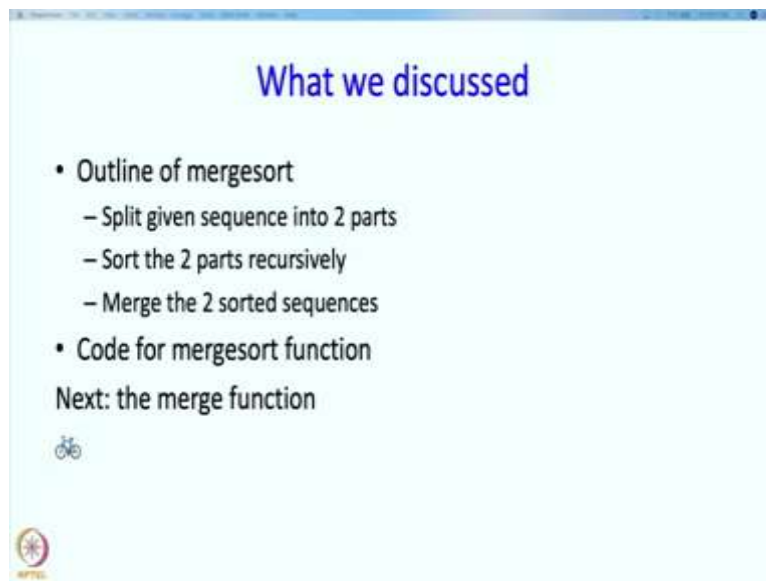


An Introduction to Programming through C++
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Lecture No. 18 Part- 4
Arrays and recursion
Merge function

Welcome back, in the previous segment we discussed the outline of merge sort and we also discussed the code for it.

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What we discussed

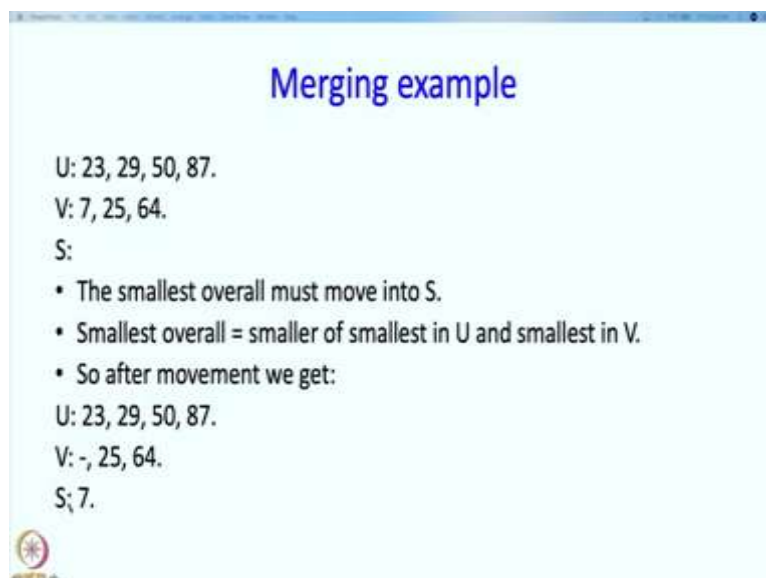
- Outline of mergesort
 - Split given sequence into 2 parts
 - Sort the 2 parts recursively
 - Merge the 2 sorted sequences
- Code for mergesort function

Next: the merge function

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In this segment we are going to discuss the merge function. So let us do an example first.

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Merging example

U: 23, 29, 50, 87.
V: 7, 25, 64.
S:

- The smallest overall must move into S.
- Smallest overall = smaller of smallest in U and smallest in V.
- So after movement we get:

U: 23, 29, 50, 87.
V: -, 25, 64.
S: 7.

So let us take the U sequence as 23, 29, 50, 87, so this is the sorted the result of sorting. That U that we had got by splitting our original sequence and V is the other part again sorted. So how do we merge this? So we have to produce the sequence S. And if you think about it what do we want in S? Well the first element should be the overall smallest. Now how do we get the overall smallest? We need to look at both these sequences in great detail. Well no. Because they are sorted it is the smaller of the smallest which is present in the first position and the smallest over here which is also present in the first position.

So if I want the smallest in these two sequences I just need to look at these two positions. So if I look at these two positions then I know that the smaller is 7, so that 7 must come down over here. So what we get after this is this picture, so we started over here we looked at the two elements which are (respects) respectively smaller in U and V. And we do not have to look at the rest rest of the elements to decide which is the smallest in this entire entire set. So we picked this 7 which is the smallest over here and we move it to S.

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What do we do next?

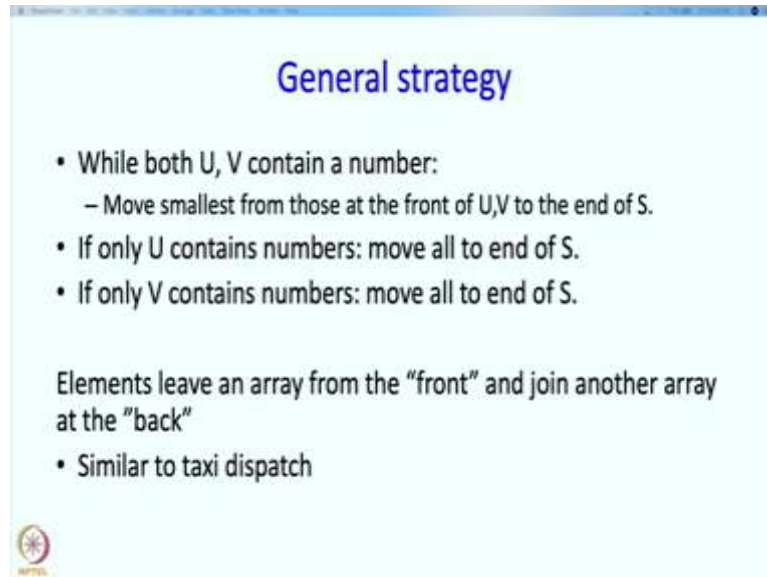
U: 23, 29, 50, 87.
V: -, 25, 64.
S: 7.

- Now we need to move the second smallest into S.
- Second smallest:
 - smallest in U,V after smallest has moved out.
 - smaller of what is at the "front" of U, V.
- So we get:
U: -, 29, 50, 87.
V: -, 25, 64.
S: 7, 23.

And this we are going to continue, so what do we do next? So this is our situation, we again want the smallest of these two. So we really want the second smallest, in what was originally U and V. But now that S has moved out we can say look what is the current situation of U and V and we want the smallest amongst those. So how do we get that? So the second smallest is the smallest in U, V after the smallest has moved out. So which is exactly this position over here? And to get that we simply have to ask what is the smaller among what is at the front of U and V? So the front of U is this so here there is 23 the front of V is now this because this element has moved out. So we have to now pick the smaller of these two

elements and move that at the end of S. So if we do that we get this, so the smaller has moved to the end of S or the back of S and the fronts, the elements which were originally at difference have gone away and the fronts have sort of advanced.

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General strategy

- While both U, V contain a number:
 - Move smallest from those at the front of U,V to the end of S.
- If only U contains numbers: move all to end of S.
- If only V contains numbers: move all to end of S.

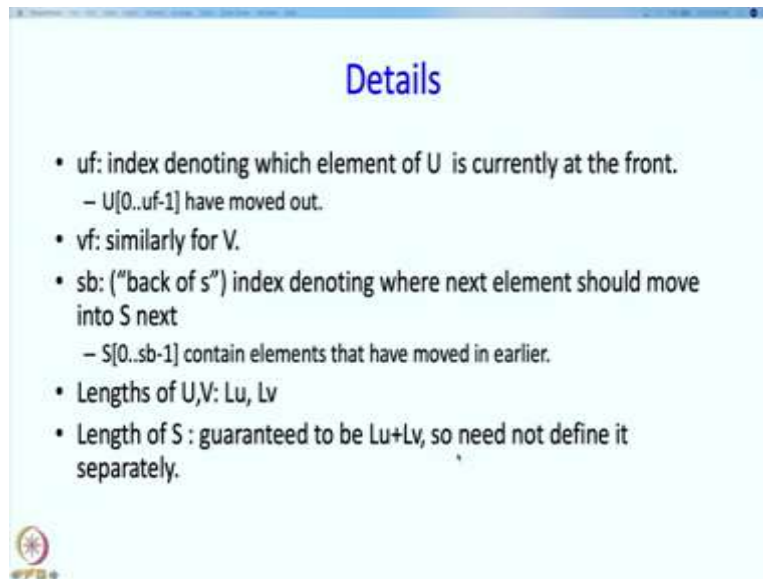
Elements leave an array from the "front" and join another array at the "back"

- Similar to taxi dispatch

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So what is the general strategy? So while both U and V contain a number, move the smallest from those at the front of U, V to the end of S. If U contains only U contains numbers, then that means everything in V has already been moved to S and so therefore move everything in U to the end of S similarly for V. So basically elements are leaving the arrays U, V from the front and joining the array S at the back. Now you have encountered this, this is really what was happening in this taxi dispatch problem. So the code is also going to be somewhat similar.

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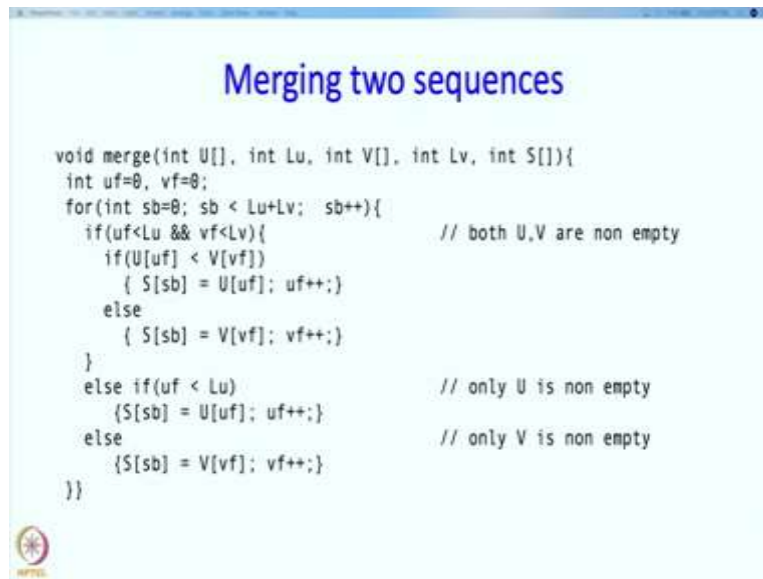
Details

- uf : index denoting which element of U is currently at the front.
 - $U[0..uf-1]$ have moved out.
- vf : similarly for V .
- sb : (“back of s ”) index denoting where next element should move into S next
 - $S[0..sb-1]$ contain elements that have moved in earlier.
- Lengths of U, V : L_u, L_v
- Length of S : guaranteed to be $L_u + L_v$, so need not define it separately.

So what are the details? So we are going to maintain a variable uf which is going to denote the the element of U , the index at which that the front is currently. Similarly, so what that means is that element 0 through $uf-1$ have already moved out and therefore the front has now advanced to index uf . Similarly, there is vf and then we also want to have an index sb to the back of s . So this is the index denoting where the next element should move into s .

So again 0 to $sb-1$ contain elements that have already moved in earlier. So this is this is really pretty similar to the taxi dispatch as you might be noting. So we also need to keep track of the lengths we need to know the lengths of U and V and so let us say we use the variables L_u and L_v to denote the lengths. We should we could have a variable to denote the length of S but we know that S is going to be exactly equal to the lengths of L_u and L_v because we created U and V in that manner. So we need not define a variable for it separately.

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```
void merge(int U[], int Lu, int V[], int Lv, int S[]){
    int uf=0, vf=0;
    for(int sb=0; sb < Lu+Lv; sb++){
        if(uf<Lu && vf<Lv){ // both U,V are non empty
            if(U[uf] < V[vf])
                { S[sb] = U[uf]; uf++;}
            else
                { S[sb] = V[vf]; vf++;}
        }
        else if(uf < Lu) // only U is non empty
            {S[sb] = U[uf]; uf++;}
        else // only V is non empty
            {S[sb] = V[vf]; vf++;}
    }
}
```

So that is basically it, we can get started on our function for doing the merging. So again let me explain U is the first sequence that we want to merge its length is Lu, V is the second its length is Lv. And S is the array into which the result is supposed to go. And uf and vf are the positions of the fronts. So initially the front is at the 0th index here and the front of V is also at the 0th index.

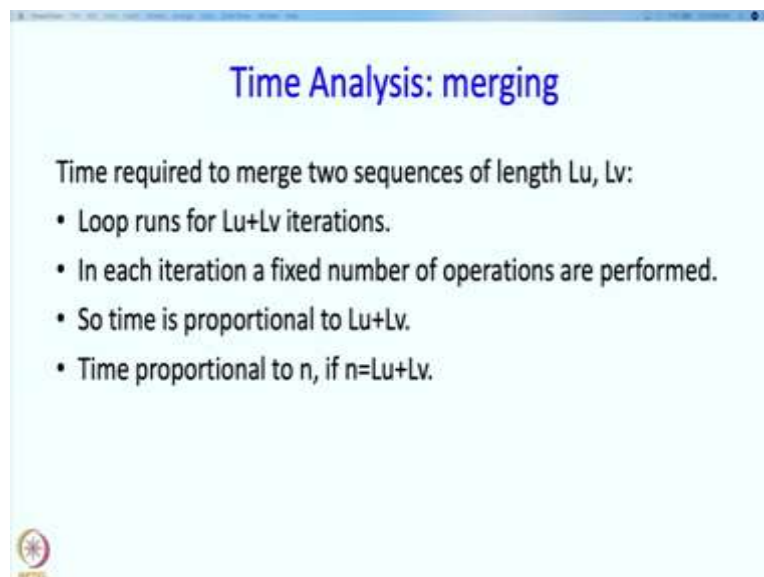
We need an element for the back but that will be a part of our main loop. So we are going to move elements at the back and so the back is going to keep on advancing, so it is going to be the 0. It will become it will keep on increasing and when we have moved Lu plus Lv elements then we are going to stop. So that is how the whole overall structure is going to be. So how do we do this movement? Well if both U and V are non-empty and when will they be non-empty? Well if uf is smaller than Lu so the front is still pointing to a valid element in U. And if the front is pointing to a valid element in V, then that means both U and V are non-empty.

In this case what should we do? We should check which one is smaller. If the U side has the smaller element than the V side what should we do? Well we should move the U side element to the back of S. And we should advance the pointer the front of uf. You should advance the back of uf as well but that will get advanced at the end of the loop anyway because of the statement over here. So we are not going to do that explicitly here. If, on the other hand, this V had the smaller element, then we should do the same thing but with V rather than with U.

So the front element of V is going to move to the back of S and the front for V is going to advance. So if one of those U and V is empty so let us say U is not empty. So if is smaller than U, so that means V is now empty. So in that case, we do not have to do any comparisons we just move whatever is at front of U to the back and then we just advance the front for U. Otherwise, it means that only V is non-empty, so then we are going to do the same thing whatever is at front of V is going to be moved to the back of S. And we are going to advance the front of V, so that is it I should have yeah so this brace closes this brace and this brace is closing this brace over here.

I do not want to put it later on because I just want to keep it otherwise the font becomes too small. But anyway this is this is the code for merging these two sequences. Now you can check okay so yeah so we already discussed what exactly happens. And so now we are we can analyze the time it takes okay. So how much time does this take? Well it has this main loop so this is the main loop and how many iterations is the main loop going to run? It is going to run L_u plus L_v iterations and therefore its time is going to be proportional to L_u plus L_v since in each iteration we are going to do some fixed amount of work, the work may be different, depending upon whether both queues are non empty or only one is non-empty. But it is going to be some fixed amount of work.

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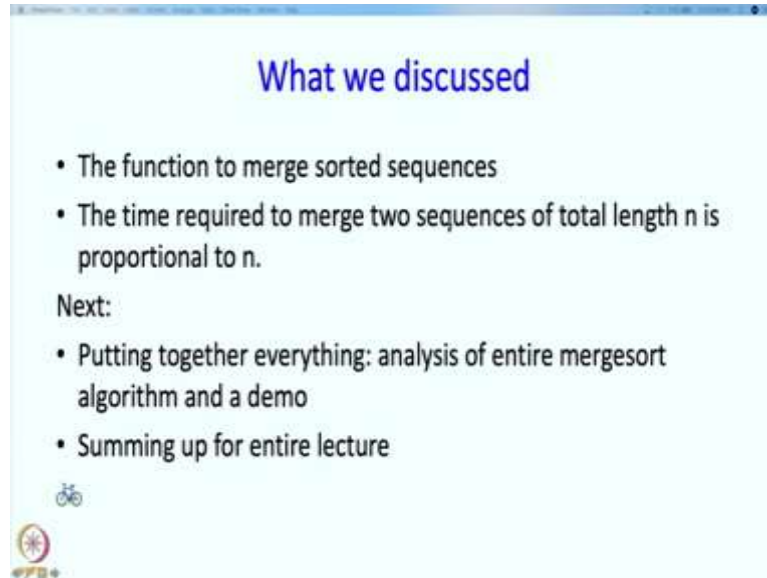


The slide is titled "Time Analysis: merging" in blue text. Below the title, it states "Time required to merge two sequences of length L_u , L_v :" followed by four bullet points: "Loop runs for L_u+L_v iterations.", "In each iteration a fixed number of operations are performed.", "So time is proportional to L_u+L_v .", and "Time proportional to n , if $n=L_u+L_v$." There is a small logo in the bottom left corner of the slide.

So if we are merging two sequences of length L_u , L_v the loop runs for L_u+L_v iterations. We do a fixed amount of work in each iteration and so the time is proportional to L_u+L_v . So I could say that the time I can say that the time is proportional to n if L_u+L_v is equal to n or if

the final sequence has length n then the time is proportional to $L_u + L_v$. So now we have decided what the time taken for merging is.

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So what have we discussed in this segment? We discussed how to merge sorted sequences and we have discussed that the time taken to merge two sequences of total length is proportional to n . In the next segment we are going to put together everything and we are going to do the analysis of the entire merge sort. And we will also have a demo of the merge sort and then we will also conclude for this entire lecture sequence, so we will take a quick break.