Logic for CS
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Lecture - 10
The Completeness Theorem

Today we will do an important theorem called the Completeness Theorem. Which, you will see some of the consequences of compactness and it something that this compactness is actually very closely related to that topological notion of compactness. And it is possible to say it, possible that it is possible to show that this, compactness is really the same as the topological notion of compactness either, defined through matrix spaces or through neighborhood spaces whatever. So, this compactness theorem but we will look again independent of topology wewill just look at it as a in it is isolation.

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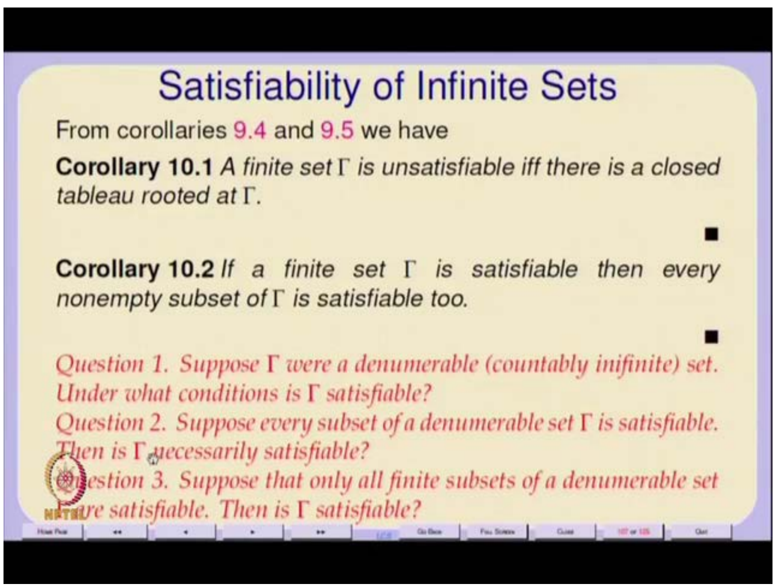

And so and it is we will see the importance of that. So, main thing is what does supposing you have an infinite set of sentences. And you want to know essentially by infinite of course we are talking about a logical system it is we mean only countably infinite. So, denumerable we are looking at denumerable sets of sentences and essentially if, you look at this Satisfiability of Infinite Sets say essentially from these corollaries 9.4 and 9.5 which essentially says that.

So now, when you are talking about infinite sets there is no guarantee that your tableau construction would be finite. So but, we can always take the notion of that tableau to infinite a tableau is just a tree. We take the notion to infinite trees thatis as simple as that .

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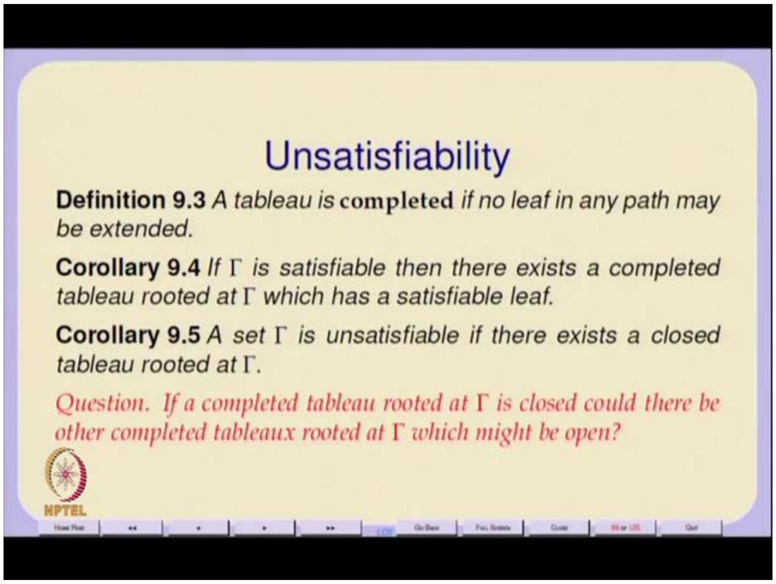

So, this corollaries 9.4 and 9.5 say that. if gamma is satisfiable then there exists a completed tableau rooted at gamma which has a satisfiable leaf. And a set gamma is unsatisfiable if there exists a closed tableau rooted at gamma but, a the corresponding notion for infinite sets would be that. If, an infinite set is unsatisfiable then you should definitely get a closed tableau or there should be at least one closed tableau. But, if it is satisfiable then there would be essentially be some kind of infinite tree but the problem with any finite point of that infinite tree is you do not know whether itis going to close at a later point .

So, there is a problem when dealing with these issues for infinite sets. And so what we would like to do is we would like to look at this notion of satisfiablility of infinite sets. And satisfiability as far as we are concerned is the same thing as consistency of an infinite set of sentences . So, essentially so these are the questions that it raises suppose gamma were a denumerable set under what conditions is gamma satisfiable. Suppose every subset of a denumerable set gamma is satisfiable then, can you say whether gamma is necessarily satisfiable.

So whole point about dealing with any kind of infinity is to either get a finite representation or to get approximation in terms of finite tree objects.

So, the construction of let us say irrational numbers or real numbers by deleting cuts is essentially as looking at finite tree intervals or at least intervals with end points and trying to represent letus say real numbers as limits of those. So, limits and continuity are the most important notions when it comes to infinite sets in any branch of mathematics here. So one question is whether you can always approximate something infinite by a possible infinite set of finite tree approximations. So that this infinite tree set each approximation is finite tree but, the set itself might be infinite so essentially you have some limit construction which tells you that if this limit exists then so it is then this infinite tree object is essentially being captured.

And that is exactly what has been happening throughout the history of mathematics ever since the start of the subject of analysis. So, this question to essentially is asking whether this infinite tree object this infinite set of sentences it satisfiability can somehow be expressed in terms of finite tree approximations satisfiablility of finite subsets. Then, of course the converse question is that is related to the topological notion of continuity and limit does a limit exist sometimes under discontinuity those limits do not exist. So here, suppose only finite subsets of a denumerable set gamma are satisfiable then what can you say about the satisfiability of the whole of gamma that is.

So, these are the kind of questions that actually come up when you have in any branch of mathematics when you have some infinite tree object. And this also includes all branches related to mathematics like this also happens in the case of trying to find the semantics of recursion in programming languages for example. So, it has a wide kind of applicability the kinds of methods that are used have a wide kind of applicability.

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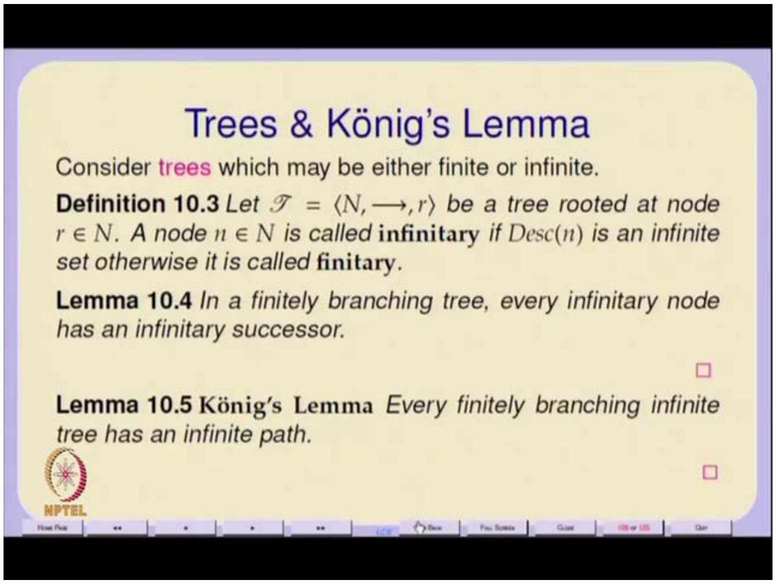

So now, we have to consider trees in some generality. We have to be prepared to deal with infinite trees throughout our data structures and algorithms kinds of course trees are always finite. But now, for example you have actually come across infinite trees when you do define this SOS semantics of a programming language with values what an SOS semantics generates for a program is for the dynamic behavior of the program is essentially an infinite tree. And so the program itself is a finite object thatis a finite abstract in text tree but, the behavior of the program the execution behavior of the program is essentially infinite tree. So, we letus look at trees and let us also specify some of our notation and terms.

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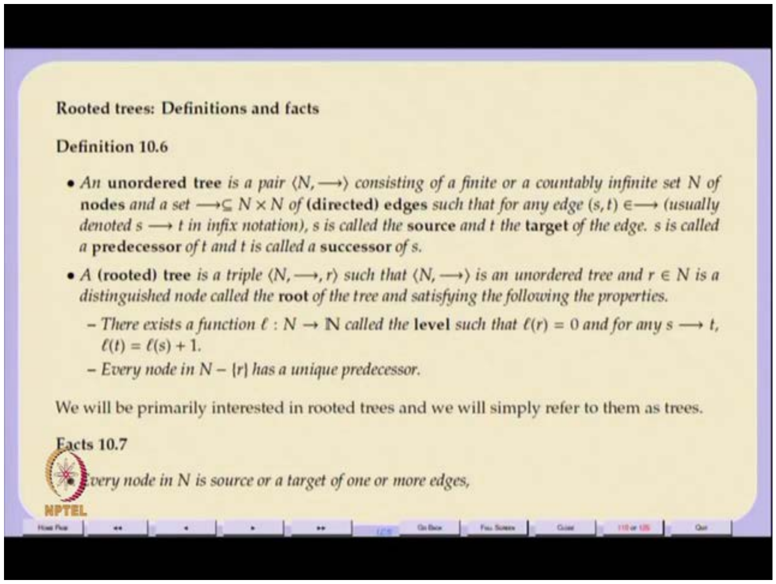

So, we are interested really in not un ordered trees where so Iam distinguishing between un ordered trees and rooted trees. Unordered trees are that kinds of trees you get in graph theory which, is not necessarily the same as the kinds of trees you get in let us say structures and algorithms so all the trees that we deal with in computer science usually are rooted trees. So, our abstract in text trees are rooted trees execution behavior of programs are all rooted trees they start from a starting point and then more over all the edges in these trees are all directed edges and they are not undirected edges. So, wewill look at a rooted tree.

So, a rooted tree essentially consists of a set of nodes end and an edge relation which Iwill represent by an arrow. And of course it distinguished node called the root such that of course every note other than the root has some what might be called a unique predecessor. And there exists a function called the level function which for the root gives you a 0. And for any given an edge from s to t where s and t are both nodes then the level of t is one more than the level of s. So, this ensures I mean these things are fact that l is function, and the fact that you have an exact notion of successor here and a predecessor here essentially make it clear that. This rooted tree is for example a cyclic and it does not have any self loops nodes do not have self loops.

And then this function essentially guarantees that the nodes are not sort of repeated here so, there you cannot have multiple copies of nodes. So, it is like the standard notion of trees that you encounter in data structures in algorithms like binary trees and so on so forth. And of course, only difference here is that this set N could be infinite so, you can have a tree with an infinite set of nodes it need not be finite. Otherwise it is a standard set of standard notion of a binary tree or a multi way tree that we have.

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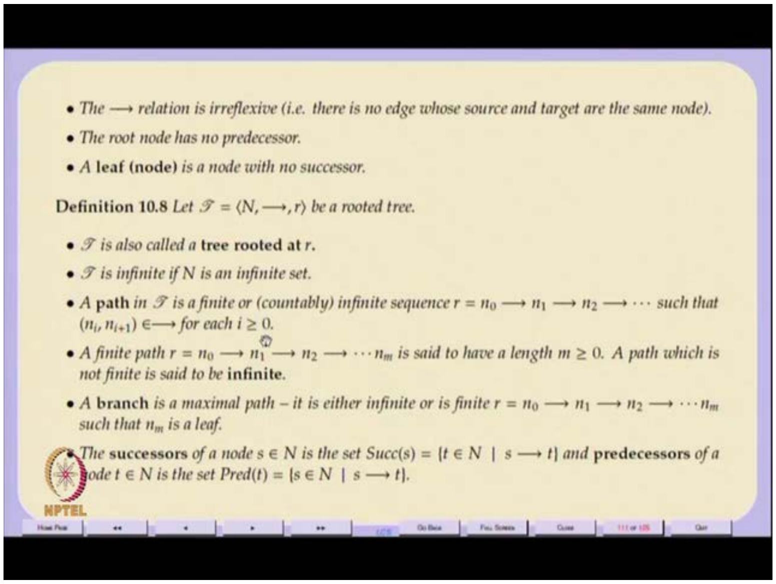

So, the basic facts are that so this edge relation is irreflexsive and the root has no predecessor and there is a notion of a leaf node which are in the case of finite in the case where trees are finite there is a the leaf nodes do not have successive that is. So, we also talk about trees rooted at some r and the usual notion of a path in this case we define a path as always starting from the root here. If, it is does not start from the root we can talk about a segment of the path or we can talk about a path starting from a the root of a sub tree rooted at some node and so on so forth.

But, otherwise in general when we talk about a path wewill just talk about a path starting root so essentially you traverse the arrows you traverse the nodes so that each you go through successor nodes some sequence of successor nodes. So, since our trees can be infinite so your path can also be finite or infinite. And wewill talk about branch as a maximal path so, it means you do not start any where you do not stop any where you go through the full path from the starting from the root. And then we have this usual notions like successor of a node predecessor’s of a node.

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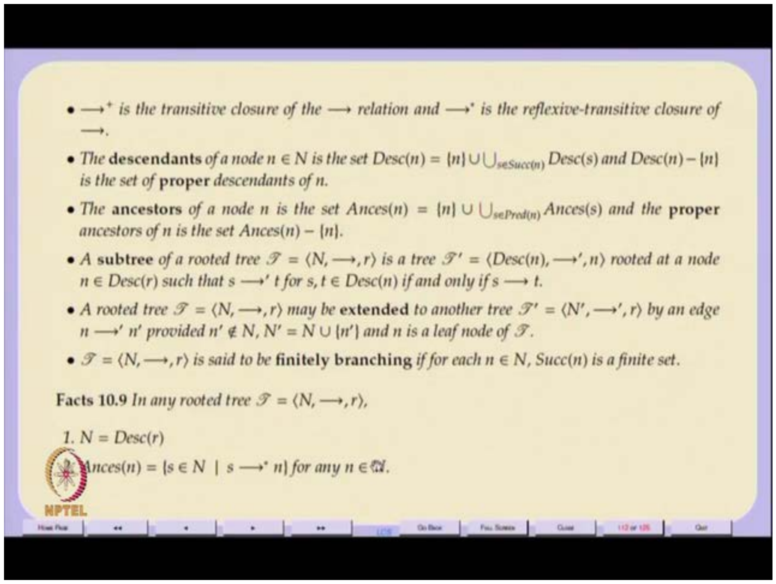

And since this arrow the edge relation is irreflexive we can consider the transitive closure of the edge relation. We can consider the reflexive transitive closure of the edge relations those will just be represented as arrow plus and arrow star respectively. And you can talk about the descendants of a node which I will represent by Desc. We can talk about ancestors of a node and of course, the root node has no ancestors we can talk about sub trees rooted at some node and we can also talk about tree being extended at the leaves to another tree.

So, what I have in particular I have said that when we talk about a tree we are talking about this edge relation in such a way where that tree is well in graph theoretic terms itis weakly connected. So, there are no isolated nodes there are no sub trees disconnected from other sub trees and so on and so forth. So, I have so a sub tree rooted at a node also means that we are essentially restricting the set of nodes from n to all the descendants of that node n. So, that is why the sub tree t prime here that is why so I can take a sub Ii can take the tree rooted at r and i can take a tree t prime rooted at some node in t. And just look at all itis descendants so and then there is a essentially a new edge relation arrow prime. This arrow prime is just a subset of the old arrow relation the edge relation except that itis restricted to only the descendants of n. So, by the way my definition of descendants includes n also so there is a I made it reflexive and similarly for ancestors also I have made it reflexive.

So, this is the notion of a sub tree rooted at something then, we can also talk about extending the tree with a new node such that the new tree has the same root as old tree. So it is the distinguished root is still the same except that there is an extra edge for a new node n prime which was not there in the original set of nodes. So, you are the new set of nodes capital N prime is just the old set of nodes n union the new n prime. So, you can extend it one leaf at a time so and this is an extension that i have tableau construction naturally allows so this is a way. And finally we will say that a tree is finitely branching. So now, when you talk about infinite trees there is possibility that you might have an infinite number of branches you might have an unbounded number of branches but finite and you might have infinite paths also.

So, we can talk about trees being finitely branching. If, every node has only a finite set of successors immediate successors here then we will say that the tree is finitely branching. So the tree could be infinite and a finitely branching tree could still have an infinite number of paths. So, in that sense the number of different branches could be infinite paths could be of infinite length, the number of different branches could be infinite. But, any particular node has only a finite number of immediate successors and so that is it is important the other thing of course is that something that I did not mention but which we will take for granted is that this set of nodes n can be at most countably infinite. I mean we are not taking set of nodes which might be uncountable or anything.

So, it could be only countably infinite set of nodes. But, then even thought the tree is finitely branching it could actually have a infinite number of different paths distinct paths each it could have both finite and infinite length paths branches. And so those we are not putting any restrictions on that. So, one thing of course these are some simple facts as a consequence of these definitions if the for any tree rooted at r the descendants of r are exactly the set n set of nodes ancestors and so on.

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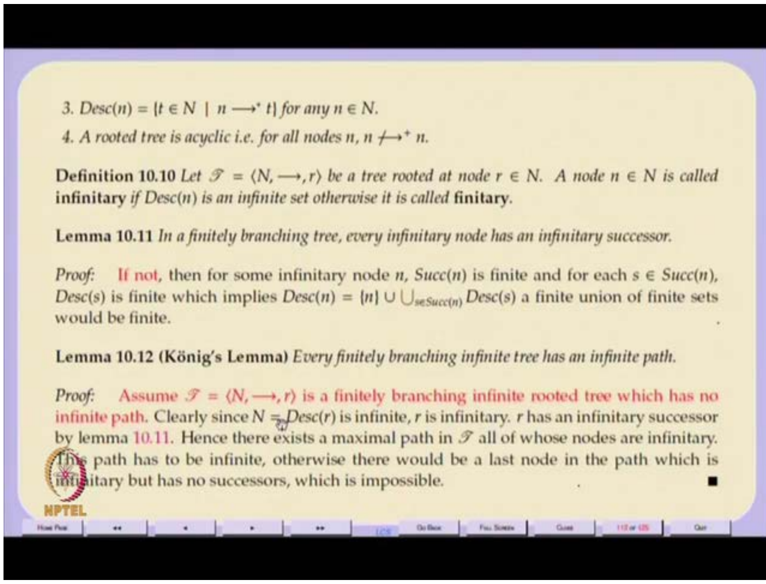

And these are I mean these are usual things and then a rooted tree is usually a cyclic and basically for all nodes there is no path of length one or more from a node tree itself. So, it is thatis the basically condition so the notion of self loop is something we eliminated at the start because, we said that l of t is equal to l of s plus 1. And that is not possible unless I mean unless you exclude self loops. But now, we have to this also consequence of all that is also that trees have to be basically. The other thing is we will take so, we will letus take any tree and wewill call a node to be infinite tree if the set of descendants of that node is an infinite set. So, a descendants means you are taking all possible finite length paths starting from that node. All possible so descendants is synonymous with this reflexive transitive closure of the edge relation so this reflexive transitive closure does not go through infinite path infinite length paths it only looks at all finite length paths thatis important.

So, the a node is called infinite tree if all itis descendant if the set of itis descendant's is infinite otherwise the node is called finite tree. But, still so a finitely branching tree could have infinite tree nodes. So all that we are saying is that for any node itis the set of itis immediate successor's is a finite set. But, the set of itis descendants need not be finite. So, nodes could be infinite so one basic one simple Lemma that we have is in a finitely branching tree every infinite tree node has an infinite tree successor. You can think of this the next two these two lemma as essentially some kind of pigeon hole principle generalized to infinite trees. Means the analogy with pigeon holes is letus look at the standard pigeon hole principle. Itis usually defined for finite sets and you essentially say if there are n plus one elements n plus one balls to be distributed in n boxes. Then, there is at least one box which has two balls.

You can start generalizing this for all kinds of finite ends and n plus all kinds of ends so on and so forth. But, you can also go into an n infinite set if I have an infinite collection of balls and I have a finite number n of boxes. Then, an extended pigeon hole principle would essentially say that there is at least one box with an infinite number of balls in it. But, these when you look at so finitely branching trees are a further weakening of that because even though a tree might be finitely branching. The number of different branches might still be infinite. So, actually you have so the branches corresponding to the infinite boxes and think of putting the individual nodes in the individual boxes distributing the nodes in the boxes. So it is a kind of infinite tree pigeon hole principle for finitely branching trees.

So, in a finitely branching tree so what we are saying this every infinite tree node has an infinite tree successor is essentially like saying that there is at least one branch. So actually that it becomes more clear with the next lemma which is going to use this lemma every finitely branching infinite tree has an infinite paths and this is Konig's Lemma.

And this essentially says that if I take all these branches as boxes and the nodes as balls. Then, there is at least one box with an infinite number of balls in it. The main weakening here is that a finitely branching tree need not correspond to only a finite number of boxes. Itis finitely branching but the total number of distinct branches might be infinite. So, itis still so you are you are looking at a case of an infinite number of balls being put in an infinite number of boxes. But, there is this constraint of finitely branching which somehow seems to indicate that there should be at least one box with an infinite number of balls.

So, letus look at the proof of the first lemma. So, all we are saying is that so we can label the nodes as being each we can associate the finite tree or infinite tree property with each node. And we are saying that you know finitely branching tree every infinite tree node has an infinite tree successor. Supposing not suppose thatis not true then for there is some infinite tree node which of course is a finitely branching tree so the successors of that node the set of successor's of that node is a finite set. So which means and if it is if there is an infinite tree node which does not have any infinite tree successor. Then, all the successors are finite tree. If, all the successors are finite tree then they each of the successors has only a finite number of descendants. And therefore I can actually add up all those sets and I get only a I get that then original node therefore must be finite tree which contradicts assumption that the original node was infinite tree. So, which means that among the successors of this infinite tree node n there is at least one node which is also infinite tree yes is it clear.

So, now letus go to Konig's Lemma proper which just says that every finitely branching infinite tree has an infinite path. So, again we prove by contradiction assume that t is a finitely branching infinite rooted tree which has no infinite path. Then, firstly the root that descend the set of descendants of the root is entire set of nodes itis an infinite tree. So, the root is clearly infinite tree by the previous lemma the successors of the root there must be one node which is infinite tree at least. So which means that so r is the root node r is infinite tree and r has an infinite tree successor by this previous lemma which means there exists a maximal path end t all of whose nodes are infinite tree. Because, an infinite tree and every infinite tree node is guaranteed to have an infinite tree successor so you take this. So, from r there exists a maximal path in which all of the nodes are infinite tree which means that, path has to be an infinite path.

So, even though the tree might have an infinite number of different paths there is itis not that so the tree can never if it is finitely branching so if the tree is infinitely branching then all paths could be finite. So, for example if I were to right. so for example here is a so you can just take a tree with root node 0.

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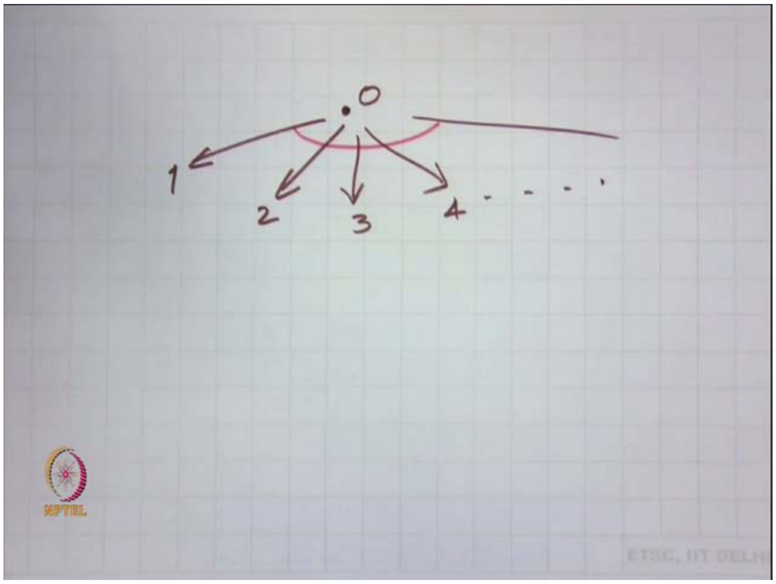

And i can have infinite number of branches basically 1, 2, 3, 4. Say in infinitely branching tree where every path can be finite. Every path has as link of 1 this is an infinitely branching this is an infinite tree which as a single infinitery node 0. And all other nodes are a leaf node is finitery cannot be infinite. So, what quensclimer says is that so now if I have finitely is but unfortunately this thing is in infinitely branching node this node 0 is an infinitely branching node. So, this does not satisfied quensclimer. If, I made sure that every node has only a finite number of success words and the tree is still infinite. Then, even thou their might be a infinite number of different branches there is at least one branch which is an infinite paths.

It is actually a beautiful generalization of a pigeon over principle for finitely branching trees. So, in fact what you can do is so this this is actually in the form of an implication. It is actually you can I can reword quensclimer as every infinite routed tree is either infinitely branching or is finitely branching and has at least one infinite path. That is your pigeon over principle for all router trees if you like. So, this see it is actually is surprising the lemise is very basic but some entity is actually widely known. But, it is never thought usually in any under graduated course as have you guys uncounted this lima before is a very simple lima. But, it is some never someone never people never teach it any under graduated course. But, it becomes important for the for our notion of compactness

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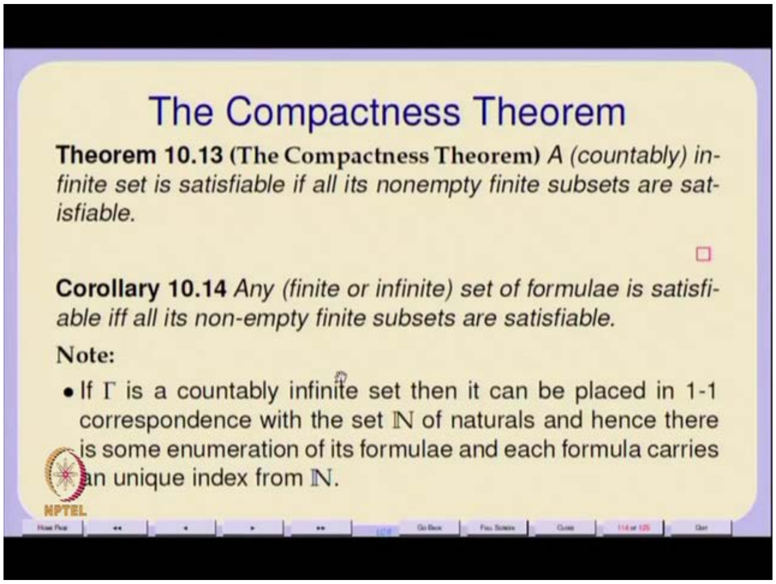

So, which gives us brings us to the compactness things. So, take a countable infinite set a sentence satisfiable. This infinite sentences is satifiable if all non empty finites sub sets of the set are satisfied. The first thing to realize in this theorem is that any count ably infinite set has an unaccountably infinite number of different sub sets. But, of course out of that uncountable number of different sub sets. The number of finite subsets is only countable all the other sub sets or all infinite sub sets I mean some of this thing give you a severe headache thinking about it. But, the analogy is with real numbers let us take the real numbers a real numbers basically consists of the rationales and the air rationales. The rationales are a countably infinite set. The real's are an unaccountably infinite sets their fore clearly the set of irrationals this is unaccountably infinite. And its exactly the same whatever, argument you might have applied to the real's to. E ssentially we are looking here essentially here looking at an extension of numbers in and cadnalities to infinite sets.

So, what we are saying is that the number of rationales is really the same as the number of the number of integers or number of naturals. But, the number of irrationals is really the same is the number of real's. And its and it is notch higher. In the case of your in countably infinite sets also you take any count ably infinite set it what we do. So one thing is clear no set no no set weather finite or countable or uncountable no set has the same number of elements as it is power set. Is a simple diagnolisation argument which will show that in case a set a can we place to in one to one correspondence with its power set 2 raise to a. And then i can actually create a diaganolisation argument which shows that any any kind of the that there is their exits are sub set. Which you are not accounted for. So, which destroys the notion of the 1 to 1 corresponding so this is this a very standard thing that we can show.

So, one thing we can show is that no set can be placed in 1 to 1 correspondence with its power sets. Which means no set the cardinality of no's of any set cannot be the same as the cardinality of its power set. But, then ones you got that the cardinality of a set is different from the cardinality of its power set. We can defiantly ask the questions, What is the cardinality of this set of finite sub sets of the set? And what is a cardinality of the set off infinite sub sets of this sets? If the set is the original set is infinite we can ask this questions. So, in the case of this and just like, you go through these proofs for real numbers and rationales and irrationals. You can go through similar proofs to show that the number of finite sub sets of a count ably infinite set is actually countable. And the number of infinite sub sets is actually uncountable.

And therefore the total number of sub sets of an infinite sets of a countable infinite set is actually uncountable infinite. So, I do not want to get into that but this is so that this. The compactness theorem says that i just required the all the finite sub sets to be satisfied. And I can guaranty that original infinite set will be satisfied.

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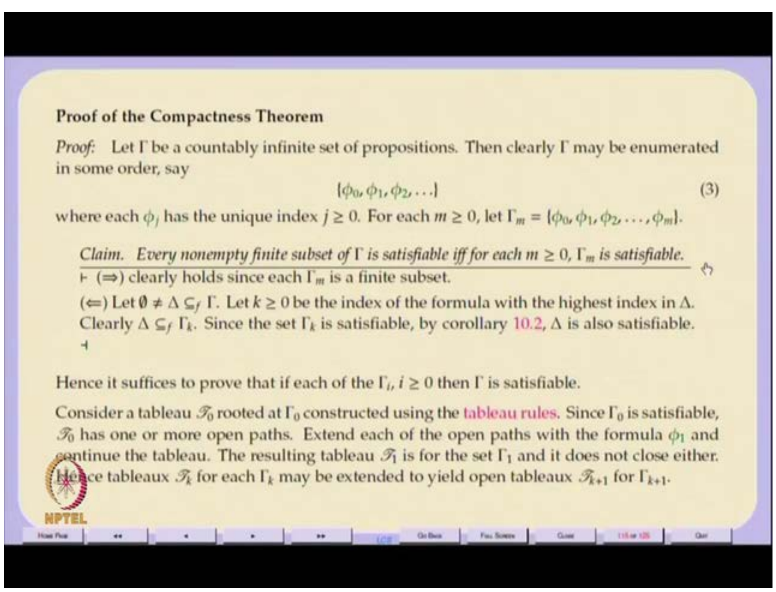

So, let us look at the proof of this theorem. So, let us, so if let me start with a set gamma. Which is countably infinite and it has. If it is count ably infinite then basically it can be placed in 1 to 1 correspondence with the naturals.

Which means I can essentially enumerate the set of formulae and gamma and index them with naturals. So, I can talk about phi0, then phi1, phi2 all the elements in gamma can be a enumerated corresponding to whatever 1 to 1 correspondence with the naturals you may choose to define. So, typical for any natural j phi j is essentially is the j plus 1 the element in this a numeration like. And now what I can do is take this a numeration I can consider all this finite sub sets gamma j for example. So, gamma j just consists of the first j sentences in the enumeration. So, where i have written gamma m should be this m minus 1 it this lets look at gamma m may be its may be its does not matter.

But, lets think of gamma m as consisting of phi not phi m minus 1 the first m elements in the enumerations. So, one thing is every non empty finites I have a pequrearway of sometimes writing proofs. Which is that I state claims and a create approves. So, this that you see here there is a claim, which is under line and in italics.

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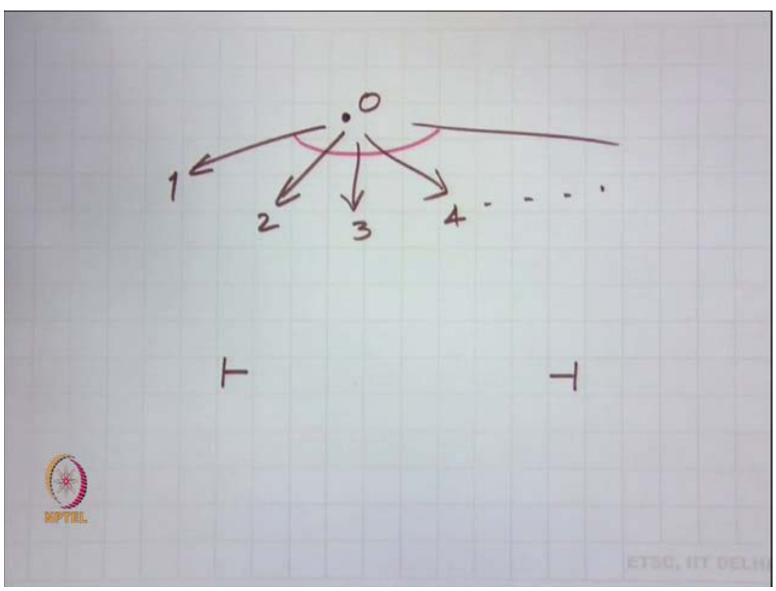

And then there are these two symbols turn style and left turn style and turn style which actually bracket the proof of only this claim. So, that so it is this is like a lot like program structuring in any kind of structure programming thing. You would like to divide up your program into various functions and then use those functions in some name way. So, this claims are like some functions in a program. And we will see the analogy with programming in the proof theory also that we do. But, essentially possible to have a nested structure of theorems and proofs especially of proofs. So, that you can have a individual claims sub claims and so on so forth and very much of like programmers. So, here is a claim which actually is not very important. It just says that every non empty finite sub set of gamma by the way and it also follows the usual spoke rules of programming I mean. So, gamma as already been declared in the outer spokes. So, it is available in the innerscopia un less it is renamed.

So, it follows all the structure programming a rules. So, every non empty finites are and basically so all the properties that you have defined before are available in here, fallows exactly the program structure that we normally would like to employ. Every non empty finite sub sets of a gamma is satisfiable if an only if for every m gamma m is also is satisfied. So, this is one thing of course is clear if every nonempty finite sub set of gamma is satisfied will than clearly each gamma m being a finite sub set also a satisfible. Other thing is given this particular order of gamma ms given that there is a particular. So, take can you claim make the same claim that all all the finite sub set are also satisfied. So, will just go from finitery.

So, you take any finite sub set delta by the way this is my notation sub set with sub script f means it’s a finite sub set. So, and of course this is non empty finite sub set we are only looking at non empty finite subsets. So, take let delta be any nonempty finite sub set then clearly there is an m in which such that delta is also a subset of m. So, delta contains these various formulae from the enumeration phi naught, phi1 phi2, phi3 etcetera and they all index by the naturals. So, there is a highest index in delta. So, now i just take gamma m or gamma m plus 1 their.That will include every sentence chosen in delta. So, delta is a subset of some gamma m or some gamma k.

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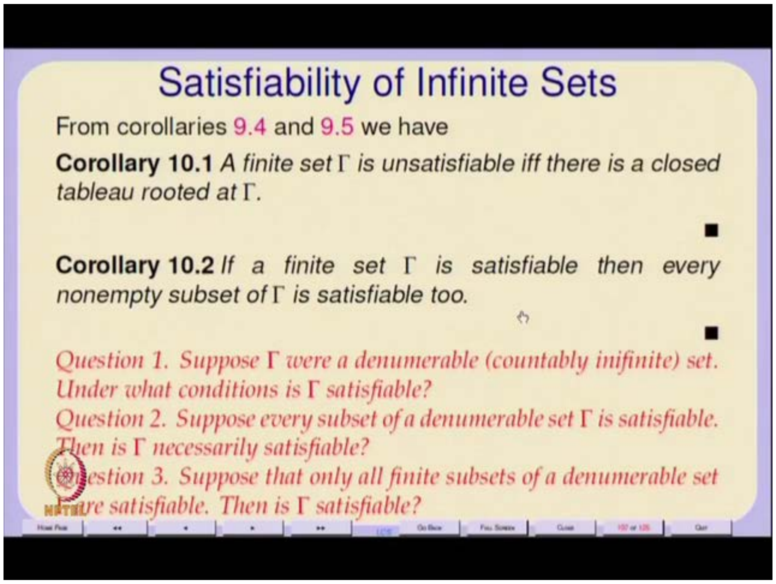

So, then gamma k is satisfiable we know that any finite subset of gamma is gamma k is also is satisfied. Because the same truth assignment which was used to satisfied all the sentences and gamma k can be used for all these sentences in delta and will be true. So, now the our theorem proof of the compactness theorem reduces to essentially showing that if each of the gamma eyes is satisfiable then gamma is also satisfiable. So, what do I do, I start I am going to use it tableau method. So, I start with gamma not so gamma naught just consists of that formulae phi naught create a tableau for it. The assumption is that gamma is a count ably infinite set of propositions in which each subset of gamma is satisfiable. So, a single ten set is also a subset and so their fore what does is mean it means that this tableau rooted at gamma naught this tableau at gamma naught. The essentially a tableau rooted at this at the formulae phi naught does not close. Since that means phi naught is satisfiable.

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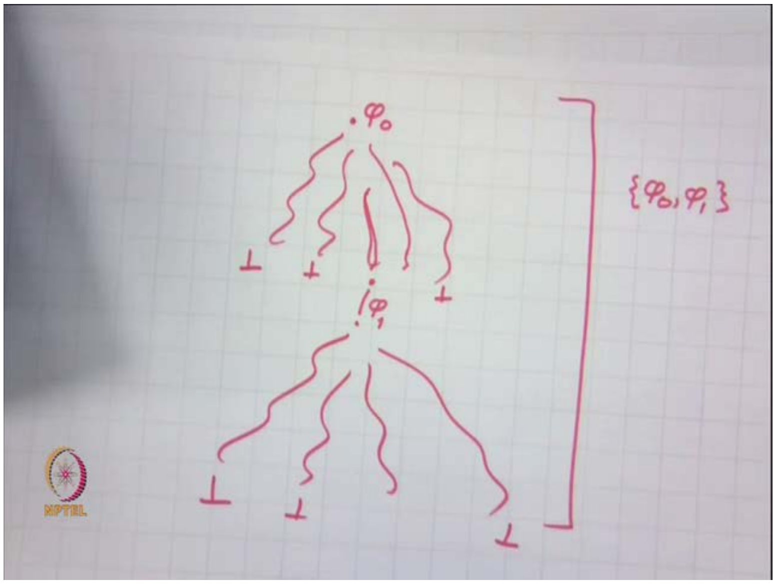

So, I have this tableau which so I have this tableau which essentially starts with phi naught. And it might have some close branches but, the fact that phi naught is satisfiable means at there is at least one open branch. Now, what do I do in this open branch I also had phi1. So, I extend it to phi1 so now what do I do I continue the tableau phi1 is some complex formulae. And I continue this tableau this will again have some close branches. But, this is this whole thing is essentially a tableau for this set phi naught phi1. And that is a finite set and it is satisfiable and their fore there must be one open branch. I add phi3 there and so on. I essentially and then basically then I get an extended tableau. Which is a tableau for a subset gamma 3 phi naught phi2 for gamma 2. And then gamma 3 and so on and so far.

So, now so essentially i start with a tableau t not extended to tableau t1 extended to tableau t2 and so on so fore. And since each of these gamma ms is a finite set each of them is satisfiable. So, for each m tableau tm has at least one open path. Then by quenisclamer I and this this tableau is a finitely branching tree. So, their exists an in an infinite tree such that all the formulae is phi i in the set gamma accruate some stage on open paths. So, basically all I am saying is.

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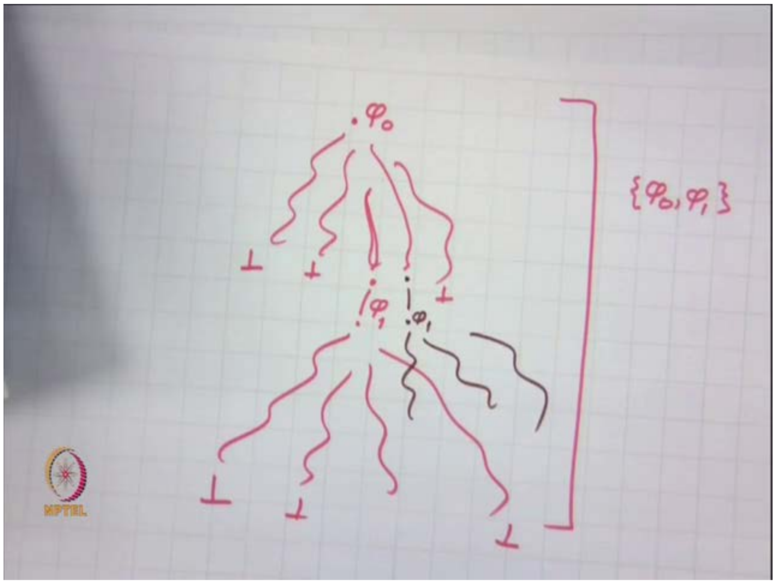
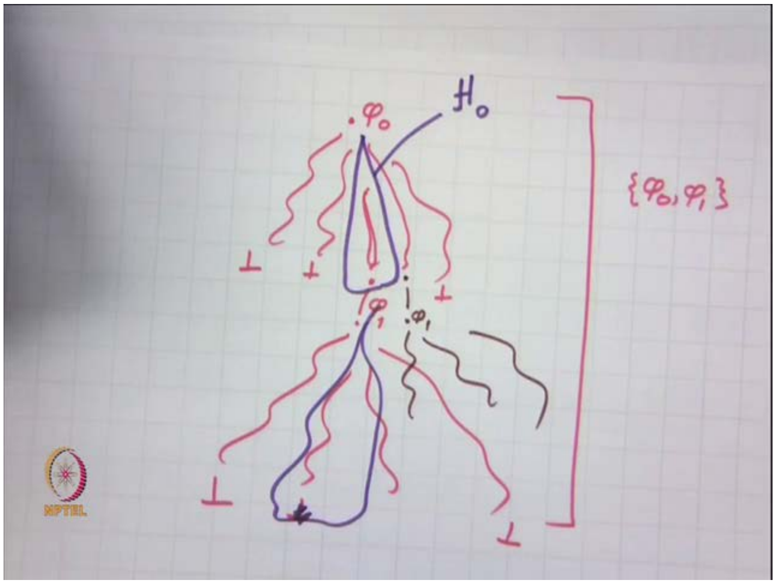


So we take all the open paths and I should have mention that in every open path we put phi1 here. And and when you put phi2 you put it on every open path. Because you do not know which one might close and which one not close.

But, what the satisfiability of each finite assures you that there is always going to be at least one open paths. And that open paths will have an occurrence of each of the phi i at some stage or the other. At and that will an infinite path by quenisclamer. And it will never close so every one of this formulae phi1 to phi n occur in at least one path. So, the fact that there an infinite set collection of formulae the entire gamma occurs in this path. Which is not closed and therefore this gamma must be satisfied. Why must gamma must be satisfiable? Your tableau construction creates hintikka sets, so take (Refer Slide Time: 44:22)



So, supposing you take you take this open path starting from phi naught what we were shown last yesterday is that. If, you add if you look at all the formulae their this is actually a hentikka-set. H naught and the addition of phi1 and an open path here, is and an open path here. Essentially is a hintikka set for phi naught phi1 and for each gamma m therefore if i just collect all the formulas in the path I get hintikka set for gamma m. For every m for every gamma m there is a hintikka set if i just follow the path and what we know is that is hintikka sets or always satisfiable.

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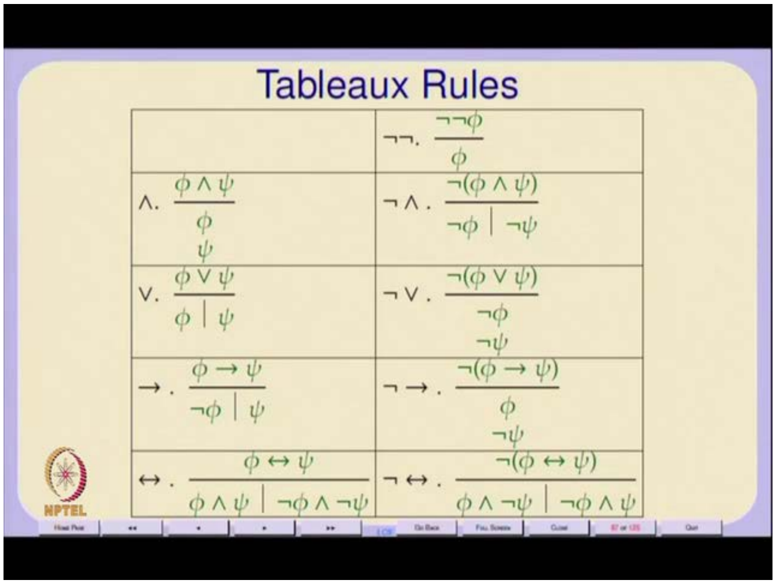

So, you this that application of the the first ser of the tableau rules and you think of it collecting all this formulas along the open path. And and what we showed last time was we showed that, every hintikka set is satisfiable here. A hintikka set every hentice set is satisfiable.

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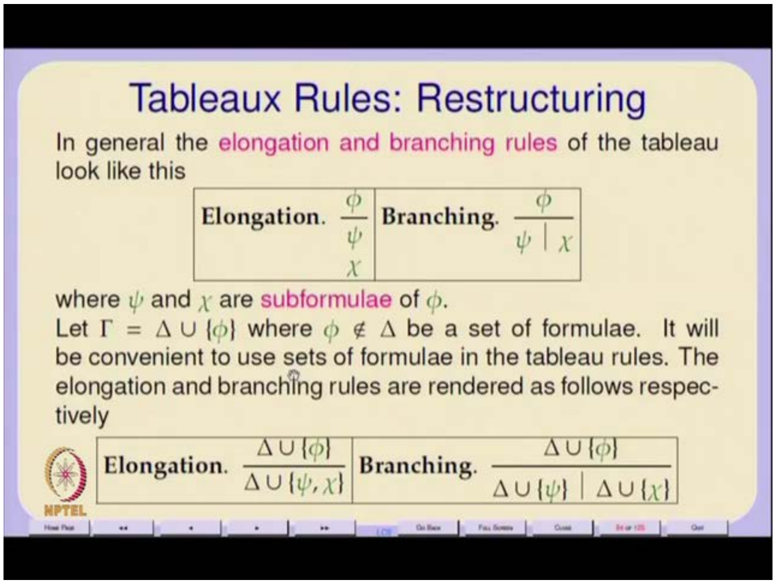

So, we showed this last time and what we are and what this open path contains a only hintikka's sets the path is infinite because it does not close. And by quenisclamer and of course every element of gamma appears in it is an open path and it Is.

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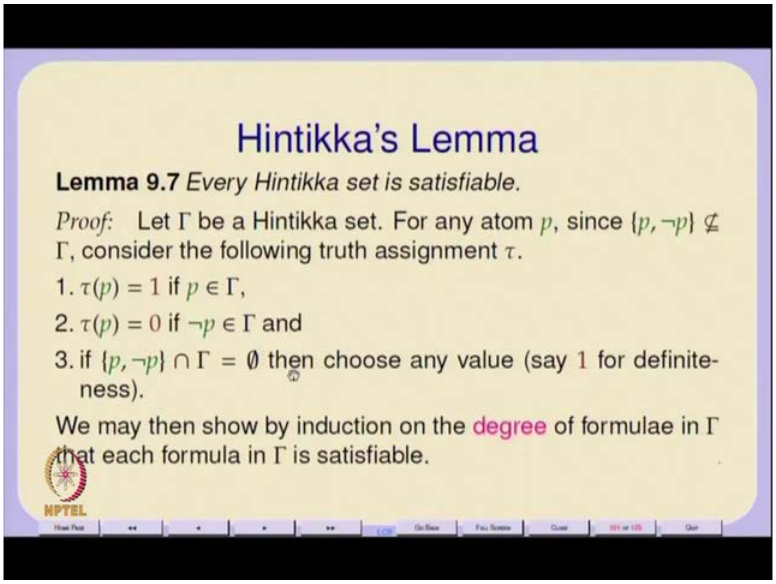

A Hintikka's set remember that the fact at its still its an infinite path. So, I cannot conclude therefore that it is directly satisfiable. What allows me to conclude their it is satisfiable is the fact that what this path has is an hintikka set. And all hintikka sets are satisfiable by this lemma. So, say essentially the tableau for T m has a hintikka se in it for each m. By queanisclama sense there is an infinite path. And it is not closed at but, that infinite path if I collect all the formulas in that infinite path. The set of all formulae that accrue in that hintikka path in that infinite path is a hintikka set. And all hintikka sets are satisfiable and their fore this entire path infinite path is satisfiable so the formulas in this path are satisfiablethe which means that gamma which is just a subset of this original hintikka of this hintikka set is also satisfiable.

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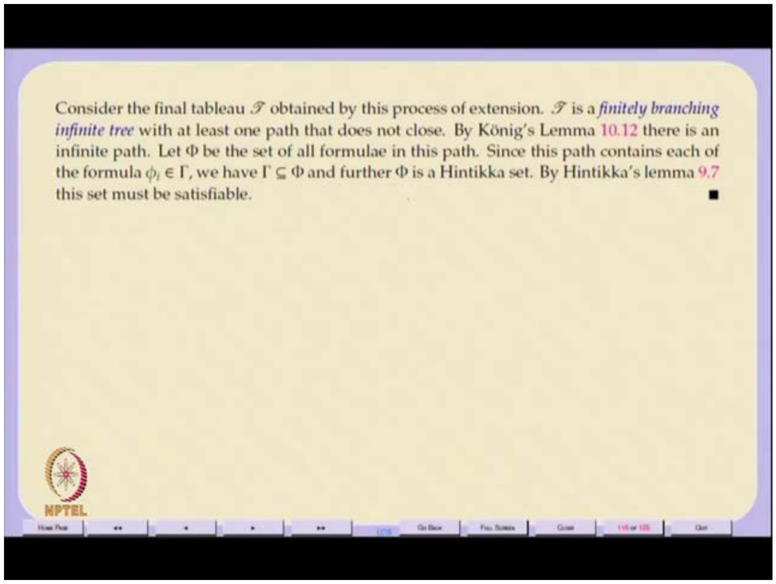

So, I think we are formely started logic at a point where we probably do not think of this. And so, we have this compactness theorem and it as some fairly serious concequences one is the notion of inconsistency.

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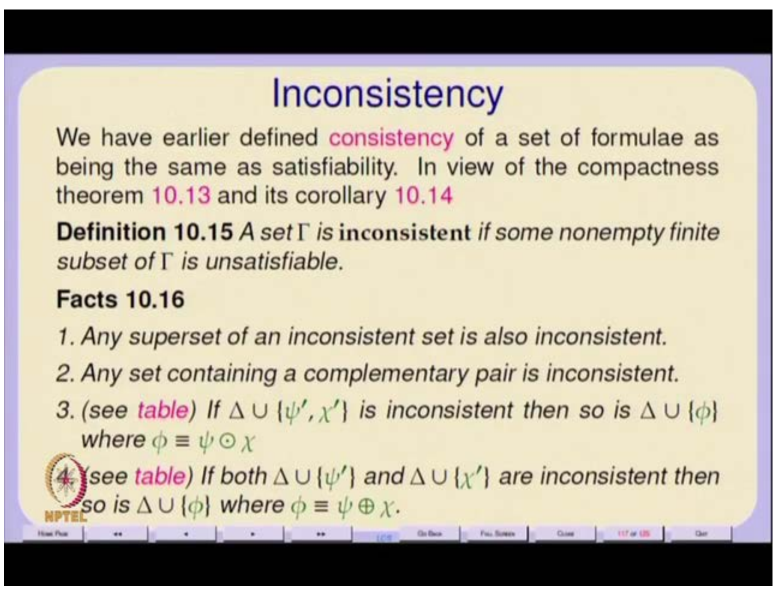

So, we defined Inconsistency as this being this same as satisfiability. So, inconsistency is just lack of consistency. But, the compactness theorem essentially says that if the set gamma is if you have an infinite set gamma. Which is consistence if every set finite subset of it consistent then this infinite set gamma is also consistence.

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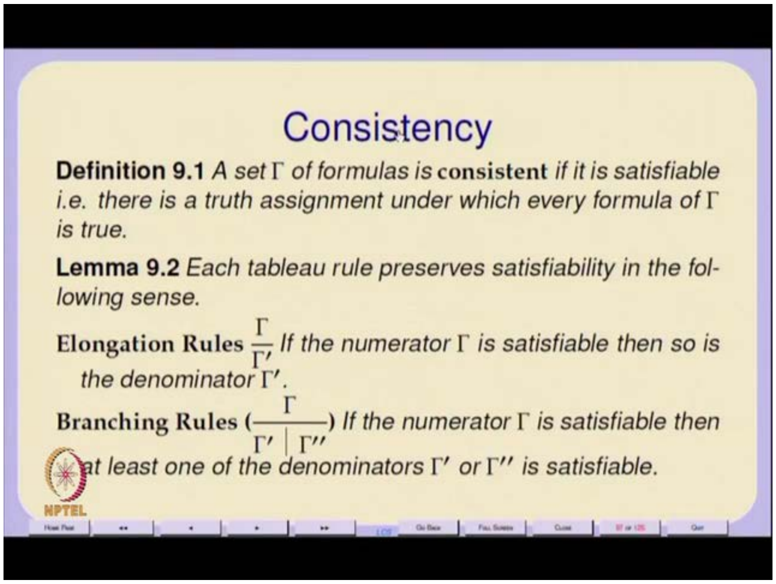

So, now inconsistency reduces to essentially taking the converses of both sides. Firstly the it is abuse that you have this corollary.

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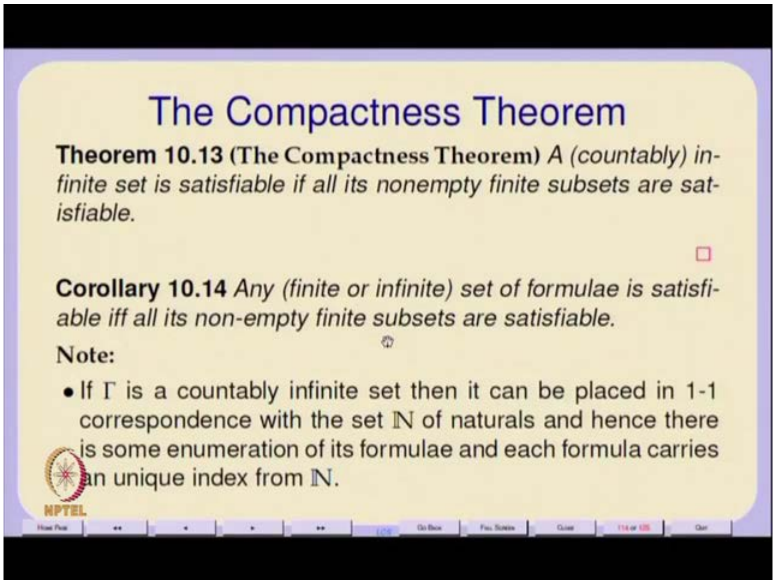

So, any finite set finite or infinite set of formulae is satisfiable or consistant. If an only if, all it is non empty finite subsets are satisfied. This is a consequence of this compactness theorem. So, what this also means for inconsistency is at any finite or infinite set of formulae is inconsistence. If an only if, their exist at least one nonempty finite subset which is inconsistence. So, inconsistency therefore reduces so this so essentially why so we can think of this definition of a inconsistency. Now, that we have the compactness theorem so we just say that a set gamma is a consistence if there is some nonempty finite subset which is unsatisfied.

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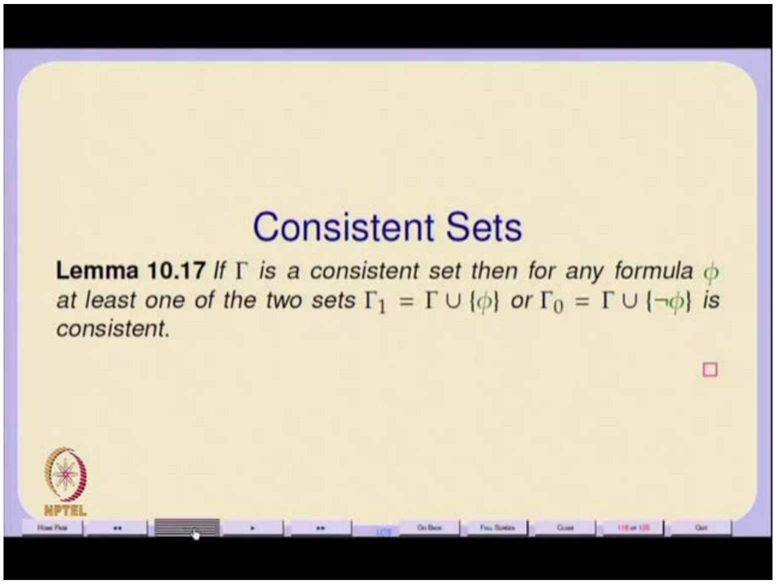

So, onething is its clear satisfiabality or consistency if gamma consistant the any subset of it is also consistant. In the case of inconsistency it works out all together all you take inconsistence set I had a few more formulae into it it is still remain inconsistant. So, any subsetp of an inconsistence set is going to be inconsistant. Because, the tableau which was use to prove let us say the inconsistency of the original set can be used without any change to show that this set is also the subsetp is also inconsistant So, the other the simple facts so that any set containing at complementary pair is obviously inconsistence it want gonna be its never going to be become a hintikka set.

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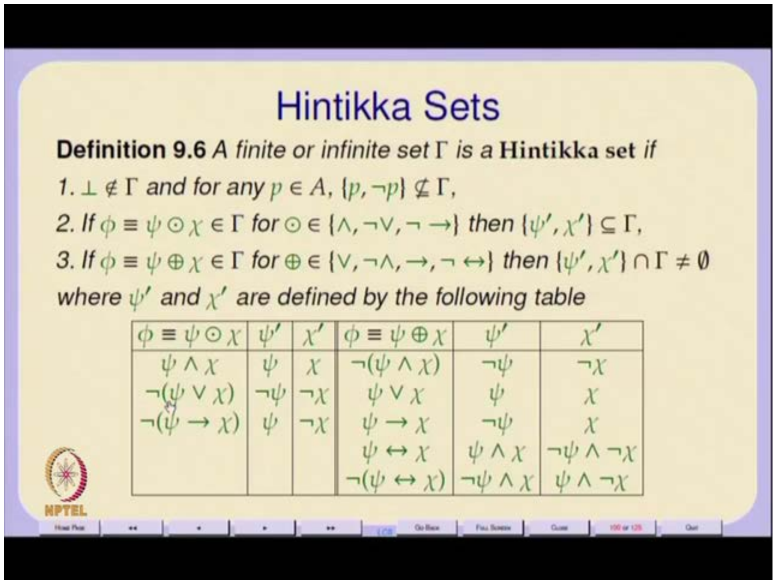

And of course our motions of multiplicated and additive operators our give us this facts. So, we had these notions of multiplicative and additive operators. So, for each minary operator multiplicative additive operator of the form I mean. So, with component say and kay we had a corresponding say prime and kay prime. So, you take this if you so the notions of inconsistency essentially say that if I take say prime and kay prime and additive said delta and that that is set is inconsistence. Then just adding phi to delta also makes the set delta union phi inconsistency. In in the case of the multiplicative operators in the case of additive operators you had only say prime or you only phi prime. If both of them are inconsistence then, adding phi makes the certain consistence like an this is justan doing the converses with negations starting from the characterization of consistency as finite or infinite set gamma is consistant. If and only if all it is finite nonempty finite are consistence.