Water Resource Systems

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Module No # 08

Lecture No # 39

Crop Yield Optimization (2)

Good morning and welcome to this lecture number thirty nine, of the course water resources systems, modeling techniques and analysis.

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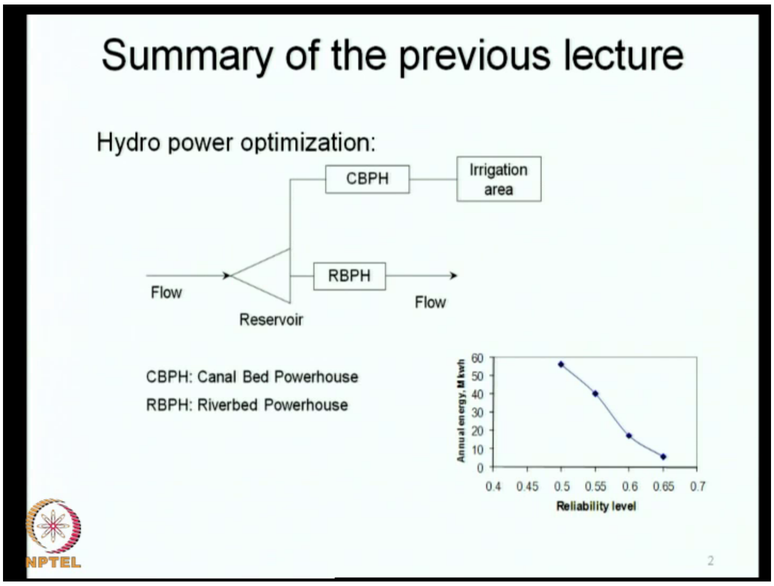

In the last lecture, we essentially looked at hydro power optimization problem where you have a reservoir and then you have a canal bed power house and from the canal bed power house, the water goes into irrigated area. You also have a riverbed power house. And, the idea here is to maximize the power generated at the riverbed power house, subject to meeting the irrigation demands at pre specified reliability. So, we will say that the irrigation has to be met with a minimum reliability of let us say 80 percentage, 70 percentage, 90 percentage and so on. And then, subject to that condition, we maximize the hydro power.

And, we do this for different levels of reliability and then generate the associated power. So, these are the reliability levels of meeting the irrigation demands. And, the associated maximized power is what we get, as a result of the model solution. Now, this is what we did in the previous class. Remember, we did not consider the canal bed power house optimization in the model; however, you can also include that in some sense. But, in the model that I discussed in the previous class, we essentially looked at maximization of the riverbed power subject to the conditions that the irrigation demands are met to a given reliability level. At least to that reliability level, the irrigation demands are met. And, we formulated the chance constraint reliability, chance constraint optimization problem, which was the linear programming problem. And then, we obtained a trade of such as this. That is, as you want to increase the hydro power, your reliability of meeting the irrigation demands will become smaller and smaller as it is seen here.

In today's lecture, we will discuss another important problem namely the crop yield optimization. If you recall in the earlier lectures where I discussed the reservoir optimization problems, we lumped all the demands of the irrigations. And then, we talked about D t as the demand during the time period t. Now, these were the lumped demands, the total demands at the crop level to be met from reservoir releases. Now, these are the operational problem that we have discussed earlier. That is good for planning purposes, where we want to plan for reservoir operation over a period of one year or long term operation over a period of ten years and so on.

So, they are actually planning models where we will lump all the demands. And then, look at the operating policy for the reservoir or the canal operating policy, where you will meet the lump demands. However, the actual problem is one of crop yield optimization; that means you want to apply water to the crops in the quantity that is demanded during different time periods of the crop growing season. And, this is where we are talking about crop yield optimization. That means, the value of the water that you put to the crop in terms of the yield that you get at the end of the season and at the end of the crop growing season.

Remember, here because we are talking about deficit water supply and then you want to get the best out of water that you apply. It is, as if almost every drop of water that you apply to the crop, you want to get the best out of that. And, that is where we start talking about crop yield optimization.

And then, we will today look at some simple models where we can optimize the crop yield, subject to the several constraints that we have on the water availability. You may also have multiple crops in which multiple crops are competing for a given amount of water which is in deficit supply. By deficit supply I mean, the total amount of water available is less than the total demand. This may happen during different intra seasonal periods. In some of the periods, you may have excess of water, but in most of the periods you may have deficit water supply in which the crops are actually competing for that available amount of the water. Now, that is the problem that we will look at now. Now, there are few concepts that you must understand before we go to the crop yield optimization.

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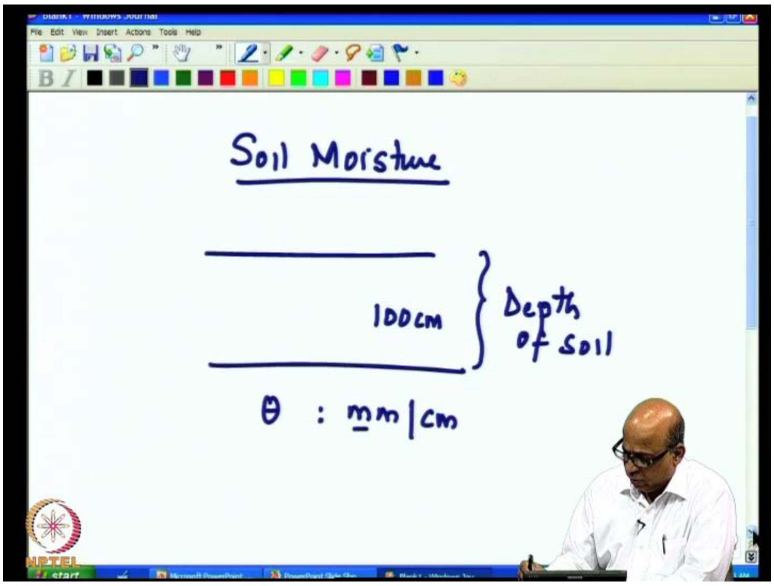

So, we just will look at the soil moisture, some preliminary definitions first because now we are looking at the crop water demands and the crop water utilizations. Typically, we express the soil moisture in a depth of soil, let us say, this is the depth of soil and typically we take this as the root depth. So, if my scale is along the depth of the soil, we will say the soil moisture will indicate as theta. We, majority in terms of millimeters per centimeters, centimeter of depth; what we mean by that is, let us say this is a root depth and this may be of the order of 100 centimeters and so on. So, per centimeter you may have so much millimeters of water. So, depth per unit depth of soil is what we express. And, this can be also expressed as percentage.

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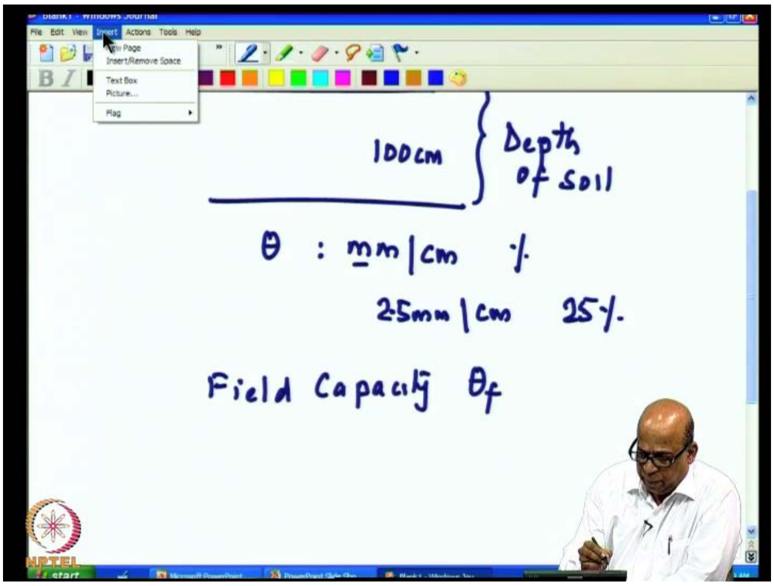

For example, we may say the soil moisture is 25 millimeter per centimeter or it may 2.5 millimeters per centimeter or it is 25 percentages. Then, this is all the depth scale is what I wrote. Now, there is a concept of field capacity. And, field capacity, we denote it as theta f in our notation. The field capacity is the maximum soil moisture that the soil can hold maximum moisture that the soil can hold against gravity.

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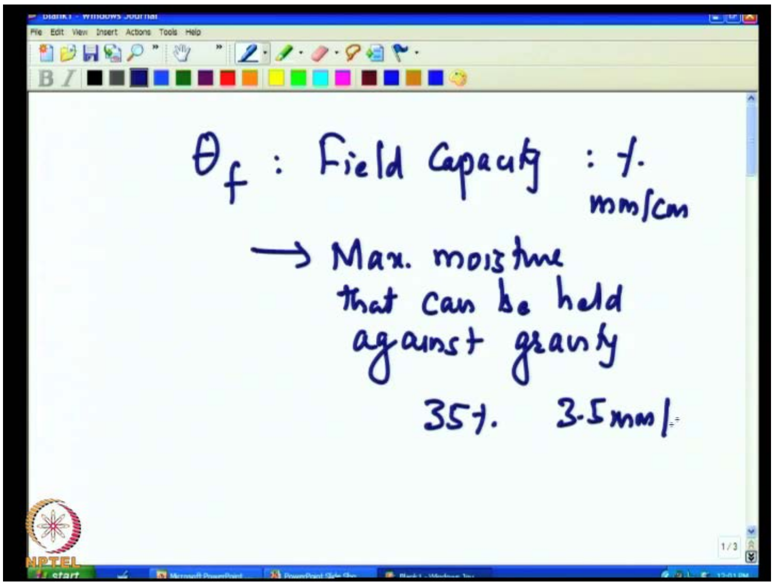

So, the field capacity also we express as, theta f is the field capacity. This is again expressed as either percentage or in terms of millimeters per centimeter. This is the maximum of soil moisture, maximum moisture that can be held against the gravity. What I mean by that is, you take the root depth of the crop and then you start applying water. Let us say it was in the dry condition, you start applying water. Now, the moisture will be held by the… and then as it exceeds the field capacity, it starts going down as deep percolation. And then, depending on the soil characteristics it may rise above the field capacity, temporarily it may go up to saturation. But, eventually the field capacity is the maximum moisture that the soil can hold against gravity. And, typically for example, if you are looking at black cotton soil and so on. You may have this as 35 percentages; which means 3.5 millimeter per centimeter and so on. So, these are the order of figures that you get here.

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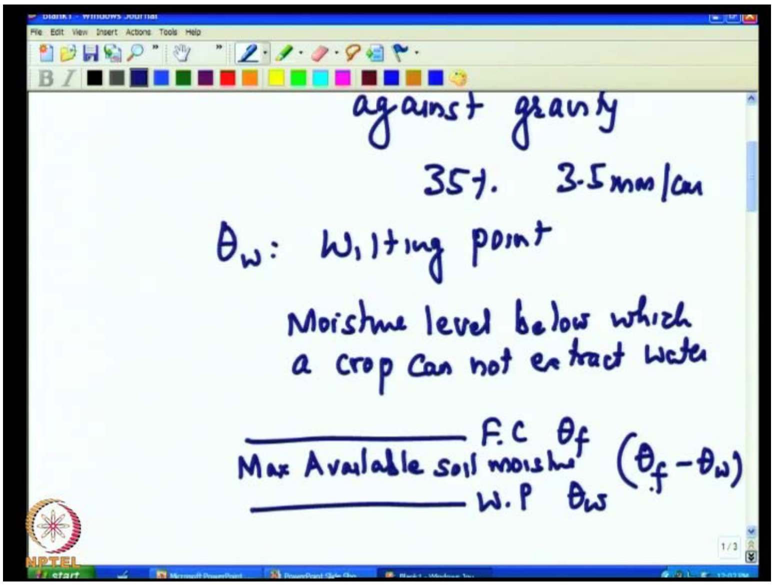

Then, you have what is called as the wilting point. This is denoted as theta w. Now, wilting point is the soil moisture level or moisture level I will say, below which the crop cannot extract water, moisture level below which a crop cannot extract water. Now, these are related to moisture stress level. So, moisture stresses and so on. So, we do not too much worry about the details of this.

What happens is, if we look at the moisture scale now, so we have a concept of field capacity F C, theta f, I denote and we have a wilting point; we denote it by theta w. The moisture within this range between the wilting point and the field capacity is available for the crop. And, it is called as the available soil moisture. And, in fact between the field capacity and wilting point there is the maximum available soil moisture.

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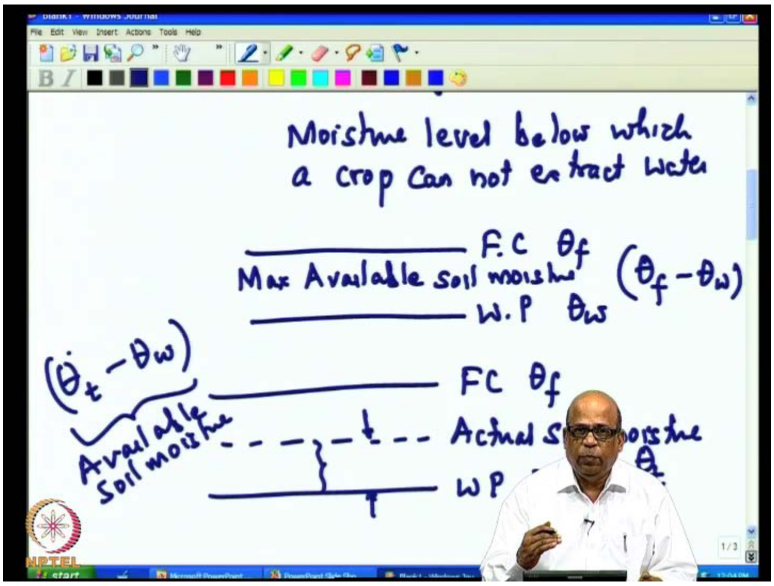

So, we denote this as maximum available soil moisture, which is theta f minus theta w. The actual soil moisture can be anywhere within this. So, we may have field capacity here and you may have wilting point here, this is theta f and this is theta w. The actual soil moisture may be here. And, this we denote it as theta.

And, in some, in the models, when we are looking at the moisture variation from time period to time period, we may put a subscript here theta t to indicate that, that is the soil moisture at the beginning of the time period t. Corresponding to the actual soil moisture, what will be the available soil moisture? The available soil moisture corresponding to the actual soil moisture will be the moisture level above the wilting point. So, theta t minus theta w or the actual soil moisture minus the wilting point gives the actual available soil moisture or we will say simply the available soil moisture.

By this, I mean the plant can extract this level of soil moisture. From the actual soil moisture to the wilting point, this much is available for the plant to extract and use it for its growth. So, that is the available soil moisture. So, these are the preliminary concepts of various definitions with respect to the soil moisture. Now, when we are looking at the crop yield optimization, we are applying the water at the crop level. And therefore, we must know how much is the available soil moisture for the crop is and then to what extent should be able to; to what extent we should raise the moisture for that particular crop. And as I said, there are several crops in the irrigated area. All of which are competing for every deficit water supply and we are optimizing the water allocation to various crops such that, the total yield that we get out of the water allocation is the maximum.

Now, this is the problem of crop yield optimization. There are two levels of problems. One is a known amount of water available for the entire season, to be allocated to a single crop. Let say there is a single crop of cotton, wheat, jowar or some particular crop, whose sowing date is known, whose length of the season is known and then the total amount of water available for the entire season is known. We want to distribute the total amount of water among intra seasonal periods for this particular crop such that, the yield that you get at the end of the season is, maximum. That is the first level of the problem.

The second level of the problem is the total amount of water available that will be allocated to a number of crops across the entire season is known and then you want to allocate the water among all this crops, among intra seasonal period. So, there is the competition for water among different crops.

The third level of problem is that the water available during intra seasonal time periods fixed is known. That, you want to integrate crop yield optimizations with the reservoir operation; which means, the problem becomes one of allocating a known amount of water during individual intra seasonal periods, which itself varies; that means a known amount of water itself varies among different crops. So, there are three different levels of problems. We will consider one by one. But, the basic of this is to do with the soil moisture. That is, how the soil moisture varies with respect to the amount of water that is available.

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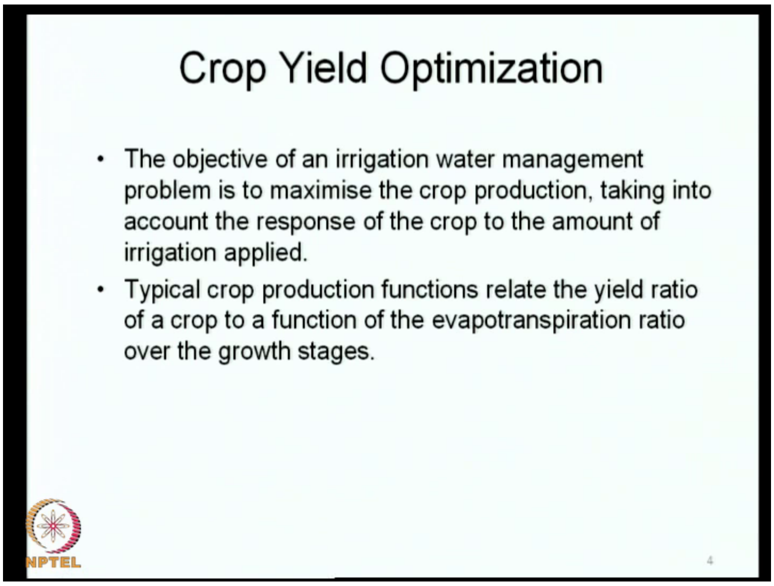

So, we do the crop yield optimization with an objective of maximizing the crop production function. Now, when we are maximizing the crop production function, we take in to account the crop response to the water deficit that occurs during different time periods. The crop response to a known amount of deficit will be varying across time periods. For example, the crop, let say it is sown during the first week of June, the response of the crop to a given deficit in the month of June or during certain period in June will be much different from the response of the crop after it has grown let say for two weeks, four weeks and so on.

So, the response of the crop to the given deficit of moisture will be different at different time periods and we need to model this. We also use, what are called as the crop yield production functions or simply crop production functions as majors of the crop yield, as a function of the amount of water that is applied. And typically, the crop yield production functions relate the yield ratio; that means, what is the maximum accepted with respect to the moisture only. We are assuming that all other conditions are conducive only with respect to the water applications. What is the maximum crop yield you can get with respect to the ratio of actual yield that you get with the maximum crop yield; that means, if you are able to supply water at the optimum rate all through the crop season and then that is what you will result in the maximum yield.

But, if you are not able to supply the water at optimum rate, but you are supplying at deficit rate during certain periods, you will get the actual yield which will be less than the maximum yield, the ratio of the actual yield to the maximum yield. This ratio is related to the ratio of the actual evapotranspiration and the potential evapotranspiration in most of the crop production functions. So, we use such crop production functions and then optimize the crop yield. As we progress, these things will become clearer.

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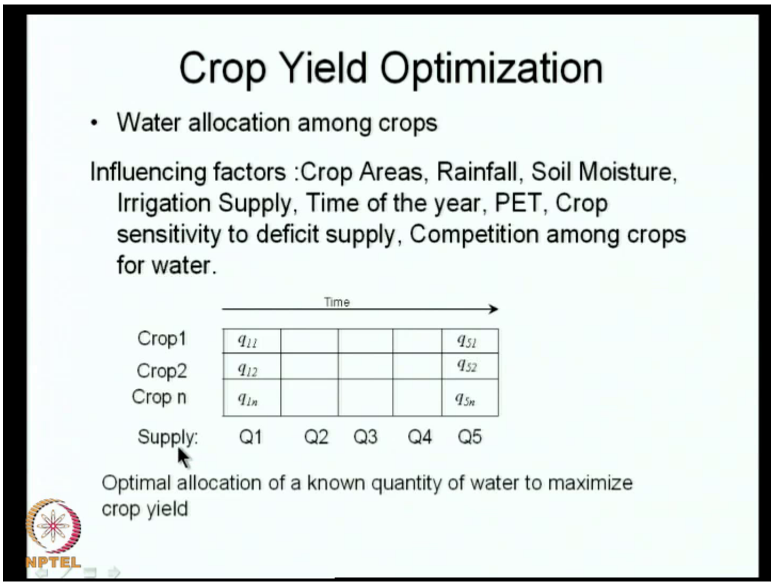

So, when we are applying crop water, let us say that the supply during this various time periods are known. Time period one, time period two, etcetera, Q 1, Q 2, Q3 etcetera, are all known. If these supplies are known, then you want to allocate among various crop during different time periods. So, crop one will get q 1 1, q 1 2, q 1 3 etcetera q 5 1 etcetera.

There are q1 1, q 2 1, q 3 1, q 5 1 five time periods. Similarly, the nth crop will get q 1 n to q 5 n like this; such that, the total yield that we get out of all the crops is maximized. Now, when we are doing this, obviously the influence in factors will be the crop areas, the relative areas. For example, crop one may have much larger area compared to crop n. This may be cotton, this may be wheat, and this may be jowar or some other things. It will also depend on the rainfall during this various time periods. It will depend on the soil moisture during the various time periods. That is, the crop yield will also depend on the irrigation supply that how much amount of supply that you have provided, time of the year, what is the deficit across the during various time periods in the year, the potential evapotranspiration across the time periods, the crops sensitivity to deficit supply. as I said, the crop response or the crop yield response to a deficit supply will vary across the time for the same crop.

The crop may be extremely sensitive to deficit during certain time periods, but it may not be as sensitive during certain other time period. In which case, you can afford to have a deficit during those time periods, in which it is not extremely sensitive. So, we need to understand which are the time periods in which the crop, in which a particular crop is sensitive. And, this time period in which the crop is extremely sensitive will vary across the crop sensors. For example, one crop may be sensitive during this time period; the other crop may be extremely sensitive during some other time period and so on. So, we need to account for such varying sensitivities or varying responses of the different crops across the time periods for a deficit supply. Then, there is a competition among the crops for a given amount of water.

What do I mean by the competition? There is a known amount of water that is available, all the crops are competing for the same amount of resource that is available and you need to look at the competition among different crops and get the maximum eight yields out of the water that is available. So, that is the broad problem that we are looking at.

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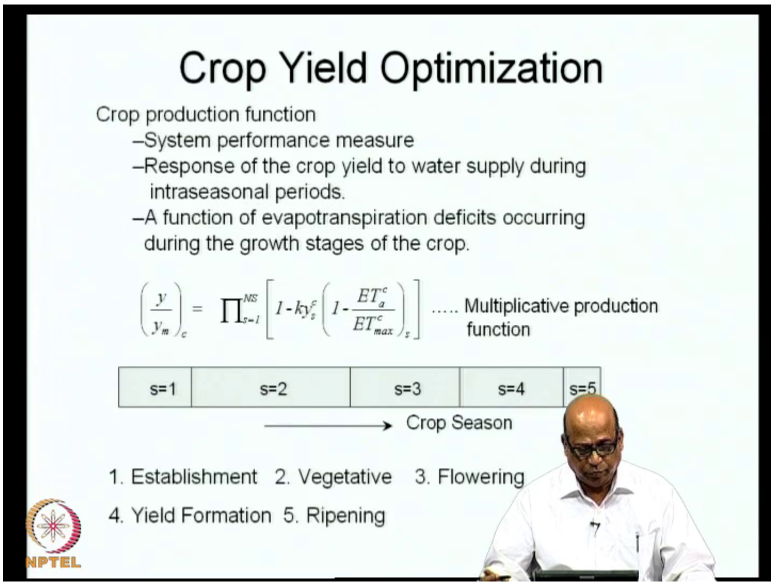

As I said, we use the crop production functions as major of the crop yield. Now, typically the crop production functions as we use in water resources are relating, are related to the actual evapotranspiration, potential evapotranspiration ratio. So, this is called as the ratio of the actual evapotranspiration to the potential evapotranspiration. The potential evapotranspiration, as you know from your basic hydrology, is the maximum evapotranspiration that occurs when all the conditions are conducive for the crop growth.

When the soil moisture as far as the water resources are concerned, we will concern ourselves only with the soil moisture. When, the soil moisture starts falling below the field capacity, up to a certain point the crop is able to maintain the evaporation at the potential evaporation rate. Remember here, the crop yield is directly related to the evapotranspiration. So, the potential evapotranspiration indicates that the crop is using the water to its potential needs. And, we would like to maintain preferably the evapotranspiration rate at the potential level all across the crops season so that, you get the best yield. And, that is the crop production functions indicate. Now, typically the crop production function relates the ratio of the actual yield to the maximum yield for a particular crop c with the ratio of actual evapotranspiration to the maximum evapotranspiration or the maximum or the potential transpiration. So, this is the actual, this is the potential.

As I said, there is different response of a particular crop across the growing season. And, these growing seasons of a particular crop is typically divided into several growth stages, such as for example, the establishment stage, the vegetative stage, the flowering stage, the yield formation stage and the ripening stage. The response of the crop during different stages will be different for a given deficit of water.

Let us say a unit deficit of water occurs in this stage, the response is much different. What do I mean by response? The deficit yield that you get because you supplied a deficit amount of water in this stage, will be different from the deficit evapotranspiration ratio that you get here for the same deficit amount of water applied at certain other stage. And, these responses are provided by what are called as the yield factors. So, this Ky here indicates the crop yield factor. And, these crop yield factors, in fact are the sensitivity, they indicate the sensitivity of the crop c, that is, the c there is a crop. In the growth stage s, for example, s is equal to 1, 2, 3, 4, 5 in different growth stages for different crops, you will have this Ky c is available from the agricultural science studies.

So, we will assume that these are available. Now, what does this production function do? The production function indicates or relates the ratio of the actual yield to the potential yield to the deficit of AET by PET; that is the actual evapotranspiration by potential evapotranspiration. Now, this is one type of production function, where we are using a multiplicative for. This is multiplicative production function. Similarly we have additive production functions, which we will also introduce later. So, typically what does it do? We would like to have a potential evapotranspiration that is the actual evapotranspiration equal to the potential evapotranspiration.

If you are able to achieve that during all time periods that is during all the growth stages, what happens? This becomes 1 for all the time period; this becomes 0. And therefore, 1 into 1 into 1 into etcetera; so, you get the maximum yield. So, y by y m will be equal to 1 and therefore, the actual yield will be equal to the maximum yield.

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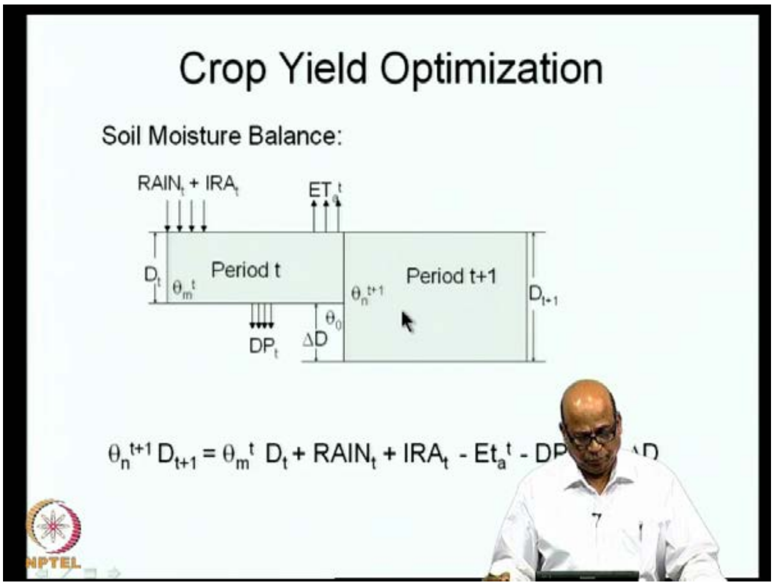

So, similar such production functions are available. We will make use of that. Then, there is an important concept of soil moisture balance. Do not look the physical picture. What we are looking at is, there is a cropped area and for one particular crop, there is a certain area in which the particular crop is grown and the type of soil is known. Now, we are doing the soil moisture balance from one time period to another time period. Typically, this has to be done on a daily scale, but for the irrigation management and so on, we use typical time scale of doing daily or weekly scales.

The crop root growth itself is, going across time periods. And therefore, the soil depth that we consider from time period to time period itself is increasing. Typically, the crop season will be known. Let say that we are talking about a four months crop, which is one twenty days crop or ninety days crop and so on. So, typically the seasons are known. In the growing season, the crop root growth keeps on growing into the soil. And therefore, the net depth of the soil that you need to consider will be increasing from one time period to another time period. So, from onetime period to another time period, we do what is called as the soil moisture balance. This is simply accounting for the initial soil moisture. And adding to it, whatever is the external application and taking out from it, whatever is going out of those particular roots of the soil zone. This is what we do.

Look at this figure now. This is the soil depth, what I am showing is the soil depth. At the beginning of the time period t, these are two time periods: t and t plus 1.This is the time period t and this is the time period t plus 1. The root depth is D t. And, you have the soil moisture theta M t. Now, theta M t is expressed in millimeters per centimeter and this is in centimeters. So, you start with the given soil moisture theta M t. And, in terms of millimeters, it will be theta M t into D t; D t is in centimeters. Then, you have a rainfall that is occurring. So, we add rainfall plus there is an irrigation allocation. So, this is the irrigation application or allocation and this is in depth unit; … we will take this as millimeter. So, both rainfall as well as the irrigation application is in millimeters. So, you started with the particular soil moisture, you added the rainfall to that, you added the irrigation allocation to that and you takeout the evapotranspiration. This is the actual evapotranspiration. Now, actual evapotranspiration can be determined based on the soil moisture for a particular crop in a particular time period t. Then, there is also a deep percolation that is taking place. This is the deep percolation. This deep percolation is in fact that particular soil moisture, which is in excess of field capacity that goes down the root depth.

Now, that is the deep percolation. As a result of this, you end up with the particular soil moisture at the end of the time period. But, at the end of the time period the root growth has extended to another delta D. If we are looking at discrete time periods, let us say one day to the next day and within the discrete time period, you will assume that the root depth is constant. So, when we go to the next time period, the root depth extends by delta D. And, this additional soil moisture has now added to the root zone, had original soil moisture of theta naught. So, theta naught into delta D gets added here. So, you get the end of the soil moisture as theta n t plus one. So, starting with theta m t, this will also be in millimeters per centimeter. Whatever theta I am using, they are all in millimeters per centimeter.

So, this is how you do the soil moisture balance. So, I will write theta n t plus 1 in to D t plus 1; which is the total soil moisture, soil depth there, is equal to theta m t D t plus rain t plus IRA t which is the irrigation, minus Et a t that is the actual evapotranspiration minus D P t which is the deep percolation plus this small moisture that gets added; theta naught into delta D. So, this is how you incorporate the soil moisture balance. Now, in this soil moisture balance, you can see that the actual evapotranspiration and the deep percolation, both of these are dependent on this soil moisture itself.

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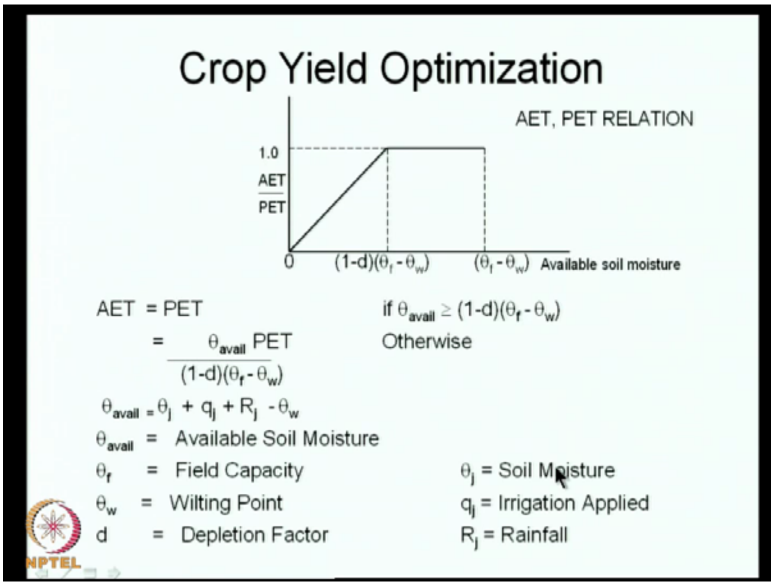

So, we must able to determine the actual evapotranspiration and the deep percolation. The actual evapotranspiration for a crop depends on the available soil moisture. As I mentioned right at the beginning, we talk about the available soil moisture. So, this is the maximum available soil moisture. So, if you plot the AET by PET ratio against the available soil moisture, so the available soil moisture will be zero corresponding to the actual soil moisture being at the wilting point. The available soil moisture will be maximum and equal to theta f minus theta w; where theta f is the field capacity, theta w is the wilting point when the actual soil moisture is at the theta f. In general what happens is, as the soil moisture starts depleting from the field capacity, their actual evapotranspiration will still be equal to potential evapotranspiration until a certain critical soil moisture level. That means you can afford to deplete the soil moisture, until critical soil moisture is reached without sacrificing on the evapotranspiration; which means, the evapotranspiration will still be at the potential evapotranspiration until it reaches a critical level beyond which it starts depleting. And, we assume that it depletes in a linear way. Actually it is the the non-linear relationship, but we can approximate this to be a linear relationship.

So, this factor here d is called the soil moisture depletion factor. And, it depends on the particular crop and the particular soil and in fact, it may change from season to season. That means, from growth stage to growth stage; that means, in a particular time period for a particular crop, you can afford to deplete the soil moisture from the field capacity to this particular level; 1 minus d theta f minus theta w, without sacrificing on the evapotranspiration. So, the evapotranspiration will still be equal to potential evapotranspiration. That is the idea there.

And, typically the d value may be of the order of 0.35, 0.4 and so on. Alright. So, this expression, now we write it as AET is equal to PET that is in this range, if the available soil moisture is greater than or equal to this level; 1 minus d theta f minus theta w. Otherwise, it will be simply theta available into PET that is, this level I am writing now; theta available by PET divided by 1 minus d theta f minus theta w. So, this how we write the PET expression. Where, theta available is the actual soil moisture plus, whatever irrigation that has been applied plus, whatever rainfall that has come minus theta w. So, this is the actual soil moisture minus theta w that is, theta available. So, this is the relationship that we add to the soil moisture balance.

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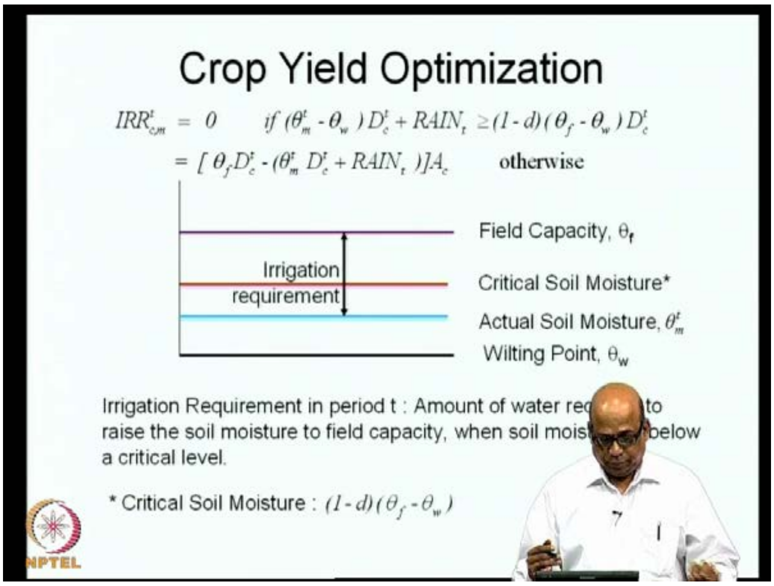

Then, we look at again the moisture scale now. So, this is the field capacity theta f, this is the actual soil moisture theta m in time period t and this is the critical soil moisture that is, theta f minus theta w 1 minus d as we just saw. What we do typically is, as we are progressing from time period to time period, we first shoot up the requirement, shoot up the available soil moisture to field capacity and allow it to keep on going down up to critical moisture. And then, from the critical moisture, we again shoot it back to the field capacity. So, it may go in a particular time period below the critical moisture; which means, the actual evapotranspiration will be less than the potential evapotranspiration.

From this level, we shoot it again to the field capacity. And, again allow it to deplete, again shoot it up to field capacity and so on. So, this is how we determine the irrigation requirement of the particular crop. And, irrigation requirement will depend on the crop area here. So, if we are in this zone, the irrigation requirement will be zero. The moment it falls below the critical zone, we shoot it back to the field capacity. And, that is how we calculate the volume. And, c is the crop and t is the time period and m is where we talking about the moisture requirement. And, reuse the area of the crops to convert it in to volume, volume units.

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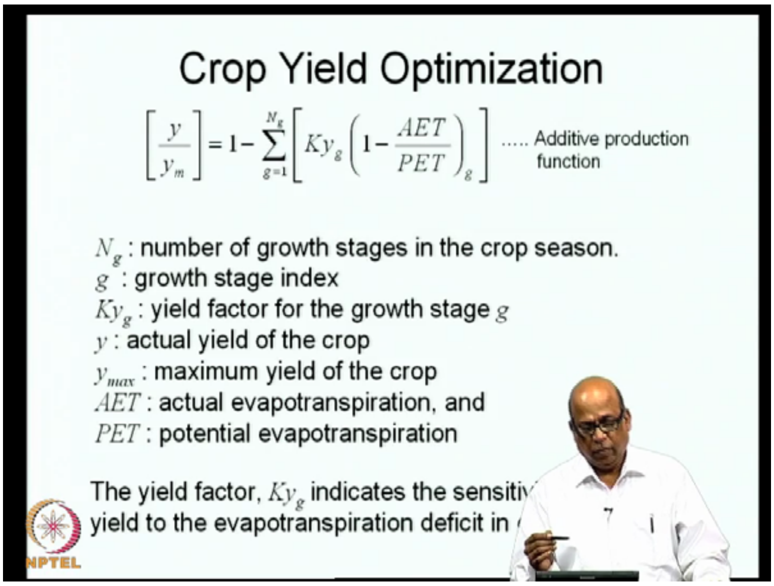

So, all these concepts, we use in the optimization problem now. Just, recapitulate what we did so far. We talked about the soil moisture balance, in which we are considering varying soil depths across the time. As the crop root grows, you include that amount of soil into the soil moisture balance. You start with the particular soil moisture at the beginning of the time period t and then go across times. Every time, including whatever has been added to the soil moisture in terms of the rainfall, in terms of the irrigation applications and take out what has been utilized from the soil moisture in terms of the evapotranspiration, that is an actual evapotranspiration as well as in terms of the deep percolation that goes out of the soil type. So, like this you will do the soil moisture balance. Then, we also looked at the actual evapotranspiration as a function of the available soil moisture itself. So, typically you can approximate it by a linear function. So, the ratio of the actual to potential evapotranspiration is expressed as the linear function of the available soil moisture.

Now with all of this, now we should able to write an optimization problem in which, we allocate a known amount of water across different time periods. So, we use an additive production function. For this demonstration, I will use an additive production function; you can also use the multiplicative and several other types of production functions depending on the type of the optimization problem that you can solve. Now, look at this. If AET is equal to PET, g is the growth stage here. If AET is equal to PET in all the growth stages, what happens? This becomes 1 and therefore, this becomes 0. Therefore, the yield becomes 1. If AET is 0 in all the time periods, your Ky g will be such that, this yield becomes 0. So, this is the type of production function that we use now; g is the growth stage and N g is the number of growth stages.

And, y and y m are actual and maximum yield. And, AET is the actual evapotranspiration and PET is the potential evapotranspiration. Just incidentally, you may know how to compute the PET or how to estimate the PET. You can use ten months relationship and many other empirical relationships to get the potential evapotranspiration or a simple way is to use the crop factors. In fact, the ten months relationship and other relationships also give not the exactly the PET, but they give what is called as the reference evapotranspiration. This is the reference evapotranspiration. So, from the reference evapotranspiration, you apply the crop factors during the time period t to get the potential evapotranspiration during the time period t.

There are the various ways of estimating the reference evapotranspiration. You can go through some basic hydrology course to get the different methods. For example, you can use the aerodynamic method, energy balance method; you can use empirical relationship such as ten months method and so on. So, once you know ET naught or simply you can get ET naught as, K pan into E pan. If you have major evaporation rates, you can get K p into E pan. E pan is the pan evaporation. So, you know how to get the PET in time period t for a particular crop c. So, that is what we are using here.

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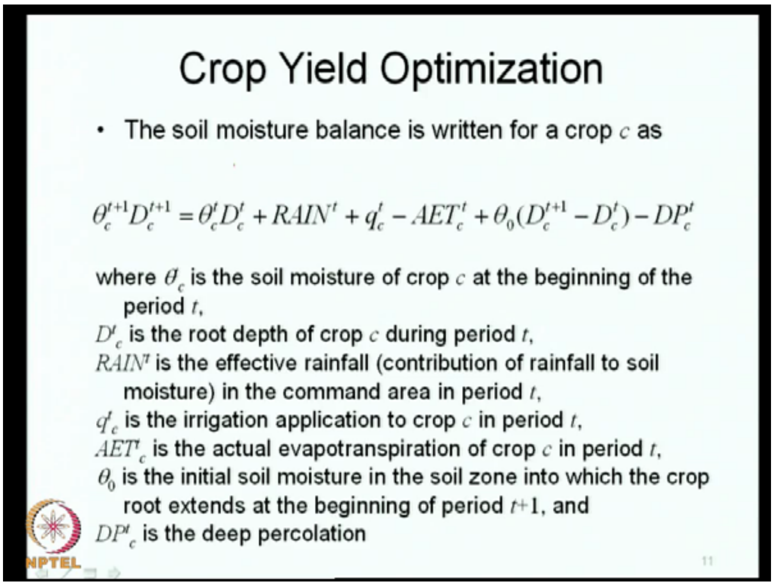

First, we will start with an optimization problem, where a known amount of water has to be supplied to a particular crop across time periods t. Now, this is the soil moisture balance, this is at the beginning of the time period t plus 1, the root depth is D t plus 1 for the crop c.

You started with the soil moisture theta c t, remember all the thetas are in millimeters per centimeter. Now whenever I use theta, it is in millimeters per centimeter. And, your Ds are all in centimeters. So, when I multiplied these two, I get it in millimeters. And, all these other terms are in millimeters. So, this is the irrigation application. And, this is the actual evapotranspiration. This is also in millimeters. Irrigation application; this is in millimeters. And, this is theta naught into some depth. Therefore, this is in millimeters. This is the deep percolation, this is in millimeters. So, like this, you get the soil moisture balance. When you have multiple crops, we use the index c to indicate that it is being written for a particular crop c and for that particular time period t.

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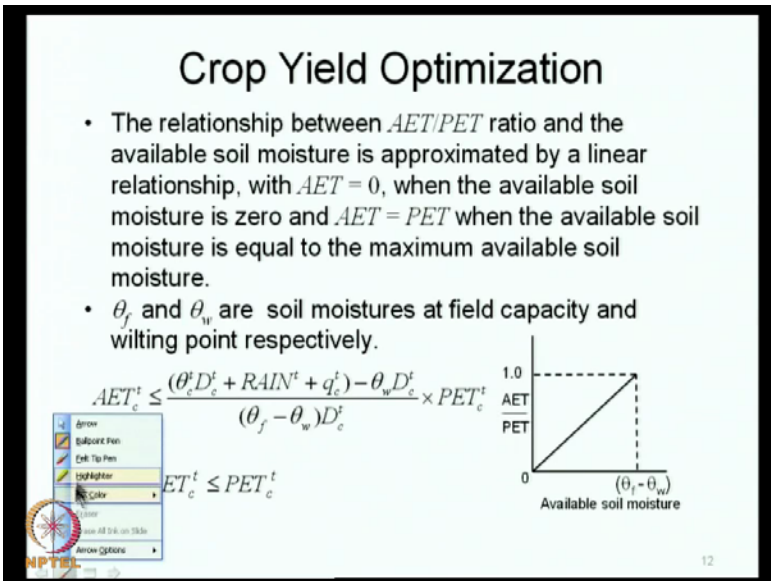

Then, we write the actual evapotranspiration. We can use the simple linear function like this. Where we are saying that AET by PET will be 0, when available soil moisture is 0; which means that, actual soil moisture is at the wilting point. And, AET by PET will be equal to 1, when the available soil moisture is maximum; which means, the actual soil moisture is at theta f. So, instead of using that…, we will, the actual relationship will be something like this because we use the soil moisture depletion. But, we approximate this to this relationship; which means, we are saying that whenever the actual soil moisture is at the field at the field capacity, the actual evapotranspiration will be equal to the potential evapotranspiration. As the soil moisture falls, the AET by PET starts falling down and it reaches 0, when the soil moisture will be at the wilting point or available soil moisture will be at 0. Now, this is how, we express the relationship between the actual evapotranspiration and potential evapotranspiration using this square. For using the linear programming models, we also write this constraint AET is less than or equal to PET because you want to apply this relationship as the linear constraint. So, you have to restrict AET to be less than or equal to PET.

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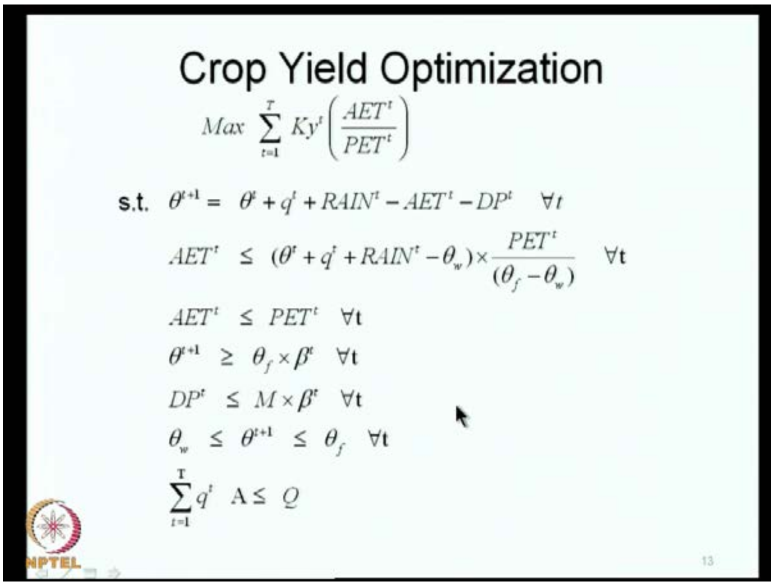

We write the optimization problem as follows. In this problem, we are allocating water to a single crop and known amount of water. So, the total water available across the time periods is known. And, that is Q. So, this is the total amount of water that is available. We write the soil moisture continuity because we are taking single crop, I will avoid the index c. So, we write the soil moisture continuity, we write the PET expression, this expression now, we write this expression for that single crop, and this we will write for all t and this again AET should be less than or equal to PET. So, just to make sure that this condition, we will apply such that, AET is always less than or equal to PET. Now, look at this now. We introduced the integer variables beta t; which means 0 or 1; when beta t is equal to 1. This is to make sure. You refer to the way we accounted for spills in the reservoir operation problem, where we use the integer variables to make sure that the spills occur only when the reservoir is at the full capacity.

Much the same way, what we do here is that we use the integer variables to ensure that the deep percolation term DPT that is occurring here will have non-zero values, only when the soil moisture is at the field capacity. So, this is what we make, this is how we ensure that theta f is the field capacity and beta t is 0 or 1. So, when beta t is 0, then what happens? Theta t plus 1 must be greater than or equal to zero. That is fine. When theta t is equal to 1, theta t plus 1 will be, that is when beta t is equal to 1, it will be greater than or equal to theta f. But, because of this condition, these two conditions together, it ensures that theta t plus 1 will never be greater than or equal to the theta f.

So, it will be utmost work to theta f. And, deep percolation, this is the large number, M is the large number. It makes sure that deep percolation is always penalized because the moment you put any deep percolation value, what happens? Here, your soil moisture balance governs that and theta t plus 1 must be greater than or equal to 0. And therefore, deep percolation is, it occurs only when the soil moisture reaches the field capacity because of these two constraints together. These two constraints ensure that the field capacity, the deep percolation occurs only when the soil moisture becomes equal to the field capacity. This is, the exactly the same way as we accounted for the spill in the reservoir problem. So, this is, you understand the various terms here.

Now, Q t is the decision variable now. Q t is the irrigation application out of the amount of do that this is available. This is the total amount of water. This is the area of the crop. Now, all the other variables, we are talking in terms of either depth or depth per unit depth. Now, a is the area, so area multiplied by the depth, you get the volume and this is the total amount of volume, this is in the volume units. And, this summation is from t is equal to one to capital T; that means, over all time periods the water that is applied, the total water that is applied across all the time periods must be equal to, must be less than the total water that is available. That is what this constraint is. Now you look at the objective functions, we would like to make the actual evapotranspiration equal to potential evapotranspiration in all the time periods. Because of the irrigation application that you are putting here, the actual soil moisture evapotranspiration gets determined by the soil moisture balance.

So, the soil moisture balances, along with the relationship for the actual evapotranspiration determines how much the actual evapotranspiration for that level of irrigation application is. And, we would like to make the actual evapotranspiration close to potential evapotranspiration. And therefore, we would maximize this. Then, we use the yield factors Ky, which are varying from time period to time period. These are the yield factors. As I have mentioned, the yield factors indicate the sensitivity of the crop to a given, to the deficit of water supply in time period t. And, they are different for different crops and different for different time periods.

So, we use these yield factors as a weightage to the term AET by PET. What does this ensure? This ensures that, in those time periods in which the crop is extremely sensitive; which means, the Ky values are very high. In such time periods, the priority is given to make AET as close to PET as possible; which means that, there is a very serious deficit. The deficit is distributed such that, in those time period in which the crop is extremely sensitive to the deficit, the priority for those time periods and therefore, the AET becomes close to PET sacrificing the other time periods in which the crop is not as sensitive. So, the yield factors are used as weightages. And then, when we solve this, you get all of these as the outcomes or the results.

You get Q t, Rain t is the data. And, you have not used D t here. That is the root depth. This solves for a constant root depth, this particular model. And therefore, I do not indicate the root depth. So, we get Q t as the solution and because of that, you get theta t as the solution, AET as the solution, and PET is known and you also get DPT, which is the deep percolation. That has the solution.

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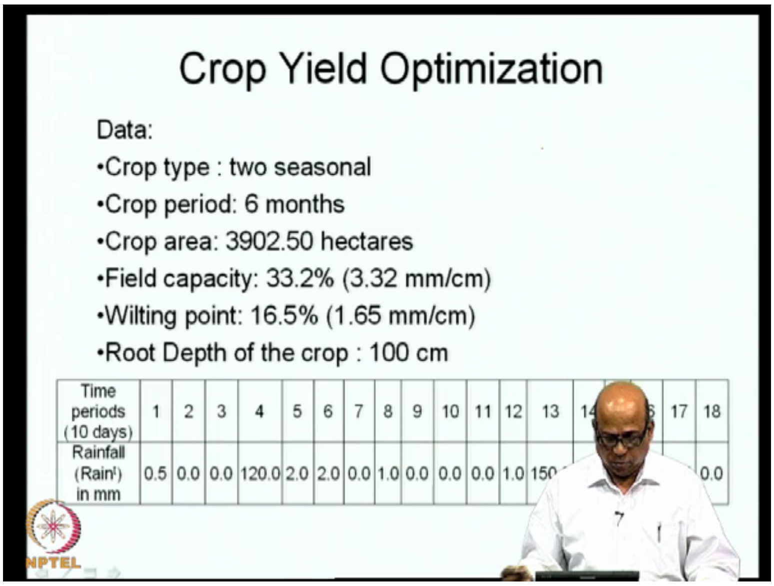

So, if you look at the solution, we will do this for a simple example now. We have the rainfall given for all the time periods. There are eighteen time periods. Now, that is we have one hundred and eighty days. So, each time period is of ten days interval. Crop area is given, field capacity is given as 3.32 millimeters per centimeters, wilting point is given, and root depth of the crop is given.

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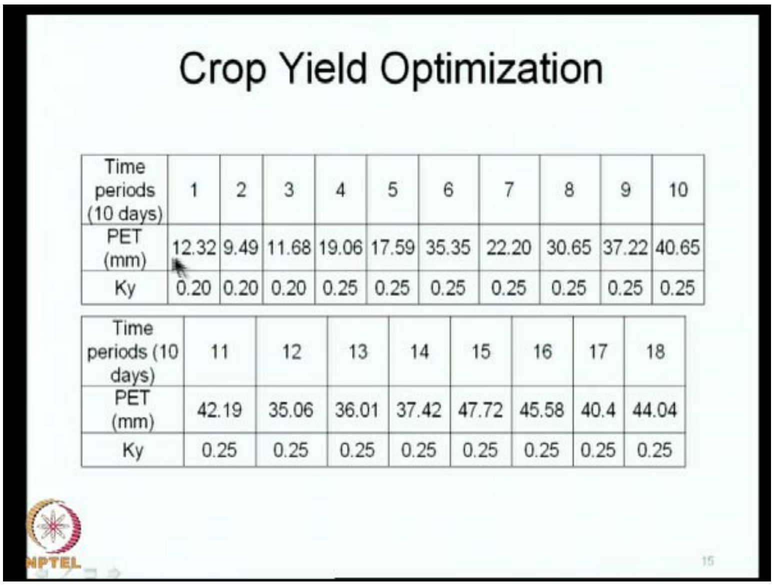

And, we have the potential evapotranspiration values for all the eighteen time periods. We have the Ky values or the yield factor values for all the time periods. When we solve this, you get these kinds of allocations for these levels of total water available.

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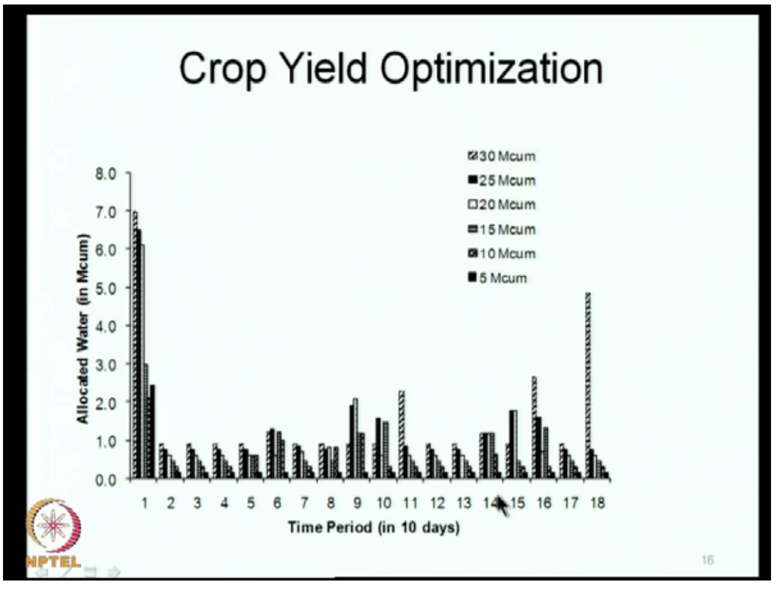

So, these are the Q, which is the amount of water available. So, we solve this for various levels of amount of water available. So, if 30 million cubic meters is available, this is the kind of allocation; if 25 is available, this is the kind of allocation and so on. So, now there are lots of analysis that are possible here. That is, what it does is that it first allocates a large amount of water during the first time period, then allows the soil moisture to deplete, then allocates whenever it is necessary. So, that is a kind of allocation that you ensure here.

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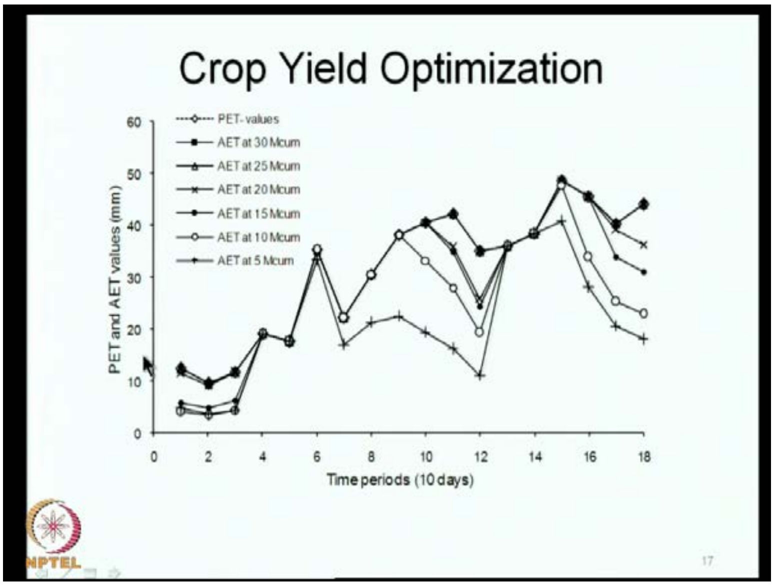

And, this is the PET versus AET values. As we can see, the upper most curve here is the PET value. When you have adequate amount of water which is the 30 million cubic meters, the AET is equal to PET in all the time periods. Similarly, it may happen for 25 and 20. But, as you start depleting the amount of water, so at 30 million cubic meters is when the total amount of water is 30 million cubic meters. As it starts depleting, the actual evapotranspiration becomes lower and lower. And therefore, the yield will suffer.

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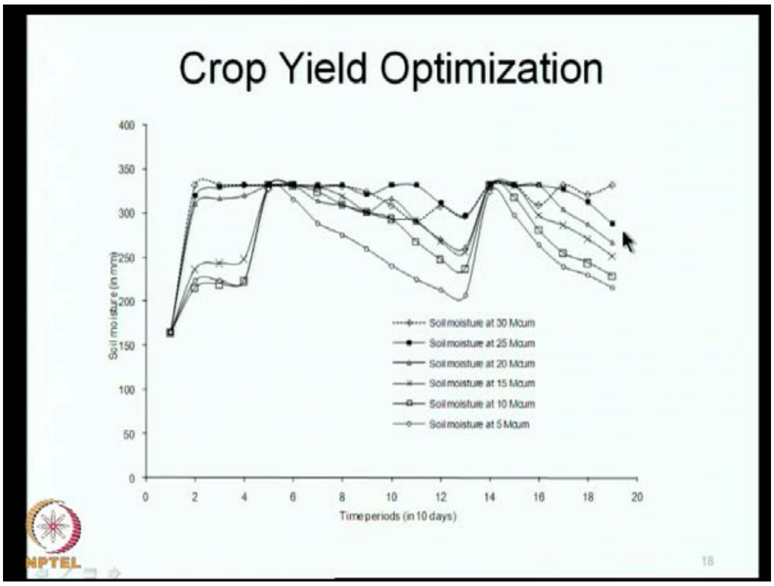

Similarly, you can have the soil moisture because soil moisture also comes as an output; the field capacity is here, 332. So, it will not exceed the 332. And, the soil moisture varies with respect to different amount of total water that is available.

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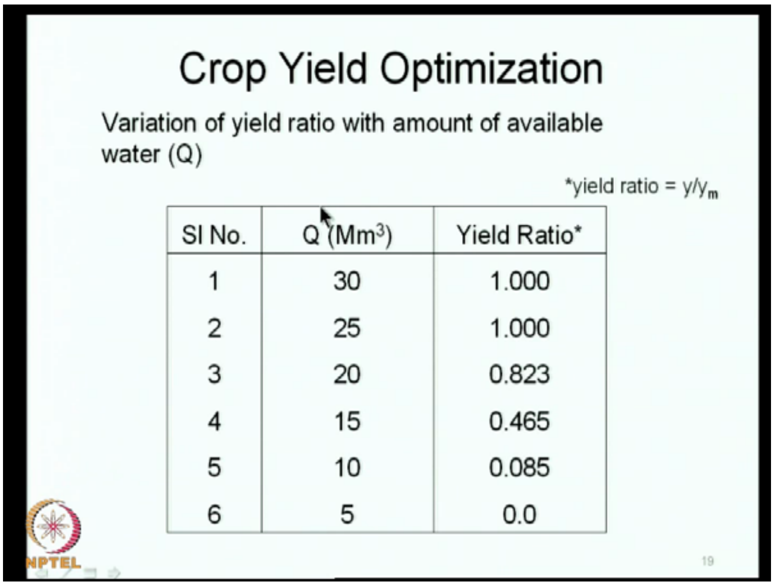

And, the yield themselves, yield ratio as you can see with various levels of water available, the yield ratio will be like this. What does this mean? That, this means that if you have a minimum of the 25 million cubic meters of water, then you can maintain the yield ratio to be 1, which means, the actual yield will be equal to the maximum yield. But, as it starts reducing even with optimal allocation, your yield ratio starts coming down.

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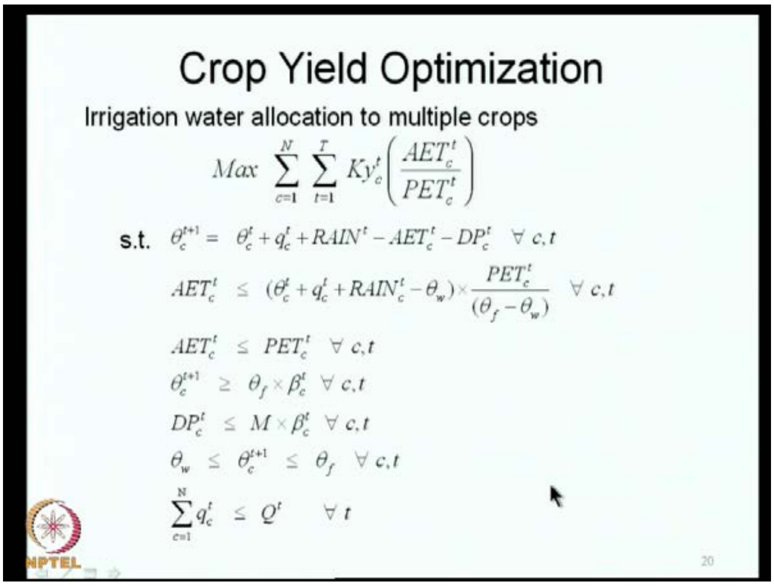

Now, this problem now we extend to multiple crops. So, what we did for the single crop was only this summation we took. Now we add another summation for multiple crops c is equal to 1 to n. Now, this Ky c t then will be for different crops for different time periods. And, similarly we write for all of these constraints for various crops and various time periods for all c and t. Recall that, what we did earlier was for a single crop we wrote only for all t, all of these constraints. Now, we write all of these expressions for multiple crops and write them as for all c and t. So, the soil moisture is written for all c and t. And, similarly all the constraints are written for all time period t.

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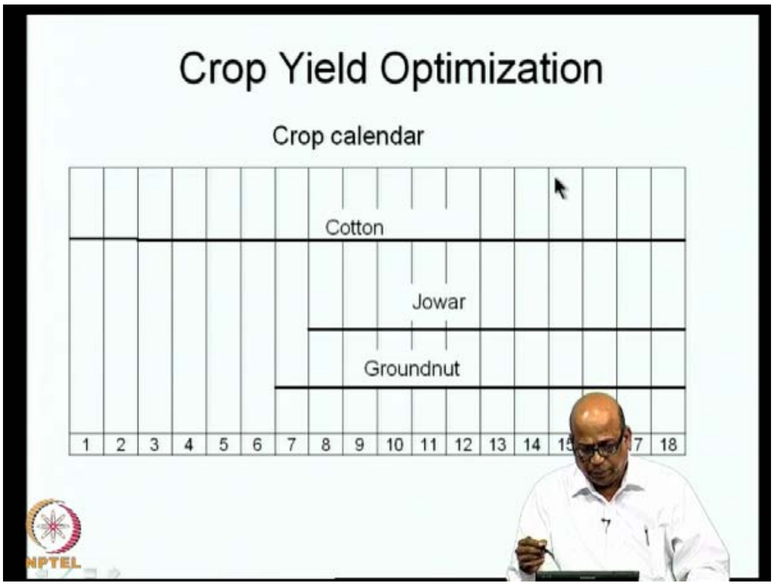

Then, we do this for the multiple crops now. For example, you have three crops here; cotton, jowar and groundnut. You have, these are the time periods. So, this is called as the crop calendar. So, these are called as, these are the time periods. So, you may start the cotton at the beginning of the time period is equal to 1 and it may extend to all the eighteen time periods. Jowar may start at the beginning of the eighth time period and groundnut may start at the beginning of the seventh time period and so on. We also have all the other details required. For example, the potential evapotranspiration values are known, the yield factors are known and whatever data that we have used earlier they are all known, similarly the rainfall is known.

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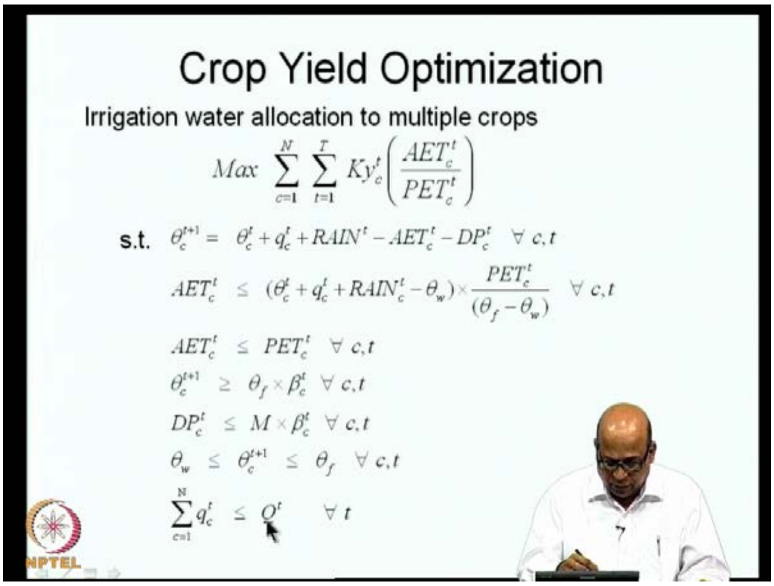

In this particular problem, what we do is look at this constraint, instead of saying that the total amount of water to be allocated across all the time periods is known, what we will say is that the water available in time period t. So, this water, we want to allocate among various crops. So, across all the time periods, the total water available is known. And, that is what we need to allocate. So, that is available irrigation supply. Now, this may come from your reservoir. So, these are known and then we allocate this amount of water among the three different crops such that, the total yield is maximized. And, this is the solution for various levels of Q. That is, whatever Q is available here, we will solve for that, and then we start reducing Q across all the time periods and then look at how the allocations will be varying for each of the crops. We will see for cotton, we will see for jowar and we will also see for groundnut. So, this is how we get the allocated water.

This allocated water also has the associated soil moisture balance. That is, when you allocate this water, how, what is the kind of yield that you get, what is the kind of soil moisture that you get, and so on. Remember here, what we did in this particular example is that, across the time periods we varied the water available and then allocated the water among three crops such that, the yield is maximized across all the crops, across the season, that is, at the end of the season.

Now, we go one step further and see where from this available irrigation supply is coming from. And then, we relate this or integrate this with the reservoir operation itself. Essentially, what we are doing is, in the previous problem now, I solved this problem that is for a given amount of water here, how much to be allocated across the time period. This is what we solved. Now, what we do is we go step upwards, upstream, and then look at this problem that is, if there is inflow sequence, then how much should I release such that, after integrating with the optimization problem that you have solved, you will know the release policy; that means during each of the time period, how much should be made available here such that, at the end of the year for example or at the end of the season, the crop yield is optimized.

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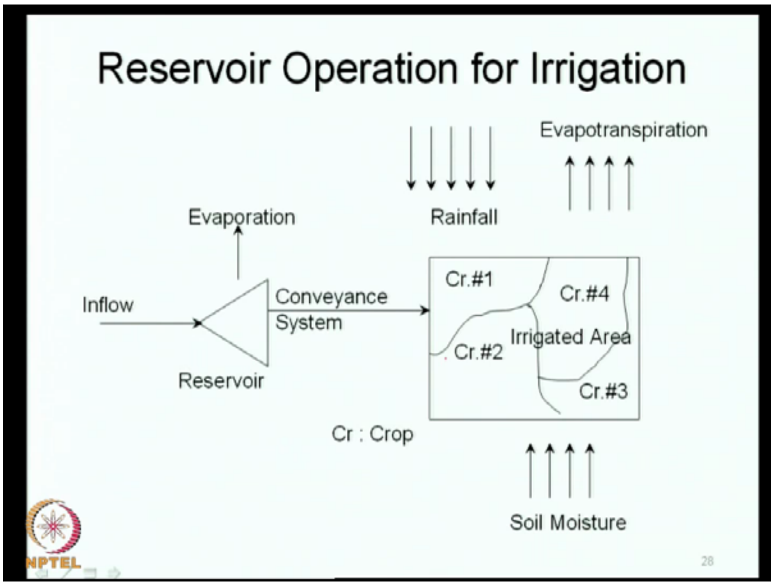

So, essentially what we are saying now is that, in this particular problem that I have solved, we said a known amount of water in each of these time periods to be allocated optimally. This is the problem that you have solved here. Now we say that, what should be this known amount of water itself? What should be the known amount of water, which is released from the reservoir after accounting for the losses etcetera. This is actually, what is available for application at the field level. What should be this amount such that, the crop yield is optimized. That is the question we ask in the reservoir operation for irrigation. Now, all these details are available in the certain literature, papers and so on. Typically, you can refer to the textbook by Vedula and Mujumdar 2005, which I have given the references right at the beginning of the lecture number five, lecture number one.

So, in essence, in today's lecture what we did is that, we talked about the crop yield optimization and specifically we talked about three different levels of problems. One is known amount of water to be allocated to a single crop, second is known amount of water across different time periods to be allocated to the different a numbers of crops. All of which are competing for this amount of water, which is available during different time periods. Then towards the end I just mentioned, how you integrate this with the reservoir operations itself by looking at the optimal allocation at the crop level and looking at the optimal reservoir operations; that means, you also account for the uncertainties in the inflows and then integrate the water allocation at the crop level with the reservoir operation itself.

So, this how we do, we use the systems, models for optimal allocation of water for irrigation. In the earlier lecturers, I had talked about optimal operations, optimal hydropower generations and earlier I have also talked about conjunctive use of ground water as well as the surface water. All of these can be combined. For example, you may have reservoir where you have hydro power, you may want to use conjunctively ground water as well as the surface water. All of these can be combined in an exhaustive and comprehensive, yet elegant optimizations problem. So, we will continue this discussion in the next class. Thank you very much for your attention.