Logic for CS
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Lecture - 01
Introduction

In introductory lecture and of course, you all done courses on architecture.

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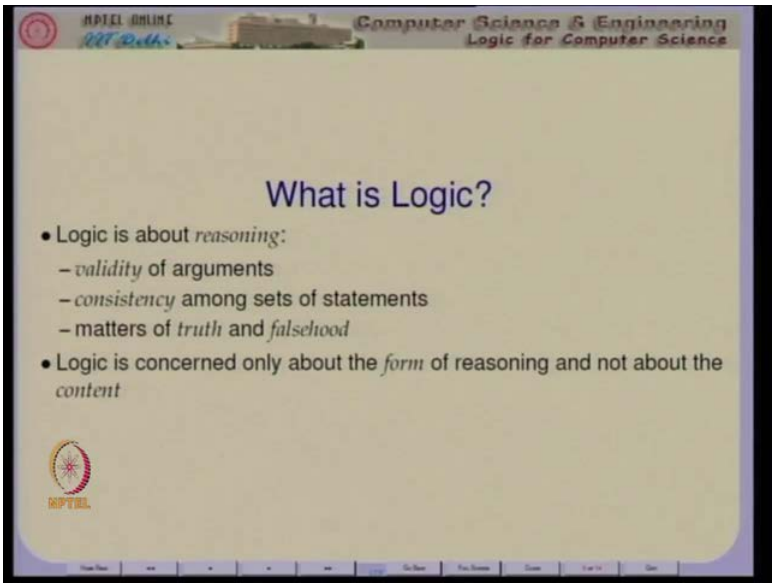

So, logic is about reasoning essentially and that is what it has been for a long time since the time of Aristotle, but now a day's logic is a maximum formal discipline. A large amount kind of logic that was actually prevalent once related to reasoning, but we had a more a philosophical, we had a more blend because it had to reason about essentially mans position in the universe.

Initially it was mans position on earth, but now the specialness become much larger it is about mans position in the universe itself and the notions of reasoning and understanding, the regular mathematical validity is as universal as we where is it related to only human or does it move beyond human formation. And in some sense logical reasoning isn't absolute that is it is an important question, which in your the moments when you are not busy with anything you can think about I mean that is it is an important question about you and your position, and what your influences are about the universe.

So, it essentially the whole business of logic was to essentially establish principles of valid reason, and we will look at it essentially from that point, what was it mean to say that there are principles of valid reasoning. What does it mean to say that second statement is consistent, what is consistency what is truth and what is falsehood is there any absolute truth, is there any absolute falsehood? We will we will look so we will look upon truth and falsehood as essentially the basic tools, based on which based on which most of our logic is logic and especially mathematical logic discussed.

So, say the whole point about logic and that is this is very important and it does not and it is hidden very often when you, but it still holds and that is that logic is only concerned about the form of reasoning and not about the content that is an important issue. In the sense that there are forms of reasoning, which are invalid and forms of reasoning which are valid. And those forms they have a certain shape to them which unfortunately can be separated out form the context, and the invalidity of the reasoning is not necessarily related to the content. So, this is a fairly I mean the fact that things are more found than content is actually quit a joke to many people, who say they are being very logical.

But by being logical essentially what you are saying is that you are confounding to certain founds of valid reason and those founds of valid reasoning from the computer science perspective, they are essentially same category. We do not have those founds valid reasoning not necessarily have a semantic content, and the question of semantic content of this founds of reasoning is primarily a matter of interpretation and that. So, those are those are some aspects that we will look at in a fair amount of detail.

So, there is some notion so all notions have validity of reasoning have to be based on some notion of truth or false. And therefore, they are often therefore, there are often levitated to what are known as symmetrical interpretations. So, very often the actual truth or false of the statement is not absolute it is relative to a certain notion of interpretation. And that interpretation is the important point our questions themselves will be concerned more with the validity of the reasoning with the forms, that we use for reasoning rather than really with the content of the arguments and so.

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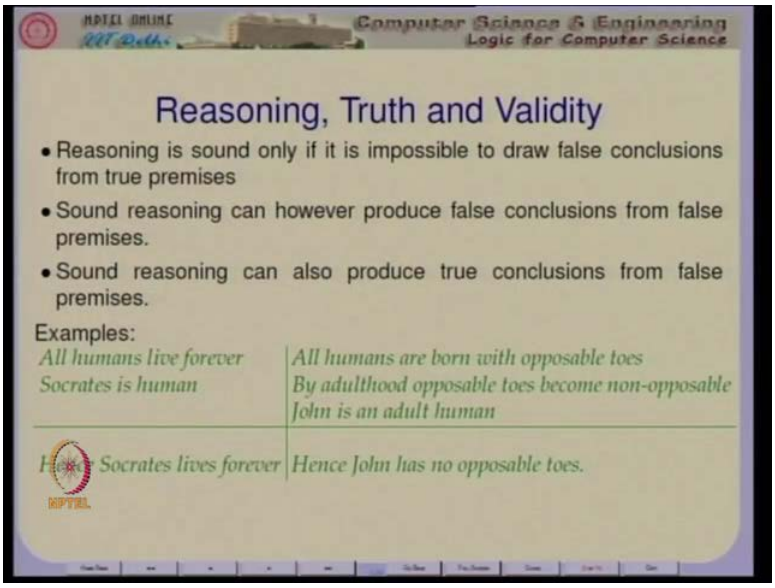

So, what we really want to know are sound reasoning, so the main thing is there is some version of truth and there is some version of falsehood. And we will not concern ourselves with truth and false hood as a continue. Do you want to continue on this the real form a continue where the rational numbers are discreet. So, as you take any real interval like 0, 1 that is also a continuum, but the actual numbers the integers and so on are discreet. In fact whatever is discreet is essentially something that is capable of being counted. Whereas whatever is continuous is not really counted, you can only you can associate or rational numbers with them, but they are not you cannot do combinatory manipulations on them.

So, our notion of truth and false hood will also be discussed, so what it means is that things like fussy logic, where there is a continuum of truth and falsehood, and notions like hard troughs white lines and so on will not interfere with our notion of reasoning because our notion reasoning is re discreet there are just two values true and falsehood, and they are discreet, there is nothing in between and there is nothing beyond that. So, essentially what we are saying our basic model of truth and falsehood will just talk about reasoning from those notions of truth and falsehood.

And we will say some reasoning is sound only if it is impossible to draw false conclusions from true hypothesis. So, what it means, is that it is possible to draw through sound reasoning it is possible to draw false conclusions from false hypothesis. It is also

possible to draw true conclusions from false hypothesis right, but our notion or reasoning should ensure that you can never draw a false conclusion from all true hypothesis, or premises hypothesis are also called premises.

No, whatever is the question of truth or falsehood right so the first thing of course, is to realize is that since we are interested in more pounds of reasoning, rather than in their actual content. And we are actually looking at all this in relationship to essentially the methodology of mathematics and computer science. And therefore, we are not particularly interested in the actual truth content of these statement in these examples. The actual truth content may be used as a basis for reasoning, but our reasoning mechanism it is elf is independent of the actual truth content of the individual statements, right? In that sense we do not we do not actually consider truth to be related, first of all first of all if truth is not related to just our version of reality.

There are realities well beyond what you can perceive and what I can perceive. So, there are mathematical realities, which are quite abstract and may not have anything to do at all with the real world. So, the point so the notion of truth and falsehood therefore, is related to a certain interpretation. So, interpreting this statements in this our common perceived reality is unimportant. So, in that sense, however if you were to look at it only within a common perceived reality that we have then of course, all humans great for ever may be considered false.

But however what are seeing is that this argument has a form which is form of reasoning which is valid. So, that the influence given that these premises are true in some reality this conclusion would also be true in that reality, if logical reasoning has a universal reach that is essential what we are planning. So, similarly, I mean the second amendment was inspired by this cartoon, do you guys read. So, there was this cartoon where camel tells horse you know so far I think my life have been good, except that I do not have a causable toes you know what a causable toes are causable, you have a causable thumbs that is what helps you grasp things and so on except for the fact that I do not have a causable toes my life has been good.

So, far that is so it has basically inspired by that if you like the whole point is that whole point of all these is that actual reality either perceived, or absolute reality is of no concern to us. So, what is what is the concern to us is whether this is a valid argument

that is it, is the form of reasoning somehow valid and is that something that can somehow be considered universally valid. For example, in some alien land would they have the same principles of valid reasoning that we have, and so far these principles of valid reasoning have been very good not just in our mathematics. You have also been good in the applications of mathematics to the other science research and so on.

So, perhaps they are so there is something universal about them that is that is a thought to worry to think about. Especially when you are in a drinking party.

With respect to the notion, but what I am saying is that truth is not absolute it is all I am saying. For example, in this case if there is a reality in which human beings evolved as opposable toes and human hand by the time human begins become adult in that reality, they lose their opposable toes of there, those become non opposable then. And john is an adult human being in that reality then john will not have opposable toes.

My question was is it possible that there is a form of reasoning there is sound under notion of truth, but not sound in other notion truth or if there a restriction on the notion of truth.

There is a restriction on the notion of truth and that is the restriction that I have already imposed that there are only two possibilities either true of false. Other than that because even in that I can have like there is possible then there is reasoning which varies under one notion of one.

We do not know that is why I am that is why I am leaving this posing this question whether the principles of reasoning the notions of truth could be related. And essentially what we will do is we will relegated them to the idea of an interpretation or of a model. So, any kind of mathematical model is possible in which you can assign truth or falsehood to various statements, but given that there are given that in that mathematical model this statements above the horizontal line are true. Then the statement below the horizontal line follows, and whether that process itself is universal is an open question.

I accept though it is an question of philosophy more than a question of logic, so that is what happens so reasoning is sound only if it is impossible to draw false conclusions from true premises. Of course, you can produce false conclusions form false premises you can produce true conclusions from false premises and so on, but so these the

standard Aristotle reasoning mechanism, what Socrates would actually all humans are mortal Socrates is human therefore, Socrates is mortal I changed. So, if you want that is that is the way of getting a producing true conclusions from true premises which I decided was too obvious I will not have any example.

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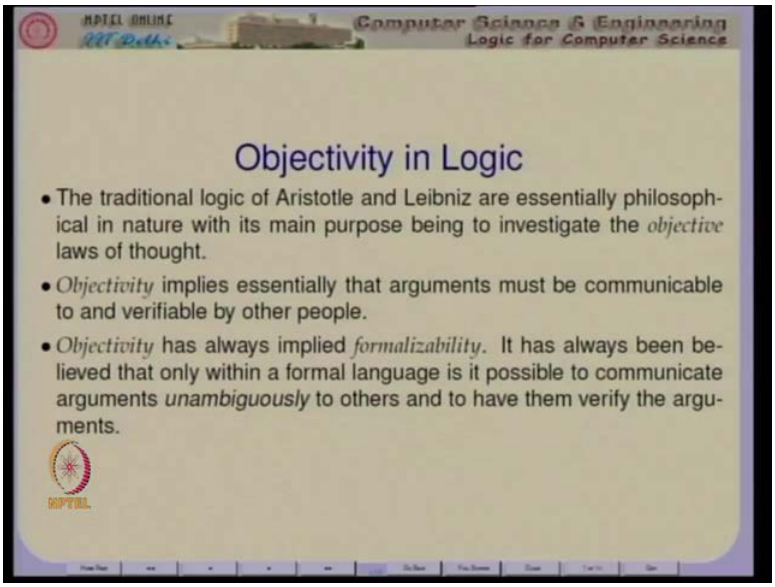

So, the traditional logic of Aristotle and there have been various schools of logic and reasoning all over the world. So, however we follow the grand tradition established by Aristotle. So, let's not distract our selves by talking about Sankya philosophy and the logics of ancient individual thoughts. So, there is something in that mathematical reasoning which still falls, falls with the domain of Aristotilian logic. So, it so the notion of logic or syllogism started with Aristotle and Leibniz it is it is a very interesting thing that Aristotle did not Aristotle was interested in reasoning principles of reasoning he was not particularly interested in another aspect. But which was very interesting to Leibniz.

And Leibniz everybody has heard of Leibniz he and Newton fought over the differential and integral calculus unlike he is also a great philosopher, he wrote the fundamental principle that which is what we call in our algebra it is that you can always replace equals there equals, where Leibniz take it that before, but likely it is actually formalized in much more general fashion one thing. But the most important about Leibniz was that that is of importance to us besides being a philosopher and a having co-invented the

differential and integral calculus was also the fact that Leibniz designed calculating machines.

And being a philosopher and a magician he actually thought about logical reasoning, itself as something, which can be recognized which can be perhaps done by machines. So, the first indication that something could be automated and actually performed by machines comes from lives, not well before our completely we designed calculating machines the same way Pascal designed a calculator. But and the calculating machines did nothing more than elementary arithmetic, but it did not prevent him from projecting it further and looking at reasoning mechanisms themselves as something that can be mechanized.

So, to this however they called their Leibniz called it objectivity and object objectivity of course, now objective for means calling a group of guys and ask them to give marks from 0 to 10 and take the average, but objective has a different science. And the main was to investigate the loss of heart and reasoning is essentially in such a way that they are communicable and verifiable by others if so they are communicable and verifiable by others. That puts in an any initial constrain itself that logical reasoning, has to be somehow synthetic image. The form the form of reasoning is somewhat more important than the content itself.

So, this objectivity essentially therefore means that you have to be able to formalize this reasoning. So, when you have when you have formalizing a reasoning mechanism, what does that mean? In the twentieth and twenty first centuries formalizability essentially means designing of formal language of algorithms. So, the formalization therefore is comes from the linguistic philosophy. So, what we are going to study is really what is know as formal logic.

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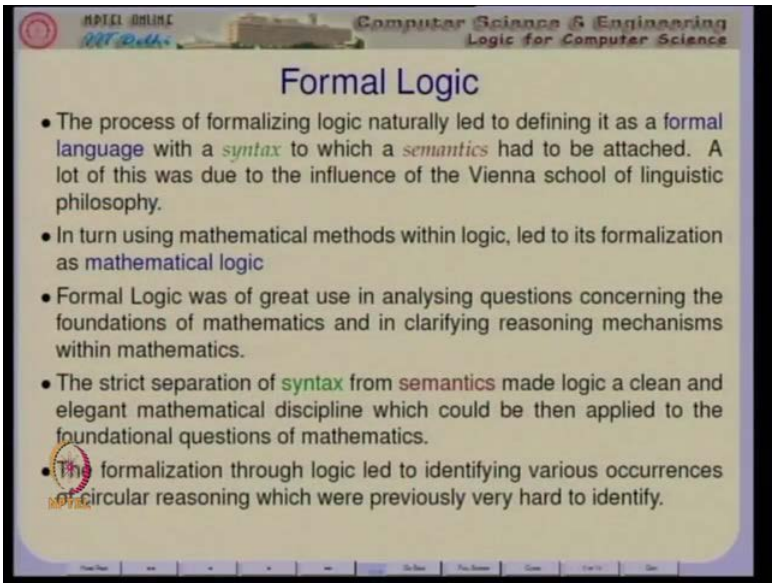

And the process of formalizing logic naturally leads to defining a language, a formal language. Thus perfectly symbolic language in which immature reasoning mechanisms are also expressed completely in symbolic way. We apply these mechanisms to some particular world or to some particular model, then you end of them, end of the statements about that model with some meaning in that model.

And you essentially verify your if your principles of reasoning are correct, then in the model you should be able to get from all from true hypothesis, you should be able to influence only true notations. So, if the reasoning mechanisms are really sound we should be able to input true conclusion from true premises in inner model. So, now what has happened now what this does is that the notion of formalization therefore, brings you the notion of syntax, it brings you the notion of semantics which essentially this cases that are modeled, what is the problem?

So, essentially what we are saying is that so there is a formalization in terms of language, and actually now form this sand point of the twentieth and twenty first centuries, we know that what it means is that you design a formal language. And with the hundred of programming languages available designing a formal language is no great job. Therefore, the notion of reasoning can also be put in as part of you can design a formal language of logic for the good part of this course, we will we will essentially be interested in what is

known as first order logic. I will define the term first order and so on later on, but we will essentially be interested in first order logic.

And the principles of reasoning of course, start a logic are actually valid also for what might be called higher order logics. In all those cases it is a matter you are just defining as syntax and defining the principles of reasoning and more or less any higher logic can be embedded with in first order logic and so it is sufficient to essentially study of on the first order logic. There are also variety of other kinds of logics which are useful in computer science. If we have the time, we will study some of them then they are and model logics are essentially subsets of logic at least the way we look at it. And we and they are of great use in computer science and for various uses.

So, the most so the what now the effect of the historically separating out syntax sense and optics, essentially logic it is elf and the fact that the reasoning principles that you use in mathematics, themselves are used to reason about logical systems. Essentially makes logic a decision with in mathematics. So the way you look at logic is it is mathematical logic, it is this is the sub decision mathematics. And it can be applied in the same way to branches of mathematics and identifies it is applications analysis algebra so on have been formularized.

So, in all these a so, you can think of logic either, but the main formulization of logic especially to the especially in the late nineteen century early twenty. Essentially take from this a from linguistic philosophy point of view and but effect it had, was to actually to answer questions about paradoxes in the foundations of there is so the growth of mathematics. So, there is all of us science essentially brought about a huge amount of confusion because it is does not clear how in what order, the clearance are the clearance are to be proven for example.

So, there is a question of circular reasoning which has troubled mathematicians the centuries and in particular so the need by establishing a foundation based on some principles, especial principles of logic quiz founded in the negative situation. So, essential starting with Boold who did the first formulization on logic, logic has not formulized as a mathematical discipline, which uses mathematical principles of reasoning in order to formulize reasoning process is that. It is a very interesting way of

looking at so in some sense kind of formulized principles of reasoning in order not to get into what are known as circular arguments.

But the fact that you using the same mathematical principles to formulize the principles of reasoning, itself opens the question of… However there is a reasonable philosophic review, which says that it is it is totally valid because of the fact that your mathematical truths certain speed exists, where you reason about them in a certain order or not. The truth still exists I mean the question of in a Euclidean geometry, equilateral triangle is also average.

The fact that you should not use circular reasoning to prove that equilateral triangle does not distract from the truth of that statement, in that mathematical model equilateral triangle highly with the equi angle. So, the principle so there are so actually bring so, the fact that you are using mathematical principles to formulizes the reasoning and trying to ensure that logics of mathematics, which have presented in a circular way do not have are not guilty of circular things. Means they are there are two different things operating it, one is the notion of a mathematical aguish. The mathematical existence what is known as a platonic existence right is different from it is presentation.

Point is in Euclidian geometry whether you like it or not equilateral triangles are equiv angular. But if you have to communicate this fact as a inference there you have to present Euclidian geometry in a certain way and may be take in one presentation you might take equilateral triangle as to be a definition. And you will prove equiv angularity from previously established theorems about equilateral triangles, which also means you have to have previously established theorems about lines planes definitions of triangles so on so far.

And if you are going to use in this if you are going to use the conference of triangles you should have throw the conference of triangles. So, the fact that in infer a statement S from another statement E does not mean that R and S did not exist simultaneously, there existence is got nothing to do with presenting a argument. So, mathematical intrusion therefore, can be quite illogical, so when we use mathematical intrusion you come up with a brilliant idea that this must be proved, this statement must be proved in let say some model. But the question of proving it is a matter of presenting a covariant sequence of ideas, without circularity is that clear.

So, logical reasoning is about presentation after you mathematical is a different thing all together. So, you can have so the fact that your ideas have to be communicable, and verifiable by others. Means that in your presentation, there should be no circular reason, but that is not mean that they you are saying that your theorem is a cause of some other definition of the theorem, causality is not involved in that ideal of mathematics is that causality does not play any role anywhere. The exists truth just exists, all you are looking for is just suitable organization of the prosecution of this case in such a there is no circular arguments is this clear.

I mean this will be the deepest philosophy I will ever go into, but it is a philosophy means that is why there is no contradiction at all in this mathematical principles be used, in order to study the notion study the mathematical theorem reason. All those things exists, it is just exists branches of mathematics that is it and you can apply one to the other the presentation is what we are looking at. We are not looking at iteration itself. So, remember one thing, when you think logically your problem not going to come up with very great truths, only it going to come up with most driveled truths. It is mathematical aggression that makes you go to lead some imagination, later then you have the obligation in order to prove in order to show that is the truth that the is truth, we all have an obligation to present it in such a that is logically consisting.

So, intuiting about a about a mathematical property I am proving it two different things already there right, that is an important that is what you know that is the important lesson we take from formulization of a log of logics as purely syntactic exercises. So, it is only presentation, so what it means is that it is possible and this is we have an example of this kinds of things. So, you take Srinivasan Ramanujam is mathematical to come with huge numbers of integral the properties and identities. And some are never proven, right? Standing example of a person with tremendous mathematical, but with no training in the principles of these.

So, the other thing so a main uses of logic basically in formulizing the foundations of mathematics such a way that you do not attribute to any causality to the truths of mathematics, they just exist and that is the platonic existence, where interrelationships are a matter of logical reasoning. And that is where you want you want a formulization. So, the other thing is that so that is what the formulization through logic let to identifying various occurrences are circular reasoning, which are previously very hard to identify.

One example of these kinds of circular reasoning actually it takes back to the eleventh century, when people actually many people thought that parallel postulate, right? It is set of postulates five postulates and the fifth postulate was the parallel postulate, for which you know that what is that it just says that to a point not on a line on the plane, there exist a unique line pattern of the given line. Many people thought that the other axioms and basic cues that are simply formed. The other axioms who are very fundamental in this with an appeared to be, so cue essentially is very few people actually try to prove this from by trying continuously reformulate in given geometry in such a way that postulate as a theorem.

But the point is that it proves who are such that it is, it was with those proofs always adds some circularizes. So, in that presentation of that proofs they would have assumed something implicitly which, which aculeate morals from this postulate it will live on there own presentation. So, what it mean so first thing is it is possible that you can take a mathematical discipline and present it in a completely different ways completely different sets of axioms.

The same mathematical tools can be presented then that is possible that is what that is what will happening in the case of predetermined. But more importantly because of the fact that reasoning was not to formulize, many of these proofs it is very hard to debug them to find out where exactly it is the bubble the proofs. It is as hard debugging C programs large C programs to identify exactly, where it is it is diverging process very much to debarking programs.

So, you are we are asking does this follow from that does this follow from this and so on so, the need for formulization was felt to the nineteenth century. Especially because of the explosion and mathematics of various disciplines and in all these formulization you need to identify some foundation as basic truths, on which you build up the whole of mathematics. So, this and this formulization itself was raided with was raided with inconsistencies. So, you are required to use principles of reasoning in order to actually establish the foundations.

So, there are various way of establishing the foundation of mathematics in one the most popular, which we have always which we all follow is set theory as a foundation for other branches of mathematics. It is possible to not set really as a foundation it is

possible to choose something else, like the notion of the relation as fundamental call the notion of function is fundamental and build up some mathematics. It is not clear that if you able to build up the whole of mathematics that way, but that is something that is that is the challenge. Can you build up mathematics on some other foundation? You can I mean so these are the kinds of challenges. So, it is largely a question of presentation so you can see it is in that it is a matter of communicating ideas through the syntactic.

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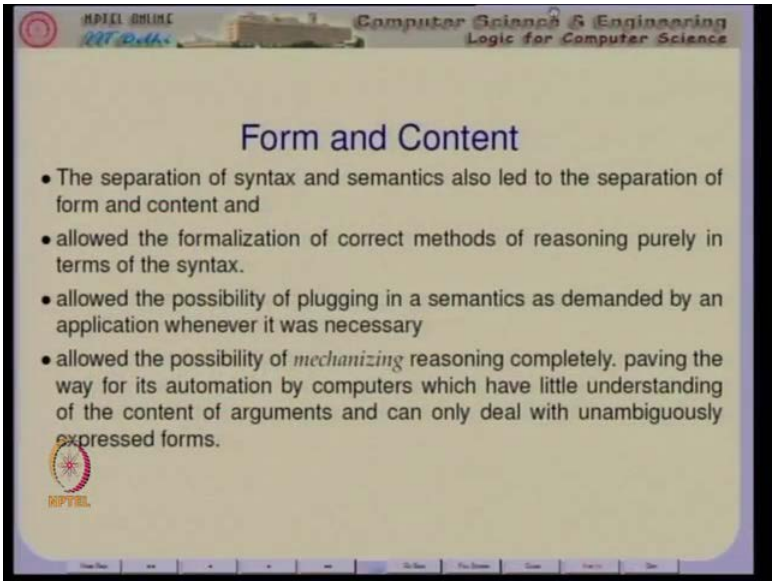

Of course, then therefore we separate form and content as two different thing. The separation of all form content naturally from our stand point, opens up the possibility of mechanizing. Mechanizing, automating and therefore, programming them by machines right and that is so that mechanized reasoning is what is one of the things. And fair amount of work has gone into the mechanization of the reasoning, some important things and this is especially useful in those cases, where there are large number of this is to be analyzed, which however much humans might want to do it, it is not possible.

The other case is that the number of different steps you have to go through in order to reasoning might be so low level and tedious that it best left to machines rather than the humans. So, it can it is possible this is what happened in the case of proof of ((Refer Time: 42:25)) graph theory, the huge number of cases were reduced to a few thousand cases and those thousand cases were checked out by machines and that's as good proof as any.

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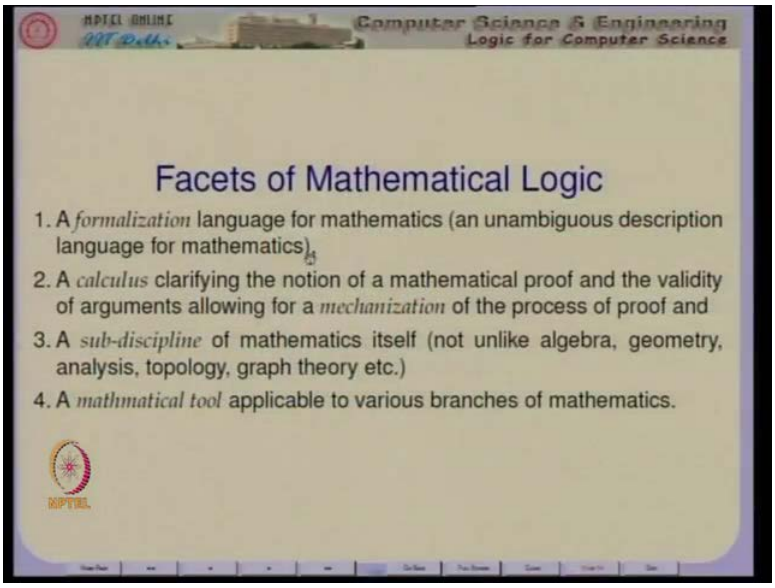

So, essentially mathematical logic has these four facets, the formulization language for mathematics. And basically it is an anonymous description language for mathematics if you write, it is also sub discipline for mathematics and the machinability bring through the motion of calculus. What is a calculus? ((Refer Time: 43:12))What is the calculus, let's go down to primary school. So, I do not no how the Romans did the long, but you take the place value system, and take your keen brother or something, who learnt and long division and you can give long division.

Essentially without understanding finding square roots by long division which, you take ((Refer Time: 44:15)) I am sure most of you do not know something that requires the proof but it can be done right? So, there is something mechanical without looking for me this another from itself and a few you mug up the multiplication tables, but they are essential which again tell you what is to find right? Essentially you do some calculation in the mechanical passion; it does not required any intelligence. So, there are some few finite well defined rules which have to be applied and apply them and that is what we call calculus, and calculus most of huge number of look at the pattern use to apply this rule apply decoration that parts or define when those 1 by x right that is that question.

Essential has a set of finite rule a finite set of rules of in such a way that each of those rules is finitely in nature, that is why in the place value system you can do it in a finite set of rules in the roman numerals you not have a finite set of rules. So, you have a finite

set of actually those of you have a finite set of decidable rules for doing pattern matching ((Refer Time: 46:45)). So, the rule by itself doing that and if you if you have a system of manipulation, it just uses a finite set of such decidable rules were each rule is also a finite in nature that is this also have a finite set of symbols and everything is done. Then what you have is a patterns.

So, you can see that such rules can be applicable in machines without any intelligence, and that is what this logical reasoning is more about form, then about content it is possible to erase it, through various kinds of fraction. There is one small difference of course, when applying when you are integration, differentiation, sometimes you need to have a strategy of whether to apply this group or that group. That requires some intelligence may be, but still based on that what should be applicable is not told to you that requires really some intelligence, some sharpness some whatever.

But the rest of it essentially and that is over so that once you have completely symbolized logic and made a calculus of it, it is come it is hopefully will be mechanize able if not completely mechanize able, for those crucial thing. Should I apply integration that part show should I apply a separation of variable or should I use parameterized form, may be that through some user interaction, the rest of the can be done mechanically. So, the modern interactive theorem prove is are essentially that kind.

So, strategy of proof often come from the user, the uses specifies what rules to be applied, what rule is to be applied now. But then the crunch work, backed up simplifying to so on which machines better than the human beings is done is done by the system. So, interactive theorem proves, essentially allow human interaction, whenever the theorem prove gates stuck. Very often the human actually provides the proofs strategy of proving a theorem. And of course, you can apply this to various branches of mathematics.

Student: ((Refer Time: 43:39))

Ya ya sure, I am stopping now, are there any questions?