Introduction to Programming through C++
Professor Abhiram G. Ranade
Department of Computer Science and Engineering
Indian Institute of Technology, Bombay
Lecture 12 – Virahanka Numbers

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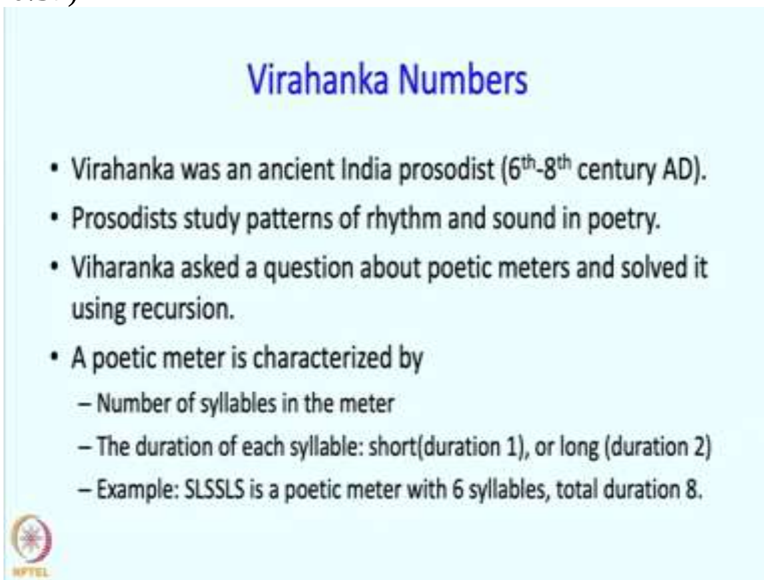

Hello and welcome to the NPTEL course on An Introduction to Programming through C++. I am Abhiram Ranade. The lecture of today is on Virahanka Numbers. Virahanka was an ancient Indian prosodist from 6^{th} or 8^{th} century and prosodists are people who study patterns of rhythm and sound in poetry. Virahanka asked a question about poetic meters and interestingly enough, solved it using recursion. So that is what we are going to study today. So let me tell you a little bit about today what a poetic meter is.

A poetic meter is characterized by the number of the number of syllables in the meter and the duration of each syllable. So syllables have either a short duration or a long duration and a long duration is just twice the short duration. So for example a poetic meter could be described by the sequence of characters SLSSLS. So that means that there are 6 syllables and the total duration is 8, the additional two counts come from the two long durations.

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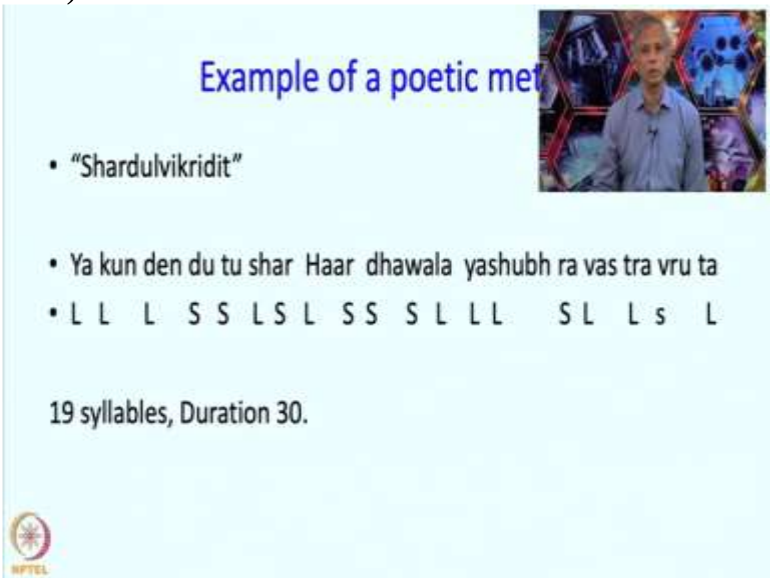

So let me give you an example for a poetic meter and actual poetic meter, and here is a poetic meter called Shardulvikridit. So this is used in many many poems and many many shlokas and perhaps here is a familiar shloka that, that is in this meter. So I am going to I am going to sing this shloka and you will note that some of the syllables are going to be short and some of the syllables are going to be long and it is the long and short, the long and shortness of the syllables which gives it, gives this shloka a certain character and that is what a poetic meter does.

So Ya kun den du tu shar haar dhawala yashubh ra vas tra vru ta. So as you can see the first Ya is a long syllables, kun is a long syllable, den is a long syllable but then you sing Ya kun den du tu, so du tu are short syllables. So over all there are 19 syllables and the total duration is 30 if you count if you count it. So prosodists were concerned with such poetic meters and they ask questions such as how many different poetic meters can there be and they sometimes even wanted to enumerate the poetic meters.

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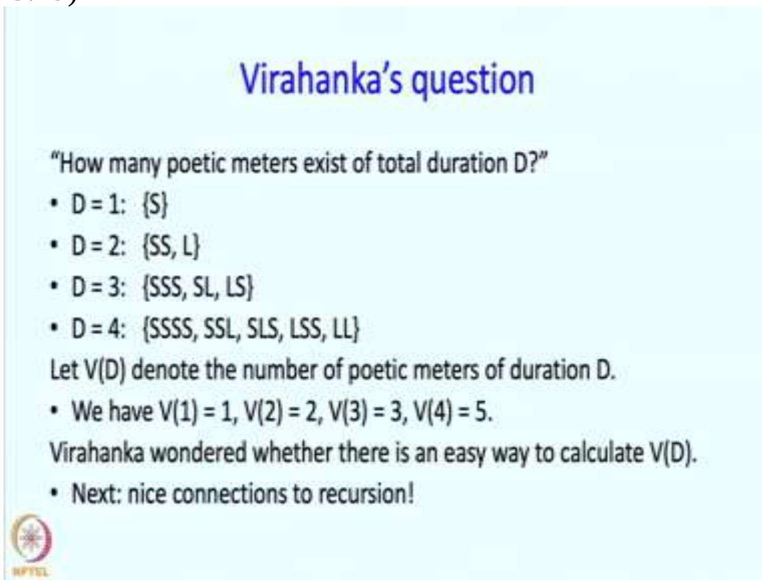

So Virahanka's question was how many poetic meters exist of total duration D? So let us try this out, let us take a few simple cases. So if D is equal to 1, so you want a poetic meter of duration 1, well you do not have a choice. The only poetic meter possible of duration 1 is sort of a very trivial thing, a poetic meter which has just 1 short syllable.

If you go to duration 2, then the duration 2 can be made either by one long syllable or it can be made by two short syllables. So at this point you have two possible meters. 3 if you work this out if you try this out and if you try to do this exhaustively, what do you get? Well, you get that duration 3 can be made up by three short syllables or a short and long or a long and a short. So there are three poetic meters of total duration D. Then for D equal to 4 you may again work this out, so there might be all four shorts or you might have SSL, SLS, LSS, or just two longs.

Now, you want to ask this question in general and so let us that V(D) denotes the number of poetic meters of duration D and what we have worked out is that V of 1 is 1, V of 2 is 2, V of 3 is 3, and V of 4 is 5. Virahanka wondered whether there is an easy way to calculate V(D). So that was his question and we are interested in this not because this is a course on poetry or prosody or anything like that, but because this question has some interesting things to say about recursion.

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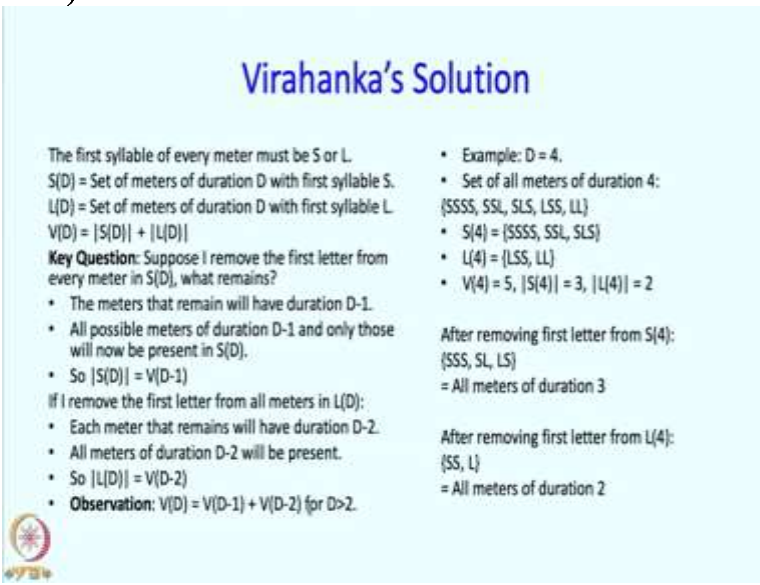

Okay, so let me tell you Virahanka's solution. So Virahanka observed that the first syllable of every meter must be S or L. Of course, I mean there is nothing much over here but that is that is just the first step. So then he said that look let me let me use S(D) to denote the set of meters of duration D in which the first syllable is in S.

So let us take an example to make sure that this notation is clear. So if you take D equals to 4 then the set of all meters of duration 4 which we just worked it out is this and of these the first three have the first syllable S. So S(4) is just those first three meters, then L(D) we are going to use to denote the set of meters of duration D with first syllable L. So clearly L(4) are the remaining, so these three belong to S(4). These two are the remaining ones from the all the set of all meters of duration 4 and they clearly belong in L(4), okay.

And clearly V(D) the total number of meters of duration D must be equal to the short meters plus long meters and since S(D) and L(D) are sets, we are just taking the sizes of those sets. So checking it for D equal to 4 what we have is V(4) is 5 which we counted over here and three of these comes from the ones with the short syllable first and two of these come from the set which has the long syllable first, so just I am just using D equal to 4 just to make sure that our notation is being well understood.

Now, here is the key question to ask which Virahanka did. So, suppose I remove the first letter from every meter in S(D). What happens? Okay, what is what remains what remains in S(D)? Okay, well if you remove every the S from anything, the duration will decrease by 1. So you started off with meters of length D, so if you remove the first meter which is going to be S in the set S(D) then you will be left with meters that have duration D minus 1. Okay, and interestingly enough, not only will we be left with meters which have duration D minus 1 but in fact you are guaranteed to have all meters of duration D minus 1 after you remove the first syllable from the meters in S(D).

So again let us go to a running example. So we have S(4) which is this and let us remove the first syllable from these. So what do we get? Well, from this four SSSS we get three SSS and SSL gives us SL and SLS gives us LS. So indeed if we look at the durations of these meters they are 3 okay and if you look back into what we did earlier, these are exactly all the meters of duration 3,.

Now, we should probably prove this. So here is very simple proof. Okay, so I am going to I am going to do it with respect to this set. But you can see that it is really a general argument. So suppose this was not, so this was not the entire set of meters with duration 3 and may be there was some additional member that we did not have in this set, so suppose I take that additional member which has to be different from this and I add an S in front of it, what do I get? So I will get a meter which starts with S and has total duration 4. But then that would have to be present over here. So either, so but that clearly is not the case, so that means this is a set which contains all meters of duration 3 and since the the number of all meters of duration 3 is V of 3, in general I can say the size of the set S(D) must be V(D) minus 1 because after I remove one letter from all the meters the number of meters in this set does not change, so that continues to be cardinality of S(D). And now we have established that that is the same as the number of meters of size D minus 1 and therefore that must be equal to V(D-1). So we can do the same thing removing the first letter from all the meters in L(D), so what do we get?

Well, if you remove the first letter from L(D), L has duration 2, so now everything that remains will have duration D minus 2. So in fact not only will the meters that remain have duration D minus 2 but in fact by the same argument all meters of duration D minus 2 will be present, okay. So again let us check this. If I look at this L4, if I remove the first L, what do I get? I get this and indeed these are all meters of duration 2, there are only two meters of duration 2 and these are here, okay.

So again the number, the the size of L(D) must be equal to V(D) minus 2. So combining this, this and this, what do we get? Well, V(D) is cardinality of S(D) but that is V D minus 1 plus L(D), that is V D minus 2, so this is what we get and I should observe that this is valid only for D greater than 2. Because if D is not greater than 2 then V(D) minus 2 would not be bigger than 1 and our formulae, all our discussion is for having at least one at least one syllable in the meters that we are talking about. So our discussion is really about so in our discussion whatever sets we are talking about should be should have say V(D), so this has to be bigger than bigger than zero. So D had better be bigger than 2, okay.

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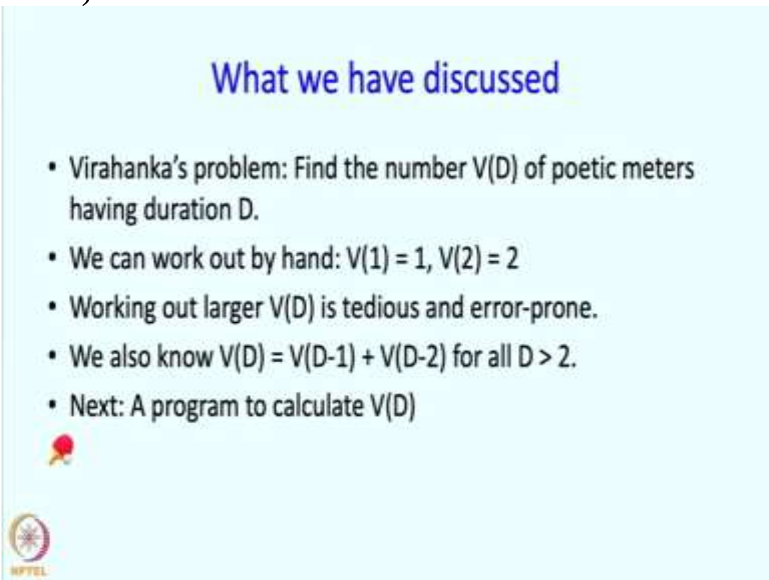

What have we discussed at this point? Well, so we have introduced Virahanka's problem and that was find the number V(D) of poetic meters having duration D and we can work out by hand that V of 1 and V of 2 equals 2 and we said that working out larger V(D) by hand is tedious and error prone and so we derived the relationship which is that V(D)=V(D-1)+V(D-2) for all D greater than 2. Next we are going to write a program to calculate V(D) but let us take a short break.