Indian Institute of Science
Bangalore
NP-TEL
National Programme on
Technology Enhanced Learning

<u>Course Title</u>
Finite element method for structural dynamic
And stability analyses
Lecture – 36
Inverse response sensitivity analysis
(continued)
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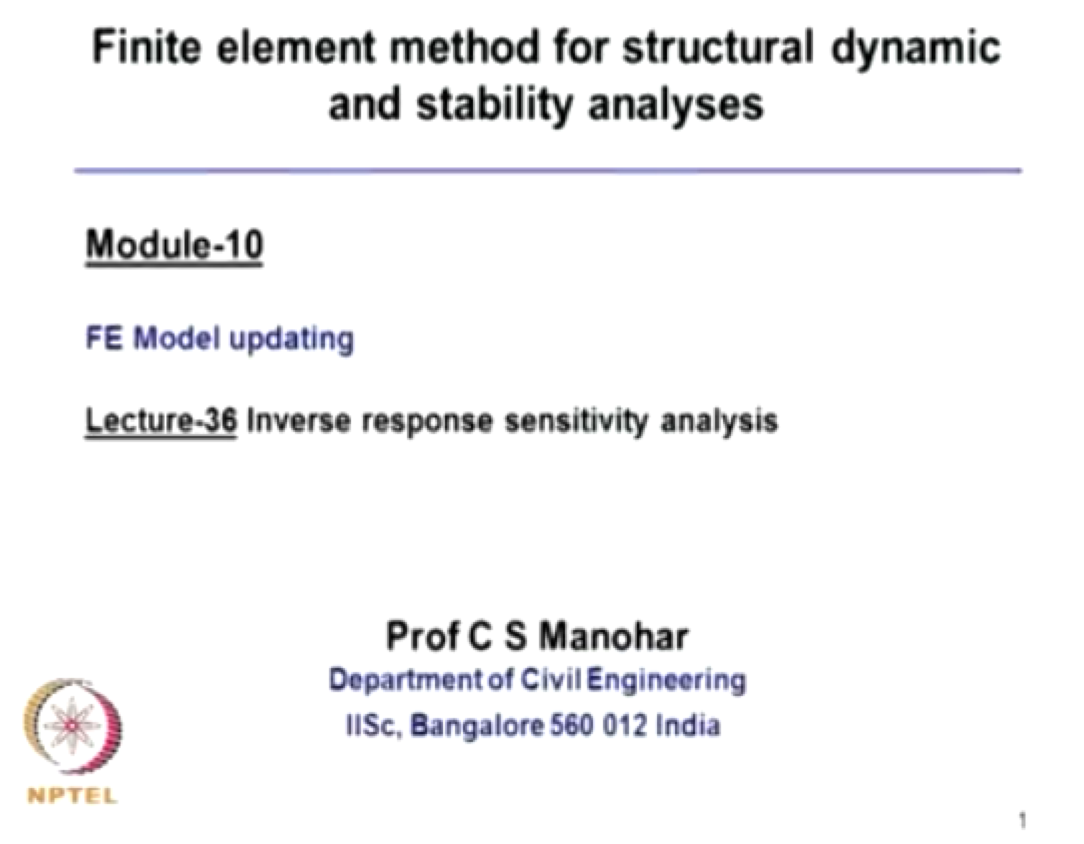
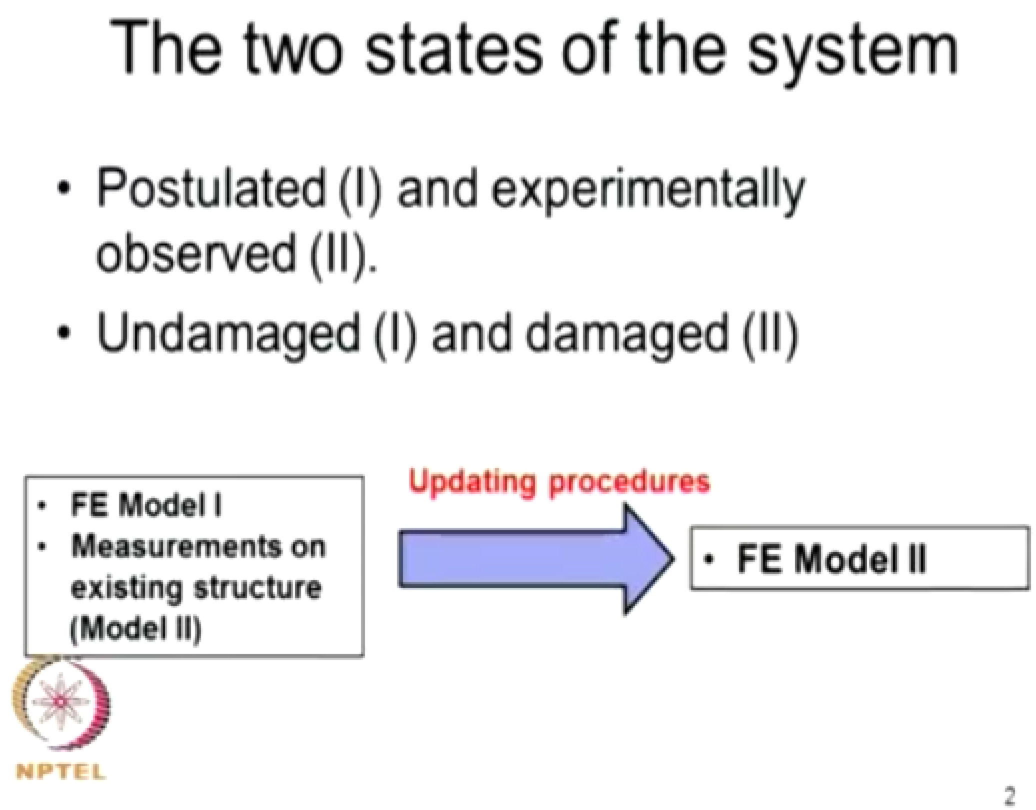


We will continue the discussion on finite element model updating, in the previous lecture we

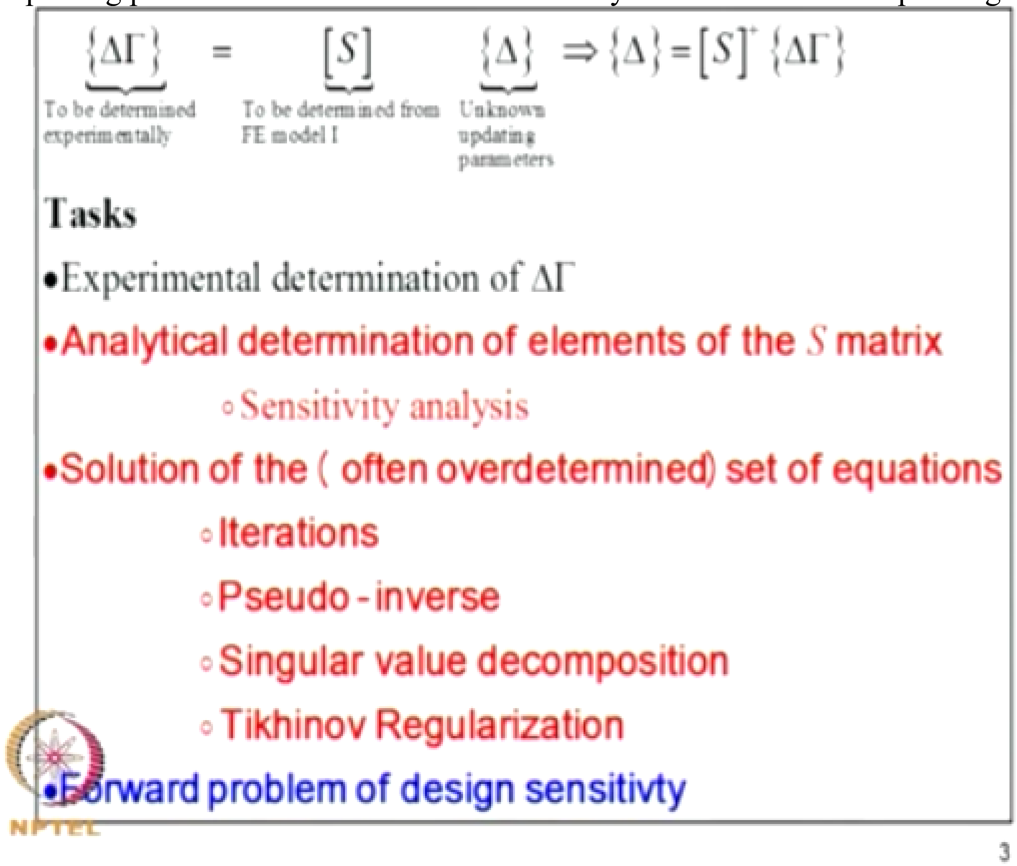


derived sensitivity of natural frequencies, mode shapes, and frequency response functions, we considered situations where natural frequencies where free vibration was done for damped systems and as well as undamped systems.

So we postulated basically two states for the system, in the context in which we discussed this in the previous lecture there was one state one which is the postulated model for the structural behavior on which measurements have been done, the second state is the model on which experimental studies have been done. Now our objective is to arrive at a finite element model for the system on which measurements have been made, there is an alternative perspective which is related to this approach, this is related to problems of damage detection, it is closely related to problem of finite element model updating here also we postulate two states, one is system in its healthy state, let us call it as undamaged state and system which is postulated to be in a damaged state. So the word damage here could mean lot of changes in stiffness, mass, or damping characteristics in the context of problem that we are dealing with, and our objective would be to, you know analyze the measurements carried out on the damaged system and infer, if the structure is indeed damaged if yes, where is the damage? And what is the quantum of damage? And more complicated questions like what is the residual life left and things like that.

Now so in the discussion to follow we will try to adopt these two alternative perspectives, so it could be that we are dealing with a system on which the object is not to do damage detection but to simply identify the parameters of the model, so the postulated finite element model is the first state of the system and our objective as I already said to derive the finite element model for the structure in the way it currently exists.

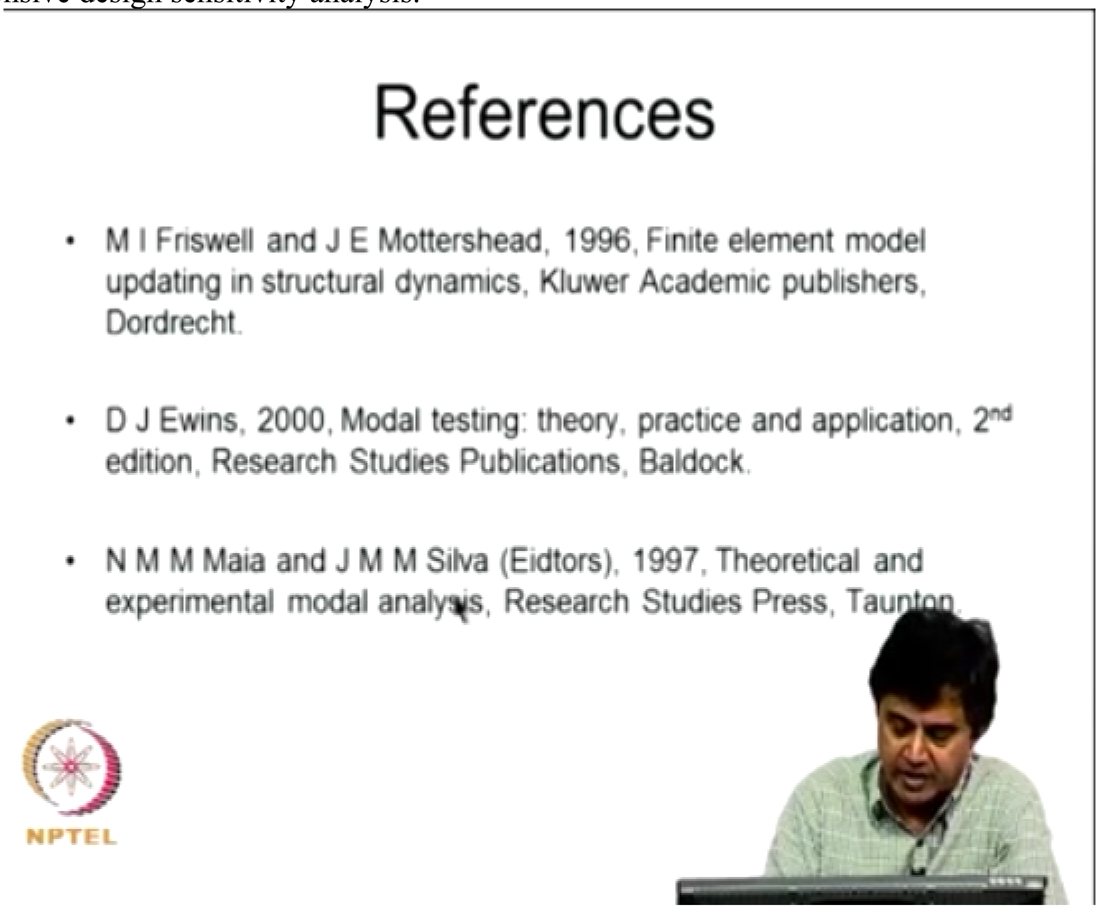
Now so we have a FE model 1 and measurements made on existing structure which is model 2, and the updating procedure leads to the FE model for system in state 2. So depending on the,



based on the various sensitivity information that we studied we got the updating equation in this form we have certain changes in response characteristics on the system that we call as delta gamma this is to be determined experimentally, part of it is measured experimentally, and part of it is postulated through the baseline the finite element model for the structure, and the difference is calculated. This S matrix is the matrix of sensitivity you know sensitivity matrix, this has to be determined from the postulated finite element model, these delta is the vector of unknown updating parameters, so this typically constitutes a set of over determined equation this is a formulation based on first order sensitivity method, and we obtain delta typically by using pseudo inverse of S into delta gamma, we can refine this procedure as I discussed in the previous lecture by using Tikhonov's regularization strategies.

So the task to be done is experimental determination of delta gamma, an analytical determination of elements of S matrix that is sensitivity analysis, then solution of the over determined set of equations either by iteration, pseudo-inverse, singular value decomposition, Tikhonov regularization etcetera.

Now this problem is notably different from the forward problem of design sensitivity in which we impart certain changes to system parameters in the, parameters of the system and we would like to know what would be the change in response characteristic that is the objective of a forward sensitivity problem, but we are using the idea of the sensitivity analysis in the inverse way, that means we have seen that certain changes have taken place in the system response characteristics and we don't know what are the changes in system parameters that have caused those changes and we use the same equation to find delta instead of delta gamma as in forward responsive design sensitivity analysis.



Now there are many research papers on this subject of finite element model updating, there are some few useful references, this is a book by Friswell and Mottershead which is entirely dedicated to problem of finite element model updating, and the book on modal testing by Ewins has a chapter on, you know topics related to finite element model updating, and in this collected papers edited by Maia and Silva, there is a chapter again on finite element model updating.

Mode Indicator Function

- MIFs are often real valued frequency dependent functions that show local maxima/minima at the system natural frequencies corresponding to the real normal modes.
- They can be used to identify repeated eigenvalues.

$$[H(\omega)]_{n\times p} = [U(\omega)]_{n\times n}[\Sigma(\omega)]_{n\times p}[V(\omega)]^{H}_{p\times p}$$

$$CMIF(\omega) = [\Sigma(\omega)]^{T}_{p\times n}[\Sigma(\omega)]_{n\times p}$$

- The peaks in the first (largest) CMIF indicate the location of natural frequencies
- Double or multiple natural frequencies are indicated by simultaneously large values of two or more CMIF values.
- Associated with ω= ωr, the left singular vector indicates the mode shape for that mode and the right singular vector represents the approximate force pattern needed to generate response on that mode only.

NPTEL

5

In the previous lecture we discussed briefly the quantity known as complex mode indicator function, so the mode indicator function are often real valued frequency-dependent functions that show local maxima or minima at the system natural frequencies corresponding to the real normal modes. They can be used to identify repeated eigenvalues. So if H(Omega) is N x P measured frequency response functions we can do a singular value decomposition of this rectangular matrix and the matrix of singular values, you know we define what is known as complex mode indicator function in terms of singular values of this matrix, and this is the definition.

Now we discuss some of properties of CMIF and we Illustrated this with respect to a 7 degree freedom system in which natural frequencies were repeating, but I would like to augment that discussion with 1 or 2 illustrations, so that's why this topic we have returned to again, the peaks in the largest CMIF indicate the location of natural frequencies, double or multiple natural frequencies are indicated by simultaneously large values of 2 or more CMIF values, associated with omega = omega R, that is the R-th natural frequency, the left singular vector indicates the mode shape for that mode, and the right singular vector represents the approximate force pattern needed to generate response on that mode only, so this is something that is a property of singular value decomposition of the FRF matrix.

EXAMPLE 1

eta1=0.01;

eta2=0.01;

eta3=0.005;

eta3=0.005;

eta4=0.001;

eta5=0.001;

eta6=0.001;

eta[illegible]=0.001;

K =

10000	-10000	0	0	0	0	0
-10000	30000	-10000	-10000	0	0	0
0	-10000	30000	-10000	0	0	0
0	-10000	-10000	40000	-10000	-10000	0
0	0	0	-10000	20000	0	-10000
0	0	0	-10000	0	30000	-10000
0	0	0	0	-10000	-10000	20000

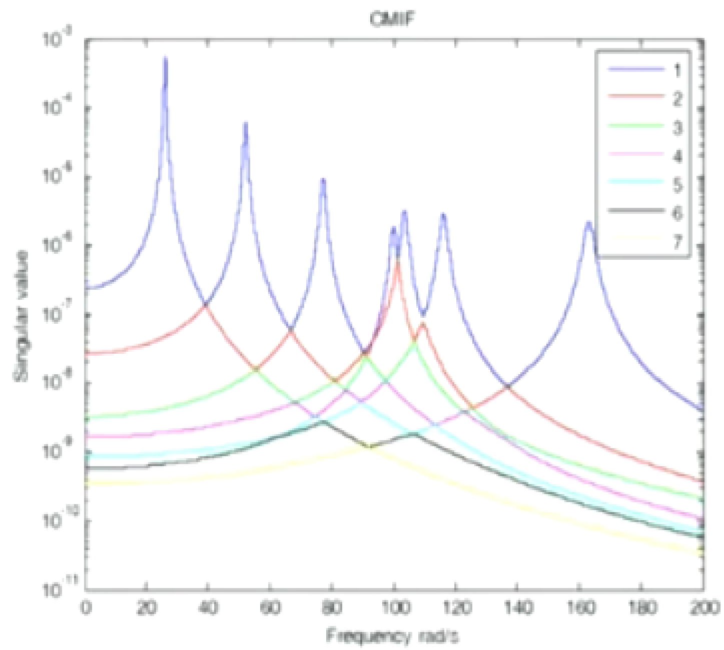
M =

2	0	0	0	0	0	0
0	5	0	0	0	0	0
0	0	4	0	0	0	0
0	0	0	4	0	0	0
0	0	0	0	3	0	0
0	0	0	0	0	2	0
0	0	0	0	0	0	1

C =

2.6531	-1.5225	-0.1756	-0.1732	-0.0575	-0.0302	-0.0150
-1.5225	6.9580	-1.5195	-1.4805	-0.2434	-0.1403	-0.0590
-0.1756	-1.5195	6.6561	-1.3073	-0.1859	-0.1100	-0.0441
-0.1732	-1.4805	-1.3073	7.4643	-1.3509	-1.0413	-0.2004
-0.0575	-0.2434	-0.1859	-1.3509	4.[illegible]015	-0.3132	-1.0074
-0.0302	-0.1403	-0.1100	-1.0413	-0.3132	4.6926	-0.8238
-0.0150	-0.0590	-0.0441	-0.2004	-1.0074	-0.8238	2.7039

Now we have considered few examples I want to illustrate this with one more example, suppose we again consider a 7 degree freedom system, this is a stiffness matrix, this is a mass matrix, and we use this as a damping ratios and C matrix can be constructed based on you know these modal damping ratios and that is given here.

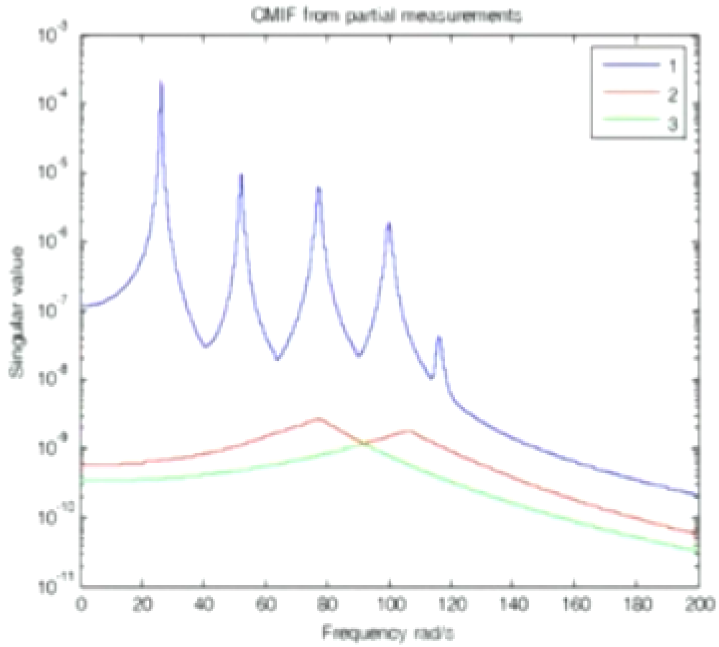


CMIF based on 7 by 7 FRF matrix

7

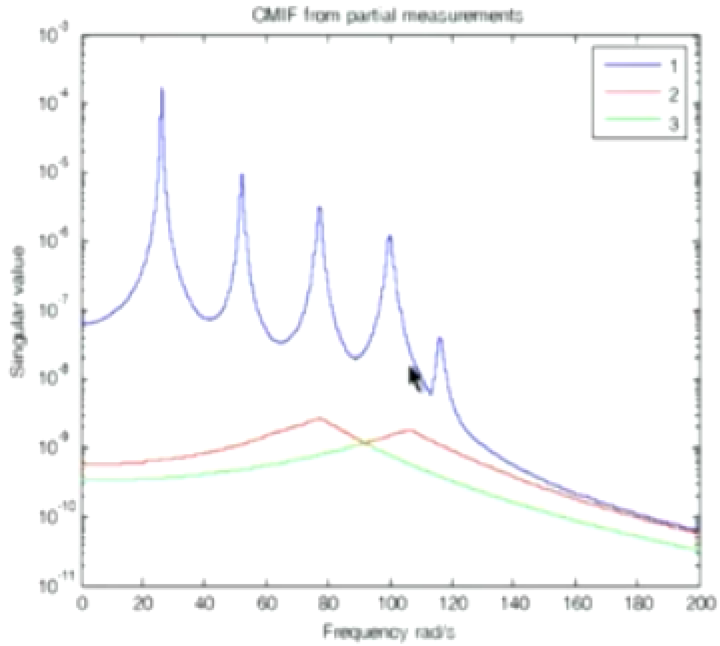
Now the question that I am trying to answer here is as you saw here we are talking about frequency response function which is not necessarily square, if you measure all frequency in a discrete multi-degree freedom model the frequency response function matrix will be DOF/DOF but often we will not measure all the elements of the frequency response function so the point that I am trying to discuss now is what happens if we have a rectangular frequency response function matrix.

So for this system if we have the full 7 by 7 FRF matrix, these are the singular, the CMIF's and the blue one is the largest you know the singular value, and we see that there are 7 peaks corresponding to frequencies at which there are system natural frequencies, so this is fair enough, this is clear.



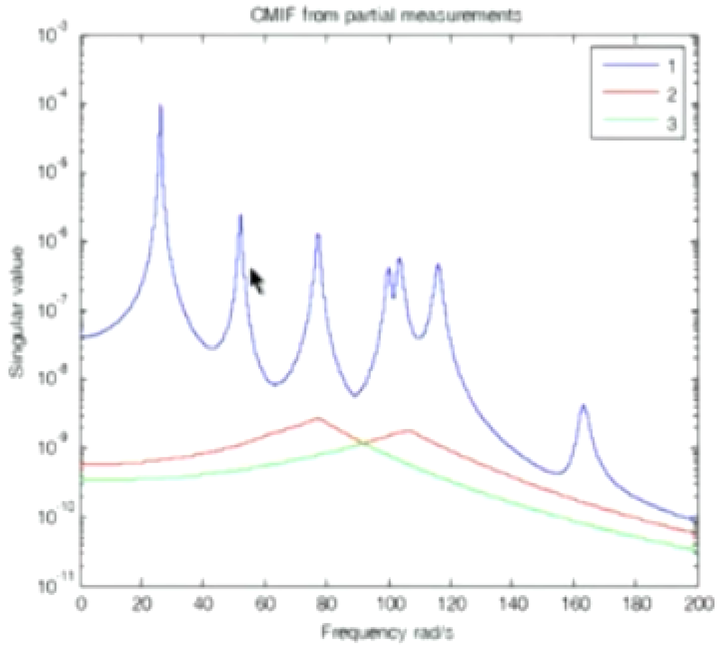
CMIF based on alpha(1:4,1:3)

Now if I now take FRF's to be 4 by 3, that means I will consider 4 rows and 3 columns, and assume that only this is available, then we are getting only 5 peaks so CMIF from partial measurements may not provide all the natural frequencies, and this is another one where I have

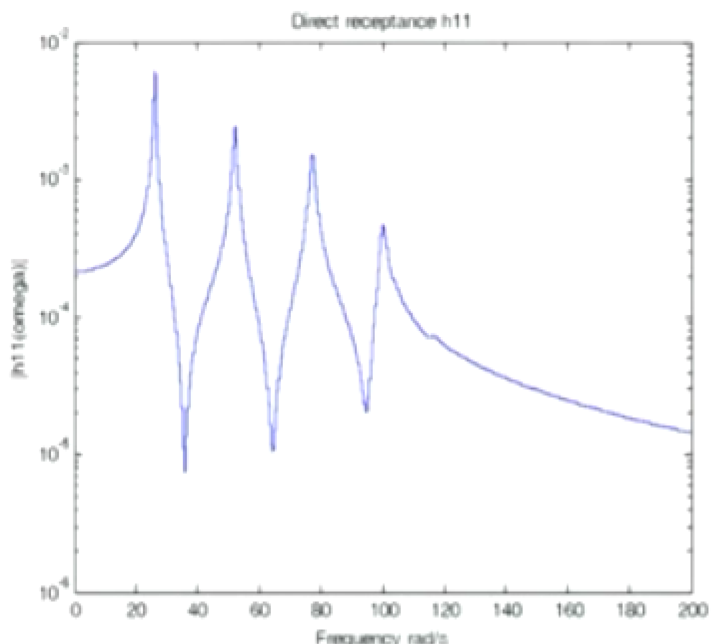


CMIF based on alpha(2:5,1:3)

taken rows 2 to 5, and columns 1 to 3. Now in some other combination we see that all frequencies are captured, so this is something that one should be alert to when using CMIF with partially measured FRF matrix.

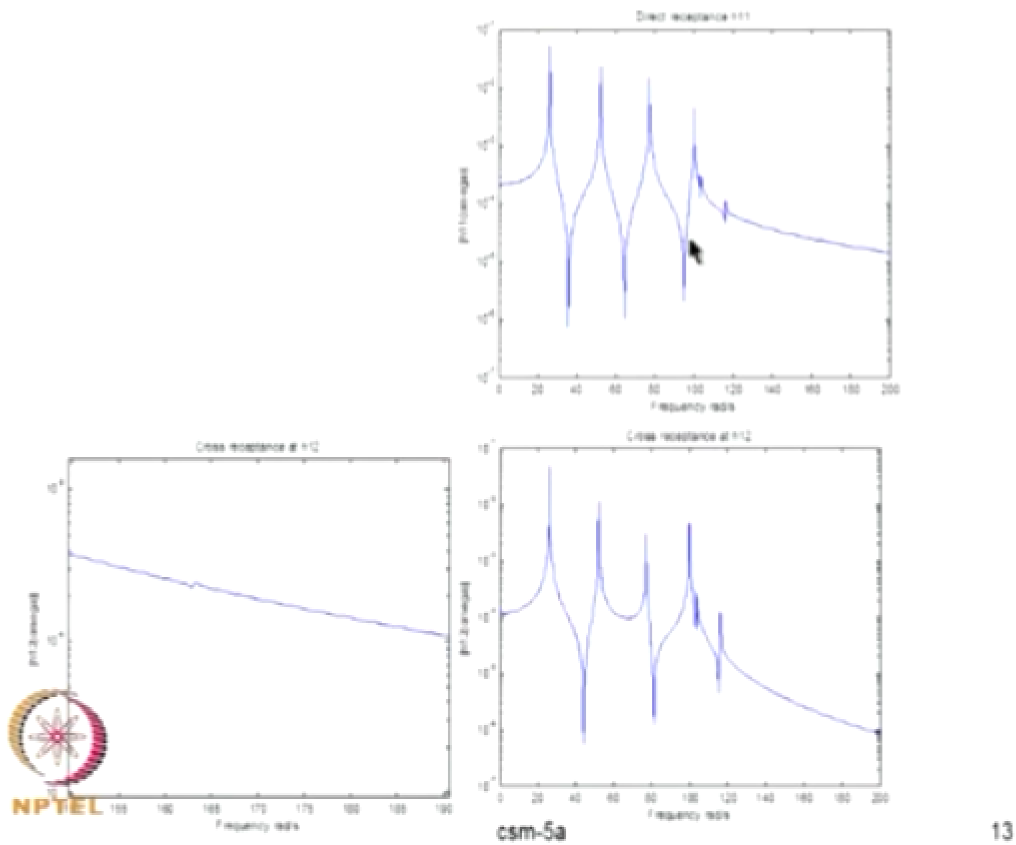


CMIF based on alpha(3:6,2:4)



Receptance h11

On the other hand the receptance functions themselves may not capture all the natural frequency, there are 7 frequencies from a mere visual inspection of this, we will not be able to see the 3 more natural frequencies, but if I look at other elements of FRF matrix we will be able to see certain other frequencies which is not, is mildly perceptible here, it becomes more pronounced here, and you know other you know by reducing damping I am trying to show that



there is another peak here between 160 to 165, this is a zoom of H12(omega), amplitude of H12(omega) that is not perceptible here, but this kind of peaks are clearly seen if you look at CMIF plots.

Remarks

- CMIF with full FRF provides clear indication of all the modes.
- CMIF with partial FRF measurement could miss some modes.
- Elements of FRF may fail to indicate (visually) the existence of resonant frequencies. The modal extraction algorithms may be able to pick them.
- "CMIF frequencies" and "MDOF frequencies" need not match

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So a few comments can be made, CMIF with full FRF provides clear indication of all the modes, CMIF with partial FRF measurement could miss some of the most, elements of FRF matrix may fail to indicate at least visually the existence of a resonant frequencies, the modal extraction algorithms may be able to pick them up, although you may not see it in the FRF plots. The CMIF frequencies and MDOF frequencies need not match, so this also is something to do with variation of mode shapes at the drive and measurement points, because of which there will be again fluctuations in the FRF plots, therefore there would be differences, okay.

How closely do the analytical and experimental models agree?

- Preliminary step: matching the size of the experimental and analytical models
  - Use either model expansion or model reduction
  - Recall discussion on model reduction using SEREP

Basis: $\Phi^t M \Phi = I \;\&\; \Phi^t K \Phi = \Lambda = \begin{bmatrix} \omega_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \omega_N^2 \end{bmatrix}$

$\phi_A^i = i^{\text{th}}$ mass normalized mode shape from the analytical model

$\phi_X^i = i^{\text{th}}$ mass normalized mode shape from the experimental model

$\psi_A^i = i^{\text{th}}$ arbitrarily normalized mode shape from the analytical model

$\psi_X^i = i^{\text{th}}$ arbitrarily normalized mode shape from the experimental model

Subscript A : refers to the analytical model

Subscript X : refers to the experimental model

15

Now we move on to a brief review of another question, how do we quantify the proximity of analytical model and experimental model, that is the question we are asking is how closely do the analytical and experimental models agree, so how do you quantify this? Now there are few matrix for this, that's what I will quickly review, there is lot of literature on this but I just want to give a flavor of what the issues are. Now the preliminary step would be the matching of the size of, matching the size of the experimental and analytical model this issue I have already mentioned in experimental work the number of sensors you have may not match the number of degrees of freedom in a finite element model, often the number of degrees of freedom in a finite element model far exceeds the number of degrees of freedom in an experimental work, simply because some of the interior nodes will not be accessible, certain responses may not be easily measurable like rotation and so on and so forth.

Now we can use either model expansion, that means the experimental model is expanded to match the degrees of freedom of the analytical model or a model reduction on the analytical model to match the degrees of freedom included in the experimental work, so for this typically we use SEREP, this I have discussed earlier in the context of model reduction in one of the previous lectures.

Now the basis for setting up measures for comparing analytical and experimental models is again something to do with orthogonality relation, so we define phi to be mass normalized modal matrix, phi transpose M phi is I, and phi transpose K phi is diagonal matrix of squares of system natural frequencies, we use the subscript A to indicate analytical model, and subscript X for experimental model, and phi we use for mass normalized mode shapes, and sai is arbitrarily normalized mode shapes, so this is the nomenclature, subscript A refers to analytical model, subscript X refers to the experimental model, so mass normalized normal modes will satisfy this condition, if it is not mass normalize this matrix will still be diagonal, but the elements will be you know the generalized masses will not be identity equal to 1.

Normalized Cross Orthogonality (NCO)

$$\text{NCO}\left[\psi_X^i,\psi_A^j\right]=\frac{\left[\left\{\psi_X^i\right\}^t\left[M\right]_A\left\{\psi_A^j\right\}\right]^2}{\left[\left\{\psi_X^i\right\}^t\left[M\right]_A\left\{\psi_X^i\right\}\right]\left[\left\{\psi_A^j\right\}^t\left[M\right]_A\left\{\psi_A^j\right\}\right]}$$

Remarks

- Defined in terms of arbitrarily normalized real normal modes
- $0\le\text{NCO}\left[\psi_X^i,\psi_A^j\right]\le 1$
- Values closer to one indicating the closeness of analytical and experimental mode shapes
- If m=number of experimental modes & n= number of analytical modes, NCO would be a $m\times n$ matrix.
- This matrix facilitates establishing the correlated modal pairs
- $\Psi_X=\alpha\Psi_A\Rightarrow\text{NCO=I}$

16

Now there is a quantity known as normalized cross orthogonality denoted as NCO, it is a quantity that is computed between 2 modes, suppose from the experiment you pick the I-th mode, and from the analysis you pick the J-th mode, we define this quantity, this is sai X transpose, MA, sai A you know analytical, MA is analytical mass matrix which will be available to you. Now what is the issue here? The issue is when I experimentally extract certain modes, natural frequencies and mode shapes and I computationally predict them, how do you order, how do you pair, which mode in analysis corresponds to which mode in experiment, so we call them as correlated model pairs, so it is very important that we establish the correct pairing between the modes of, modes predicted from analysis and experimentally measured modes, it is quite conceivable that in an experimental model we miss some of the modes, so suppose if you measure 5 modes experimentally and your analytical model has 25 modes which 5 of the experimental modes correspond to which of these 25 modes in a computational model is not a question that can easily be answered, so we define the quantities like NCO to able to answer that, so this is as you see it is defined in terms of arbitrarily normalized real normal modes, and you can see that if sai X is sai A that means the experimental and analytical mode shapes match this quantity will be 1, if they are very much different that quantity will approach 0.

Values closer to one indicating the closeness of analytical and experimental mode shapes, now if M is the number of experimental modes and N is the number of analytical modes, NCO would be M x N matrix, okay, then those elements of this M x N matrix which are close to one, provide the correct strategy to identify the correlated modal pairs.

So first mode in experiment will be purely, the NCO between first mode in experiment and say third mode in analysis will be close to 0 if these modes are correctly, nomenclature is correctly assigned to them. If you are dealing with say third model in experiment and third mode in analysis, NCO will come pretty close to 1, so this helps in identifying the correlated modal pairs. Now if sai X is alpha time sai A, NCO will be an identity matrix that means if mode shapes are proportional, okay, this NCO will be I.

Modal Assurance Criterion (MAC)

$$MAC\left[\phi_X^i, \phi_A^j\right] = \frac{\left|\{\phi_X^i\}^t \{\phi_A^j\}^*\right|^2}{\left|\{\phi_X^i\}^t \{\phi_X^i\}^*\right| \left|\{\phi_A^j\}^t \{\phi_A^j\}^*\right|}$$

Remarks

- Defined in terms of mass normalized complex valued normal modes
- This can be used in conjunction with coordinate-incomplete mode shapes; e.g., the analytical mode shape can be partitioned.
- $0 \le MAC\left[\phi_X^i, \phi_A^j\right] \le 1$
- Values closer to one indicating the closeness of analytical and experimental mode shapes
- If m=number of experimental modes & n= number of analytical modes, MAC would be a $m \times n$ matrix.
- This matrix facilitates establishing the correlated modal pairs.
- $\Phi_X = \alpha\Phi_A \Rightarrow MAC \neq I$ ($\because$ the definition does not involve weighting by M)

Now this as you see requires the mass matrix of the analytical model, we can define another quantity known as Modal Assurance Criterion, here we don't use any structural matrices, again this is defined between I-th experimental mode shape, and J-th analytical mode shape, this is a MAC of between these two modes is given by this quantity, these are all intuitively defined quantities there is no mathematical basis for arriving at these quantities.
Now you observe this here we are allowing for complex valued mode shapes, that means influence of damping in determining mode shapes is included, as I already pointed out in an experiment you will be always be measuring damped normal modes. Now this can be used in conjunction with coordinate-incomplete mode shapes, and for example the analytical mode shape can simply be partitioned instead of doing a model reduction or expansion. And MAC also lies between 0 and 1, and values closer to 1 indicating the closeness of analytical and experimental mode shapes. Again MAC would be a M x N matrix if M is the number of experimental modes and N is the number of analytical modes, this matrix facilitates again establishing the correlated modal pairs.
Now if phi X is alpha phi A in this case MAC won't be an identity matrix, because this is not, there is no mass matrix here, you recall that the mode shapes the way we are deriving by solving the eigenvalue problem associated with K and M are orthogonal with respect to mass

and stiffness matrix, so phi transpose phi is not a diagonal matrix, it is phi transpose M phi is a diagonal matrix, so this won't be equal to an identity matrix.

Normalized Modal Difference (NMD) and Modal Scale Factor (MSF)

$$NMD\left[\phi_X^i, \phi_A^j\right] = \frac{\left\|\phi_X^i - \gamma\phi_A^j\right\|}{\left\|\gamma\phi_A^j\right\|}$$

$$\gamma = MSF\left[\phi_X^i, \phi_A^j\right] = \frac{\left|\left\{\phi_X^i\right\}^T \left\{\phi_A^j\right\}^*\right|}{\left\{\phi_A^j\right\}^T \left\{\phi_A^j\right\}^*}$$

Remarks

- It can be shown that $NMD\left[\phi_X^i, \phi_A^j\right] = \sqrt{\dfrac{1 - MAC\left[\phi_X^i, \phi_A^j\right]}{MAC\left[\phi_X^i, \phi_A^j\right]}}$
- No FE matrices are used
- Mode shape at each DOF is erroneous by 10% $\Rightarrow$ NMD=0.1

NPTEL

18

Now there is another quantity known as Normalized Modal Difference and Modal Scale Factor, so again this is defined with respect to 2 mode shapes, this is given by this quantity phi X(I) - gamma phi A(J) this is L2 norm/L2 norm(gamma) phi AJ, and this gamma itself is, this is modal scale factor between the 2 modes, and this itself is defined in this manner. We can show that in this normalized modal difference is related to MAC through this equation, I'll leave it as an exercise. Again here no FE matrices are used, but the utility of this notion is that mode shape at each DOF, for example is erroneous by 10% then this NMD will become 0.1, so it gives you directly a measure of error in a more direct way.

How to locate spatial regions where the differences between analytical and experimental mode shapes are the most pronounced?

$\Rightarrow$ Coordinate Modal Assurance Criterion $[\text{COMAC}(i)]$

Let N_{CMP} = number of correlated modal pairs

$\phi(i,j)$ = value of the mode shape at the i^{th} coordinate in the j^{th} mode

$$\text{COMAC}(i) = \frac{\left[\sum_{j=1}^{N_{CMP}} \left|\phi_A(i,j)\phi_X^*(i,j)\right|\right]^2}{\sum_{j=1}^{N_{CMP}} \left|\phi_A(i,j)\right|^2 \sum_{j=1}^{N_{CMP}} \left|\phi_X(i,j)\right|^2}$$

Remarks

- $0 \le \text{COMAC}(i) \le 1$
- Has no physical basis
- Can be displayed as a contour plot over the domain of the structure
- Regions of low COMAC represent the regions where the consequences of difference between analytical and experimental modes are felt most pronouncedly. Such regions need not be the regions where actual errors are located. (*e.g.*, errors in modeling fixed end conitions in a cantilever beam)

Now this MAC and NCO compare 2 mode shapes and give a measure of, which mode has to be paired with from experiments with, which mode in analysis, but they do not tell where is the location of difference between analytical model and the experimental model, so the spatial information is not included there, so to do that we ask the question how to locate spatial regions where the differences between analytical and experimental mode shapes are the most pronounced, okay that is where you may like to update the model parameter, that is a hope, but there is a catch in that which we must understand carefully, so with that in mind we introduce what is known as coordinate modal assurance criterion, here let NCMP be the number of correlated model pairs which has been established by some criterion, then phi IJ I define as value of mode shape at the I-th coordinate in the J-th mode, so I is the coordinate, J is the mode count, so the coordinate modal assurance criteria for the I-th coordinate, okay it is defined with respect to coordinate not with respect to a mode, and all the modes are summed here, this metric is obtained by summing over the mode shapes at that value of the spatial coordinate or the degree of freedom that is defined in terms of the analytical mode shape and the experimental mode shape through this relation as shown here.

This COMAC also lies between 0 and 1, it has no physical basis it is intuitive, it can be displayed as a contour plot over the domain of the structure, that means you can plot the structure and you can have color coding for different values of COMAC or draw contours and where COMAC is close to 1 we suspect that the agreement, disagreement between the analytical model and experimental model is most pronounced.

Regions of low COMAC represent the regions where the consequences of difference between analytical and experimental modes are felt most pronouncedly, these need not be the region where you have actually made errors, there are errors in parameter values, it is only the place where the consequence of the difference is felt most pronouncedly, so for example in a cantilever beam if you have made an error in modeling boundary condition at the fixed end then the effect of that error will be most pronounced at the free end of the cantilever, you might have done modeling at the free end appropriately, but the error done elsewhere, the consequence of that will be felt somewhere else, so that precaution you have to take, but typically we tend to use COMAC to identify those spatial regions where we want to correct the parameters, but while doing so that is a hope that COMAC indicates that, but it is not guaranteed to do that but it gives you kind of a hint that there could be something wrong with properties in those regions where COMAC is low, okay, so that is how this is done.

Dynamic force balance method

$$\{f\}_j = \left[K_A - \omega_{Xj}^2 M_A\right]\{\phi_{Xj}\}$$

Remarks

- $\{\phi_{Xj}\} = \{\phi_{Aj}\} \Rightarrow \{f\}_j = 0$
- Requires coordinate complete measurements
- At anti-nodes the errors tend to be high
- Helps in locating regions in which the residual errors are high
- The errors could be in stiffness or mass

Remark

On the lines of MAC and COMAC one can define

Frequency Domain Assurance Criterion (FDAC) and

Frequency Response Assurance Criterion (FRAC)

wherein one uses measured and analytical FRF-s instead of mode shapes

Now there are few other metrics I will just for sake of completeness briefly mention them, suppose you have determined mode shape, suppose J-th mode shape has been determined and J-th natural frequency has been determined, and if I now plug it back into the analytical model K into phi - omega square MA into phi must be 0 that is the eigenvalue problem that you need to solve, but if there is an error there will be a residual force that again tells you where is the trouble, okay, so if these 2 are equal that is phi XJ = phi AJ it is automatically satisfied because KA and MA are analytical matrices, and phi AJ is analytically determined mode shape that has to satisfy the eigenvalue, statement of the eigenvalue problem.

Now to implement this idea you require coordinate complete measurements, that means this phi XJ should be, the size of phi XJ should be equal to the degree of freedom in analytical model, so you may have to expand the experimentally measured mode shapes using a suitable strategy, so the answers will be affected by the strategy that you employ, so the aspiration is it would

help in locating regions at which residual errors are high, the errors could be in stiffness or mass that issue doesn't get resolved here.
Now this is the matrix that I have talked about are in terms of natural frequencies and mode shape they can be damped or undamped, we can also define similar quantities, these frequency response functions, analytically we can predict the frequency response function and experimentally also you can measure the frequency response function. Now on the lines of MAC and COMAC one can define frequency domain assurance criteria called FDAC, and frequency response assurance criteria called FRAC, wherein one use is measured and analytical FRF's instead of mode shapes, so in the definition of MAC and COMAC instead of using mode shapes you use the measured frequency response functions, so you get FDAC and FRAC, so they again serve the same purpose as MAC and COMAC by enlarge, and it doesn't require extraction of mode shapes, and natural frequency.

Illustrative Examples

- A hypothetical MDOF system
- Studies on beams and simple building frame models

NPTEL

21

Now this is just a brief overview of the model correlation methods again you need to go back to the references that I provided especially the book by Evins to you know completely understand what the issues are, so here in these 2 lectures on finite element model updating I am trying to provide a flavor of how to approach these problems and what are they issues and in some simple illustrations. So the illustrative examples that I am going to present now are with respect to a class of hypothetical multi degree freedom systems, and we have done some studies on simple beams and building frame models experimentally, and the presentation of some of these results will now follow.

Study 1: Inverse eigensensitivity analysis for undamped systems

Objectives 1. Evaluation of mass and stiffness parameters

2. Study the effect of cross orthogonality relations

Summary of solutions of equations of motion

1	Equilibrium equation employed for determination of the modal properties	$M\ddot{x} + Kx = 0$
2	Assumed solution	$x(t) = \Phi e^{i\omega t}$
3	Eigenvalue problem	$K\Phi = \omega^2 M\Phi$
4	Eigensolution	$\omega_1^2 \le \omega_2^2 \le \cdots \le \omega_N^2, \Phi = [\Phi_1 \Phi_2 \cdots \Phi_N]$
5	Orthogonality relations	$\Phi^t M\Phi = I, \Phi^t K\Phi = \chi,$ $\chi = diag[\omega_i^2];\ i = 1, 2, \cdots, N$

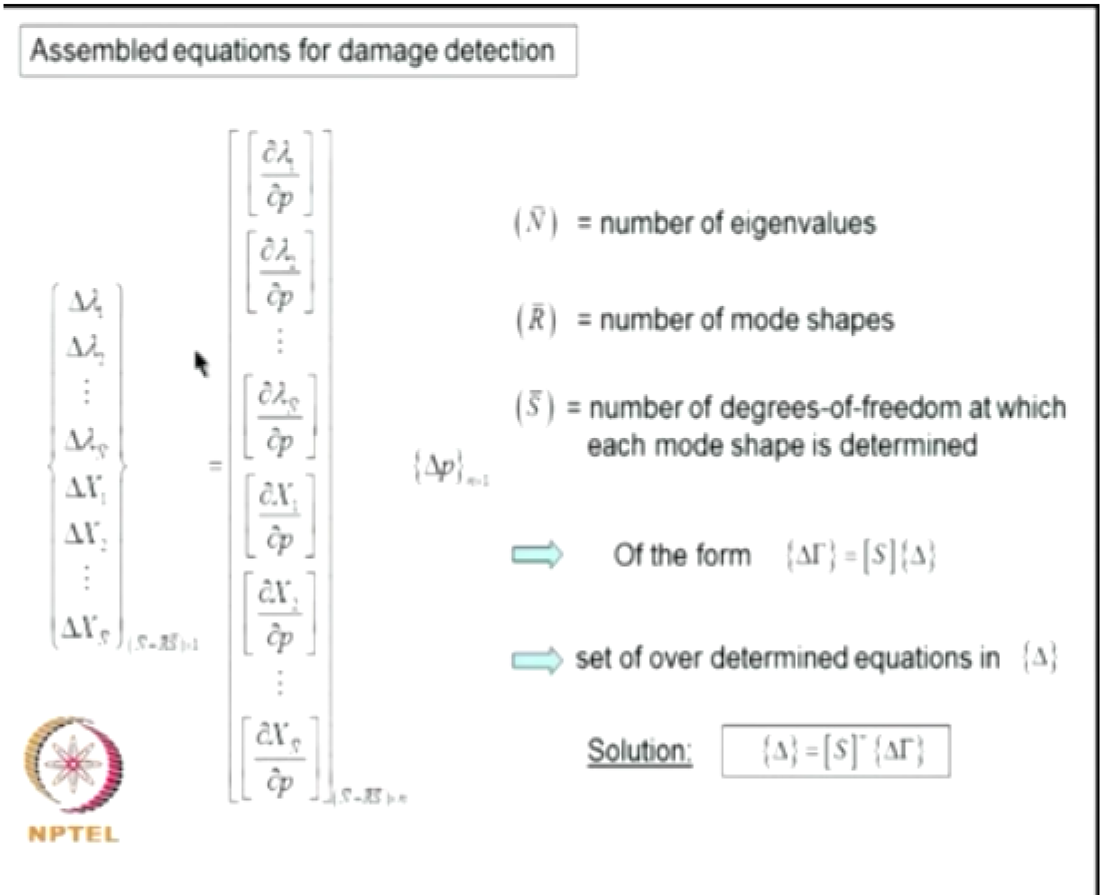
First order sensitivities

$$\frac{\partial \lambda_i}{\partial p_j} = X_i^T \left[\frac{\partial K}{\partial p_j} - \lambda_i \frac{\partial M}{\partial p_j} \right] X_i$$

$$\begin{bmatrix} F_i & 0 \\ 0 & F_i \\ X_i^T M & X_i^T M \\ X_i^T K & X_i^T K \end{bmatrix} \begin{Bmatrix} \frac{\partial X_i}{\partial \delta_j} \\ \frac{\partial X_s}{\partial \delta_j} \end{Bmatrix} = \begin{bmatrix} -\frac{\partial F_i}{\partial \delta} X_i \\ -\frac{\partial F_s}{\partial \delta} X_s \\ -X_i^T \frac{\partial M}{\partial p_j} X_s \\ \frac{\partial \lambda_i}{\partial \delta_j} \delta_{is} - X_i^T \frac{\partial K}{\partial \delta_j} X_s \end{bmatrix}$$

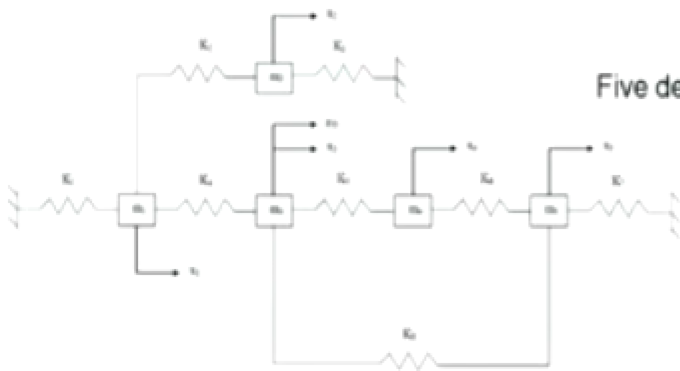
NPTEL

In the first study we want to you know apply the inverse Eigen sensitivity analysis for undamped systems, so the objective of this illustration is evaluation of mass and stiffness parameters and study the effect of including cross orthogonality relations in deriving the updating equations, so this is quick recall of the governing equation, this is a equilibrium equation, this is assumed solution, this is eigenvalue problem to be solved, these are the natural frequencies and mode shapes, these are the orthogonality relations.



Now in the previous lecture we have derived the equations for sensitivity of eigenvalues and eigenvectors and based on that we form the updating equation, and we got the solution as delta is S pseudo inverse delta gamma, now we will apply this on a 5 degree freedom system as shown here.

Numerical illustration



Five degree of freedom system

Baseline model

Mass parameters M (kg)	$m_1=10,\ m_2=20,\ m_3=15,\ m_4=30,\ m_5=20$
Stiffness parameters K (N/m)	$k_1=2000,\ k_2=3000,\ k_3=1000,\ k_4=4000,\ k_5=3000,$ $k_6=1500,\ k_7=2000,\ k_8=2000$
Undamped natural frequency ω_n (rad/s)	$\omega_1=6.4027,\ \omega_2=11.7416,\ \omega_3=17.1085,\ \omega_4=22.3806,$ $\omega_5=33.9492$
Mass normalized modal matrix Φ	$\begin{bmatrix} -0.0717 & 0.0812 & 0.0126 & -0.1396 & 0.2619 \\ -0.0676 & 0.1961 & -0.0204 & 0.0696 & -0.0412 \\ -0.1032 & 0.0077 & 0.0344 & -0.1916 & -0.1344 \\ -0.1348 & -0.0691 & -0.0908 & 0.0446 & 0.0127 \\ -0.0873 & -0.0322 & 0.1904 & 0.0700 & 0.0142 \end{bmatrix}$

Now these are the masses, these are the stiffnesses, and for this system these are the natural frequencies, and this is mass normalized modal matrix. we call this as a baseline model and I am going to alter some of the properties of mass and spring and call it as system in damaged state, so if we are not talking about damaged and undamaged systems it could be a postulated finite element model and the model unknown model from which we have taken measurements, in the illustration that I am presenting in this part the experimental results are synthetically generated, that there is no true experiment but numerically we are doing an experiment. The idea here is to see how the updating equations can be implemented, and what if any are the

We denote $m_i^d = \alpha_i m_i^u$, $k_i^d = \beta_i k_i^u$ and $c_i^d = \gamma_i c_i^u$

Parameters $\alpha_i, \beta_i, \gamma_i$ denote the damage indicator factors

If the structure suffers no damage $\alpha_i = 1, \beta_i = 1 \,\&\, \gamma_i = 1$

Method 1 – no cross orthogonality relations

Method 2 - cross orthogonality relations included

Case 1: Damage scenario with $(\alpha_1 = 0.7, \alpha_2 = 0.55, \alpha_3 = 0.85, \alpha_4 = 0.8, \alpha_5 = 0.6)$

$(\beta_1 = 0.5, \beta_2 = 0.5, \beta_3 = 0.45, \beta_4 = 0.5, \beta_5 = 0.8, \beta_6 = 0.75, \beta_7 = 0.35, \beta_8 = 0.85)$

Case 2: Damage scenario with $(\alpha_1 = 0.3, \alpha_2 = 0.55, \alpha_3 = 0.65, \alpha_4 = 0.4, \alpha_5 = 0.6)$

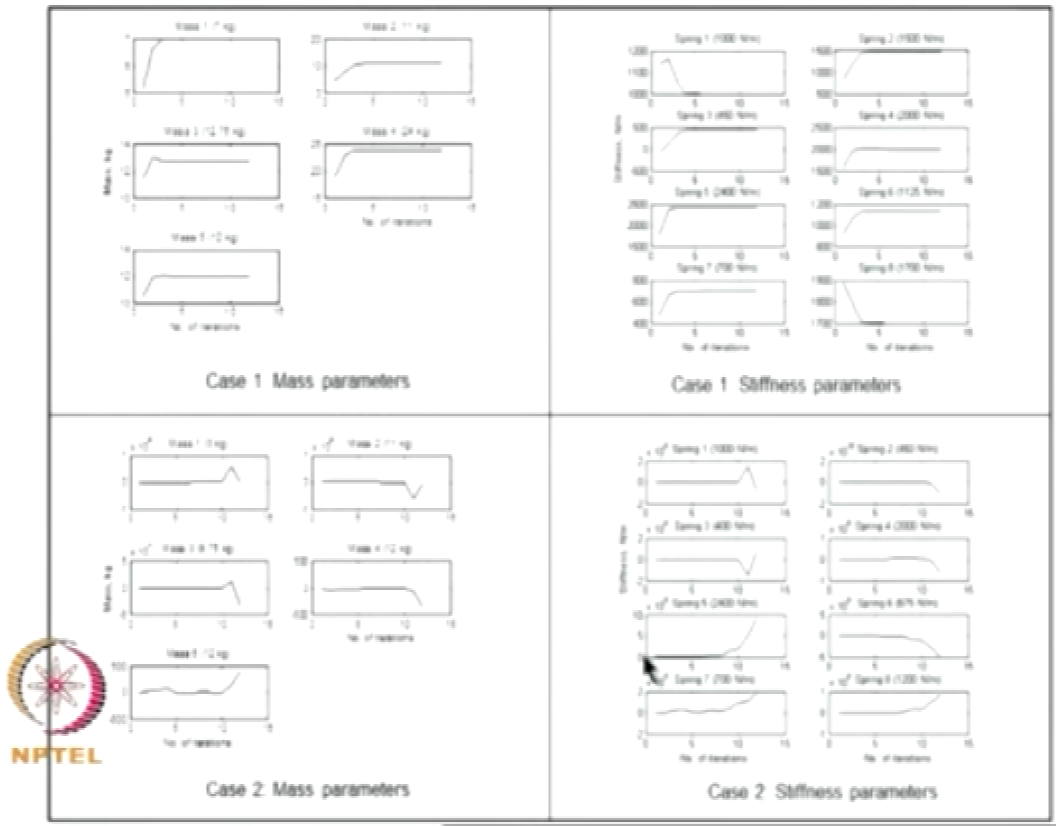
$(\beta_1 = 0.5, \beta_2 = 0.15, \beta_3 = 0.4, \beta_4 = 0.5, \beta_5 = 0.8, \beta_6 = 0.45, \beta_7 = 0.35, \beta_8 = 0.6)$

NPTEL

pitfalls in using that, this is a exercise that is worth doing before you actually start working with experimentally observed structural properties, so what I will do is I will use the language of damage and undamaged systems, so MI in damaged state is taken as alpha A into MI in undamaged state, this MI, KI, and CI are the discrete elements shown in this figure, so we introduced alpha I, beta I, and gamma I and the updating parameter is consequently are these non-dimensional numbers alpha I, beta I, and CI, if the structure suffers no damage all these quantities will be equal to 1.

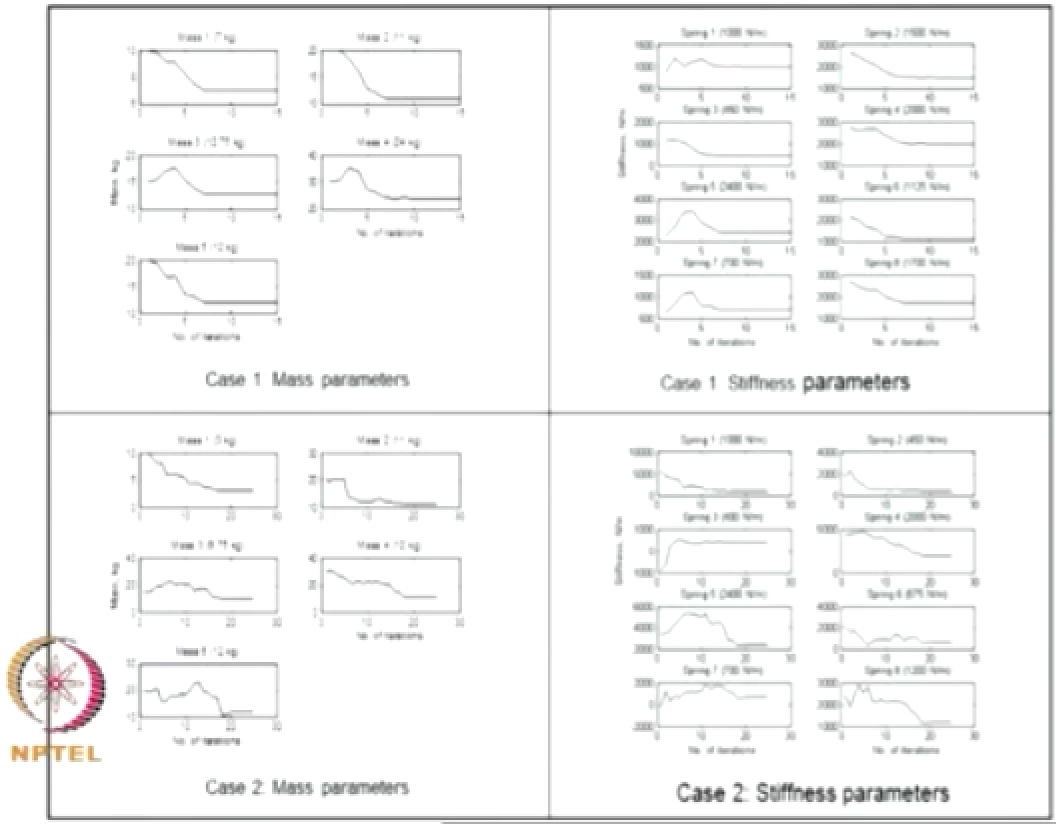
Now we will adopt two methods, in first method we will not include cross orthogonality relations. In the second method we will include the cross orthogonality relations and see if there is any advantage in doing including cross orthogonality relation, now there are 2 damage scenarios with various levels of severity, the notes for this presentation will be available with you, you can study this in greater detail I will now present the main features of the result without getting into the base all the intricate details.

Results: Method 1



Method 1 where cross orthogonality is not included predicts the 5 masses in acceptable manner, and stiffness parameters also in an acceptable manner for the method 1, that is case 1, for case 2 what happens, the method does not perform acceptable, for example on X-axis in all these plots is the iteration, the global iteration step that I mentioned in the previous lecture you start with initial guess and improve upon that successively through an iterative process, here you can see the algorithm is not converging, where as you can see here all the system parameters have converged and have become constant after about 10 iterations, whereas here they seem to be diverging here. So method 1 does not perform for case 2 of the so-called damage scenario.

Results: Method 2



Now you include now the cross orthogonality relations we see that case 1 the method works, case 2 the method indeed works, so this is an example where the objective is to illustrate that by including the cross orthogonality information in driving sensitivity information we are able to achieve better solutions in terms of determining finite element model updating parameters.

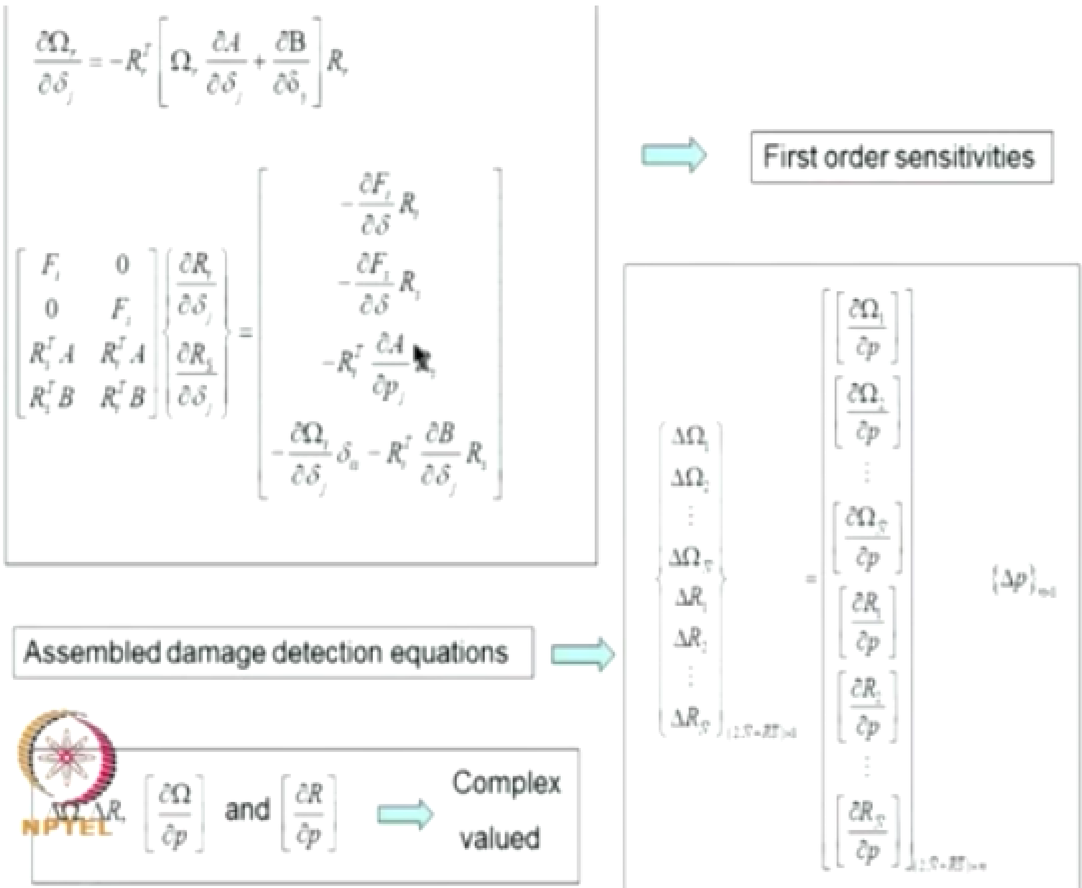
Study 2: Inverse eigensensitivity analysis for damped systems

Objectives
1. Evaluation of mass, stiffness and damper parameters
2. Study the effect cross orthogonality relations
3. Analysis with complex modes

Summary of solutions of equations of motion

1	Equilibrium equations	$\begin{bmatrix}0 & M\\ M & C\end{bmatrix}\begin{Bmatrix}\ddot{x}\\ \dot{x}\end{Bmatrix} + \begin{bmatrix}-M & 0\\ 0 & K\end{bmatrix}\begin{Bmatrix}\dot{x}\\ x\end{Bmatrix} = \begin{Bmatrix}0\\ f(t)\end{Bmatrix}$ $A\dot{y} + By = F(t)$ $y(0) = y_0$
2	Equilibrium equations for eigenvalue analysis	$A\dot{y} + By = 0$
3	Assumed solution	$y(t) = \theta e^{\Omega t}$
4	Eigenvalue problem	$\Omega A\theta = -B\theta$
5	Eigensolution	$\{\Omega_1\Omega_2 \cdots \Omega_N\ \Omega_1^*\Omega_2^* \cdots \Omega_N^*\}$ $\{\theta_1\theta_2 \cdots \theta_N\ \theta_1^*\theta_2^* \cdots \theta_N^*\}$
6	Structure of eigensolution	$\Lambda = \begin{bmatrix}\Omega_1 & 0 & \cdots & 0\\ 0 & \Omega_2 & \cdots & 0\\ \vdots & & \ddots & \\ 0 & 0 & \cdots & \Omega_N\end{bmatrix}$ $\Phi = [\theta_1\ \theta_2\ \cdots\ \theta_N]$ $R = \begin{bmatrix}\Phi\Lambda & \Phi^*\Lambda^*\\ \Phi & \Phi^*\end{bmatrix}$
7	Orthogonality relations	$R^t AR = I,\ R^t BR = -\begin{bmatrix}\Lambda & 0\\ 0 & \Lambda^*\end{bmatrix}$

This study was for undamped system, in the next study we are considering the same approach inverse eigensensitivity analysis, but now for damped systems, so the objective here is to evaluate mass, stiffness, and as well as damper properties, then again study the effect of cross orthogonality, and how to carry out analysis with complex modes that is the objective, so we rewrite the equation of motion in this form AY dot + BY = F(t) and we get the mode shape, modal matrix in this form, and eigenvalues are N pair of complex conjugates and these are the mode shapes both are complex valued and the orthogonality relations are as here, so this we



have done the sensitivity analysis in the previous lecture and this is the using first order sensitivity method, this is the updating equation that we need to solve, so these are complex

Numerical illustration

Baseline model

(damping matrix is proportional)

Mass parameters M (kg)	$m_1=10,\ m_2=20,\ m_3=15,\ m_4=30,\ m_5=20$
Stiffness parameters K (N/m)	$k_1=2000,\ k_2=3000,\ k_3=1000,\ k_4=4000,\ k_5=3000$ $k_6=1500,\ k_7=2000,\ k_8=2000$
Damping parameters $C_d=[2\eta_n\omega_n]$ (Ns/m)	$C_{d1}=0.1281,\ C_{d2}=0.2113,\ C_{d3}=0.2737,$ $C_{d4}=0.3133,\ C_{d5}=0.4074$
Undamped natural frequency ω_n (rad/s)	$\omega_1=6.4027,\ \omega_2=11.7416,\ \omega_3=17.1085,\ \omega_4=22.3806,$ $\omega_5=33.9492$
Mass normalized modal matrix Φ	$\begin{bmatrix} -0.0717 & 0.0812 & 0.0126 & -0.1396 & 0.2619 \\ -0.0676 & 0.1961 & -0.0204 & 0.0696 & -0.0412 \\ -0.1032 & 0.0077 & 0.0344 & -0.1916 & -0.1344 \\ -0.1348 & -0.0691 & -0.0908 & 0.0446 & 0.0127 \\ -0.0873 & -0.0322 & 0.1904 & 0.0700 & 0.0142 \end{bmatrix}$
Modal damping ratio η	$\eta_1=0.01,\ \eta_2=0.009,\ \eta_3=0.0080,\ \eta_4=0.007,\ \eta_5=0.006$

Damped natural frequency Ω_n (rad/s)
$-0.0640+6.4024i,\ -0.1057+11.7411i,\ -0.1369+17.1079i,\ -0.1567+22.3800i,\ -0.2037+33.9486i$ $-0.0640-6.4024i,\ -0.1057-11.7411i,\ -0.1369-17.1079i,\ -0.1567-22.3800i,\ -0.2037-33.9486i$

Damped normal modal matrix t
[illegible]

valued and we separate the real and imaginary parts and arrive at appropriate simplified updating equations.

Now in the numerical illustration I again consider a 5 degree freedom system, these are the masses and stiffnesses, and these are the 5 damper elements, and undamped natural frequencies are this, and this is the modal matrix, and I am applying this method now on a proportionally damped system, so we can use complex normal mode analysis on a proportionally damped systems also, it doesn’t prevent us from doing that, so these are the natural frequencies and these are the damped natural frequencies, clearly you will see that there is a relationship between this damping ratios and the quantity shown here.

Study 2: Inverse eigensensitivity analysis for damped systems

Objectives 1. Evaluation of mass, stiffness and damper parameters

2. Study the effect cross orthogonality relations

3. Analysis with complex modes

Summary of solutions of equations of motion

1	Equilibrium equations	$\begin{bmatrix} 0 & M \\ M & C \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{x} \end{Bmatrix} + \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix}\begin{Bmatrix} x \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ f(t) \end{Bmatrix}$ $A\dot{y} + By = F(t)$ $y(0) = y_0$
2	Equilibrium equations for eigenvalue analysis	$A\dot{y} + By = 0$
3	Assumed solution	$y(t) = \theta e^{\Omega t}$
4	Eigenvalue problem	$\Omega A\theta = -B\theta$
5	Eigensolution	$\{\Omega_1 \Omega_2 \quad \Omega_N \; \Omega_1^* \Omega_2^* \quad \Omega_N^*\}$ $\{\theta_1 \theta_2 \quad \theta_N \; \theta_1^* \theta_2^* \quad \theta_N^*\}$
6	Structure of eigensolution	$\Lambda = \begin{bmatrix} \Omega_1 & 0 & 0 \\ 0 & \Omega_2 & 0 \\ 0 & 0 & \Omega_N \end{bmatrix}$ $\Phi = [\theta_1 \; \theta_2 \quad \theta_N]$ $R = \begin{bmatrix} \Phi\Lambda & \Phi^*\Lambda^* \\ \Phi & \Phi^* \end{bmatrix}$
7	Orthogonality relations	$R^t AR = I, R^t BR = -\begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda^* \end{bmatrix}$

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Now this is the modal matrix you can understand this right now it has this structure, that phi lambda, phi*, lambda*, phi and phi* so you can see that such a structure exists here, now the

Method 1 – no cross orthogonality relations

Method 2 - cross orthogonality relations included

Case 1: Damage scenario with

$$(\alpha_1 = 0.7, \alpha_2 = 0.75, \alpha_3 = 0.85, \alpha_4 = 0.8, \alpha_5 = 0.7)$$

$$(\beta_1 = 0.5, \beta_2 = 0.55, \beta_3 = 0.65, \beta_4 = 0.6, \beta_5 = 0.55, \beta_6 = 0.7, \beta_7 = 0.75, \beta_8 = 0.5)$$

$$(\gamma_1 = 0.4, \gamma_2 = 0.35, \gamma_3 = 0.5, \gamma_4 = 0.55, \gamma_5 = 0.6)$$

Case 2: Damage scenario with

$$(\alpha_1 = 0.7, \alpha_2 = 0.75, \alpha_3 = 0.75, \alpha_4 = 0.7, \alpha_5 = 0.7)$$

$$(\beta_1 = 0.45, \beta_2 = 0.5, \beta_3 = 0.45, \beta_4 = 0.55, \beta_5 = 0.5, \beta_6 = 0.45, \beta_7 = 0.6, \beta_8 = 0.45)$$

$$(\gamma_1 = 0.25, \gamma_2 = 0.3, \gamma_3 = 0.25, \gamma_4 = 0.25, \gamma_5 = 0.3)$$

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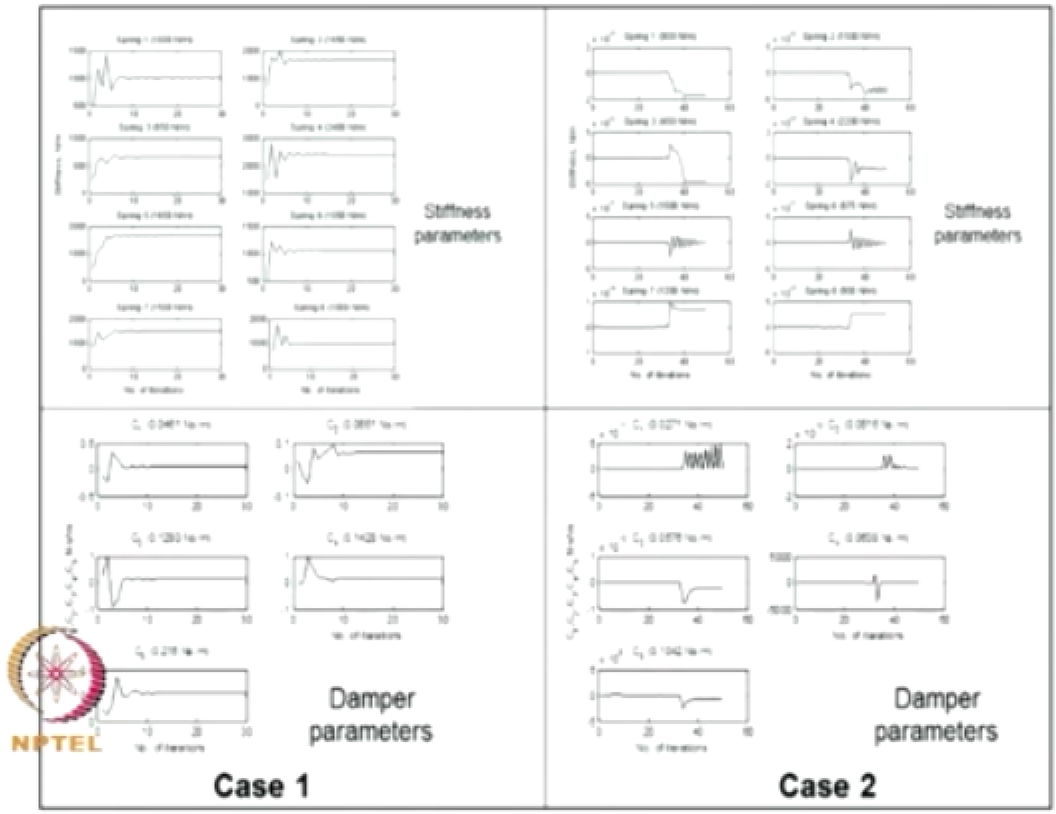
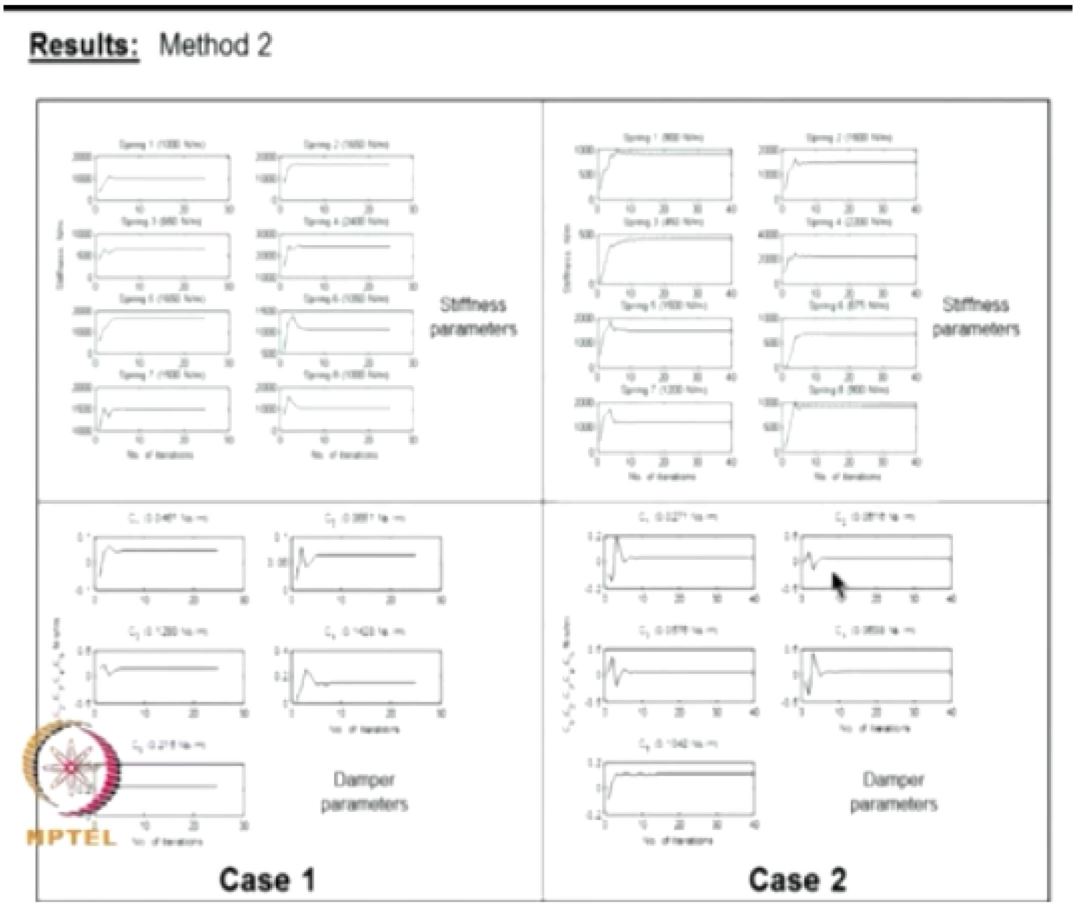
problem is we will introduce 2 damaged scenarios, we will simultaneously change the mass properties, damper properties, and stiffness properties, and there are 2 such scenarios that we are adopting, and the objective of this study is to see how the first order sensitivity method performs by including cross orthogonality relations and by excluding cross orthogonality relations.

Results: Method 1



So for method 1 the first case that seems to perform reasonably well, the second case shows certain instabilities it is not satisfactory, there are certain perturbations and so on and so forth.



So we use method 2, the solutions are much well behaved all the stiffness, parameters, damping, mass and damping parameter for both the cases are show stable behavior as iterations proceed.

Study 3: Inverse sensitivity analysis of frequency response functions

Objectives

1. Evaluation of mass, stiffness and damper parameters
2. Compare first and second order sensitivity methods
3. Assess their performance for partial measurements of FRF matrix

Receptance FRF matrix

$$[\alpha(\omega)] = \left[-\omega^2 M + i\omega C + K\right]^{-1}$$

Dynamic stiffness matrix

$$[D(\omega)] = [\alpha(\omega)]^{-1} = \left[-\omega^2 M + i\omega C + K\right]$$

Identity

$$[\alpha(\omega)][D(\omega)] = I$$

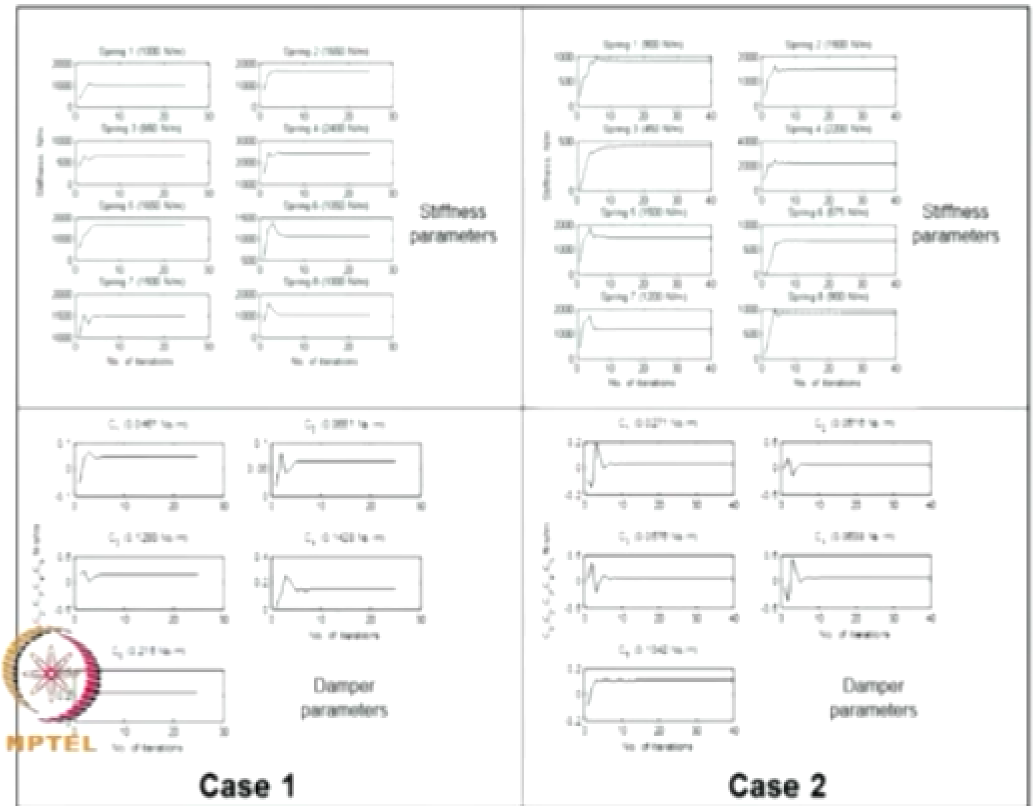
First order sensitivity

$$\frac{\partial \alpha}{\partial p_j} = -\alpha\left[-\omega^2\frac{\partial M}{\partial p_j} + i\omega\frac{\partial C}{\partial p_j} + \frac{\partial K}{\partial p_j}\right]\alpha$$

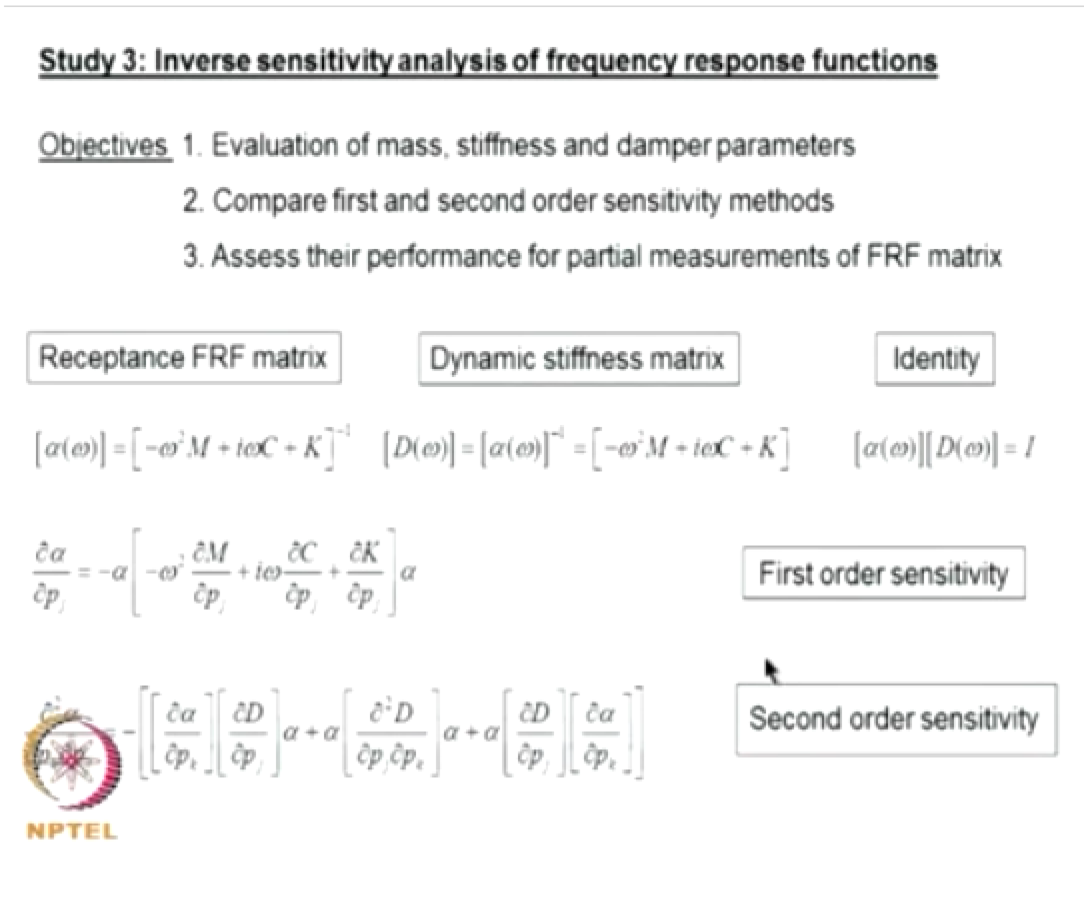
Second order sensitivity

$$\frac{\partial^2 \alpha}{\partial p_j \partial p_k} = -\left[\left[\frac{\partial \alpha}{\partial p_k}\right]\left[\frac{\partial D}{\partial p_j}\right]\alpha + \alpha\left[\frac{\partial^2 D}{\partial p_j \partial p_k}\right]\alpha + \alpha\left[\frac{\partial D}{\partial p_j}\right]\left[\frac{\partial \alpha}{\partial p_k}\right]\right]$$

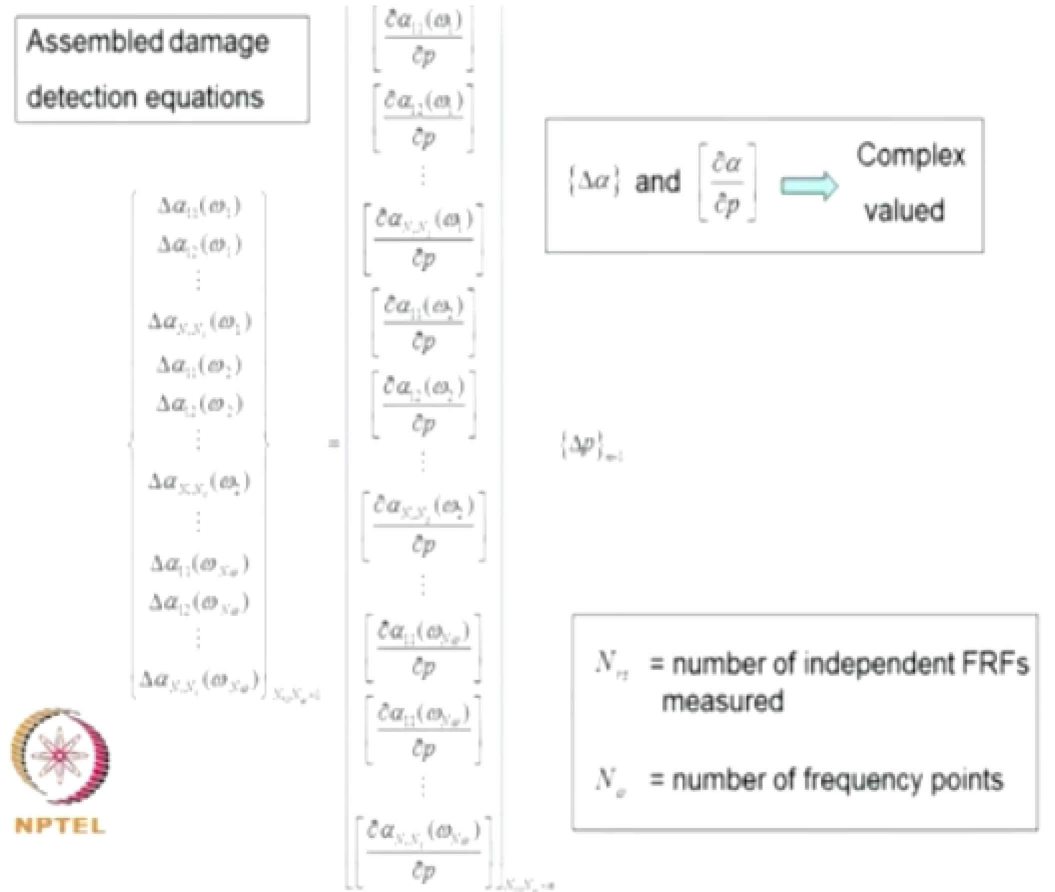
Results: Method 2



Okay here again we reach the similar conclusion that including cross orthogonality relations is helpful in getting better solutions. Now we also seen that we can perform inverse sensitivity



analysis using frequency response functions themselves, so we need not have to extract natural frequencies in mode shapes and do this analysis, we can directly do the analysis with the measured frequency response functions, this we have derived in the previous lecture so we will not repeat that. In a first order frequency response based sensitivity analysis, this is the finite



element model updating equation and we will illustrate that now, we will take a baseline model,

Numerical illustration

Baseline model: the damping matrix is taken to be diagonal

Mass parameters M (kg)	m_1=10, m_2=20, m_3=15, m_4=10, m_5=20
Stiffness parameters K (N/m)	k_1=2000, k_2=3000, k_3=1000, k_4=4000, k_5=3000, k_6=1500, k_7=2000, k_8=2000
Damping parameters C (Ns/m)	$\begin{bmatrix} 15.0000 & 0 & 0 & 0 & 0 \\ 0 & 30.0000 & 0 & 0 & 0 \\ 0 & 0 & 22.5000 & 0 & 0 \\ 0 & 0 & 0 & 15.0000 & 0 \\ 0 & 0 & 0 & 0 & 30.0000 \end{bmatrix}$
Undamped natural frequency ω_n (rad/s)	ω_1=7.7831, ω_2=12.6003, ω_3=20.0225, ω_4=25.2090, ω_5=34.1945
Mass normalized undamped modal matrix Φ	$\begin{bmatrix} 0.0988 & 0.0498 & 0.0590 & 0.1431 & 0.2526 \\ 0.1063 & 0.1811 & -0.0440 & -0.0493 & -0.0391 \\ 0.1276 & -0.0436 & 0.1066 & 0.1316 & -0.1407 \\ 0.1401 & -0.0962 & 0.1393 & -0.2203 & 0.0564 \\ 0.1085 & -0.0995 & -0.1676 & 0.0093 & 0.0110 \end{bmatrix}$

The available FRF matrix is taken to be of the size 5×3

Method 1: First order sensitivity analysis

Damage scenario: $\alpha_1 = 0.945, \alpha_2 = 0.925, \alpha_3 = 0.925, \alpha_4 = 0.9, \alpha_5 = 0.95;$
$\beta_1 = 0.925, \beta_2 = 0.92, \beta_3 = 0.925, \beta_4 = 0.935, \beta_5 = 0.93, \beta_6 = 0.95, \beta_7 = 0.945, \beta_8 = 0.94;$
$\gamma_1 = 0.95, \gamma_2 = 0.95, \gamma_3 = 0.95, \gamma_4 = 0.95, \gamma_5 = 0.95$

Method 2: Second order sensitivity analysis

Damage scenario: $\alpha_1 = 0.89, \alpha_2 = 0.6, \alpha_3 = 0.6, \alpha_4 = 0.6, \alpha_5 = 0.6;$
$\beta_1 = 0.6, \beta_2 = 0.89, \beta_3 = 0.89, \beta_4 = 0.89, \beta_5 = 0.89, \beta_6 = 0.89, \beta_7 = 0.89, \beta_8 = 0.89;$
$\gamma_1 = 0.85, \gamma_2 = 0.85, \gamma_3 = 0.85, \gamma_4 = 0.85, \gamma_5 = 0.85$

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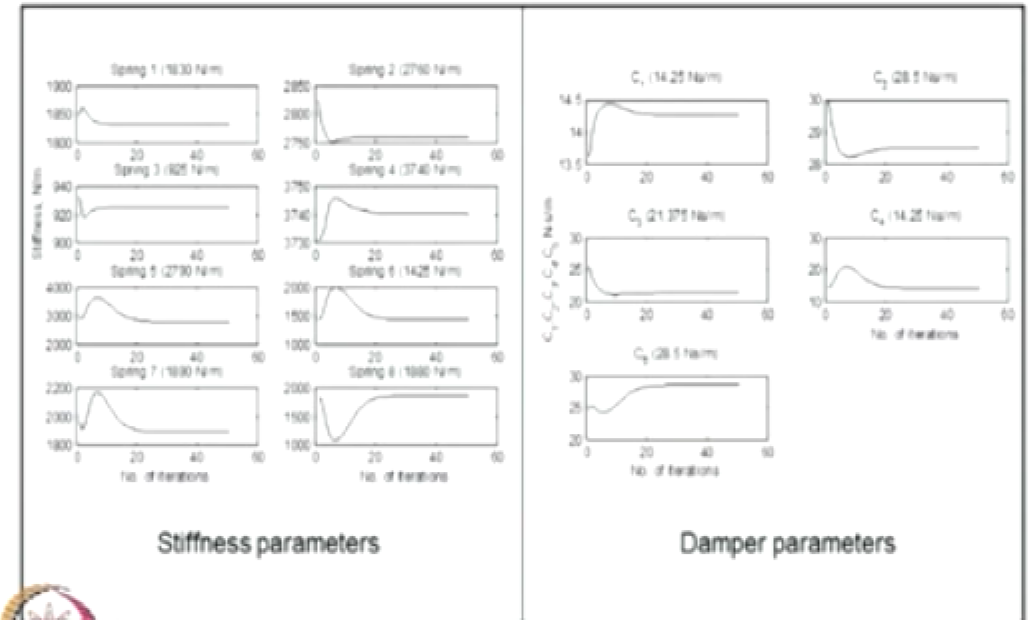
the damping matrix is simply taken to be diagonal and this is for again for illustration, these are the undamped natural frequencies and this is a mass normalized modal matrix.

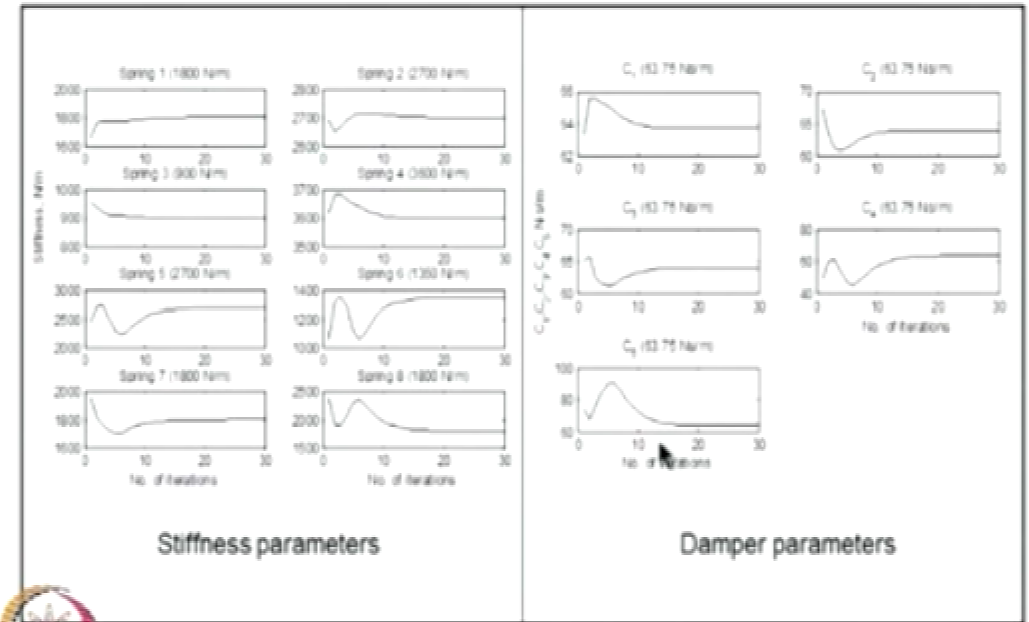
Now there is a damaged scenario as depicted here, and we are going to use first order sensitivity analysis and also we are going to illustrate the functioning of second order sensitivity analysis where we have increased the level of damage to see whether we gain any advantage in using second order methods.

Results: Method 1 (first order sensitivity)



So for the method 1 and damage scenario 1, the first order sensitive method seems to provide

Results: Method 2 (second order sensitivity)



reasonably good answers so this is fine, and second order method also provides good results, both the methods seem to perform well here.

Study 4: Inverse sensitivity of singular values of FRF matrix

Objectives:

- Damage detection in systems with repeated/closely spaced modes
- Identify damage parameters in systems
  - a) case 1:Distinct modes (before damage) and repeated modes (after damage)
  - b) case 2:repeated modes (before damage) and repeated modes (after damage)
  - c) case 3:repeated modes (before damage) and distinct modes (after damage)

Definition:

Let $B_{N_r \times N_r} = \alpha\alpha^H$ and $Q_{N_i \times N_i} = \alpha^H \alpha$

Here, $[\alpha]$ ⇒ Receptance matrix of size $N_r \times N_i$; H ⇒ conjugate transpose

Eigenvalue problem: $BX = \mu X$ for every $\omega, \omega = \omega_1, \cdots, \omega_{N_\omega}$

- the nonzero eigenvalues of B and Q can be shown to be equal
- the spectrum of these nonzero eigenvalues $\{\mu\}$ is termed as the CMIF
- their square root provide the singular values of matrix $\alpha(\omega)$
- the CMIF can be used as response feature for damage detection

Now we talked about inverse sensitivity of singular values of FRF matrix, now here we will select an example where our objective is to do damage detection in systems with repeated or closely spaced modes, and identify damage parameters in the systems, so we synthetically simulate a few situations where before damage the structure as distinct modes and because of damage the system will have repeated modes, this is an artificially simulated scenario but it is useful to see in these limits how the system performs and the solution strategy performs, so before damage the system has repeated modes and after damage also it continues to have repeated modes, and third case is repeated modes before damage and distinct modes after damage, so how these methods perform? So in state 1 there are either there is a repeated natural frequencies or not in the state 2 the similar thing with different combinations are used, so we have gone through this definition of singular values and things like that I will not repeat that, so we consider this system now, so this is again a 5 degree freedom system, now it is supported

Baseline model (damping matrix is proportional)

Mass parameters M (kg)	$m_1=300,\ m_2=300,\ m_3=300,\ m_4=400,\ m_5=100$
Stiffness parameters K (N/m)	$k_{1g}=30000,\ k_{2g}=30000,\ k_{3g}=10000,\ k_{4g}=40000,\ k_{5g}=10000$ $k_{12}=k_{13}=k_{14}=k_{15}=k_{23}=k_{24}=k_{25}=k_{34}=k_{35}=k_{45}=10000$
Damping parameters $C_d=[2\eta_n\omega_n]$ (Ns/m)	$C_{d1}=2.0000,\ C_{d2}=3.0672,\ C_{d3}=3.2660,\ C_{d4}=3.2660,$ $C_{d5}=4.6107$
Undamped natural frequency ω_n (rad/s)	$\omega_1=10.0000, \omega_2=15.3361,\ \omega_3=16.3299,\ \omega_4=16.3299,$ $\omega_5=23.0536$
Mass normalized undamped modal matrix Φ	$\begin{bmatrix} 0.0267 & -0.0181 & 0.0408 & -0.0236 & -0.0083 \\ 0.0267 & -0.0181 & -0.0408 & -0.0236 & -0.0083 \\ 0.0267 & -0.0181 & -0.0000 & 0.0471 & -0.0083 \\ 0.0267 & 0.0419 & 0.0000 & -0.0000 & -0.0054 \\ 0.0267 & -0.0047 & -0.0000 & 0.0000 & 0.0962 \end{bmatrix}$

3rd and 4th modes repeat

Method 1: Inverse CMIF sensitivity analysis

Method 2: Inverse eigensensitivity analysis of real modes without cross orthogonality

Method 3: Inverse eigensensitivity analysis of real modes with cross orthogonality

Undamaged structure with repeated eigenvalues $\quad \omega_n=(10.0000\ \ 15.3361\ \ 16.3299\ \ 16.3299\ \ 23.0536)$ rad/s

Damaged structure with repeated eigenvalues $\quad \omega_n=(9.8907\ \ 15.1685\ \ 16.1515\ \ 16.1515\ \ 22.8017)$ rad/s

Damage scenario: $\alpha_1=0.92, \alpha_2=0.92, \alpha_3=0.92, \alpha_4=0.92, \alpha_5=0.92;$

$\beta_1=0.9, \beta_2=0.9, \beta_3=0.9, \beta_4=0.9, \beta_5=0.9, \beta_6=0.9, \beta_7=0.9, \beta_8=0.9,$

$\beta_9=0.9, \beta_{10}=0.9, \beta_{11}=0.9, \beta_{12}=0.9, \beta_{13}=0.9, \beta_{14}=0.9, \beta_{15}=0.9$

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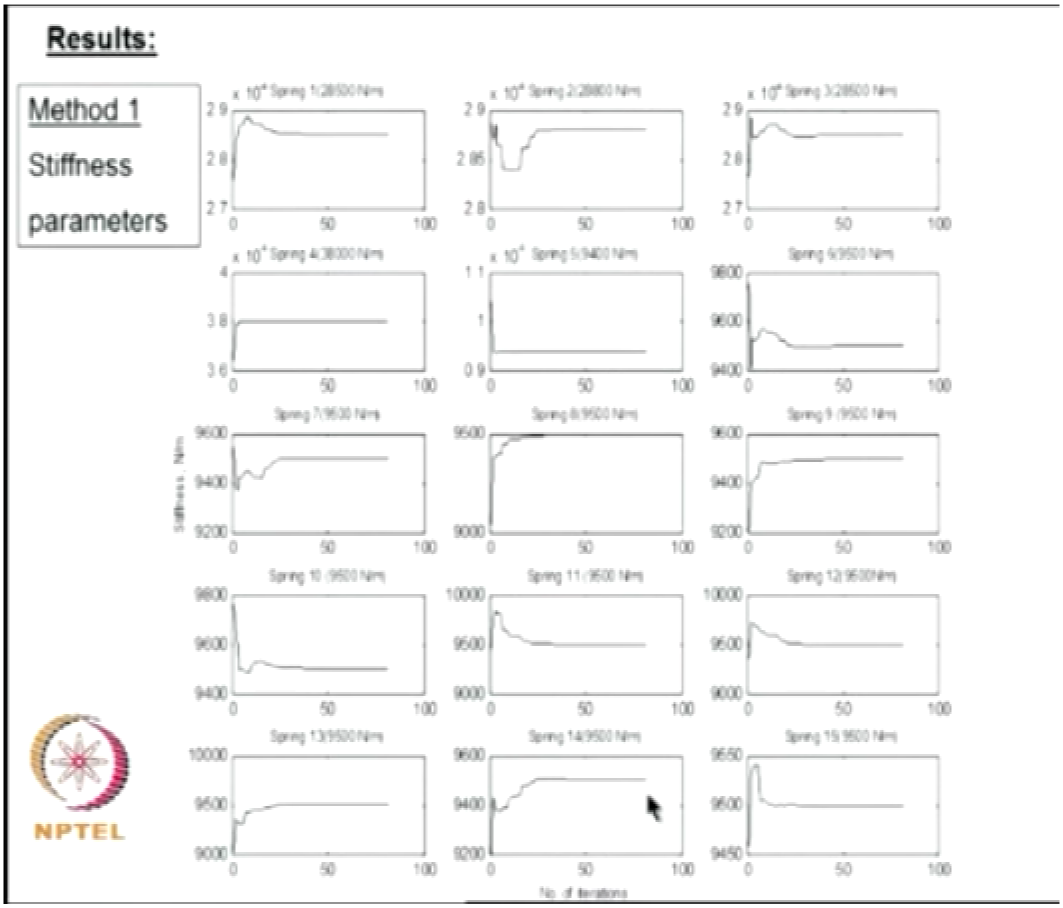
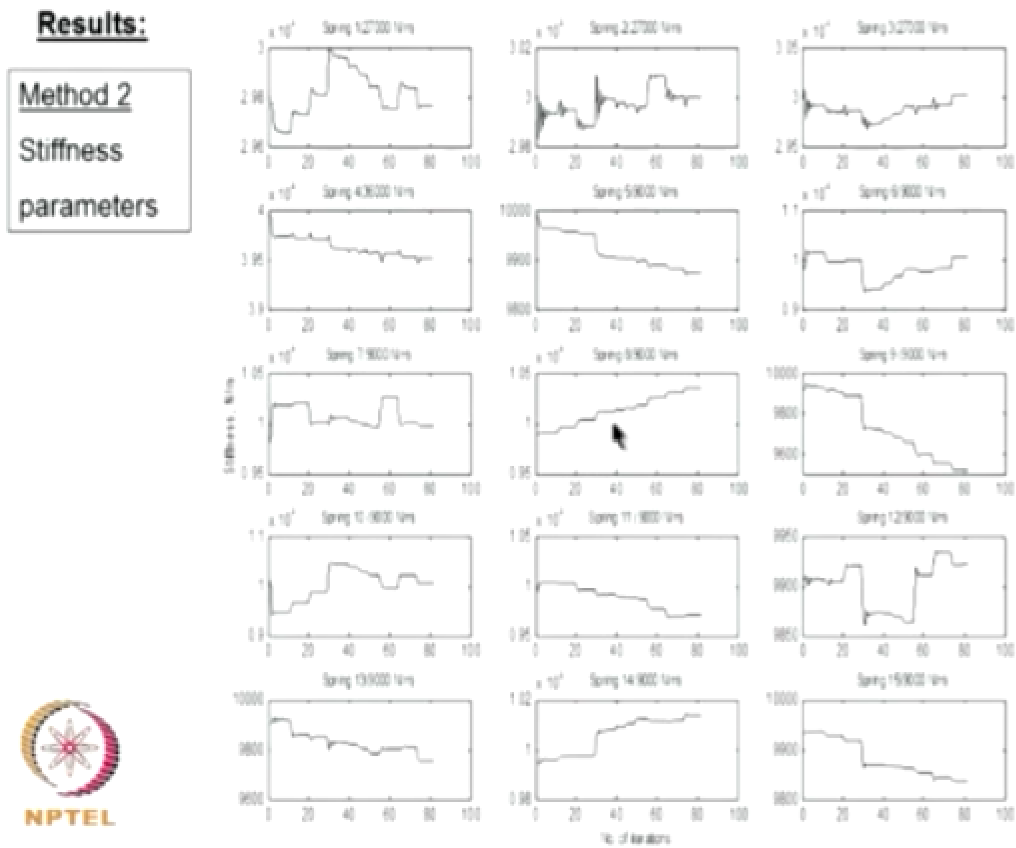
and connected in a slightly different way so we consider now the scenario where this are the properties, and in the undamped natural frequency you will see here that the third and fourth natural frequencies are repeating, okay.

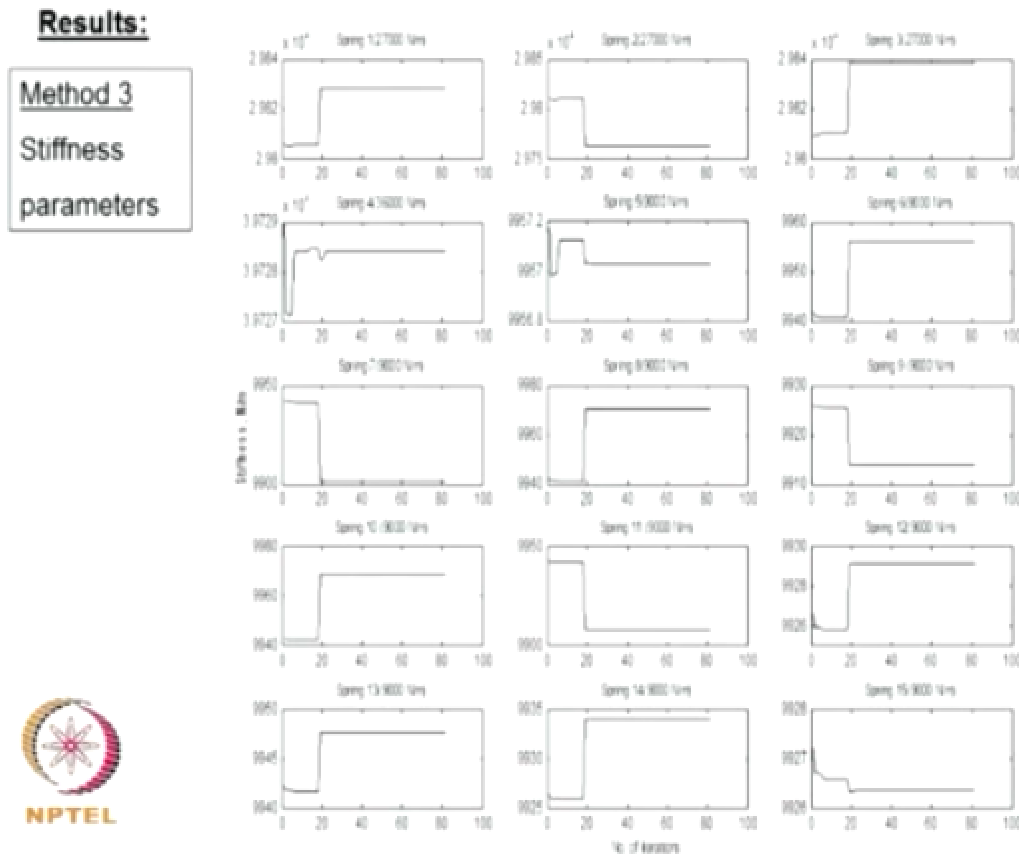
Now we will introduce a damaged scenario and here if you see, because of this damage scenario the third and fourth natural frequency still repeat but they have a different value, okay so this is in the damaged state, this is undamaged state, so we have undamaged structure with repeated eigenvalues and damage structure with repeated eigenvalue, but the natural frequencies are having different values okay, so this is how this artificially stimulated, so we



have, in method 1 it is inverse CMIF sensitivity analysis, method 2 inverse eigensensitivity



analysis of real modes without cross orthogonality and inverse eigensensitivity analysis with cross orthogonality, so method 1 seems to perform reasonably well, and method 2 there is problems here, it's not working nicely, method 3 cross orthogonality is included it seems to



converge to give the convergent answers, but we have to see whether that is right or wrong, so this is just an example so you can verify whether these results are acceptable.

Study 6: Frequency response function method

Objective: Evaluation of mass and stiffness parameters (with model reduction)

The governing equations in frequency domain for the structure in damaged state

$$\left[-\omega^2 M_D + i\omega C_D + K_D\right] X_D = F$$

If X_{DR} are the measured dofs,

$$X_{DR} = T_{0D} X_D$$

where T_{0D} is the model reduction transformation matrix given by

$$T_0 = \begin{bmatrix} I \\ -[K_{ss} - \omega^2 M_{ss}]^{-1}[K_{sm} - \omega^2 M_{sm}] \end{bmatrix} \qquad \text{Dynamic condensation}$$

$$T_0 = \begin{bmatrix} \Phi_m \\ \Phi_s \end{bmatrix} \left[\Phi_m' \Phi_m\right]^{-1} \Phi_m' \qquad \text{SEREP}$$

Now there is yet another method known as a frequency response function method, this we've not discussed earlier so we can quickly review what this is, so here it is evaluation of mass and stiffness parameters with model reduction, now let's consider the equilibrium equation of the system in the damaged state and this is the equation. Now XD's are the degrees of freedom whose size is equal to the XA which is analytical degrees of freedom but we have to do a model reduction so that degrees of freedom match and we use this transformation matrix either could

Using one of the transformations, the above equation can be recast as

$$T_{0D}^T[-\omega^2 M_D + i\omega C_D + K_D]X_D T_{0D} = T_{0D}^T F = F_{DR}$$

This can further represented as

$$\left[[U(\omega)]\ \ [V(\omega)]\ \ [W(\omega)]\right]\begin{Bmatrix}\{\alpha\}\\ \{\beta\}\\ \{\gamma\}\end{Bmatrix} = F_{1\times n}$$

here

$$U_k = -\omega^2 T_{0D}^T M_{Lk} T_{0D'} X_{DR}$$
$$V_k = i\omega T_{0D}^T C_{Lk} T_{0D'} X_{DR}$$
$$W_k = T_{0D}^T K_{Lk} T_{0D'} X_{DR}$$

The above equation for $\{\omega\} = \{\omega_i\}_{i=1}^{Q}$

$$\begin{bmatrix}[U(\omega_1)] & [V(\omega_1)] & [W(\omega_1)]\\ [U(\omega_2)] & [V(\omega_2)] & [W(\omega_2)]\\ \vdots & \vdots & \vdots\\ [U(\omega_Q)] & [U(\omega_Q)] & [U(\omega_Q)]\end{bmatrix}_{(N\times Q)\times n}\begin{Bmatrix}\{\alpha\}\\ \{\beta\}\\ \{\gamma\}\end{Bmatrix}_{n\times 1} = \begin{Bmatrix}F_{DR}(\omega_1)\\ F_{DR}(\omega_2)\\ \vdots\\ F_{DR}(\omega_Q)\end{Bmatrix}_{(N\times Q)\times 1}$$

Assembled damage detection equations

This equation can be written compactly as

$$[\Omega]\{\theta\} = \{\bar{F}\}$$

$[\Omega]$ and $\{\bar{F}\}$ here are complex valued

be condensation or SEREP, we use SEREP and substituting that I get the reduced equation and this is the equation which we will use to predict the measured FRF's, so the FRF predicted from this will not match the measured FRF, so that by writing this equation at different frequencies we can adjust the parameters of these models and get adequate number of equations to solve that, so at a frequency one value of driving frequency if you arrange these terms we will get see UK, VK, and WK are this and I am putting it here I get these equations, and if I repeat this equation for a set of Q frequencies at which FRF's are measured I get this over determined set of equations, and I get the updating equation in this form, so this is straightforward conceptually there is no problem but there is a model reduction step that is involved, so this we

Numerical illustration

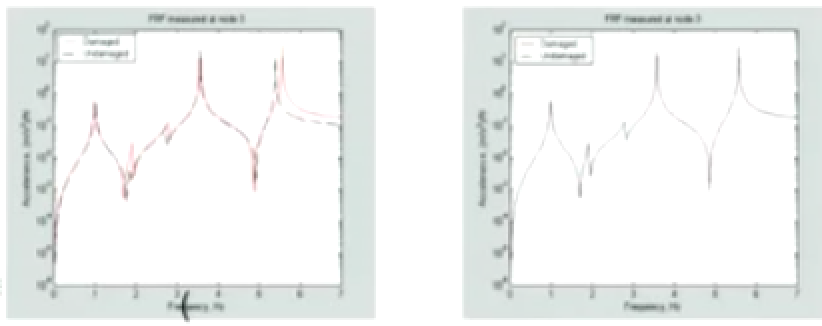
Baseline model

Mass parameters M (kg)	$m_1=10$ $m_2=20$ $m_3=15$ $m_4=10$ $m_5=20$
Stiffness parameters K (N/m)	$k_1=2000$ $k_2=3000$ $k_3=1000$ $k_4=4000$ $k_5=3000$ $k_6=1500$ $k_7=2000$ $k_8=2000$
Damping parameters $C_d=[2\eta_n\omega_n]$ (Ns/m)	$C_{d1}=0.1281$ $C_{d2}=0.2113$ $C_{d3}=0.2737$ $C_{d4}=0.3133$ $C_{d5}=0.4074$
Undamped natural frequency ω_n (rad/s)	$\omega_1=6.4027$ $\omega_2=11.7416$ $\omega_3=17.1085$ $\omega_4=22.3806$ $\omega_5=33.9492$
Mass normalized modal matrix Φ	$\begin{bmatrix} -0.0717 & 0.0812 & 0.0126 & -0.1196 & 0.2619 \\ -0.0676 & 0.1961 & -0.0204 & 0.0696 & -0.0412 \\ -0.1032 & 0.0077 & 0.0344 & -0.1916 & -0.1344 \\ -0.1348 & -0.0691 & -0.0908 & 0.0446 & 0.0127 \\ -0.0873 & -0.0322 & 0.1904 & 0.0700 & 0.0142 \end{bmatrix}$
Modal damping ratio η	$\eta_1=0.01$ $\eta_2=0.009$ $\eta_3=0.0080$ $\eta_4=0.007$ $\eta_5=0.006$

Damage scenario:

$$\alpha_1=0.9, \alpha_2=0.8, \alpha_3=0.7, \alpha_4=0.85, \alpha_5=0.6$$

$$\beta_1=0.75, \beta_2=0.8, \beta_3=0.9, \beta_4=0.95, \beta_5=0.8, \beta_6=0.7, \beta_7=0.65, \beta_8=0.6$$



a (b)

Accelerance frequency response function (drive point) (a) before damage detection (b) after damage detection

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have applied on one of the examples I am just flashing the results, I implore you to study this and maybe verify the results shown here.

So this is before updating, this is before damage detection and updating you see there are differences, and after the updating process is completed you see that the matching is perfect thereby indicating success of the updating procedure.

Summary

- Methods based on sensitivity of undamped eigensolutions can detect changes that occur in the stiffness and mass parameters.
- The other inverse methods are all capable of detecting changes in not only mass and stiffness, but also, the damping characteristics.
- Inclusion of sensitivity information with cross orthogonality relations helps in identifying systems which have higher damages
- Inverse CMIF sensitivity method successfully identifies damages in system with repeated modes while inverse eigensensitivity method is found to be unsuccessful.
- The numerical investigations have revealed that the method based on inverse eigensensitivity that includes complex nature of the eigensolutions and information on cross orthogonality, seems to perform most satisfactorily.

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Now we can summarize what we saw, so what we have seen is methods based on sensitivity of undamped Eigen solution can detect changes that occur in stiffness and mass properties, the other inverse methods are all capable of detecting changes not only in mass stiffness but also in damping characteristics potentially they are capable, inclusion of sensitivity information with cross orthogonality relations helps in identifying systems which are higher levels of damages, inverse CMIF sensitivity method successfully identifies damages in system with repeated modes, while inverse eigensensitivity method is found to be unsuccessful in such cases, the numerical investigations have revealed that the method based on inverse eigensensitivity that includes complex nature of the Eigen solutions and information on cross orthogonality seems to perform most satisfactory, except in situations where you have repeated natural frequencies and such you know exceptional situations.

Numerical and experimental investigations on structural models

Three systems considered:

1. a cantilever beam with inhomogeneous distribution of mass properties
2. a free-free beam with inhomogeneous mass and stiffness properties
3. a three-storied shear building model with inhomogeneous mass and stiffness properties.

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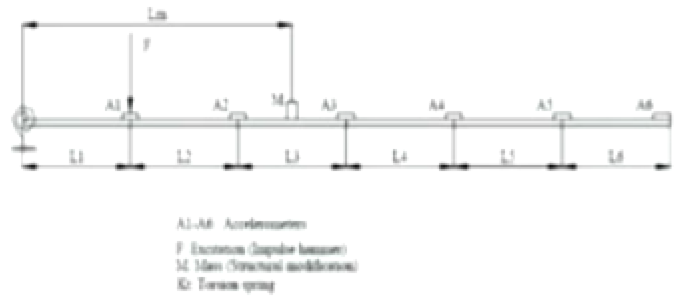
Now all these illustrations were with respect to synthetic examples where measurement was artificially simulated on a obviously overly simplified idealized system. Now finite element model updating is actually meant for studies that are actually performed in laboratory on existing structures, so to explore how the methods that we have discussed perform in a laboratory condition we have considered three problems, one is a cantilever beam with inhomogeneous distribution of mass properties, other one is a free-free beam with inhomogeneous mass and stiffness properties, and a three storied shear building model with inhomogeneous mass and stiffness property.

Structure 1: Cantilever beam



Beam properties:
L = 720 mm
E = 2.0E+11 N/m^2
ρ= 7528.9 kg/m3
Cross section: 50.6 x 6.4 mm

SCHEME	L1	L2	L3	L4	L5	L6	Lm
1	120	120	160	80	120	120	320
2	140	110	140	110	110	110	320
3	150	120	100	150	100	100	320
4	200	200	160	160	-	-	-

All dimensions in mm

Damaged structure: M is removed

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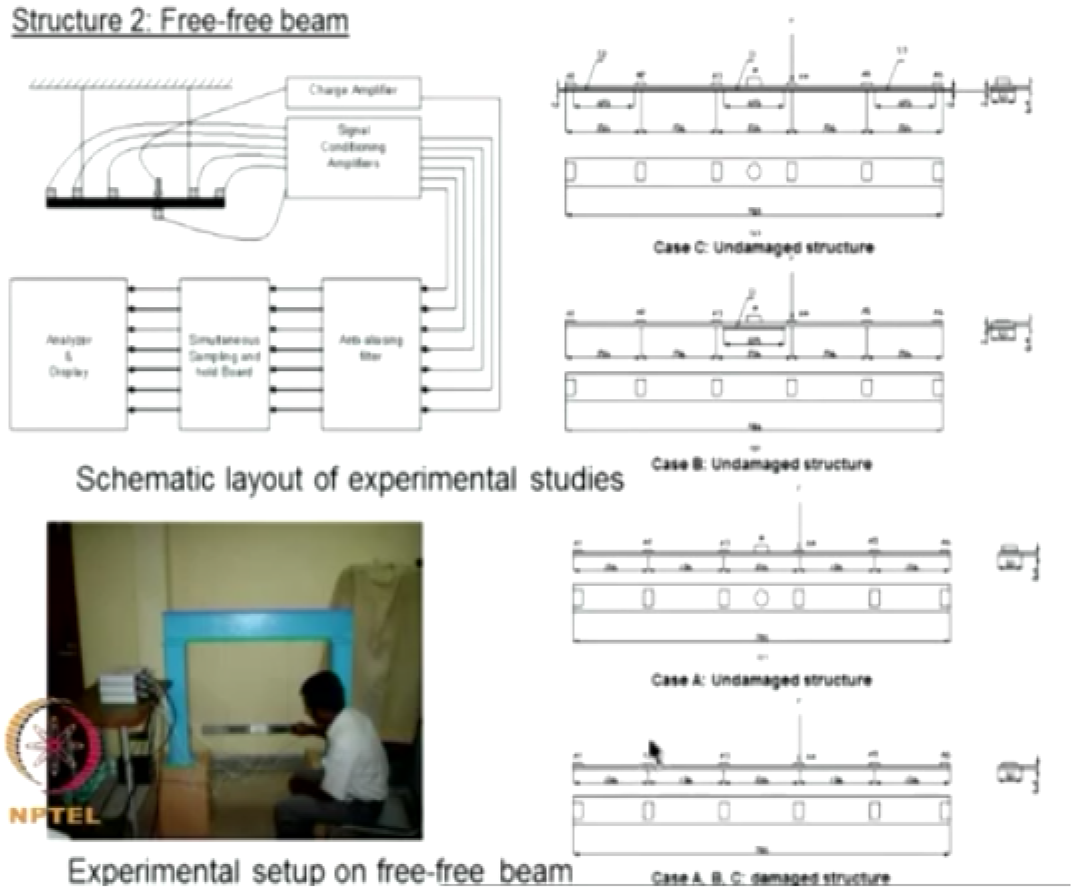
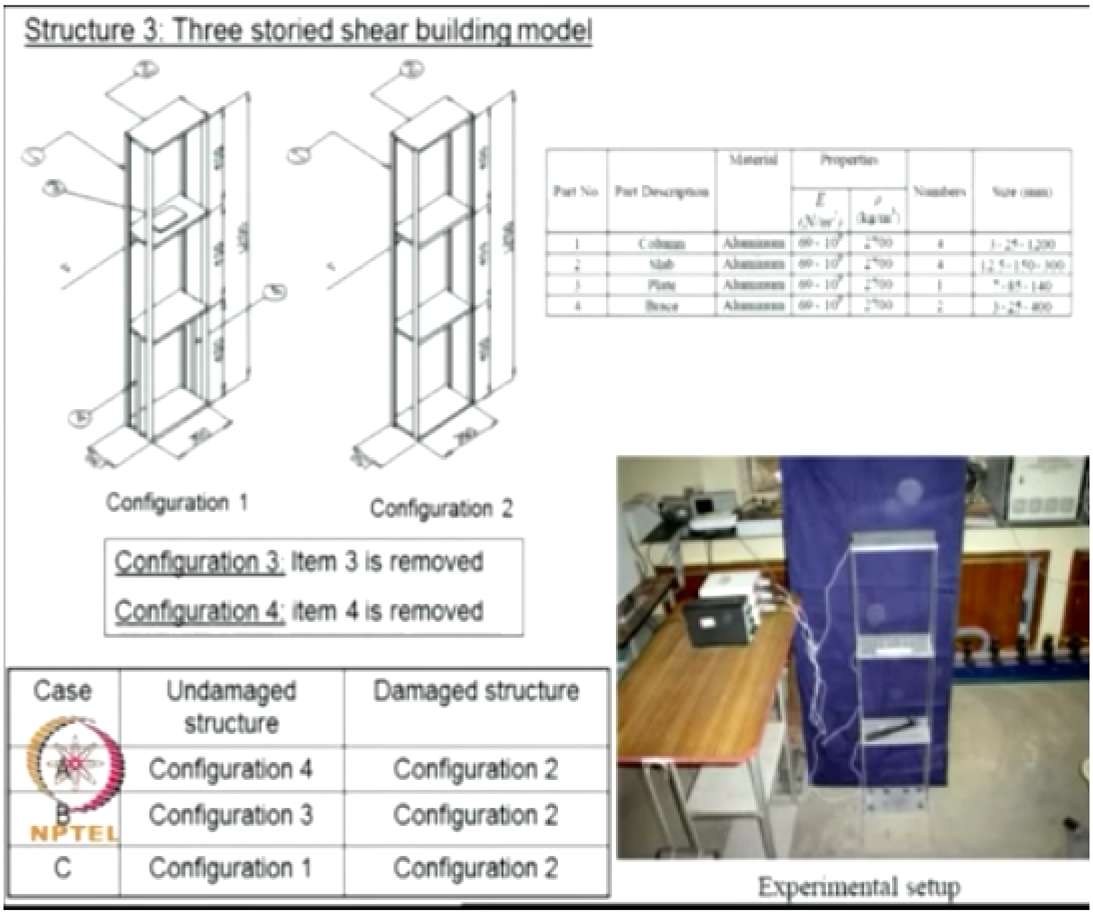
Experimental setup on cantilever beam

So in the cantilever beam, first example is a cantilever beam this is shown here you can see here this is a cantilever beam and what you see here as white you know boxes here, these are the accelerometers and the person here is hitting this beam with a instrumented impulse hammer and this is being done to measure the frequency response function and if you see carefully here the measured frequency response function is displayed here, so this is a complete experimental set up which involves the instrumented hammer, and sensors, and a computerized data processing system.

So now what we have done is there is a mass M whose position can be varied, and we assume that in the structure when it is in healthy state this mass is placed here and upon occurrence of damage this mass is removed, so we get system in 2 states and on both the, beams in both these states we perform this experiment and try to identify the properties, and the question is do we really detect whether where was the mass before it was removed, it is a self-validating exercise in the sense I know where the mass was, so it would help us to understand how the method works, there is one complication here which is invariably present in experimental work involving a fixed and pin boundary conditions, see if you see carefully see here this beam is clamped to a rigid block here with two bolts it is not clear whether this arrangement is adequate enough to deem this support conditions as being fixed. In a static sense maybe yes, but in high frequency vibration problems we are not sure whether that end is truly fixed or not, so what we do is in identification problem we add a rotary spring here to indicate that there is a partial fixity condition that we need to be aware of.



The second experiment is done on a free-free beam there is no problem of you know supporting the beam, but it is suspended through flexible wires, ropes as shown here and this person is again hitting this beam with impulse hammer to measure the frequency response functions, so there are various configurations of this beam that have been created by adding certain stiffness, so if you see carefully here there is stiffener that is added, and there is also a mass that can be moved so there are a 4 you know configurations here as shown here, and this is, A, B, C are the damaged configurations, and this is the undamaged configuration that is where all these stiffness and mass elements are shown as here.



Now the third example is on a three storied shear building frame, here there are system in two states are displayed here you can see here in this state there is a mass that is placed on this slab, and also there is a stiffener that is added in the ground floor, so by removing this we create this state, so we measure frequency response functions on these two systems and by comparing the differences in natural frequency mode shapes, FRF's etcetera we would like to identify the fact that this structure has been obtained from this structure by removing these known elements, so configuration 3 I mean we can define several configurations, in configuration 1 the stiffener and mass are present, in configuration 2 all of them are removed, in configuration 3 we can remove only one of these, either the mass or the stiffener, so we can create by different configuration, a different combination of removal of these elements, 4 different configurations, so that is outlined here and this is experimental system this is a extra mass that I mentioned and this is instrumented hammer that will be used to measure the frequency response functions.

Analytical Results: Free-free beam

Inverse eigensensitivity method

Damage indicating parameters	Case A			Case B			Case C		
	Expected vector	Detected vector	% Error	Expected vector	Detected vector	% Error	Expected vector	Detected vector	% Error
α_1	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	0.8613	0.8613	0.0000
α_2	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000
α_3	0.9335	0.9335	0.0000	0.8347	0.8347	0.0000	0.8347	0.8347	0.0000
α_4	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000
α_5	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	0.8613	0.8613	0.0000
β_1	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	0.7746	0.7746	0.0000
β_2	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000
β_3	1.0000	1.0000	0.0000	0.6881	0.6881	0.0000	0.6958	0.6958	0.0000
β_4	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000
β_5	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	0.7746	0.7746	0.0000

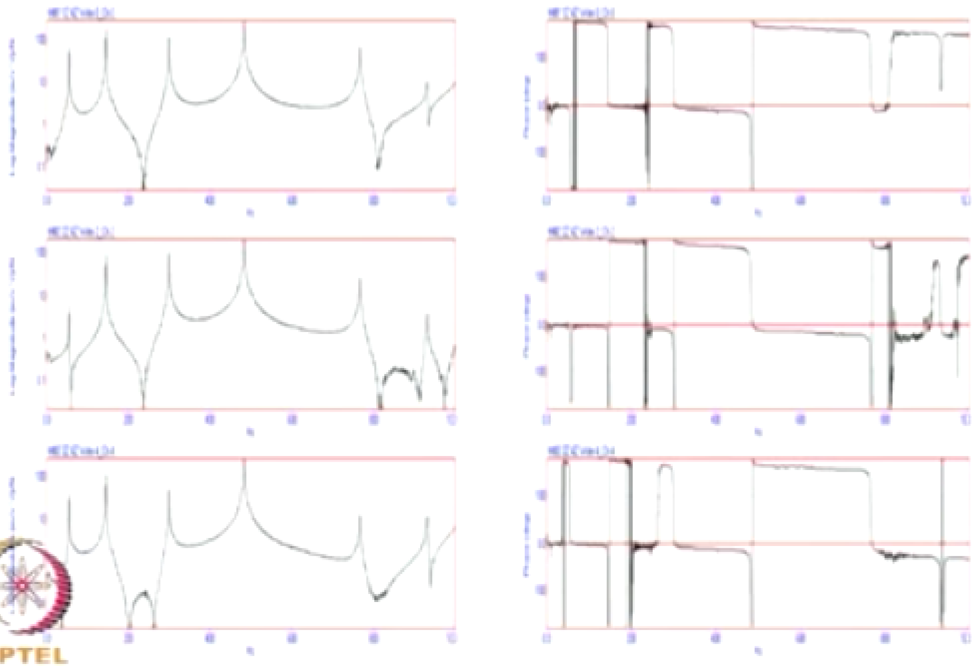
Frequency response function method

Damage indicating parameters	Case A			Case B			Case C		
	Expected vector	Detected vector	% Error	Expected vector	Detected vector	% Error	Expected vector	Detected vector	% Error
α_1	1.0000	1.0000	0.0000	1.0000	0.9999	0.0057	0.8613	0.8613	0.0006
α_2	1.0000	1.0000	0.0000	1.0000	0.9999	0.0052	1.0000	1.0000	0.0027
α_3	0.9335	0.9335	0.0000	0.8347	0.8347	0.0007	0.8347	0.8347	0.0015
α_4	1.0000	1.0000	0.0000	1.0000	1.0000	0.0048	1.0000	1.0000	0.0024
α_5	1.0000	1.0000	0.0000	1.0000	0.9999	0.0050	0.8613	0.8613	0.0003
β_1	1.0000	1.0000	0.0000	1.0000	0.9999	0.0051	0.7745	0.7745	0.0038
β_2	1.0000	1.0000	0.0000	1.0000	0.9999	0.0052	1.0000	1.0000	0.0025
β_3	1.0000	1.0000	0.0000	0.6881	0.6881	0.0011	0.6958	0.6958	0.0022
β_4	1.0000	1.0000	0.0000	1.0000	1.0000	0.0046	1.0000	1.0000	0.0025
β_5	1.0000	1.0000	0.0000	1.0000	1.0000	0.0047	0.7745	0.7745	0.0048

So we have performed the studies that I have mentioned, so I will leave this details in the nodes I leave it as a reading exercise for you to you know go through these 3 examples and see what

Experimentally measured FRFs (free-free beam)

Amplitude and phase spectrum for the 6 FRFs measured on undamaged beam at locations A1-A3



conclusions have been reached, these are the experimentally measured frequency response functions, this is the amplitude plot and these are the phase plots at different locations, and as I said I am not going to discuss all the details here we have computed the modal assurance

Details of *COMAC(i)*: Case 1: updating of baseline model; Case 2: check after damage detection

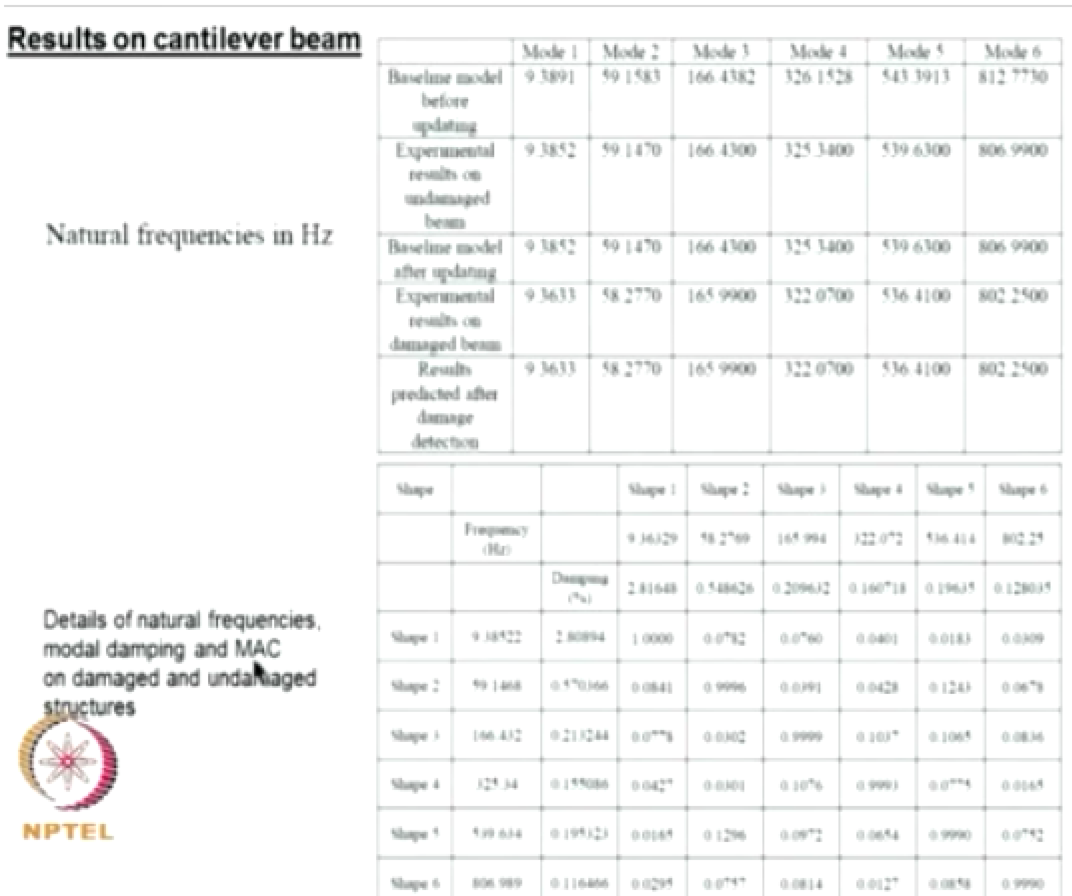
$i \rightarrow$	1	2	3	4	5	6
Case 1	0.9960	0.9972	0.9986	0.9942	0.9983	0.9926
Case 2	0.9959	0.9963	0.9982	0.9951	0.9981	0.9926

Updating of baseline model

Updating parameters	Value before updating	Value after updating	% updation
K_θ (Nm/rad)	10000	9098	9.0200
M_1 (kg)	0.3643	0.3864	-6.0700
M_2 (kg)	0.2914	0.2743	5.8800
M_3 (kg)	0.2429	0.2365	2.6100
M_4 (kg)	0.3643	0.3865	-6.1000
M_5 (kg)	0.2429	0.2325	4.2500
M_6 (kg)	0.2429	0.2380	2.0000

Results of damage detection:
cross orthogonality relations not included in computing eigenvector sensitivity

Damage indicating parameters	Scheme I			Scheme II			Scheme III		
	Expected vector	Detected vector	% Error	Expected vector	Detected vector	% Error	Expected vector	Detected vector	% Error
[illegible]	1.0000	1.0086	-0.8606	1.0000	0.9669	3.3110	1.0000	1.0121	-1.2100
[illegible]	1.0000	0.9970	0.3004	1.0000	0.9813	1.8696	1.0000	1.0017	-0.1700
[illegible]	1.0732	1.0866	-1.2512	1.0824	1.1216	-3.6171	1.1153	1.1217	-0.5738
[illegible]	1.0000	0.9521	4.7872	1.0000	0.9955	0.4541	1.0000	0.9793	2.0700
A_5	1.0000	0.9988	0.1237	1.0000	1.0498	-4.9767	1.0000	0.9899	1.0100
A_6	1.0000	1.0066	-0.6630	1.0000	0.9645	3.5514	1.0000	1.0147	-1.4700



	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Baseline model before updating	9.3891	59.1583	166.4382	326.1528	543.3913	812.7730
Experimental results on undamaged beam	9.3852	59.1470	166.4300	325.3400	539.6300	806.9900
Baseline model after updating	9.3852	59.1470	166.4300	325.3400	539.6300	806.9900
Experimental results on damaged beam	9.3633	58.2770	165.9900	322.0700	536.4100	802.2500
Results predicted after damage detection	9.3633	58.2770	165.9900	322.0700	536.4100	802.2500

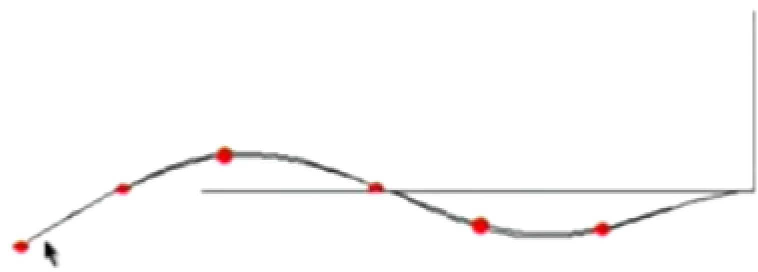
Shape			Shape 1	Shape 2	Shape 3	Shape 4	Shape 5	Shape 6
	Frequency (Hz)		9.36129	58.2769	165.994	322.072	536.414	802.25
		Damping (%)	2.81648	0.548626	0.209632	0.160718	0.19635	0.128035
Shape 1	9.38522	2.80894	1.0000	0.0782	0.0760	0.0401	0.0183	0.0309
Shape 2	59.1468	0.570366	0.0841	0.9996	0.0391	0.0428	0.1243	0.0678
Shape 3	166.432	0.213244	0.0778	0.0302	0.9999	0.1037	0.1065	0.0836
Shape 4	325.34	0.155086	0.0427	0.0301	0.1076	0.9991	0.0775	0.0165
Shape 5	539.634	0.195123	0.0165	0.1296	0.0972	0.0654	0.9990	0.0752
Shape 6	806.989	0.116466	0.0295	0.0757	0.0814	0.0127	0.0858	0.9990

criterion to pair the experimental and experimental determine normal modes and analytically determine normal modes that is also provided, the details are provided here and these are some of the results of COMAC and detection results of damage detection, so here I am showing the

Results of damage detection:

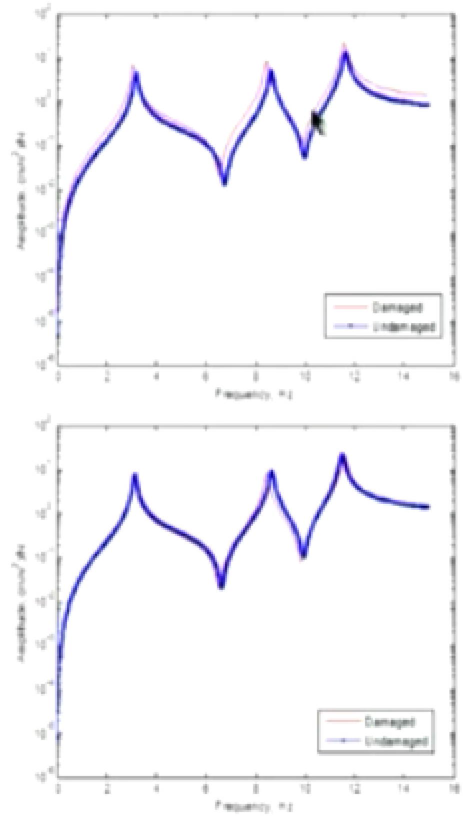
cross orthogonality relations included in computing eigenvector sensitivity

Damage indicating parameters	Scheme I			Scheme II			Scheme III		
	Expected vector	Detected vector	% Error	Expected vector	Detected vector	% Error	Expected vector	Detected vector	% Error
A_1	1.0000	1.0087	-0.8700	1.0000	0.9658	3.4200	1.0000	1.0150	-1.4997
A_2	1.0000	0.9955	0.4500	1.0000	0.9805	1.9500	1.0000	1.0032	-0.3186
A_3	1.0732	1.0880	-1.3791	1.0824	1.1233	-3.7786	1.1153	1.1201	-0.4263
A_4	1.0000	0.9516	4.8400	1.0000	0.9944	0.5600	1.0000	0.9759	2.4103
A_5	1.0000	1.0007	-0.0700	1.0000	1.0515	-5.1500	1.0000	0.9883	1.1701
A_6	1.0000	1.0052	-0.5200	1.0000	0.9630	3.7000	1.0000	1.0161	-1.6122



NPTEL Comparison of 3rd mode shape for the damaged beam obtained using experiments and predicted using detected damage parameters

experimentally determined normal mode and analytically predicted normal mode, the red one indicates the experimentally determined normal mode.



Point accelerance for the frame (Case C) before and after model updating

So these results as I said I am not going to discuss, this is for as a reading exercise for you to understand what these are, so this is for the case of the shear building model, this is how the two FRF's before you know we do damage detection and predict the behavior, appear and after the damage detection has been performed this is how it is reconciled.

Summary of results on damage detection

- The performance of damage detection algorithms when applied to synthetic data was found to be highly satisfactory
- The accuracy of damage detection when applied to experimental data varied from
  i) 0.02 to 10.7% (for beams using inverse eigensensitivity method)
  ii) 0.14 to 21.48% (for beams using frequency response function method)
  iii) 1.41 to 8.92% (for building frame model using inverse eigensensitivity method)
  iv) 0.09 to 7.7% (for building frame model using frequency response function method)

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So the summary of the findings of this investigation is that the performance of damage detection algorithms when applied to synthetic data was found to be generally very satisfactory that is to be expected, it validates the procedures developed and the way they have been coded and the method has been implemented all that is validated, this is an essential first step if you are going to do any of the implement any of these methods in practice, first you can try out all the algorithms encoding with respect to a synthetic example, where you know what is the change, where is the change, etcetera. When applied to experimental data we would not know, what is the actual, true parameters would be unknown, so whatever the method tells we have to you know except in some sense.

Now since we move in these examples what was the change that we have made to the structure, because we added a stiffener removed a stiffener added a mass remove the mass etcetera, etcetera, we have some idea of what is the changes that we are making, so based on this it is concluded that about 0.02 to 10.7% is the accuracy for beams using inverse eigensensitivity method, and this is shoots up for FRF based method, and inverse eigensensitivity for the building frame, this is the error that we observed, and this is based on FRF's, so there is no definite recommendation that we can make which method works so it seems to, it is fair enough to say that the errors that we encounter depends on the situation, all the methods can potentially lead to equally good results if applied correctly.

Other FE model updating methods

- Direct matrix method
- Eigendynamic constraint method
- Optimization approach
- Bayesian filtering

Before I close this discussion I would like to briefly touch upon a few other a finite element updating methods, we have basically discussed inverse eigensensitivity and FRF based, frequency response function based methods, but there are other strategies also so I will quickly review them, the first one is known as direct matrix method, so here what we do is for example

Direct matrix method

Find elements of M_X which minimize

$$\delta_M = \left\| M_A^{1/2} (M_X - M_A) M_X^{1/2} \right\|$$

or

$$\delta_K = \left\| K_A^{1/2} (K_X - K_A) K_X^{1/2} \right\|$$

subject to the constraints

$$M_X^T = M_X$$

$$K_X^T = K_X$$

$$\Phi_X^t M_X \Phi_X = I$$

$$\Phi_X^t K_X \Phi_X = \Lambda_X$$

Remarks

- Physical connectivity of structural model may not be honored
- $M_A - M_X$ & $K_A - K_X$ are not guarnteed to represent physically meaningful quantities

68

MX is the mass matrix for the experimental model which is not known, we assume that we propose that the elements of MX can be determined by minimizing a metric delta M as defined here, so MA is analytical mass matrix, MX - MA can be you know is defined, it is the error and

this is not known, MX is not known, so elements of MX are the variables of optimization, similar statement can be made for stiffness matrix also.
So now if you are focusing on finding elements of MX which minimizes delta M we need to put some constraints, for example mass matrix has to be symmetric, stiffness matrix has to be symmetric, and these orthogonality relations must be satisfied, where phi X and lambda X are experimentally determined. Now this is one of the earliest methods of finite element model updating that was developed, but the problem here is the physical connectivity of structural model may not be honored, if you are simply changing elements of mass matrix you may get a nonzero entry in the mass matrix which is not supported by the actual connectivity that you see in the existing structure, so that is not imposed as a separate constraint, and also the changes that we observe that is MU – MD and KU - KD are not guaranteed to represent physically meaningful quantities, so this is not quite satisfactory, a remedy to some extent to this can be

Eigendynamic constraint method

Consider the i^{th} finite element

$$[m^i]_X = (1+a_i)[m^i]_A$$

$$[k^i]_X = (1+b_i)[k^i]_A$$

$$i = 1, 2, \cdots, N_{elem}$$

$$\{\Phi_X\}_j^t [M]_X \{\Phi_X\}_j = 1$$

$$[K]_X \{\Phi_X\}_j - \Lambda_{X_j} [M]_X \{\Phi_X\}_j = 0, \; j = 1, 2, \cdots, N_{modes}$$

$$\Rightarrow$$

$$[A]\begin{Bmatrix} a \\ b \end{Bmatrix} = \{B\}$$

NPTEL

69

formulated as follows, so what we can do is we if there are N elements, that is N ELEM number of finite elements in your analytical model we can assume that each of these analytical mass matrix of the analytical financial model need to be corrected by a factor AI to obtain the corresponding mass matrix in an experimental model which is not known actually, so AI is our scalar factors associated with each element.
Similarly this is BI with stiffness matrix, now what we can do is we can formulate the experimental mass and stiffness matrix, in terms of these unknown AI's and BI's and would have measured the normal mode say phi X is measured so we can substitute them into this equation, and see we will get a set of equation from the 2 orthogonality relations and they can be recast in this form, that means we are actually constraining these AI's and BI's by the orthogonality relations of the normal modes and the relationship between natural frequency and

mass matrix, sorry stiffness and mass matrix and the eigen solution, so this leads to by suitable manipulation a set of over determined equations, and this is what you have to solve by doing maybe pseudo inverse seeing with regularization and so on and so forth.

Optimization approach

Objective function

$$J = \sum_{i=1}^{q}\left[\omega_{iE} - \omega_i(p)\right]^2 + \sum_{l=1}^{R}\sum_{i=1}^{s}\left[\Phi_{iE}(l) - \Phi_i(p,l)\right]^2$$

l = location

q=number of experimentally measured natural frequencies

s = number of experimentally measured mode shapes

R=number of dofs at which mode shapes are measured

p=parameters to be determined

In a more direct approach suppose you have measured Q number of natural frequencies and R number of mode shapes, or S number of mode shapes at R number of spatial coordinates, so we can define a metric of differences, so this is the sum of squares of observed differences between the experimental and analytical natural frequencies, this is the sum of squares of difference, observed difference between experimental model and analytical model summed over all modes, summed over all the space, so J is a positive quantity because we all square at this. Now this J will be function of the unknown system parameters so what we can do is we can optimize J, we find P which minimizes J, so unsuitable constraints that can also be imposed in solving these equations, those constraint reflecting the basic dynamical characteristics of the structure, so the

Other FE model updating methods

- Direct matrix method
- Eigendynamic constraint method
- Optimization approach

- Bayesian filtering

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other method that I briefly mentioned here the Bayesian filtering method I remarked in the previous lecture, this is one of the most powerful methods, this is based, this has roots in probabilistic methods, we apply Bayes theorem we treat the mathematical model and the measurements as being imperfect, and we use probabilistic arguments and derive the posterior probability density function of system parameters conditioned on the measurements made, so in this approach all the system parameters are treated as random variables, so this can be applied to linear problems, nonlinear problems, and in fact the resulting equations can be solved using Monte Carlo simulations, but as I already said the scope of this method is outside the per view of this course, so we will not be elaborating further on this.

So with this brief overview we will close our discussion on finite element model updating, and in the next remaining part of this course we will look at some problems of nonlinear structural dynamics problems, so how to deal with non-linearity especially in the framework of finite element formulations, what really happens? Can we still formulate element matrices? Can we still assemble? How do we solve the equations? All these questions can be posed and we will find suitable answers to those questions. So that we will take up in the next lecture, we will close this lecture at this point.

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