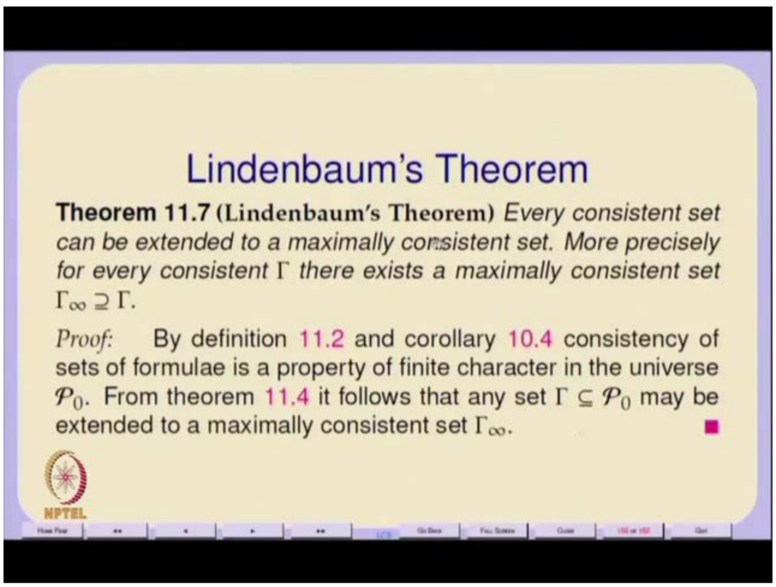

And, therefore any set of formulae can be extended to a maximally consistent set. So, Linden Baum's theorem essentially says that every consistent set can be extended to a maximally consistent set. And maximally consistent in the sense that you cannot add any more new propositions into this to the maximal set. So, you take any set gamma there is a maximally consistent set gamma infinity which is a superset of gamma.

And actually by Tukey's lemma but, we by the way this thing here, says is a another proof also I mean. So, what we can do is we can completely forget about Tukey's lemma and do everything that is there in Tukey's lemma. And, also in the proof of Linden Baum's theorem. So, if you feel more comfortable you will go through this proof. But, essentially if you were to take Tukey's lemma for granted and consider the fact that compactness is a property of finite character. Then, every set gamma every consistent set gamma can be extended to a maximal consistent set gamma infinity.

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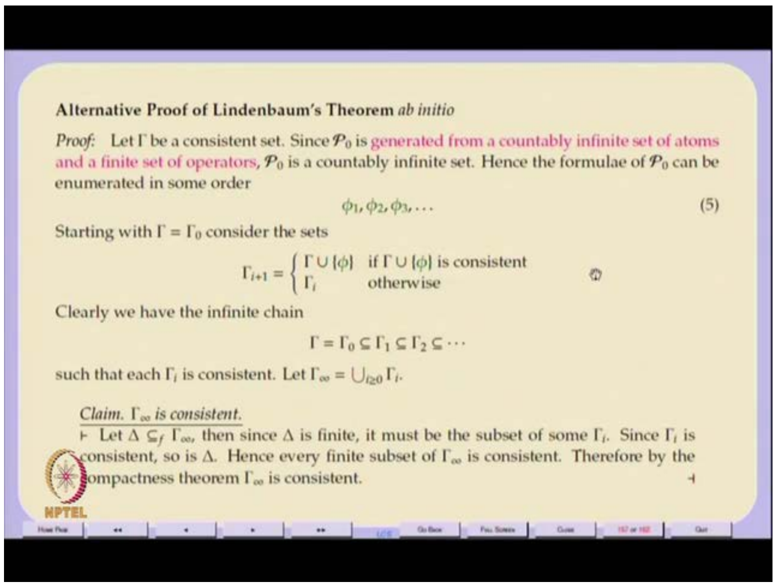

And, but if you are if you if you want to do the proof dependently It is essentially like this. So, you what we are saying is you just since, naught the set of all propositional formulae is finitely generated from an infinite collection of atoms using the operators the standard Boolean the proposition connectives. So, therefore the resulting set p naught is accountably infinite set. Therefore, it is denumerable which means that all the formulae of p naught can be extended can be enumerated in some form phi naught phi 1 phi 2 etcetera. I, start with a consistent set gamma it is a, consistent set. So, now I, means it is not looking at I, mean so we are already looking at consistency as a property of as the property of finite character. But, now we can start with starting with a consistent set gamma call that gamma naught just go through this enumeration and, add all the formulae which, in the enumeration and, keep building gamma I gamma i plus 1 and so on and so forth. So, that you maintain consistency.

So, you get this infinite chain of consistent sets all I am saying is this union of an union chain of consistent sets is also consistent that is all we are claiming. So, this gamma infinity is just this union of this chain of consistent sets. And you can prove that gamma infinity is consistent because you take any finite subset delta of gamma infinity. Then, since delta is finite it must be a subset of sum gamma i and since gamma i is consistent delta is also consistent. Hence, every finite subset of gamma infinity is consistent since gamma i is consistent every finite subset of gamma i is consistent therefore, delta i must also be consistent. So, which means this delta was arbitrarily chosen so every for every possible finite subset of gamma infinity it is true that the every such delta will also be consistent. By compactness it follows that gamma infinity itself is consistent.

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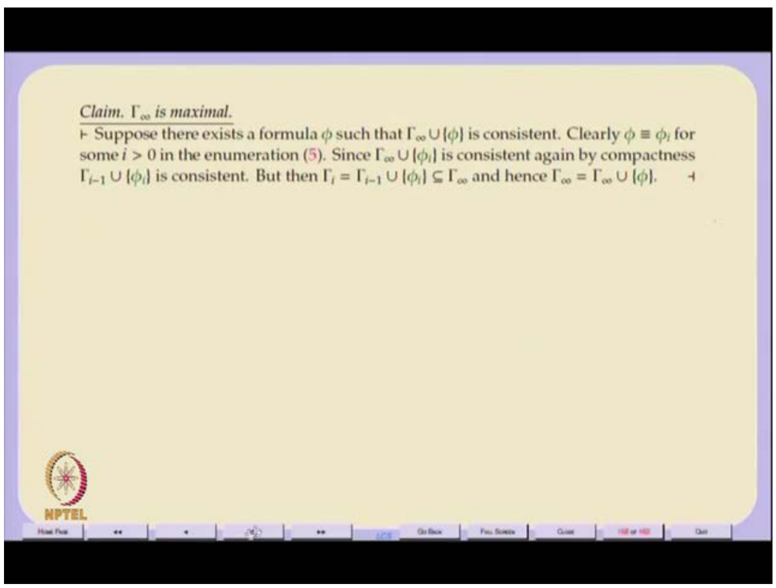

And, the fact that you cannot add any more you suppose a there is some formulae phi such that gamma infinity union phi is consistent. Then, phi is some phi i for some i greater than greater than or equal to 0 in the enumeration. And, since gamma infinity union phi is consistent again by compactness gamma i minus 1 union phi i is consistent. Which, is a because its subset of gamma infinity union phi i. And, but gamma i minus 1 union phi i is the same as gamma i. And gamma i is consistent and therefore you not added any new element. So, it does not matter even if you do not follow Tukey's the proof if, Tukey's lemma it is possible to just work with compactness itself. And, in fact the original proof by Linden Baum actually was this was something like this i do not think he had access to Tukey's lemma. He, just thought it out that an infinite union of consistent sets would actually give a consistent set that requires proof here, correct. So, that is what Linden Baum's lemma is by the way.

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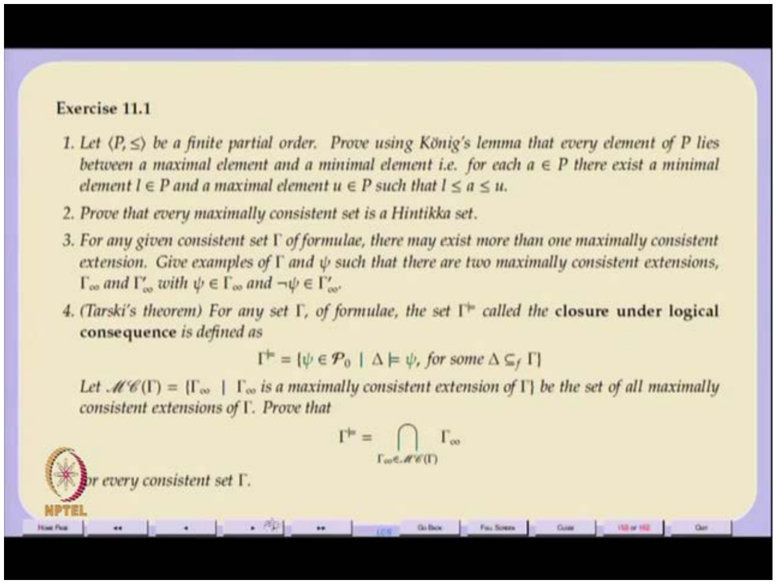

So, I have but, so there are there are a whole lot of excises we have to do which when I, put this up you will be able to see. There are some interesting excises which require a fairly I, mean require fair amount of thought I mean they are all set purely set theoretic excises part. They, are not very easy. So, please do this excises there is some there is a notion of closure under logical consequence in particular which, is related to maximal consistency. So, this is an important thing with so you one of the things we started with notion of arguments validity of arguments.

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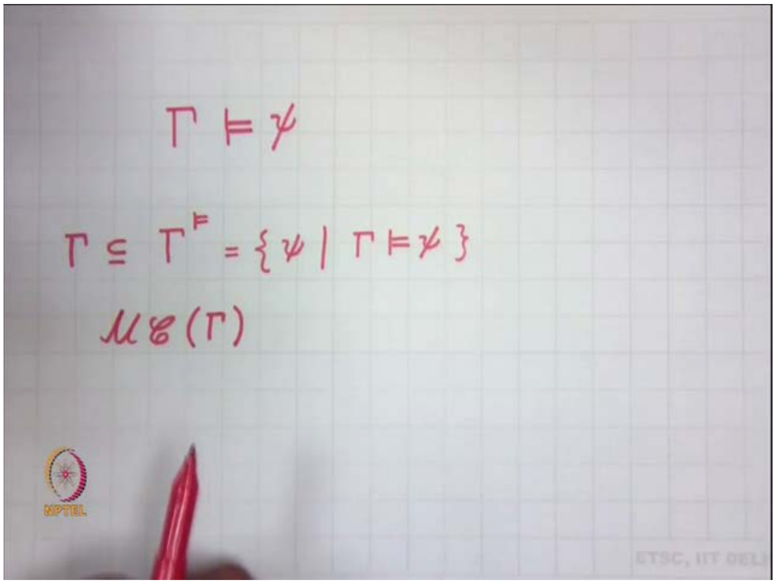

The fact that you have a what we have a set of hypothesis and you want to prove that, some conclusion is a valid logical consequence of this set. We, transformed it into well tautologies and contradictions and then we came into consistency. Now, the question is, What is the relationship between consistency and logical validity or logical consequence? So, there is an interesting theorem by Tukey. So, what I, can do is I can take this gamma and I, define gamma superscript this to be this set of all psi such that psi is a logical consequence of gamma. So, this is I call this closure under logical consequence. Remember that, if gamma is a consistent set actually does not matter whether gamma is a consistent set or an inconsistent set gamma is always going to be a subset of its closure. So, the other thing is that we look at the notion of maximal consistency. And, we said that each gamma can be extended to a maximal consistent set but that does not mean that a maximal consistent extension of gamma is unique. There, could be many different maximal consistent extensions of gamma. And, in fact there will be a countably infinite number of maximal extensions of gamma.

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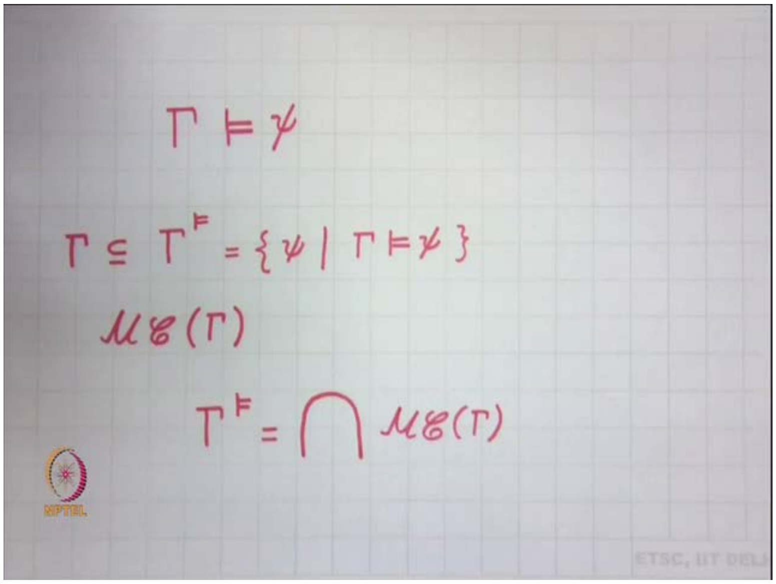

And, so what we are saying now is and what is this important thermo of says that you take this if gamma is consistent. Then, it has certain logical consequences consider all those logical consequences. In particular every formulae and gamma itself is a logical consequence of gamma. So, this is simply the intersection of all the maximal consistent extensions of gamma. And that is the relationship between mere consistency and logical consequences. So I, would suggest that you spend some time trying to prove this it looks purely set theoretic and it looks. But, the fact of the matter is that these especially these things which these set theoretic notions. Which, actually require countability and uncountability one has to be a little careful with them.

So, but please do try this is an important theorem this is what relates logical consequence with consistency. After, doing all this stuff it comes back as essentially a big intersection of all the maximally consistent extensions of gamma. So, you so every element which is, there in every maximally consistent extension of gamma is a, logical consequence of gamma. So, there are also some other interesting things the other interesting thing that you need to show is that for example every maximally consistent set is also a hinkica set. So, that means it is closed under those hinkica closure operators. So, you can take all these kinds of closure operators and play around with these sets.

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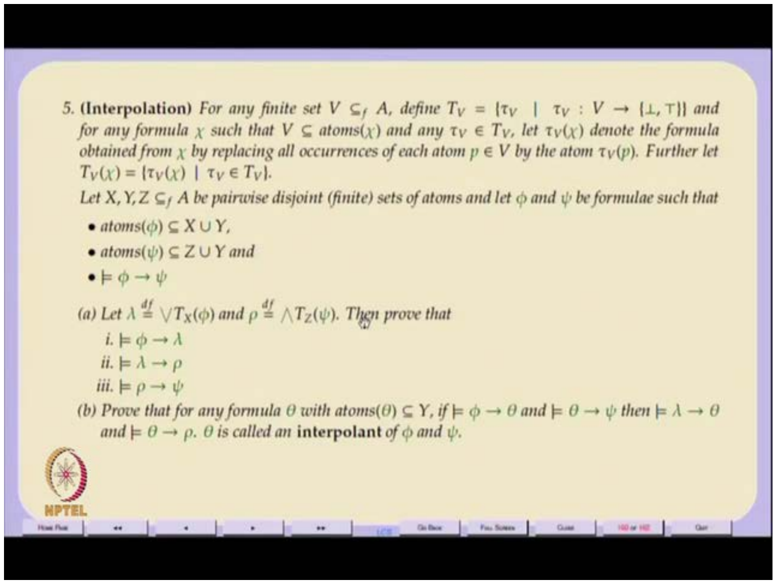

Lastly what I, have is a very fairly complicated and hard problem for you to do. Which, is that just like we can have we have interpolation in numerical analysis interpolation in numerical analysis is intimately connected with the less than or equal to relation. You are essentially talking about, given a curve given two points on the curve you finding some point in between those two points on the curve. So that so, if you look at this curve in as some if you if you just think of it some on the two dimensional plane.

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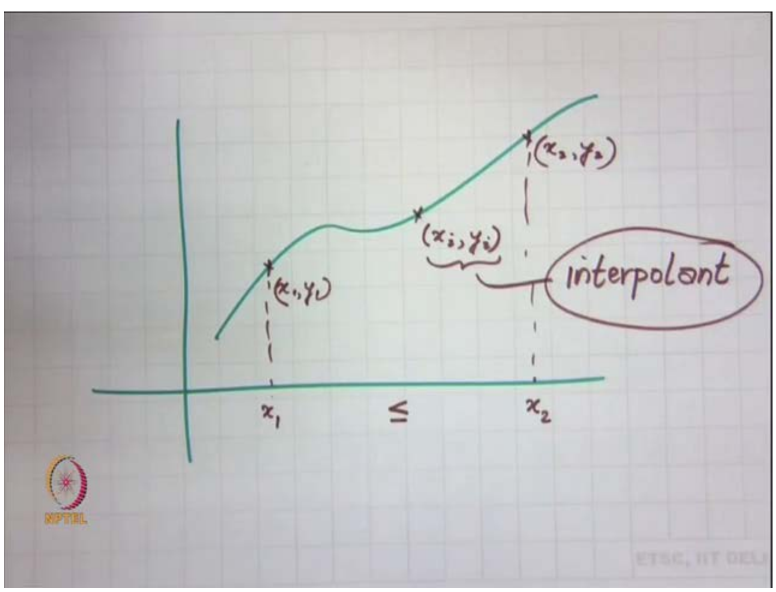

What, we are seeing therefore, is you have this curve like this, and you have to do an interpolation that means you. Let us say given some two points where let us say x1 y1 and x2 y2. Your interpolant is some point 1 or more points xi yi which lie on the curve. And, many numerical algorithms actually try to do interpolation they actually try to find points on the curve and of course these points need not be unique there many different algorithms can give you different interpolants. So, this is an interpolant if you can find this xi and underline this entire excise is the fact that your x1 is less than your x2. So, underlining all this is the fact that there is a less than or equal to ordering it happens to be a total order on the real's. But, it is actually an ordering there is absolutely no reason, Why one cannot generalize this notion of an interpolant to change of a partial order?

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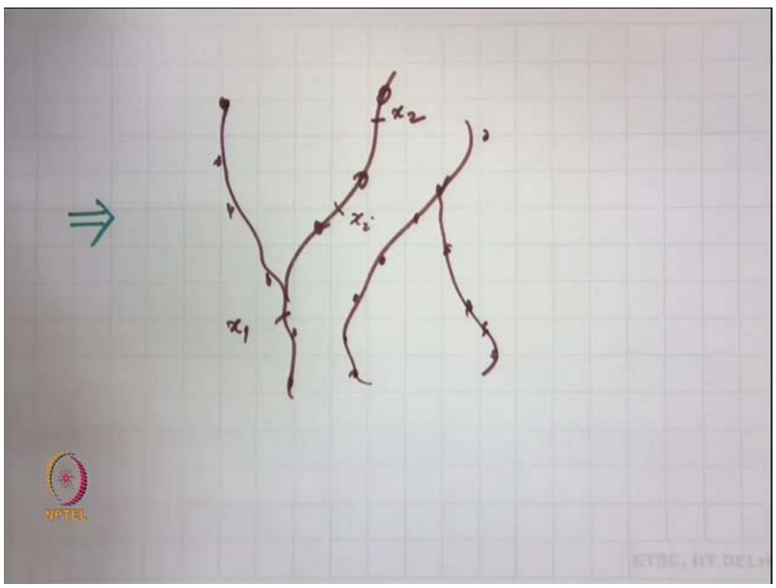

So, if I have a partial order with lots of so this is like a typical hasse diagram. Where I have various kinds of chains on this partial order. Supposing I have a partially ordered set and if I, am given two elements x1 and x2. Which lie on a chain then, How do I how does 1 find out find an interpolant xi laying on the same chain? I, mean so the notion of an interpolant can actually be generalized also to partial orders. And here, what do we have we do have a partial order. What is the partial order relation? Your logical implication is a partial order relation on propositions it is partially ordered. Because, every it is reflective because every formula implies itself logically implies itself if phi implies psi and psi implies Kai. Then, phi does imply Kai. So, it is transitive and it if, phi implies psi and psi implies Kai. Then, phi and psi need not be the same formula but they are logically equivalent. So, if I were to take if I, were to take my set p naught.

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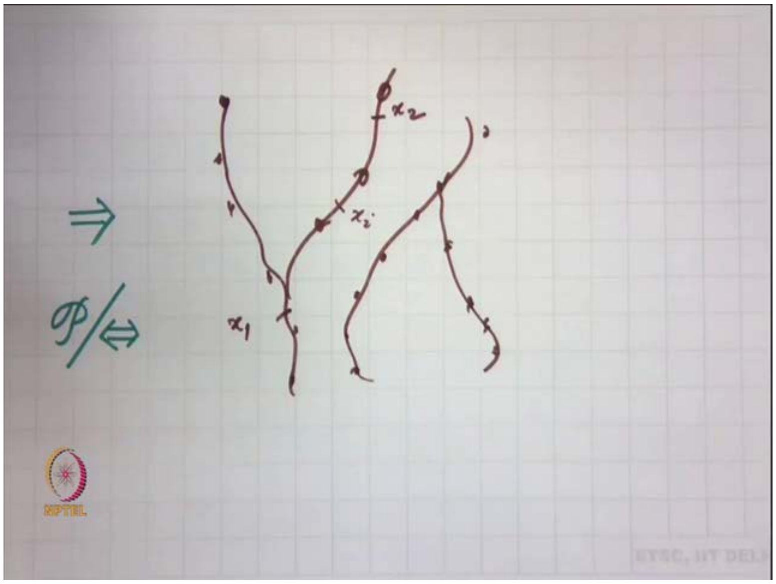

And, quotient it over logical equivalence it means I divided up into equivalence classes. Such that, logically equivalent formulae all reside in the same block. Then, what I do have is my logical implication then is a partially ordered is a partial ordering relation. Because, then what happens is you looking at equivalence classes.

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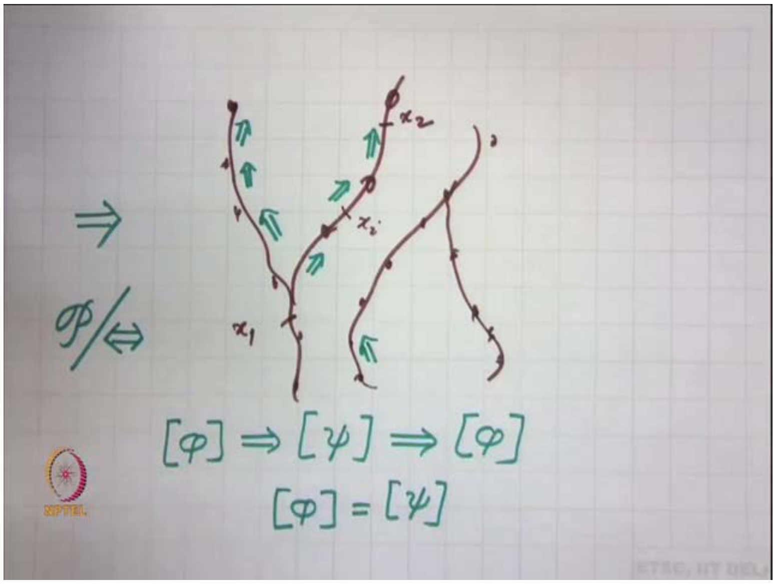

So, what you are saying is that if 1 equivalence class phi implies another equivalence class psi and psi also implies phi then, these two equivalence classes are the same. Because then, phi and psi are logically equivalent. So, your logical implication is a partial order on the equivalence classes of p naught. So, if I look at p naught it is partially ordered by the equivalence classes of p naught or partially ordered by this implication relation. So, you can think of these chains as essentially chains of implications. The standard thing therefore now, that since implications is transitive also the question is now, Can you go back and find an interpolant? So, given two formulae phi and psi can I find some interpolant and what this problem does is it actually gives a fairly complicated way of finding an interpolant and you have to prove that it is an interpolant.

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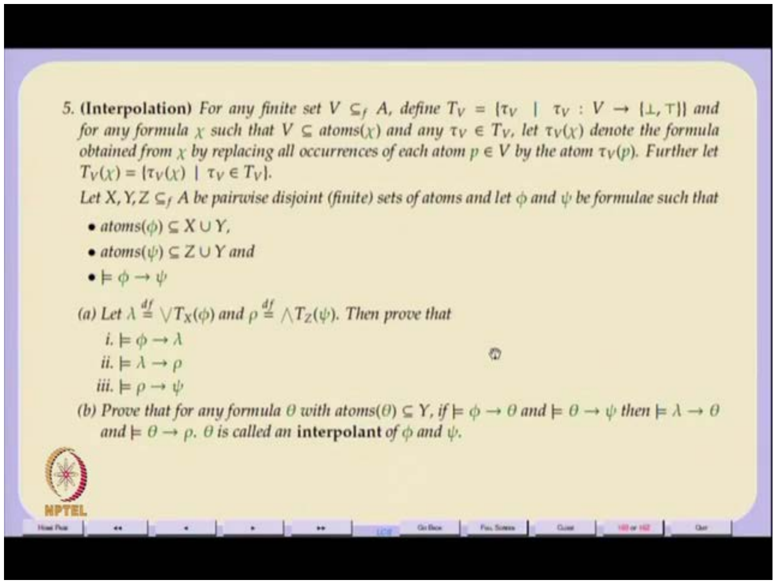

So, you can it is a funny way of defining an algorithm but it is like an algorithm for finding an interpolant. And of course interpolants need not be unique every algorithm for any pair will give its own interpolant it is just that it has to be an interpolant and, that has to be proven. So, these are some so I, put some deliberately hard excises but they have some intuition outside of logic also. Therefore it is important to know that such things exist.