An Introduction to Programming through C++
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Lecture 26 Part 3 - A graphical editor and solver for circuits - Mathematical representation of the circuit

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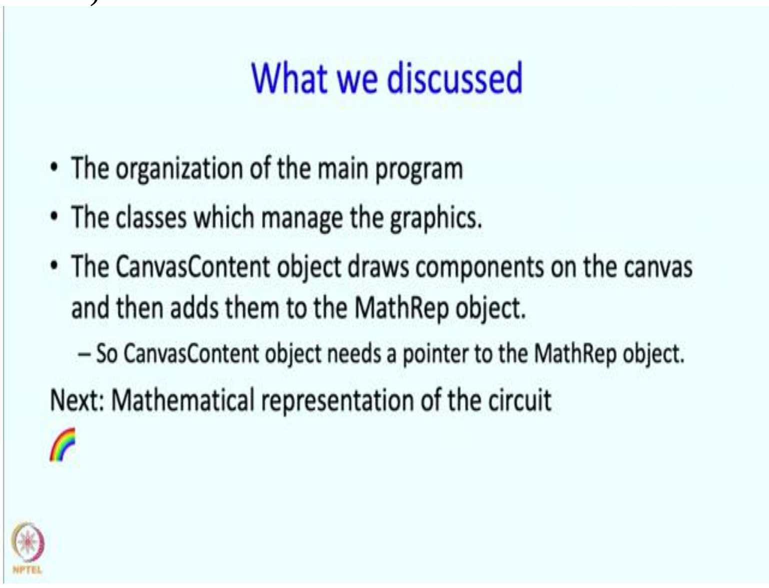

Welcome back. In the previous segment, we discussed the organization of the main program and the classes which are related to the canvas and the graphics. Now, we are going to talk about how the circuit is represented inside the computer.

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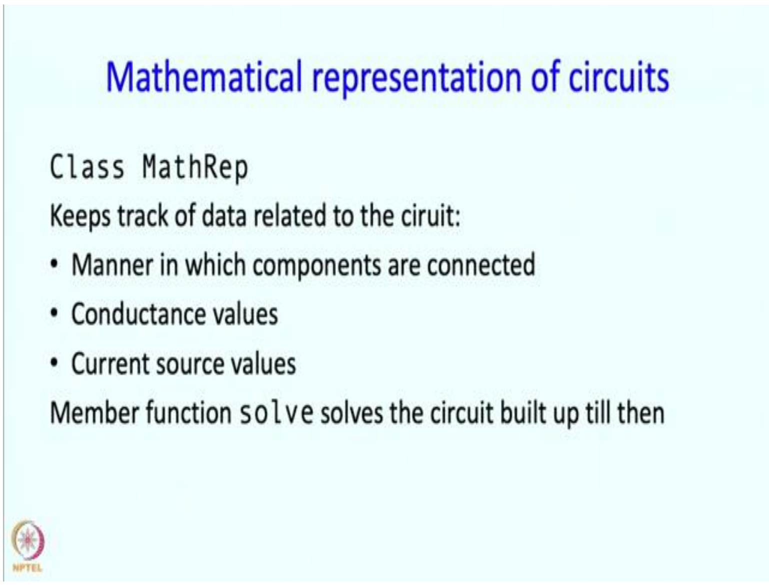

So, this is done in a class called MathRep. So, it keeps track of the data related to the circuit. So, the data consists of: the manner in which the components are connected, the conductance values and the current source values or what is sometimes referred to as current source strengths. Then the main member function in this is the function Solve which solves the circuit built up until then.

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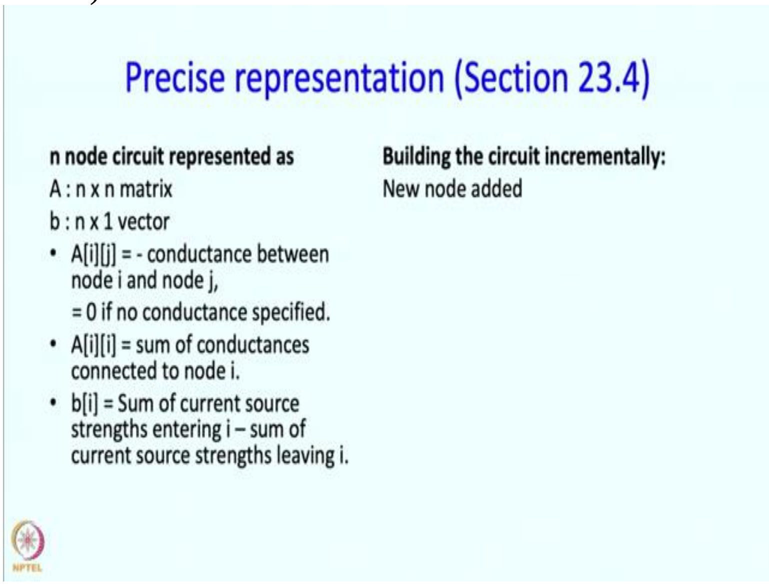

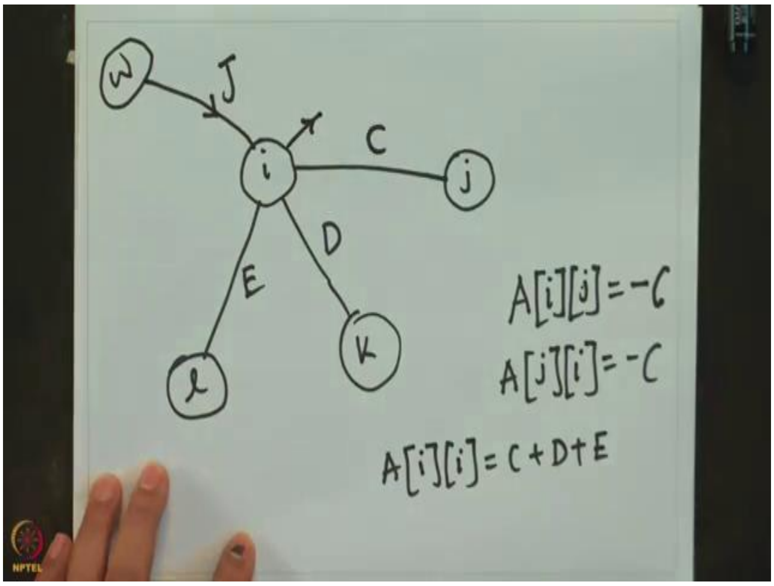

So, let me talk about the representation bit more precisely and this is discussed in section 23.4 of the book. So, suppose we have an n node circuit, then it will be represented using an n by n matrix and we are going to call this matrix A. In addition to that, we will also have a n by 1 vector b. So, these will keep evolving as our circuit gets built and then when we want the circuit to be solved, we will perform mathematical operations on this matrix and vector and we will see all of that soon. So, this matrix, A is going to be used to represent the connections and the circuit values in the following manner:

So, whatever conductance there is between node i and node j, we are going to make an entry of that value in the matrix, in the matrix element A[i][j] so, just to make sure that this is understood. So, let us say this is node i and let us say this is node j in our circuit and suppose, connecting these two we have we put some conductance C so, then we want simultaneously in the representation to have A[i][j] = -C. So, this will keep track of what the conductance is and of course it is symmetrical so, we will also have A[j][i] = -C and it is 0 if no conductance is specified.

So if there is no edge then this vector would be 0 because no edge is equivalent to 0 conductance. Conductance is indicative of how much current can flow and if there is no edge, no current can flow and therefore it is natural to say that the conductance is 0. A[i][i] is going to be the sum of the conductances connected to node i. So let me again give an example. So, this is some i j, this is some k, this is some l and let us say this conductance is C, this is some D, this is some E, then A[i][i] we want to be equal to C+D+E.

So, note that given the circuit, we can figure out what these values are and given these values, we can figure out what the circuit is. So to that extent, this is a representation of this circuit. Of course, we are not completely done yet, we have not said how voltage sources are represented. So, voltage sources are represented in the following manner, so, we not only have this matrix, but we also have that vector B.

If you have a current source which say starts off at some vertex w and enters i and it has some strength J. So then B[i] should be the sum of all such current values which are forced to enter. So we will see this in a minute and of course, if there are some currents leaving, then that is as good as negative entry. So over here, we are subtracting the some of the strengths leaving i, alright so, how does the circuit get built incrementally? Well, what happens when a new node is added? So let me draw a picture over here, a new picture.

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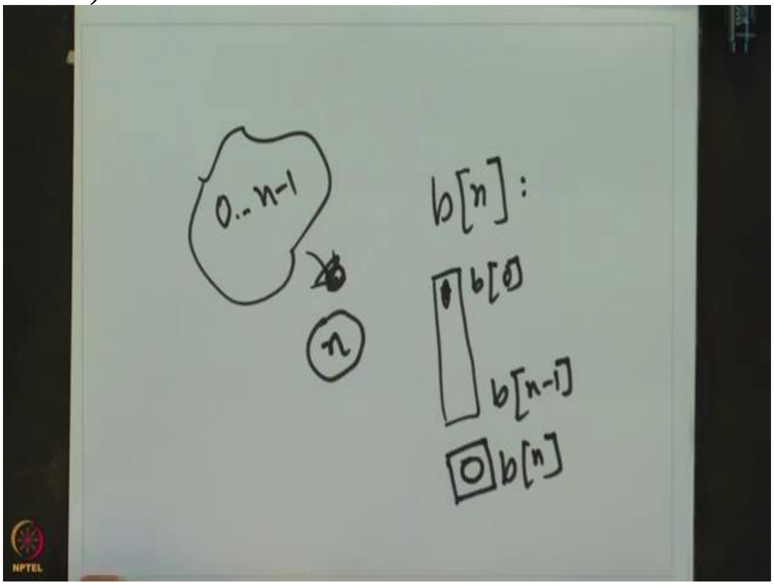

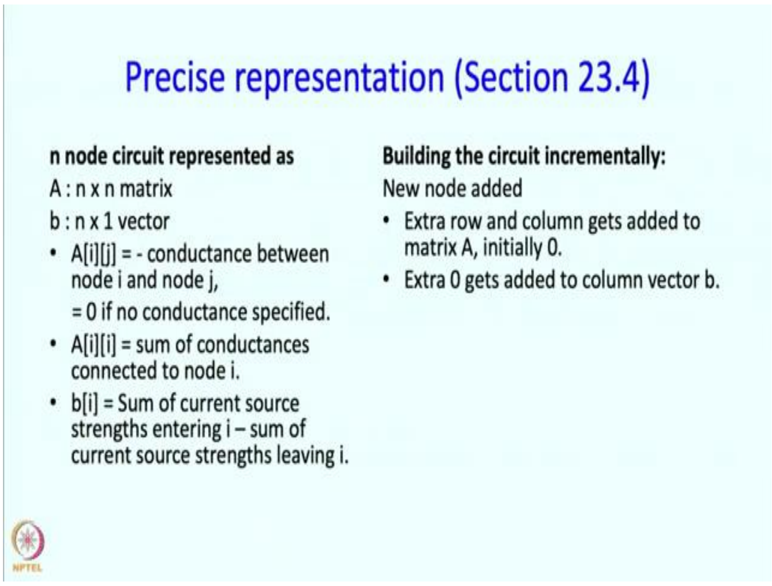

So here is some circuit that we have created and now we create a new node. Well, this new node does not have any connections at all. So what do you think will be present in its row and in its column? So, if we add so, effectively if we add a new node, we are going to have an extra column and an extra row added and since the node is not connected to anything at all, we have to make that initial row and column become 0. So again, let me remind you, how did we add a new node? Well, suppose this circuit already had nodes 0 through n minus 1, then when we created a new node, we created a new node n. So, this is now going to correspond to increasing the number of rows and the number of columns. And so the nth row will correspond to the new node and that has to be made 0. What about b[n]? So, if you remember or I guess you do not have to remember, b[n] is supposed to contain the sum of currents and currents leaving and entering through sources, but right now there are no sources connected.

So, as a result 0 gets added to the column vector b. So, originally our column vector look like this, so say b[0] through b[n-1] and we are going to add a b[n] to it and the value for this we are going to make it to as 0 because there is no current source connected to this.

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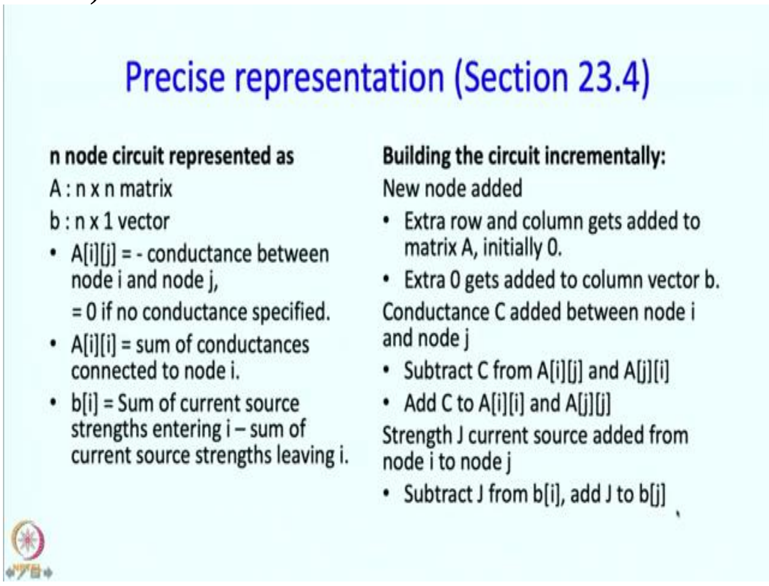

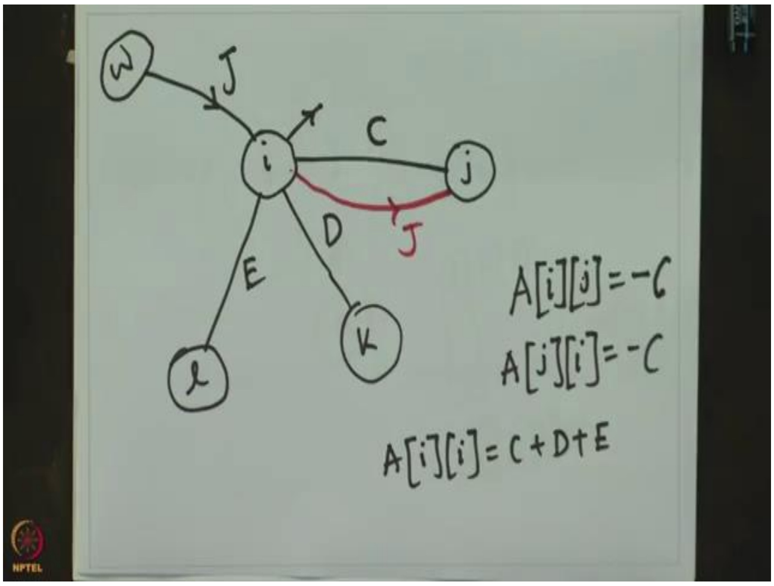

So, now what happens if conductance C is added between nodes? So, let us go back to this picture. So suppose a conductance C is added, then what happens over here? Suppose let us say we are just adding this, but if we add this A[i][j] should be -C. So as a result we are going to subtract C from A[i][j], remember A[i][j] was originally 0 so both of these were originally 0, so we are going to subtract C from them and we have to, the C has to appear over here and so to A[i][i], there might have been other entries but we now add C as well.

So that is how adding a conductance gets reflected in our data structure and if I add going from node i to node j, so let us say this is node i and this is node j. So, I can even draw it over here. So let me do it that way. So suppose I add a current source, connect going from i to j over here so, its strength is J. Well, what should happen? So b[i] should have the current sources entering i, but this is current leaving i. So as a result, I should subtract, subtract j from b[i], so which is exactly what I am doing. And this j is entering node j and therefore I am adding capital J to b[j]. So, this is how exactly that circuit gets built up.

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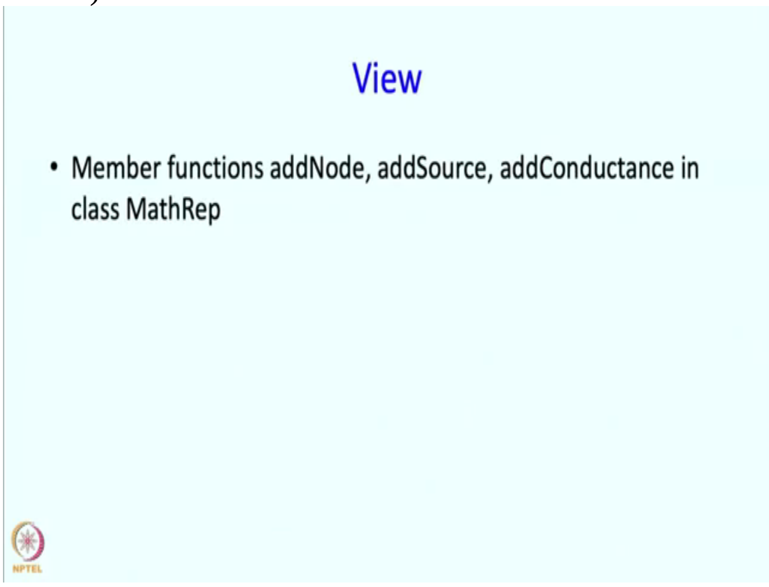

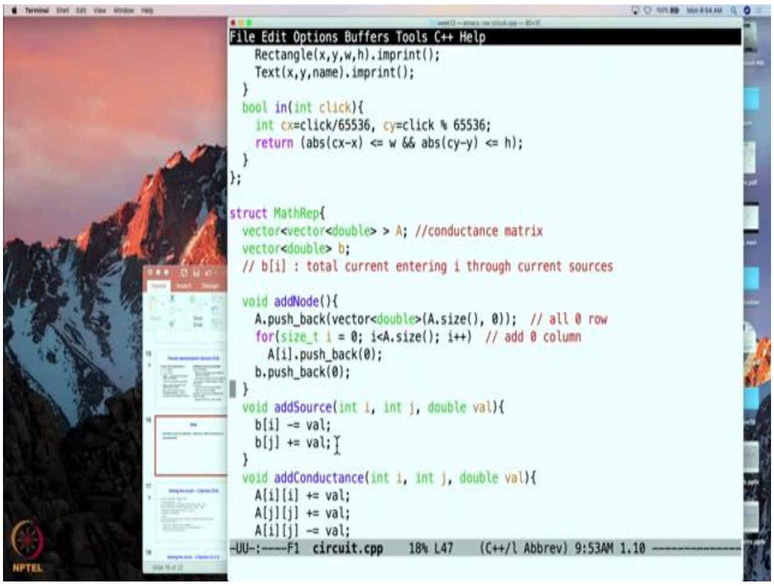

So, let us look at our code and look at the member functions, addNode, addSource, addConductance. So these are operations in MathRep, so addNode first. So we have to push back an entire 0 row, we have to append a new 0 row. So, this is what happens, this is what makes that happen. Notice that the vector class makes it very easy to append and append rows since this is a vector of vectors. So, A is the vector of vectors, see over here and therefore this will just append an entire row and then we have to add 0 column. So now what we do is we are just going to look at every row and we are going to append a 0 to it, that is it.

So this is loop over every row of that vector and we are appending a 0 and finally we want to append a 0 to our b as well and so that is what this does. What about addSource? So if I am adding a source, you remember that if a source of value V is added then going from i to j, then value has to be subtracted and to j the value has to be added. So this is what happens, what we need to do.

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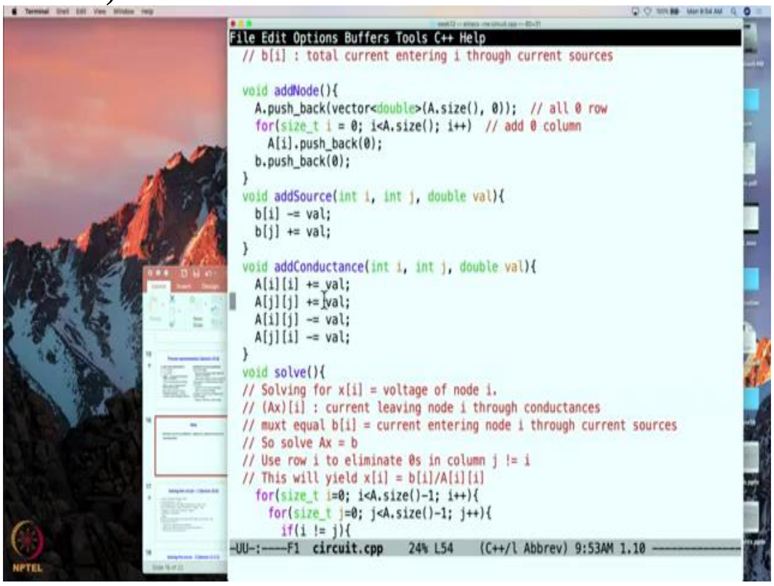

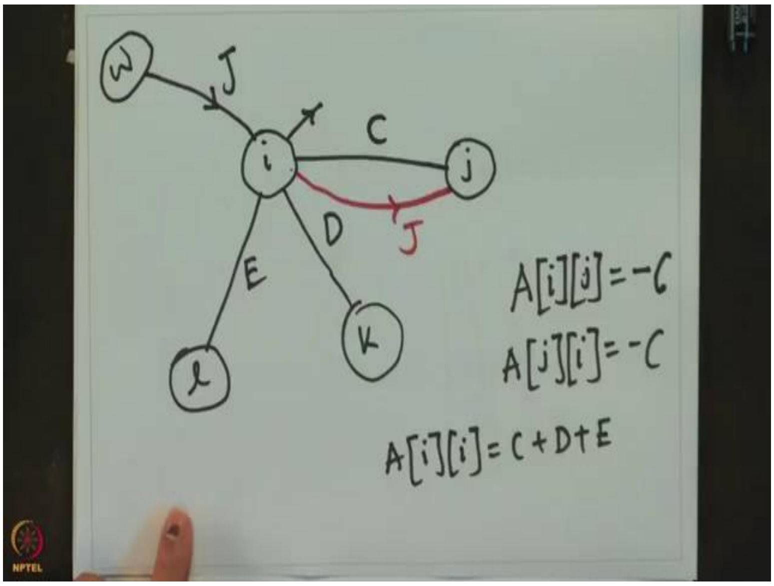

And if I want to add a conductance, then to A[i][i], the value has to be added, see this, and also to A[j][j] because the conductance is connected to and the treatment is symmetrical. And from A[i][j], the value has to be subtracted and also from A[j][i], so that is what addConductance does. So, quite simple. Alright, so let us get back to our presentation.

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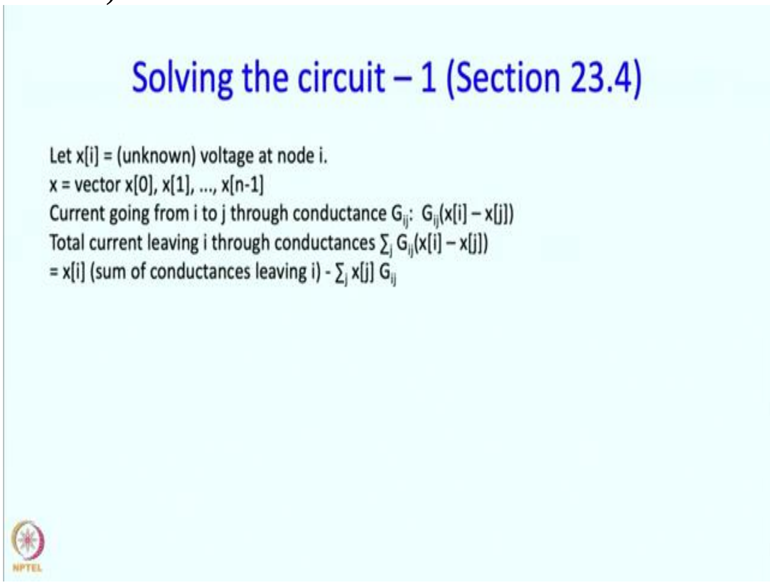

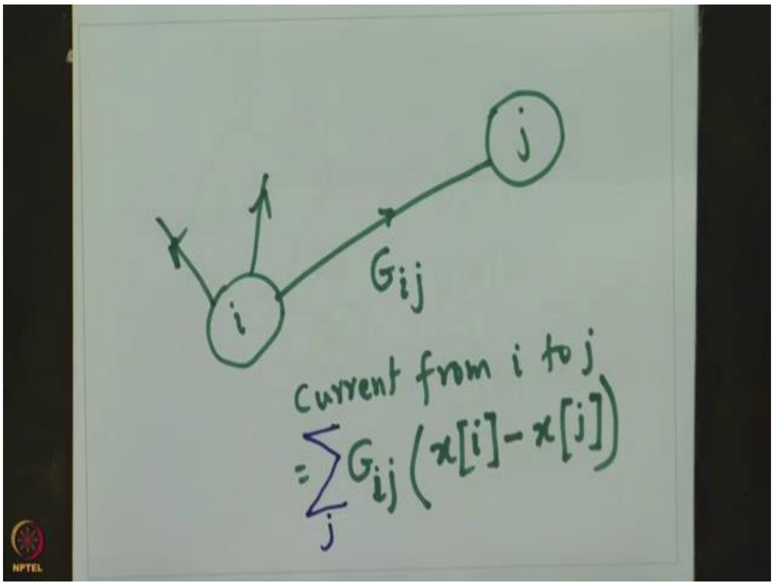

So, now we will talk about how to solve the circuit. So solving the circuit first of all requires us to find the voltages at the nodes. So we do not know those voltages and let us say we represent the voltage at node i by x[i]. So, x is going to be a vector and i is going to be the ith index in it, the value at index i in it. But right now, this is unknown to us. So now, I want to tell you a little bit about how circuits work. So let me start on that, so this is our vertex i and this is our vertex j and let us say that going from here to here, there is conductance G[i][j]. Now, what do we know about the current going from here to here? So it turns out that this current from i to j is equal to the conductance of that times the voltage of i the voltage drop or the voltage at i minus the voltage at j. So, this is the current going from i to j, now we do not know this, we only know well, yeah, so everything here is an unknown. So we do not know the current, we do not know the voltages but what we have done is we have established a relationship between the voltages at these points and the current over here. So, this is just what is called Kirchhoff's Voltage Law as you might have perhaps studied in physics.

So now I can ask, what is the total current leaving i? So there might be other edges. So how do I calculate this? Well, I just take the sum over, let me do it in a different color just to keep track of it. So I am going to take the sum over all the j, all the conductance is connected to i. So, if I just take the sum, that gives me the total current leaving.

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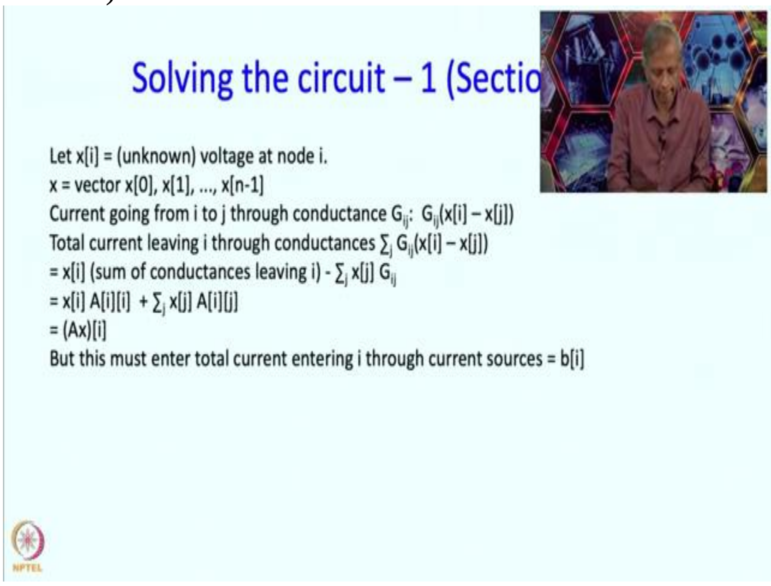

Now, I can factor this out and that factoring is important. So let me write it down a little bit carefully or I guess I can explain it on the screen, so what is happening over here? Well, there is this x[i] term. This x[i] term is common to all the terms in the series. So I am going to pull it out and what is the coefficient of the x[i] term? Well, it is going to be the sum of the conductances G[i][j]. So that is what I have written over here, some of the conductances leaving i or these G[i][j], and then I am left with this remaining term, so G[i][j]-x[i][j], and that I just pulled that minus sign out over here, so that is a total current leaving i through the conductances.

Now, this if you remember, the sum of the conductances leaving i was just A[i][i] and therefore this is x[i] times A[i][i] and G[i][j] the negative effect, is in fact A[i][j] so, this is what the entire value is. And in fact this is nothing but the ith component of the product of the matrix A and x. So, we do not know x yet, but what we have done, is we have established a relationship between the total current leaving i through the conductances and this matrix A and this unknown vector x. So, it is ith component of the product is indeed the current leaving i through the conductances and this must equal, I am sorry I should have said equal over here, this must equal the total current entering i through the conductances.

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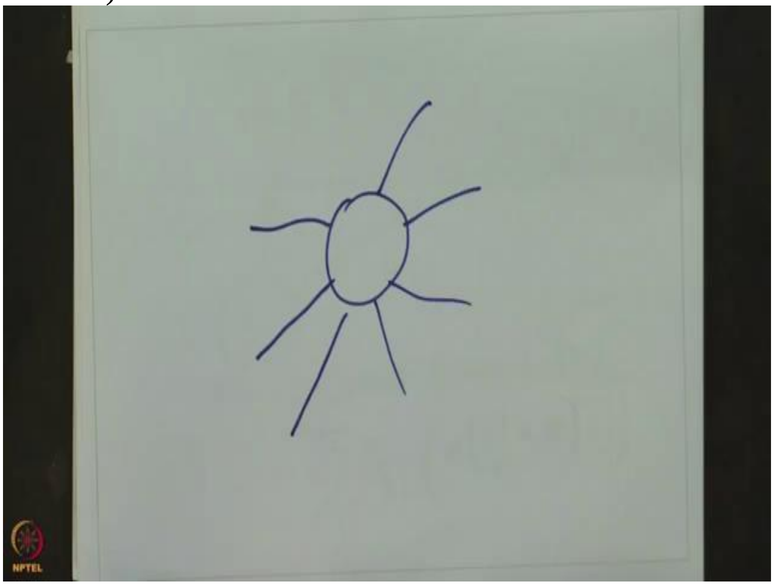

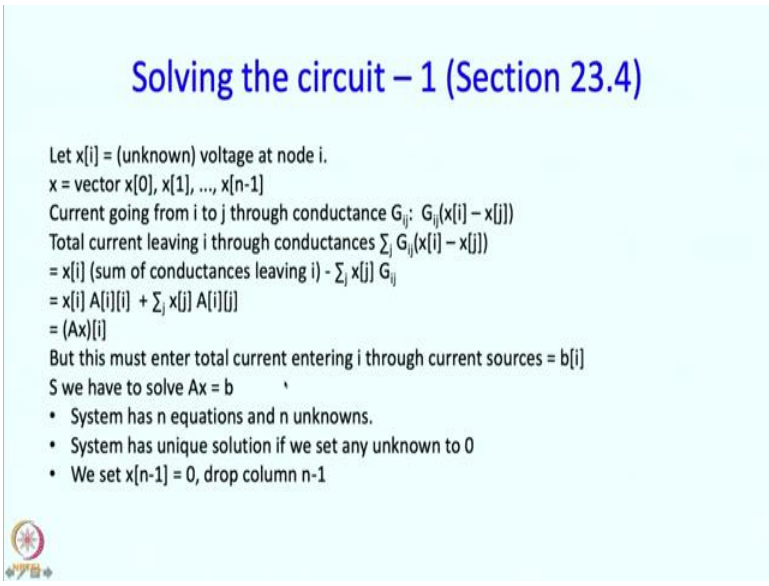

So, here is my, here is my vertex i and I am talking about current leaving through conductances but there is current entering through current sources as well. So, what is the current entering through sorry, through the current sources as well. So, what is the current entering through the current sources? Well, the way we define b it is exactly b[i]. So what does this tell us? This tells us that this must be equal to this, so the ith component of the product Ax must equal the ith component of b, but this is true for all ith components and therefore in fact this entire matrix product must exactly equal b.

So, sorry we should be, so we have to solve Ax=b because we know that Ax must equal b and now x is the only unknown, this vector of unknowns is there, but we can get that if we solve this matrix equation. So, this is a simultaneous equation which we have to solve.

Now, there is a slight hitch over here. This system has n equations and n unknowns, remember n is the number of nodes in our circuit. Unfortunately, these equations are not independent. But we will get a solution if we set any unknown to 0. So after we set an unknown to 0, we will get a solution and the details of that I am not going to talk about right now but they are discussed in this section. So, that is what we are going to do. We are going to set x[n-1] to 0 and this will effectively allow us to drop the last column as well.

So, we will get a system which has n-1 equations and n-1 unknowns and these equations will all be independent that is they will contain distinct information and then we can solve it. By the way, if you are so, many of you probably know electrical circuits a little bit and if we have voltages in our electrical circuit, then voltages are all relative. So, I can fix any voltage to be 0 and that is exactly what we are doing over here. But it relates to equations not being independent and that is discussed in this section. So, please read that.

So, anyway so, I have told you why this product Ax must equal b and I have told you that we are going to set one of the values x[n-1] to 0 and then we just have to solve for the remaining unknowns. So, we have a smaller system, well slightly smaller system which we have to solve.

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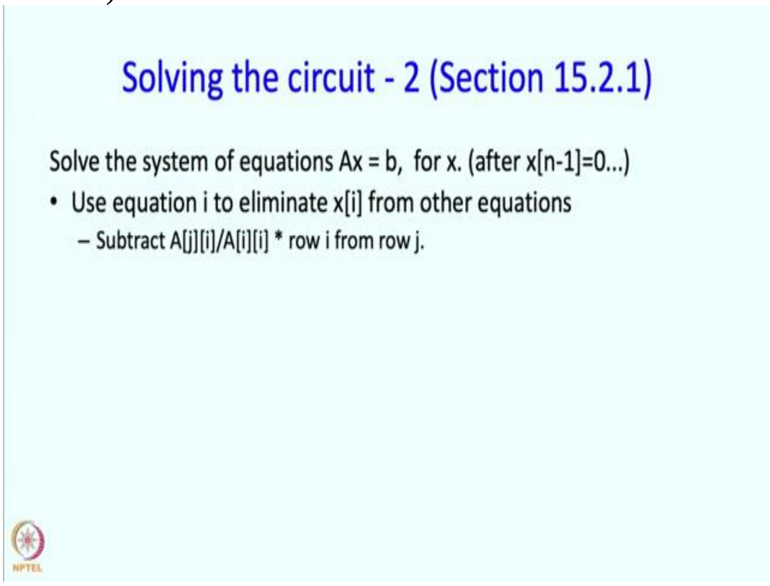

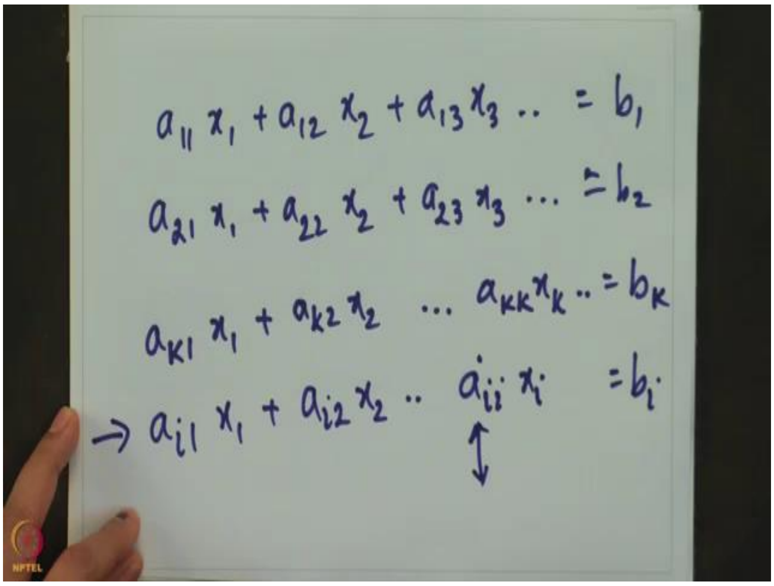

So, now how do we solve this system of equations? So, this is discussed in Section 15.2.1 and I am going to discuss it very briefly. So, we are going to solve Ax=b and we are only going to look at the first n minus 1 unknowns and first n minus 1 equations. So, in fact let us forget that we even had that n minus 1. So, basically the idea is that we are going to use equation i to eliminate x[i] from other equations. So, let me just write that down, show that to you. So our equations are a11x1 + a12x2 + a13x3 and so on equal to b 1, then a21x1 + a22x2 + a23x3 and so on equals b 2 and in general, ak1x1 + ak2 x2 and somewhere here I will get akk xk and so on equals b k.

Now, what this is saying is that I am going to use this equation to eliminate x1 from everyone, from every other equation over here. So how do I do that? So, in general if I have say ai1x1 + ai2x2 and aiixi = b i, then this equation is going to be used to eliminate this ith unknown, so this is going to eliminate the ith unknown. How do I do it? Well, so let me write down what the equations, these equations are.

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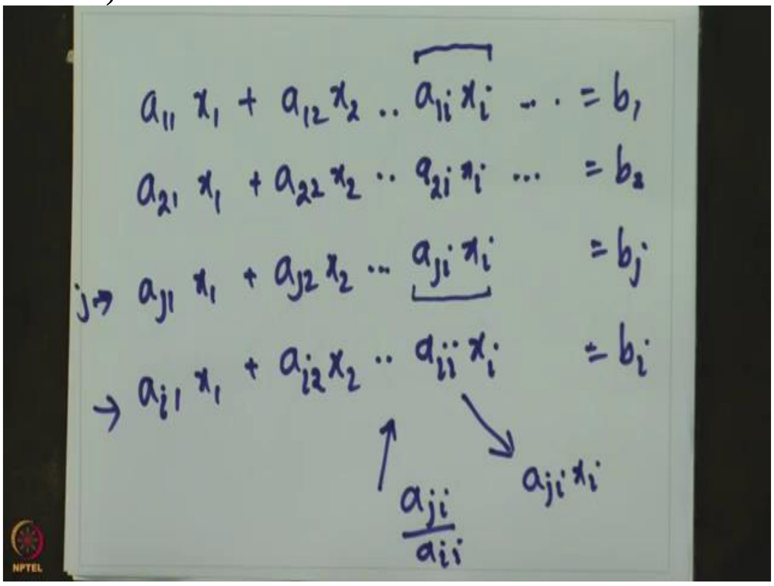

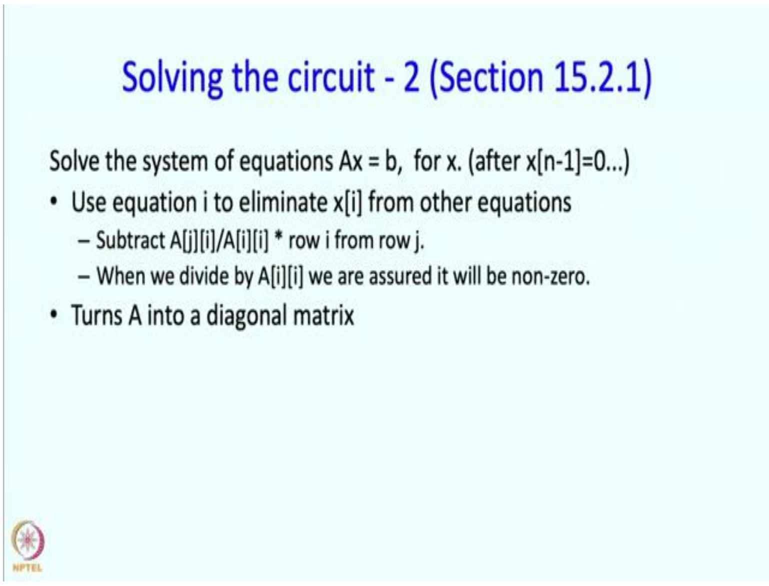

So I am going to have a11x1+a12x2 and so on and say the typical term will be a1ixi and so on equals b1, then a21x1+a22x2 and so on and the term over here will be a2ixi all the way equals b2. And maybe the jth terms is going to be, the jth equation is going to be something like aj1x1+aj2x2 ajixi=bj and let us say the ith equation is a i1x1+ai2x2 and here I am going to get a aiixi=bi. So, what our idea over here is saying is that we are going to use this equation to eliminate xi from all these other equations, so how do we do that? Well, if I want to get rid of this term over here in this jth equation, what do I need to do? I am going to multiply, I am going to multiply this by A[j][i]/A[i][i], so this entire row is going to be multiplied by A[j][i]/A[i][i]. So what do I get over here?

So, over here I will get A[j][i]x[i], if I now subtract this resulting row, which has A[j][i]x[i] from the this, I will get a 0 over here, so that is how I have eliminated this. So that is what I am going to do. So I am going to do this for this row, this row, this row and also all the rows underneath it. And now in this, I am going to divide by A[i][i], so in general we need to worry about whether A[i][i] is going to be non-zero. But the way we have defined A[i][i], we will always be guaranteed that it will be non-zero and so we do not have to worry about it.

So another way of saying this is that we do not need to pivot in case you have seen this, heard this term pivot. So what does this do?

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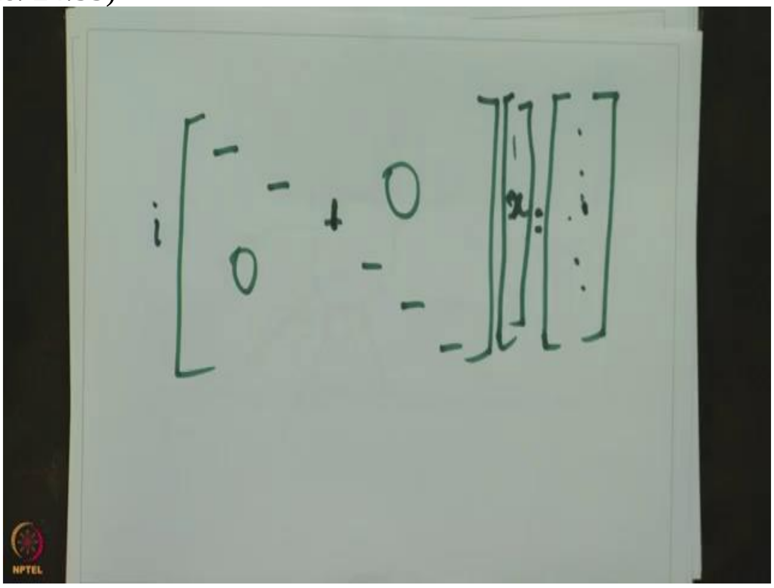

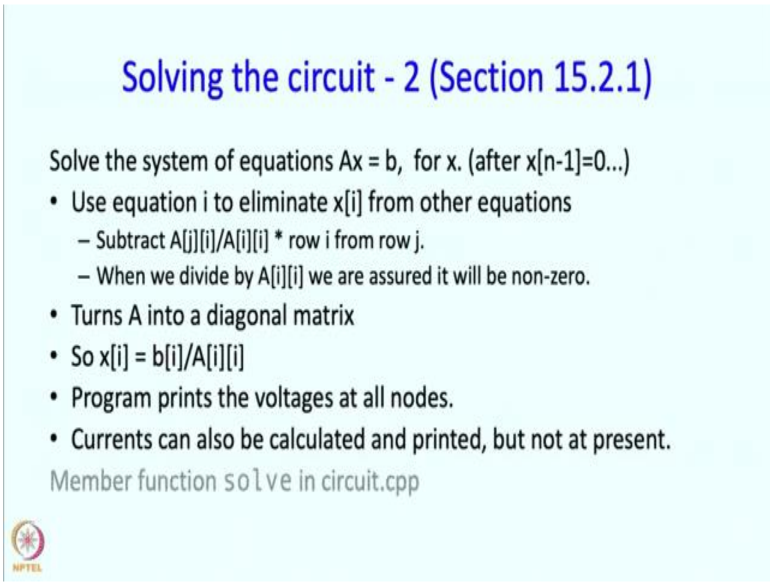

So at the end of it, once we do the elimination, our matrix A is going to look like some non-zero entries over here along the diagonal and everything else is 0 and b has changed suitably, but b may have whatever entries over here zeros and non-zeroes, in general non-zeroes. So, we have a diagonal matrix, but now if we, in fact make this 1 then this will exactly be the solution of Ax, sorry I should have x over here equal to so, if this is made 1 then this will exactly be the solution.

So, in other words if I divide this entry by this entry, I will get the solution for x[i] so, this is the ith row. So, that is in fact what we are going to do. So, x[i] is b[i]/A[i][i]. So, that is what is involved in solving the circuit.

So, at this point we will have found x[i] which are the voltages and so the program can print the voltage at all the nodes and once we know the voltages we can also of course find the currents because we already said that they can be, they are related as per this over here. But we are not going to do that, that is just a detail that you can take care of and you could also say that 'Look, I do not want the current printed on or the voltage printed on my shell window, but I want it printed in the circuit, in the canvas itself, and that also could be done but again that is a detail that you can take care of.

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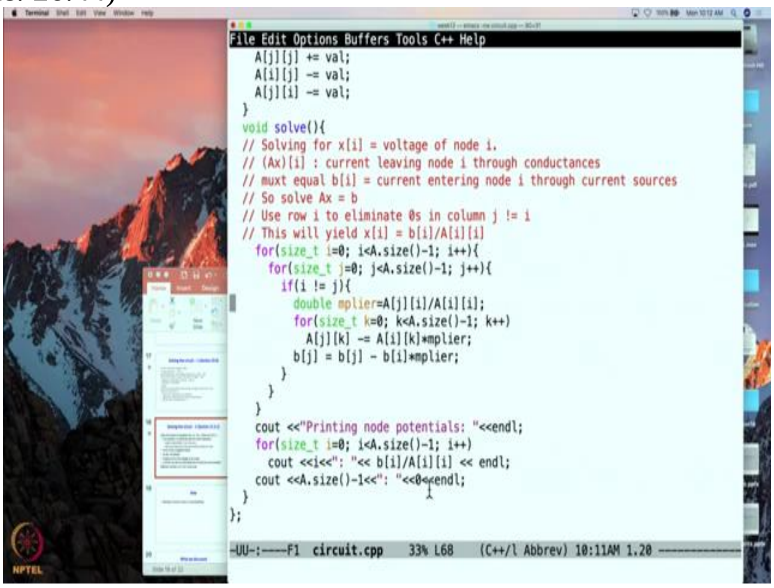

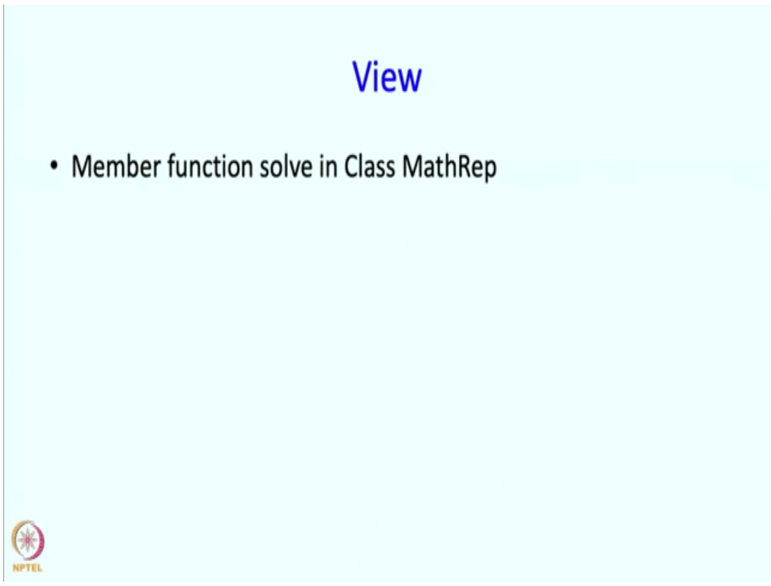

All right. So we will take a quick look at this member function Solve. So, what does Solve do? So, I have described it to you already, so we are going to use the ith row to eliminate x[i] from every jth row. So, long as of course i is not equal to j so, it is going to calculate this multiplier and then it is just going to subtract that row and that is that subtraction process, that is all. And then finally it is going to print out b[i]/A[i][i] as the final voltage values. And remember that the last, so we had thrown out 1 unknown, and we had set that to 0, so that had index A.size()-1 and that value was 0. So we are going to print that out as well, so that is it. So that is what the Solver looks like.

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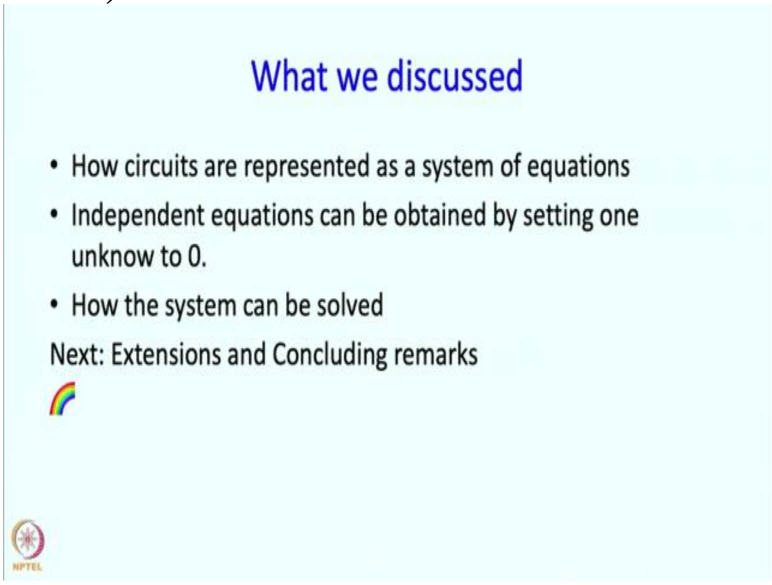

So, we have this already and so what have we discussed? So we have discussed how circuits are represented as a system of equations and we said that we are going to set 1 unknown to 0 and that gives us independent equations. Then we discussed how the system can be solved. So we will take a quick break over here and then we will discuss extensions and we will conclude this lecture sequence. Thank you.