Water Resources Systems

Prof. P.P. Mujumdar
Department of Civil Engineering
Indian Institute of Science, Bangalore

Lecture No. # 38
Hydropower optimization

Good morning and welcome to this the lecture number 38 of the course Water Resource Systems Modeling Techniques and Analysis; in the last lecture, we checked the modeling technique for conjunctive use of surface and ground water; we considered a simple model where we looked at the continuity at the surface reservoir and the continuity at the ground water reservoir, and see how the two are linked, we introduce certain linking constraints through the seepage that takes place as a water flows in the canal and contributes to the ground water storage, and the pumping that we do from the ground water subsequently also part of it comes and recharges the ground water.

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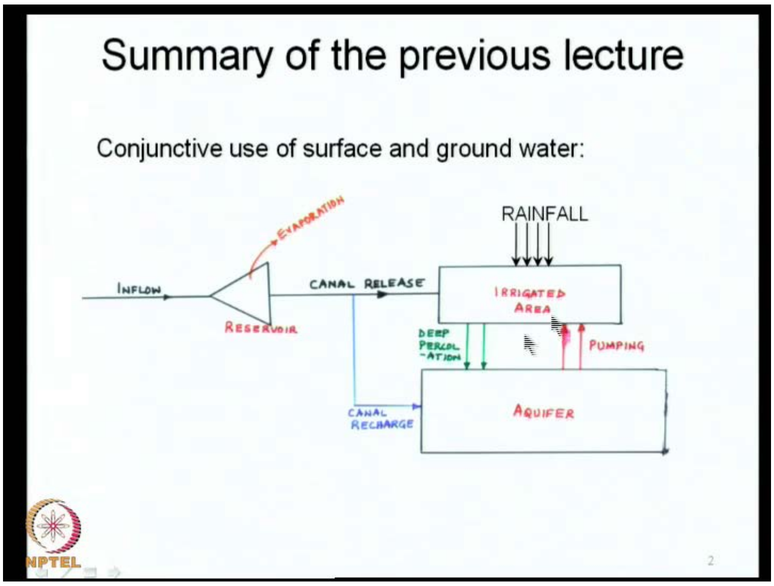

So, we considered the recharge to the ground water, we specifically look at looked at this particular type of system, where you have a surface reservoir, flood by the inflow due to the catchment rainfall, there is a evaporation that is taking place, then the canal release which is the decision variable, the part of the canal release contributes to the ground water as canal recharge, and then the water that is applied both from the surface as well as from the ground water source, part of it also comes as deep percolation, and then may contribute to the ground water aquifer. The decision that we are making in this particular problem is the rate of pumping as well as the amount of release that needs to be made from the reservoir during different time period within a year, and typically we consider monthly time periods, ten day time periods and so on, and we also solved an example, a simple example actually, where we considered lumped storage continuity here, lumped storage continuity at the aquifer.

I mention towards the end of the last lecture that you can make the ground water model as sophisticated as you desire by including, let us say the two-dimensional ground water flow equations, and then dividing this aquifer into a number of a elements and use finite element method to solve the differential equations; when you do that, you must write these balance equations corresponding to each of the element, let us say that you are discrediting the irrigated area into a two kilometer by two kilometer grid, and you write the finite element model in two kilometer by two kilometer for the aquifer, and then for every two kilometer by two kilometer grid, you read to write the balance equations; such studies are available in literature, those who are interested can go through that.

Now, we will look at another important problem where we will formulate an optimization model for hydropower generation; in our earlier lectures, somewhere on lecture number 25, 26, we have seen the simulation for hydropower, where we have seen the relationship between the power that is produced at the power house with the head available in the reservoir and the amount of release that you are going to make through that turbines; it is a non-linear relationship typically given by power being proportional to Q t into H t, where Q t is the discharges through the turbines and H t is the net head available.

Now, we will go one step further, and then look at how do we optimize the power that is generated? And typically in most of the power power generating projects especially in country like ours, we also have irrigation. So, power is in general, a secondary objective and irrigation is the primary objective in most of the systems that have in our country; in such a situation, we would like to optimize the hydropower generated subject to the irrigation demands being either completely met or irrigation demands being met with a certain minimum reliability and so on.

So, we have two major objectives in most of the systems that we have in our country; irrigation and hydro power, where irrigation takes a higher priority in general; the problem that we post now is what is the maximum power that you can generate for a given reliability of meeting the irrigation demands? So, we specify, pre-specify the reliability of meeting the irrigation demands, and then for that pre-specified minimum reliability of meeting the irrigation demand, we looked at what is the maximum power that you can generated from the system? So, we will formulate a problem of such a type, and then see how we interpret the results, how we look at the trade of between the reliability of the power with respect to the reliability of the meeting the irrigation demands and so on. So, there will be a trade of between hydropower reliability and the irrigation reliability the land that is what will examine today.

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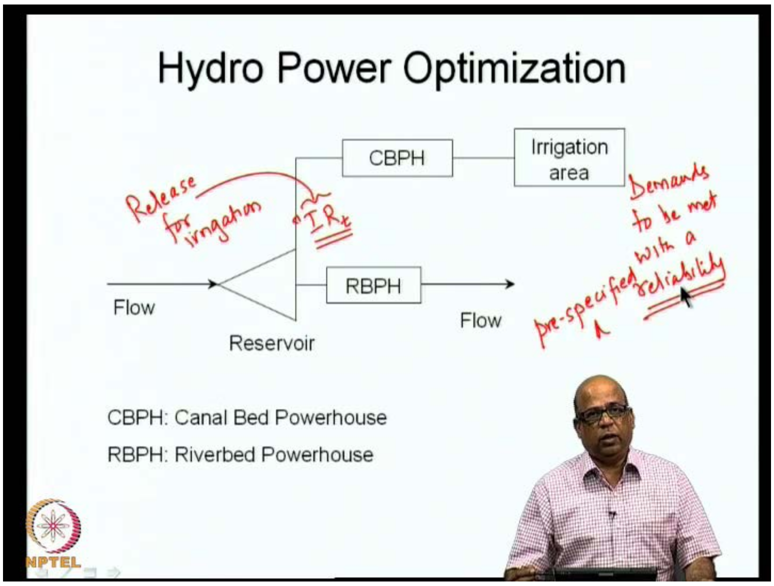

So, we will take a system, some a typical system in fact, which exists in most water resource systems in our country, where you have a surface reservoir, and then you are letting out water for irrigation through the canals, and you may have a small amount of power generated through the canal bed power house. So, CBPH that I have shown here is a Canal Bed Power House. So, it may be a very minor amount of power that is generated through the water that is led out in the canal for irrigation; however, you may have a River Bed Power House, and this is the power house that generates major power at the system at the reservoir, and release has to be made specifically for RBPH that is the River Bed Power House, in addition to the release that is made already for irrigated area.

Now, this is the power type of system that will consider now; we will look at the maximum power that can be generated through the river bed power letting out for a pre-specified reliability of meeting the irrigated demands during different time periods in the year; now this will be constraint, both of these to be constraint by the supply that is available, which is determined by the flows, inflows to the reservoir, and they are generated because of the rainfall in the catchment area now this is the typical problem that will look at, I again repeat that we will not optimize the CBPH power that is the Canal Bed Power House, because we are simply saying we are going to meet the irrigated irrigation demand with a certain level of reliability and therefore, that release gets fixed or that release gets determined based on the irrigation demands and the pre-specified reliability; however, we will optimize the power that is generated through the River Bed Power House; River Bed Power House generally will have much larger capacity compared to the Canal Bed Power House.

Now when we are looking at such a problem, we will as I said, we may say that we want to meet the irrigation demands 100 percent of the time, then what is the power that will generated? We may want to meet the irrigation demands at least 90 percent of the time, then what is the maximum power that we can generated all through the year subject to the uncertainties associated with the inflows, then we may want to look at the irrigation demands to be met with a certain reliability, let us say P, P is the probability of meeting the irrigation demands, which means the minimum probability, which is also the reliability of meeting the demand, and then we start looking at various levels of power that is generated and the associated reliabilities. So, these kind of trade of that you can generate; that means, the reliability of meeting the demands with respect to the reliability of the maximum power that we can generate here and so on. So, we will formulate first a chance constrained reservoir optimization problem for this, in which we will pre-specify the reliability of meeting the demand, and then look at what is the maximum power that we can produce.

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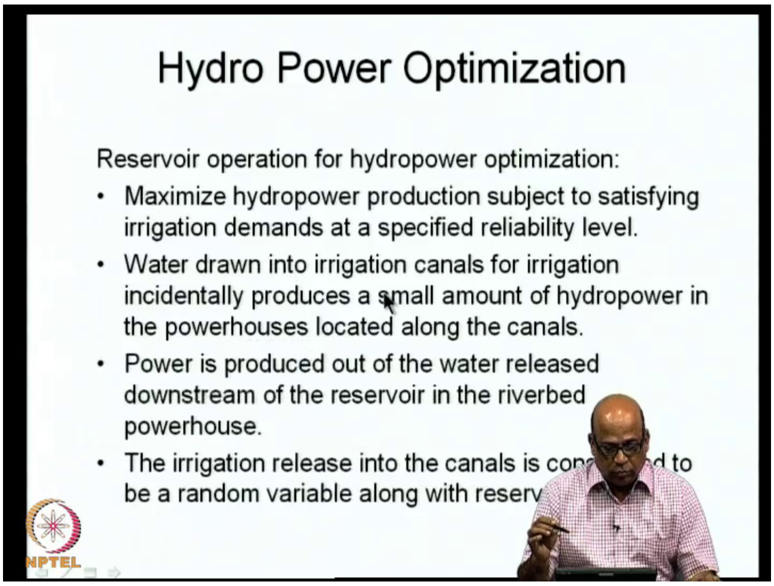

Now that is that is what, is stated here. So, we want to maximize the hydropower production subject to satisfying irrigation demands at a specified reliability level; now water drawn into irrigation canals also produces small amount of power, but we will not consider this in the optimization, but the main power that is produced out of the water that is released downstream of the reservoir into the River Bed Power House itself, and in the process, because the inflows are random the flow into the irrigated area. Now we are putting this as reliability criterion. So, we are saying that the demands to be met with a certain reliability, (10:26 to 10:39) this is a pre-specified reliability, and because of which the release here, that we will call it as IR t, let us say that it is the Irrigation Release or Release for Irrigation; now this becomes random, because it is depended on the reliability here we are specifying, and that reliability will be achieved by the flow that is available or the supply that is available, and therefore, this becomes random.

Theoretically or actually speaking, the release that we are making into the river bed power house is also a random variable, because it also depends on the storage; and the storage is a function of the flows, by using the linear decision rule in a imaginative way what we will do is, we will convert the storage into a deterministic variable, although the inflow is the stochastic variable, we convert the storage into the into a deterministic variable, there by converting the release into the power house, river bed power house as a deterministic variable; whereas, the release into the canal here, irrigation canal that becomes a random variable. So, we will formulate a chance constraint program to maximize the power produced in the river bed power house subject to certain pre-specified reliability of meeting the irrigation demands that is the purpose on.

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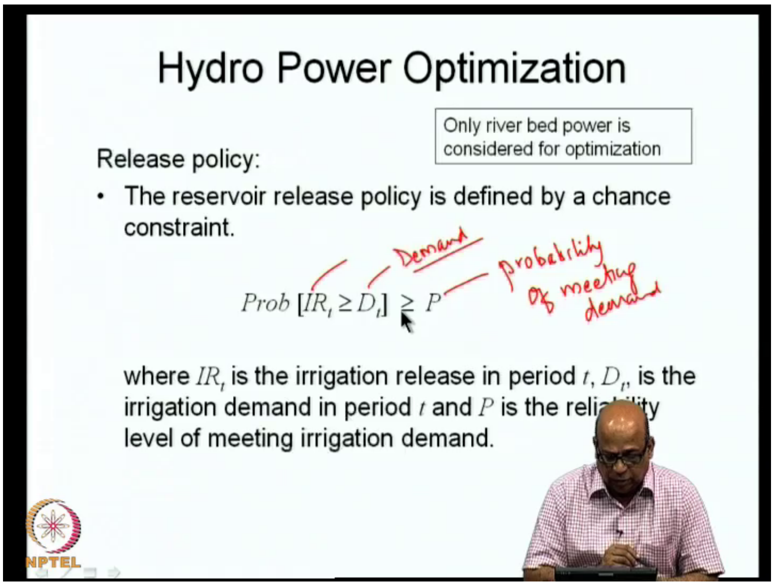

So, we will write the chance constraint first; I R t is the irrigation release, now as a said I R t depends on the supply that is available and we are specifying that that irrigation demands must met with a minimum reliability of P; now this is the probability of meeting the demand (12:55 to 13:03) meeting irrigation demands and this is the irrigation release that is made, and this is the demand in time period T; in general we can also make this as P t; that means, we may specify different levels of probability probabilities for different time periods. So, this is the chance constraint now we are going to make decisions on I R t, this is specified and this demand pattern is known, so for every time period t, you know the irrigation demands; now this is the chance constraint.

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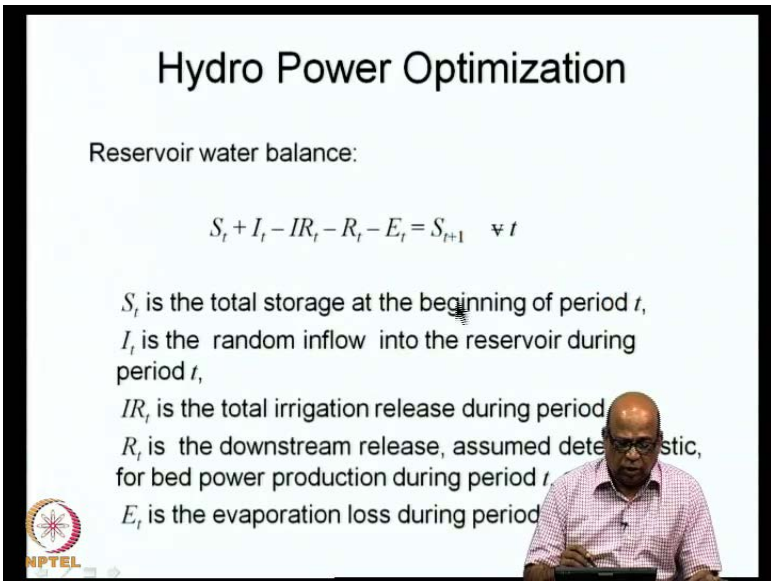

And we have the reservoir continuity S t, which is the storage at the beginning of the time period t plus I t, which is the inflow during the time period t minus IR t, which is the irrigation release minus R t, which is the power release. So, for power release, I will put it as R t and this is the evaporation. So, this is a simple storage continuity equation, we are not worrying too much about the evaporation being storage depended loss, we will include that subsequently, when we use the linear programming.

So, right now for the continuity, we just understands in this way, S t which is the storage at the beginning of the time period, you add the inflow, take out the irrigation release, this is irrigation release, take out the power release, take out the evaporation, then you will end of with storage at the end of the time period; this we have to write for all t; this continuity equation, we will use to convert the chance constraint that we have written here into a it is deterministic equivalent; I suggest that you refer to the earlier lectures on chance constraint optimization.

So, E t as you can see here, E t depends on the average storage during the time period; let us say that you are looking at the monthly at at a monthly operation of the reservoir and the E t, which is the evaporation loss in volume minutes, during that particular month will depend on the storage at the beginning of the time period, and the storage at the end of the time period.

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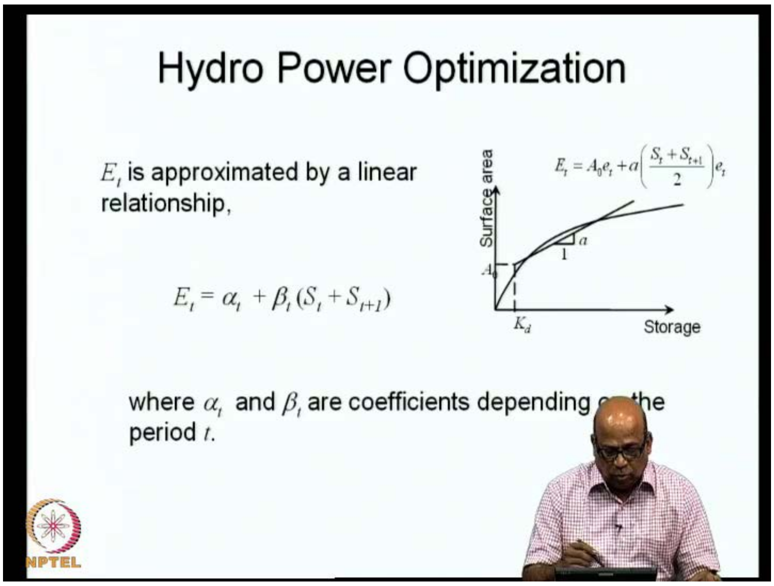

So, we will write E t, typically it will depend on the surface area that is a water spread area, during time period t and time period t plus 1, and we are looking at the live storage here. So, we approximate the storage area relationship with a straight line, refer to the earlier lecture where we derived this expression, and then write E t, which is in the volume minutes as a linear function of the storage at the beginning as well as the end. So, simply write E t is equal to alpha t plus beta t into bracket S t plus S t plus 1; now S t plus S t plus 1 by 2 is the average storage; we may have area of water spread or this is the slope of this line, and E t is the rate of evaporation all of these are observed, and then we write E t in volume minutes as a simple linear equation, because you would like to use or formulate this problem as a chance constraint linear programming problem.

And therefore, all the constraints have to be linear, all the relationships have to be linear; then the evaporation loss, once we write this E t is equal to alpha t plus beta t S t plus S t plus 1; we put that into the continuity equation, and write for IR t; from the continuity equation, I take the I R t into the left hand side, we get this as the continuity equation. So, we use this as the continuity equation for irrigation release, understand different terms here, beta t comes from the evaporation loss relationship, and S t and S t plus 1 are decision variables, which are the storage at the beginning of the time period t and beginning of time period t plus 1; I t is random inflow, this is the inflow which is random; and R T is the power release; and again alpha t comes from evaporation equation; now alpha t is known, beta t is known, S t and S t plus 1 are decision variables, R t is a decision variable, I R t is a decision variable, I t is the random input. So, this is what you get here.

Now we had the chance constraint, probability of I R t greater than or equal to D t greater than or equal to P; in this we have pre-specified the probability of meeting the demand; let us say that we want to say 70 percent of the time, the demands have to be met; 80 percent of the time, the demands have to be met; there is a irrigation demand; and therefore, we may specify P to be 0.7, 0.8, 0.9 etcetera. So, these are pre-specified probability levels. And I write the chance constraint as I R t is given like this, I take I t to the right hand side, we we are writing I R t greater than or equal to D t greater than or equal to P.

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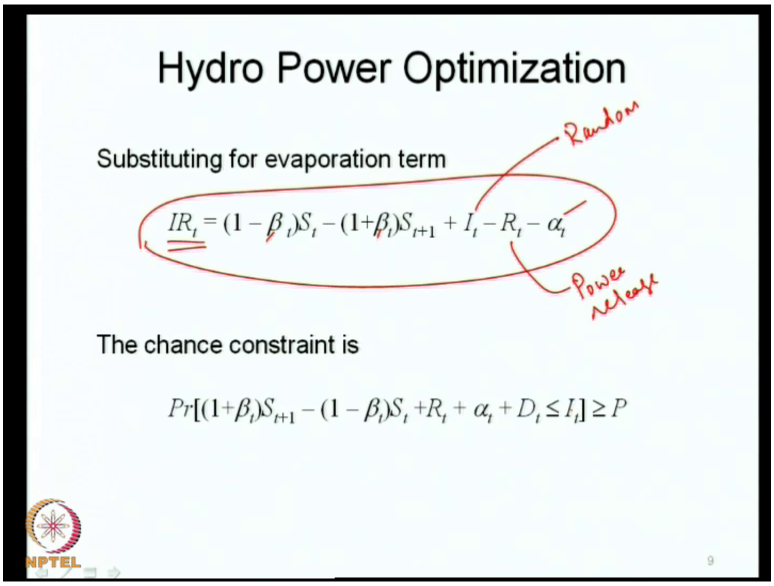

This I write it as I t, I take it on the right hand side and then rewrite this, I write probability of 1 plus beta t S t plus 1 minus 1 minus beta t S t plus R t plus alpha t plus D t, which was greater than or equal to D t was in the chance constraint there, and that would be less than or equal to I t, because I have retain these on the left right hand side, everything else I transferred it to the left hand side less than or equal I t greater than or equal to P.

Now this is the chance constraint, in which I t is a random variable whose probability distribution is known; this probability distribution we determine based on the historical data that is available, because I t is the I t is the random variable, S t that depends on I t becomes a random variable, R t becomes a random variable; however, what we will the… And therefore, it becomes complex, you cannot directly transfer the variance in I t to varies different variables. So, what we will then do is as we did indeed in our chance constrained linear programming explanation we use a linear decision rule we use the linear decision rule because you want to use a linear optimization.

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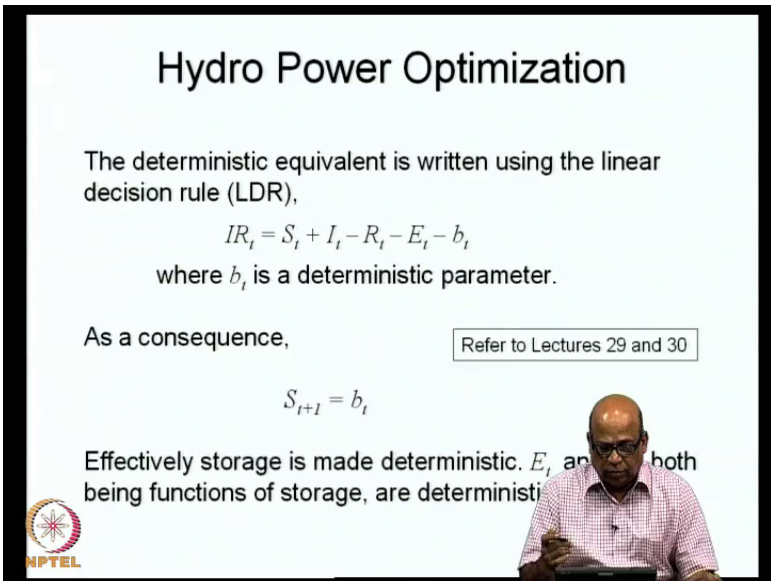

So, we use a linear decision rule you will just refer to lectures 29 and 30, where I have explained the linear decision rule, this is the irrigation release. So, the irrigation release we express as the total water that is available minus a decision parameter; so S t plus I t minus R t, which is the power release which we take it as deterministic minus the evaporation which we take it as deterministic minus b t, where b t is a decision parameter. Now remember here that R t and E t both are taken as deterministic it means that whatever is the variance in I t or whatever is the uncertainty in I t is all transferred to the uncertainty in I R t; and R t and E t they become they are retained as deterministic variables. So, there is a little understanding that is necessary here, although because S t, R t, E t all of which depend on I t, which is the random inflow, what we are saying through this particular the type of linear decision rule is that all the variance in I t or all the uncertainty in I t, we transfer it only to I R t; and S t S t, R t as well as E t, they are not uncertain, they become, they are retained as deterministic variables. So, that is the understanding that you should have.

Now when we do this and convert the chance constraint as we wrote here into a deterministic equivalent, again you refer to the these, you will get this as first of all using the continuity equation, you get S t plus 1 is equal to B t, the continuity equation is here with evaporation it is little like this, with that now we you can verify that S t plus 1 will become equal to b t, that is for R t for I R t putting this expression, and then you will get S t plus 1 is equal to B t, you go to your earlier lectures 29 and 30, you will be able to understand this. And therefore, effectively the storage is made deterministic, we remember b t are the decision may decision parameters which are deterministic; because E t, which is the evaporation loss, and R t, which is the release that is made for power both of them depend on storage alone, and storage is the deterministic variable, R t and E t also become deterministic; now that is a idea of which idea with which we use the linear decision rule.

Now you look at this now, this is the chance constraint 1 plus B t S t plus 1 minus 1 minus B t S t plus R t plus alpha t plus D t less than or equal to I t greater than or equal to P, P is the probability level, D t is known sequence, R t is deterministic, S t is deterministic; and therefore, the left hand side of this constraint is all fully deterministic and the only random variable is on the right hand side here I t, and this the distribution of I t is known, probability distribution of I t is known, and this value is pre-specified for this now, we write the deterministic equivalent.

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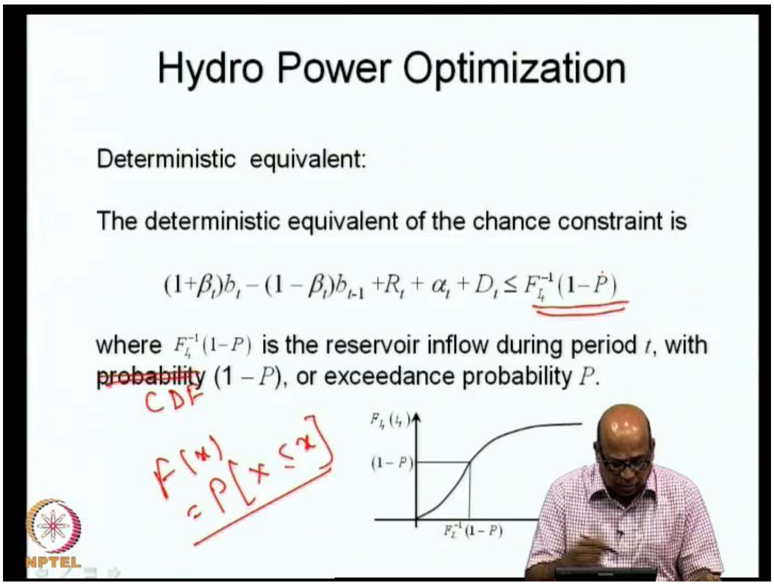

So, the deterministic equivalent turns out to be all of that left hand side of this expression 1 plus beta t etcetera, and we are writing S t plus 1 is equal to b t therefore, S t will be b t minus 1 and all other terms less than or equal to I t, and that turns out to be we write this as 1 plus beta t B t minus 1 minus beta t B t minus 1 plus R t, which is the deterministic variable plus alpha t, which is the constant plus D t, which is the deterministic known values less than or equal to F I t inverse 1 minus P; refer to lectures 29 and 30 and understand how we write the deterministic equivalents.

You must know what is this term now, F I t inverse 1 minus P; F I t refers to the CDF Cumulative Distribution Function for the variable I t and F I t inverse for a given value refers to the particular flow value I t. So, we are showing here, the CDF of I t CDF of the variable I t, for any given value of I t, the corresponding value is F I t i t; this I t, capital I t says that it refers to the variable I t, and this small i t refers to the particular value of i t, and for as specified value 1 minus P, you go to the CDF and the associated value on the x axis gives you F I t inverse 1 minus P; it is in fact, the flow value corresponding to the CDF of 1 minus P or the exceedance probability of P.

So, F I t inverse 1 minus P is the reservoir inflow during period t with probability 1 minus P, that is CDF; CDF corresponding to 1 minus P or the exceedance probability of P, that is P percentage time that flow is exceeded, because F of x remember F of x is probability of X being less than or equal to x. So, this is what you interpret from here; with CDF X will say, not probability, will say CDF of 1 minus P; so, corresponding to the CDF of 1 minus P, that is the inflow value. So, we know from the distribution of I t, we know this value, because P is specified, you go the distribution and get the F I t inverse 1 minus P; now all of these left hand side are deterministic, some of them are decision variables, some of them are constants, known constants and so on.

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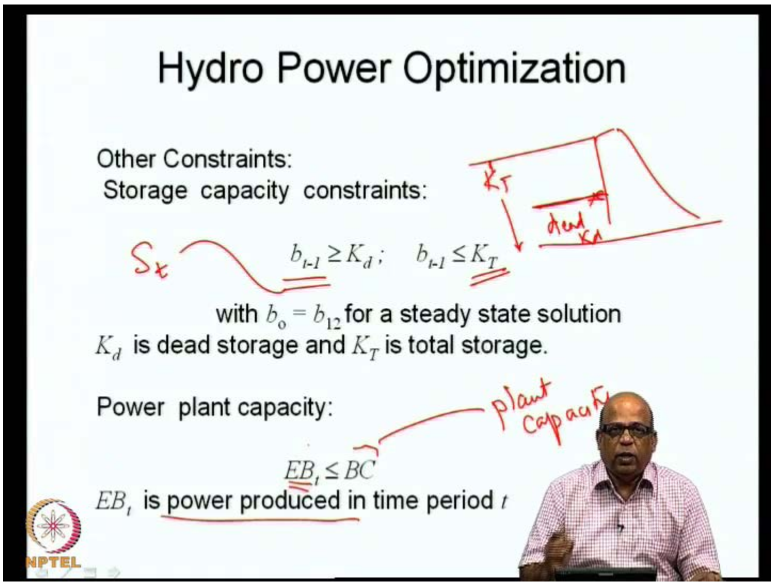

Then we put other constraints, one is the storage constraint itself; remember you have for the reservoir storage, you have a dead storage and that dead storage is denoted as K T, and you have total storage and that is dead denoted as K T, K capital T, and b t minus 1 is nothing but S t. So, we are saying that the storage at any time period t must be greater than or equal to dead storage, we do not want to allow the storage to go down, and at the same time, it should be within the total storage K t. So, these are the two constraints that we put for the storage capacity.

Then we have the power plant capacity; if you have a power produced in time period t, we denote it as EB t; EB t is the total power that is produced in time period t must be less than or equal to the plant capacity. So, this is plant capacity itself; then we need to write EB t also in terms of the release and storage; now we are writing all the constraints now, we will also write the objective function presently; now for writing E t here EB t, which is the power produced. In fact, we are going to make decisions on the power produced, we want to maximize the power produced and power produced is related to the head that is available in time period t, as well as the release R t that you are making to the turbines. So, first will look at the head that is available; now H t as I mentioned, if you are looking at monthly time periods, H t will depend on the storage at the beginning of the time period as well as storage at the end of the time period. So, we take the average of the storage is, and then look at the average net head that is available.

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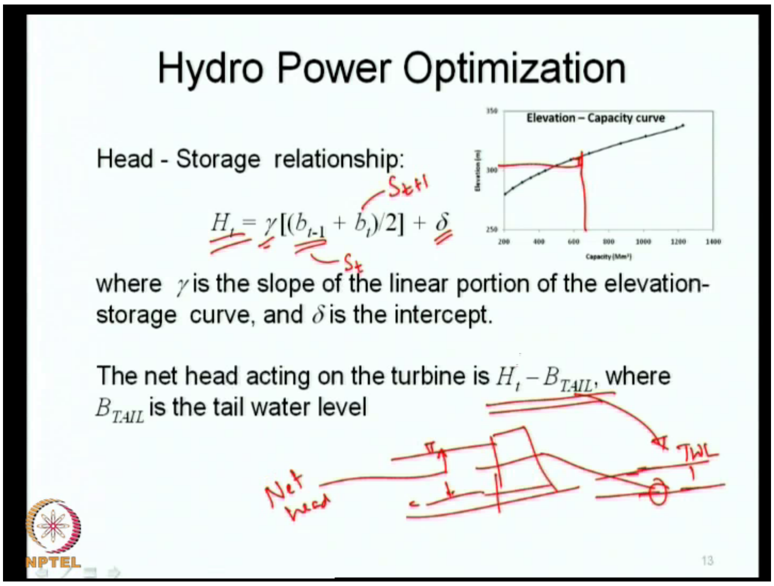

So, we will write, if you recall when I discuss hydropower generation you know, for simulation of hydropower generation, if you recall we use the capacity elevation relationship, we want to determine the elevation for a given storage, by capacity I mean the storage; and typically you will have a curve like this, this will give elevation capacity relationship; we use a linear approximation for this curve and write H t as a linear function of the storages; remember b t minus 1 is nothing but S t, and b t is nothing but S t plus 1. So, we take S t and S t plus 1 come to this level S t plus S t plus 1 by 2, in fact come to this level and then read the capacity; now that we are going to approximate by a linear relationship, we determine gamma and delta, which are the constants here through the linear relationship or linearized approximation of the capacity elevation relationship.

So, that for a given value of this storage namely S t plus S t plus 1 divided by 2, we will able to get the H t, now H t is the total head and we may have a tail water level, recall that you have the dam here, and then you have penstock, the turbines is here, and then you may have tail water level here, and if (Audio not clear here. Refer Time:31:07) here, tail water level is here, this becomes the Net head; this would be net head; and that is what we write as H t as determined from here, minus B TAIL, which is the tail water level, now B TAIL is the tail water level. So, that is how we determine the net head that is available for power house, the power production; R t is the release that you are making to through the penstock to the turbine and H t minus B TAIL is the net head that is available, and therefore, you should able to write the power that is generated.

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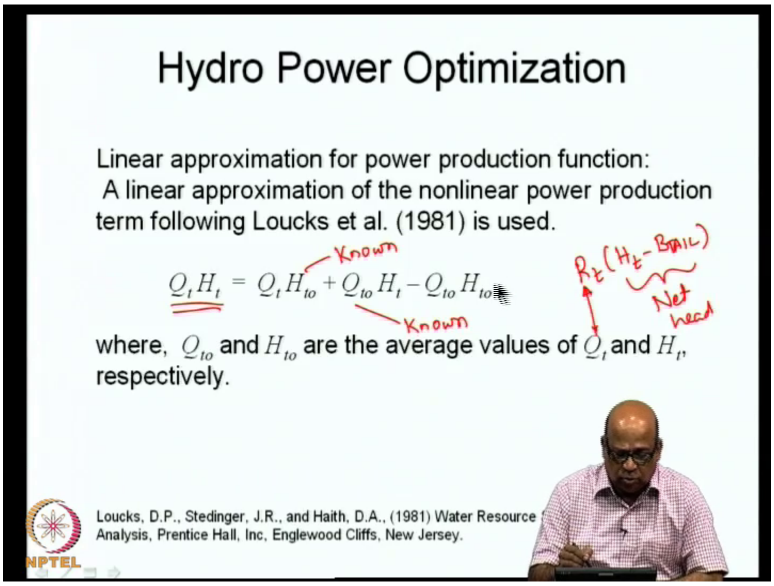

However, there is one more problem here, because you recall that the power that is generated is a non-linear function of the discharge and the head; typically we get power as proportional to Q t into H t, where Q t is the release that is made it into the penstock, and H t is the net head, and that is available. So, what we do is, we write now the product Q t into H t as a linear relationship; and this I pick it from Loucks et al. 1981 and the reference is given here, you can refer to this, what we do is, we write Q t H t, where Q t is the release that is made into the penstock, in our case it will be R t, I will just come to that; and H t is the net head, so we write Q t H t is equal to Q t into H t0, which is known, and Q t0 into H t, Q t0 is known, minus Q t0 into H t0 both of them are known. So, in this expression now, what I am saying is Q t into H t, in which both of these are unknowns; we write it as Q t into H t0, H t0 is known; and therefore, some constant into Q t plus some constant into H t minus this constant and this constant, that is how we write.

Now the question is what are the values of H t0 and Q t0, I will come to that presently, but we will replace our in our notation, it would have been R t into H t minus B TAIL, this is the net head, and this R t is nothing but Q t, that is the release that is made. So, this this is the expression that we need to express as a linear function. So, we will come to that; so essentially, what we are doing is multiplication of two decision variables both of which are unknown and multiplication of the two decision variables will render the problem to be non-linear and therefore, to overcome that problem of non-linearity we express the multiplication as Q t into H t 0 plus Q t 0 into H t; Q t 0 is known, H t 0 is known, minus Q t 0 into H t 0 therefore, this becomes a linear function.

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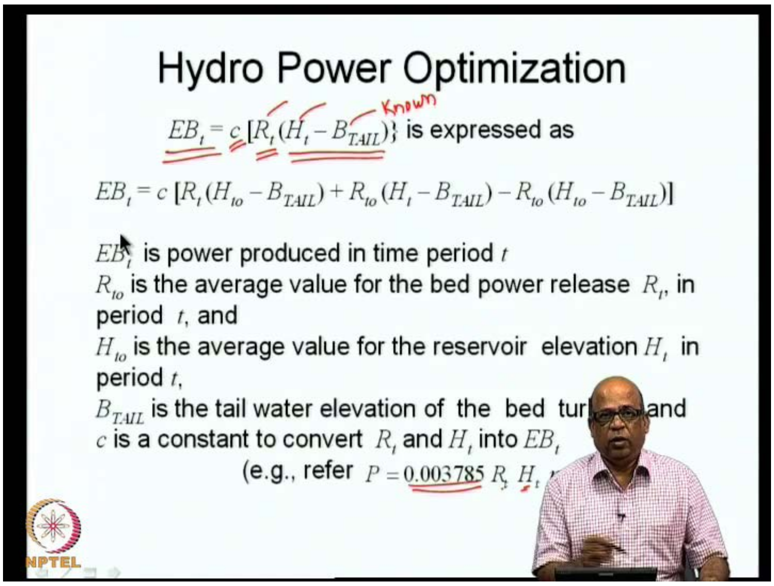

Now we are writing EB t, slightly compression notation, but just varying it further, because there are several repetitions of variables and therefore, we need to be careful about the notation. So, we will say EB 0 now; E B t is the actual power that is produced, and that will be depended on R t into H t minus B TAIL; H t is unknown head, which is depended on the storage at the beginning of time period t as well as beginning of time period t plus 1; B tail is known, this is a feature of the power house therefore, the tail water level is known, this is unknown and therefore, this is the non-linear function and there is a constant C here, which will convert their net head and their discharge, the product of that into the power that is generated. So, the constant here is you can refer to our earlier lecture, where we got something like this; P is equal to some number like this, R t into H t which is the net head into the efficiency. So, efficiency into all of this can come to this constant here C. So, the constant C here is a constant to convert R t and H t into EB t that is a power produced that can be determined based on your specific case study.

So, this will write now as using this now expression Q t H t is equal to Q t H t0 plus Q t0 H t minus Q t0 H t0. So, I will write this as C into R t into H t0 minus B TAIL that is what I am doing here is, I will retain only one of the decision variables in each of the terms either R t or H t. So R t, first I will take the H t0, which is the constant head or which is the known head minus B TAIL, B TAIL is known plus R t0 into H t minus B TAIL, in this you will have only one decision variable H t, R t0 is given; in this you have only one decision variable R t, H t0 is given; and the last term here is simply Q t0 into H t0, which is R t0 into this term here. Now what we do is, so H t0 and R t0 are assumed constant here, because we are trying to convert a non-linear expression into a linear expression; and what we do is, H t0 and R t0, we assume as the average net head average head as well as the average is during the time period t.

We solve, when we solve the model, I will write the complete model presently; when we solve the model, you will get certain values of R t as well as certain values of H t coming out as decision variables, then you need to check whether this expression is in fact, in fact satisfied for the particular decision variable that we get out of the optimization; if it is not satisfied, you do it in a iterative manner, let us say you got one value of R t and anther value of H t, and then the equation is not satisfied, the constraint is not satisfied; then you set R t0 that value of R t and H t0 is equal to that value of H t as you obtain and then resolve the problem. So, this has to be done in an iterative manner. So, all this expressions are understood, so EB t is the major variable, which we are maximizing; this is the power now, we are writing this as the hydropower generated, we are looking at maximization of the hydropower.

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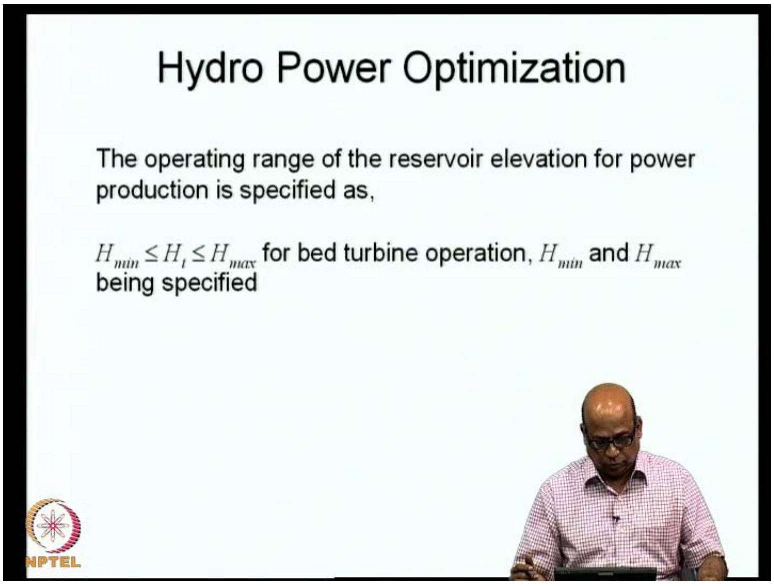

Then we also have constraints on the reservoir elevation for power generation; that means, you would like to operate the reservoir only during, only within certain range of the heads, and that is what we specify as H t must be within this range between H min and H max; some minimum elevation as a maximum elevation; which need not be always corresponding to the dead storage and the total storage. So, this range is fixed by the turbine operation, and you will know the values of H min and H max within which your head as to varied.

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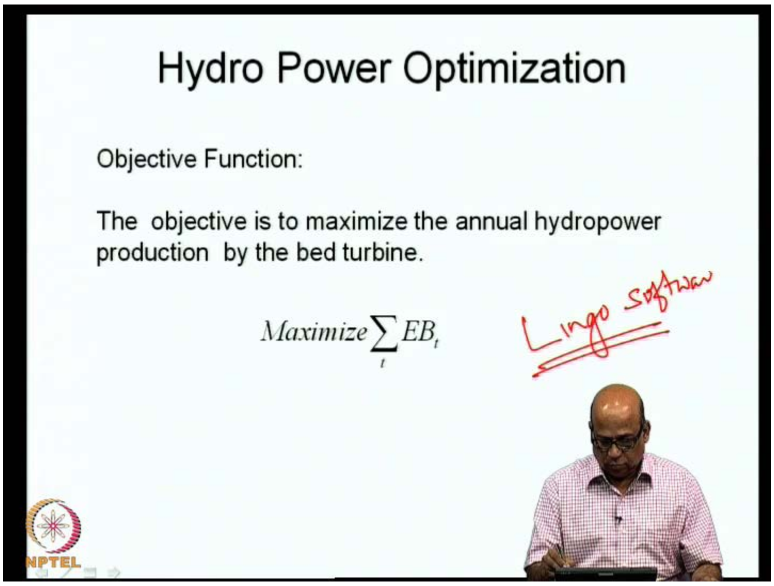

So, all the constraint you understand correctly, we started with a chance constraint, probability of I R t being greater than or equal to D t, D t being the demands, irrigation demands, and this value that is a probability value must be greater than or equal to P, which is the pre-specified reliability level; we want to meet the irrigation irrigation demands with a minimum reliability of P, and P may be of the order of 0.8, 0.9 etcetera. So, 80 percent of the time minimum, you need the demands; then you look at the the evaporation; evaporation depends on the area of water spread, and you are talking about finite length of the time period, typically of a order of one month and then you convert the evaporation into a linear relationship with respect to the storages, by using the storage elevation, storage area relationship; then you come to the head that governs the power generated the head you express again as a linear function of the storage using the storage elevation relationship; now the storage elevation as well as storage area relationship are features of the particular reservoir and these data will be available always.

Then you look at the non-linear relationship R t into H t or R t into H t minus B TAIL in our notation, where you are talking about product of the decision variable R t and the decision variable H t minus B TAIL, B TAIL is the known, but H t is unknown, and that you express as a linear function of R t as well as H t, by assuming certain R t0 and certain H t0, and this model you solve again and again until R t0 and H t0 are converse or they are such that the values the expression is satisfied, the non-linear expression is satisfied, that is a approximation of non-linear Q t into H t term or R t into H t minus B tail term is, in fact satisfied; now we look at the objective function, the objective function here is that we we are maximizing, the objective function is maximizing the total power that is generated in the year. So, we write this as maximize EB t, EB is one term, EB t which is the power generated during time period t, and that we summit our all the time periods within the year, and this becomes the the objective function.

So, we have the objective function, we have all the constraints ready with us; from the historical data, we know the CDF or the probability distribution of the inflows; we use that, and then use the deterministic equivalent of the chance constraint, and then formulate linear programming problem, use any linear programming software that is available, we used Lingo here; the Lingo software is ready available and also it is very simple to use for large linear programming problems, you go to the lingo software and then solve the problem.

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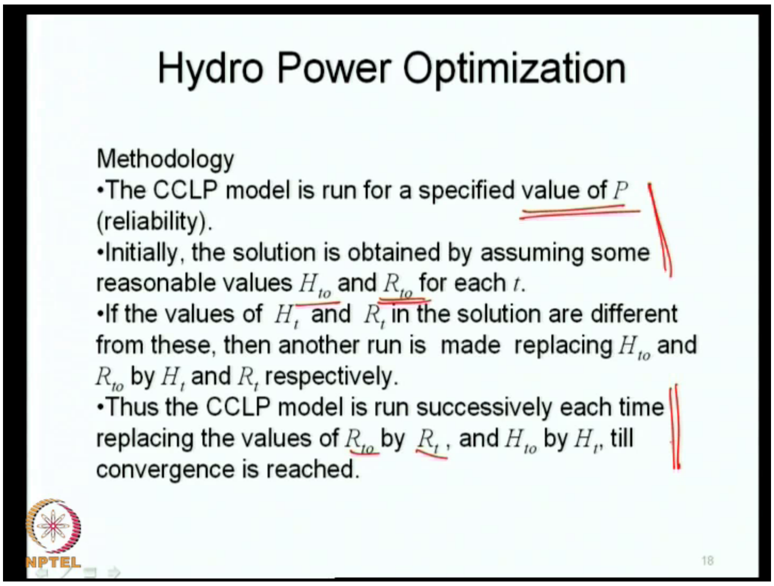

So, what we then do is, that first we specify the value of P that is, this is the reliability with which the irrigation demands have to be met; then we assume values of H t0 and R t0 and solve this problem; solve the problem, because everything else is known now; once H t0 and R t0 are assume, everything else is known or you can solve for those variables, and then solve it; then if the values of H t and R t in the solution are different from these, what I meant by that this, that if you are H t and R t are such that, they do not satisfy this expression here, corresponding to this.

Then we resolve that problem, another run is made replacing H t0 and R t0 by H t and R t respectively. So, if your H t0 and R t0 are not very close H t and R t, then you make another run by resetting H t and R t to be H t0 and R t0 respectively. So, the CCLP model, we run successively until you converge by converging I mean, R t0 becomes nearly R t, and H t0 becomes nearly H t, until that time we run this model, and then finally obtain the solution. So, that is the procedure; then what we do, let us say that we started with a value of P say 0.65 that means, we say that 65 percent of the time, the irrigation demands have to met, subject to that we are looking at what is the maximum power that can be generated. So, we get one certain power that can be generated, that is the maximum power that can be generated corresponding to the irrigation reliability being 65 percent; then we increase the irrigation reliability from 65 percent, let us say I make it 70 percent, then what happens to the power? Power has to reduced, because you are going to supply more more water to irrigation and therefore, the power has to reduced; the maximum power that can be generated corresponding to an increased level of reliability of meeting the irrigation demand will be lower.

So, as you increase the reliability of the irrigation, reliability of power will come down and therefore, the maximum power that you can generate will be smaller. So like this, we solve this problem again and again by each time increasing the reliability level until it becomes infeasible; let us say that you keep on increasing the reliability, unless the problem is, let us say that that is no limitation of water, and then there is adequate storage that is available. So, you may hit reliability of 100 percent, meeting the irrigation demand 100 percent of the time, and then you will get a maximum hydropower, but in general what happens is, as you start increasing the reliability, you will hit the certain reliability level of irrigation release, beyond which the problem becomes infeasible; that means, you may not able to increase the reliability of meeting the irrigation demands, beyond the certain point, because the inflows are limiting or the storage is limiting and at that point, you stop. So, you have generated essentially for various levels of reliability of meeting the irrigation demands, you will generate the maximum hydropower that you can generated; the solutions provide you the maximum hydropower that you can generate the corresponding to a given reliability of meeting the irrigation demands that is what is the idea there.

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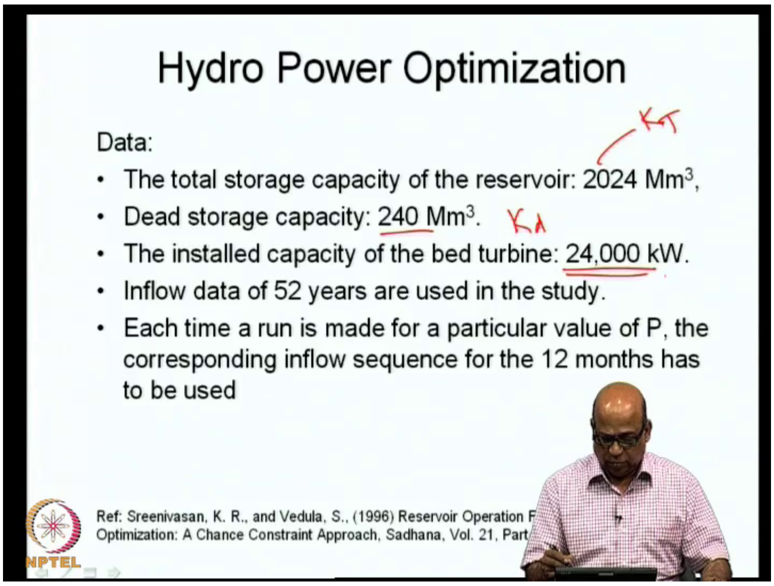

So, we will just look at some specific application in fact, this is the Bhadra reservoir and the details are available in this particular work Sreenivasan and Vedula, this is sadhana, which is the Indian academy of science is journal, you can just refer to this; but the idea here is to interpret the results. So, you have the total storage capacity 2024 in our notation, this will be K d, K t I am sorry, and this is K d, and installed capacity is known, this I thing we have used it as the variable BC, which is the installed capacity. So, we have this data BC, and inflow data of 52 years is used therefore, we know F of i t, I write this as F I t of i t, which means this is the CDF of I t is estimated, from the history historical data of 52 years; now each time a run is made for a particular value of P, and the corresponding inflow sequence for the 12 months will be used that is what what we are saying here is, this part you understand correctly, especially this is important for the new students, you are not use to the concept of reliability and associated flows here; so, this you understand correctly.

We specify the value of P which is the reliability of meeting the irrigation demands, corresponding to this 1 minus P here, you will get the inflow here, and this is for time period t, similarly for different time periods, you will get different inflow value associated with this 1 minus P value. So, P is specified, let us say 65 percent and therefore, this becomes 0.35, corresponding to 0.35 in time period t, you will get one value, time period t plus 1 you will get one value, time period t plus 2 another value like this, because you will have different CDF associated with different time periods, and for the same value of P, you will get a different flow in different time periods.

And therefore, for a given value of P, you will have a inflow sequence and that is the sequence that will use in your optimization, that is the idea here; each time a run is made; run means, run of the model, is made for a particular value of P, the corresponding inflow sequence for the 12 months has to be used, and this corresponding inflow sequence for the 12 months will come from F I t of 1 minus P, P is fixed, a F I t inverse of 1 minus P will give you that inflow value, and that is what we provide here.

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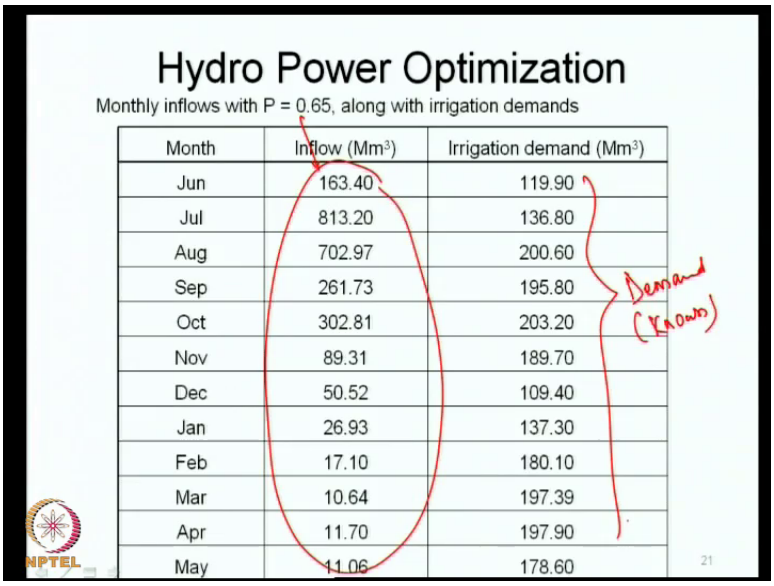

So, for month flow this is for P is equal to 0.65 as an example I am showing, P is equal to 0.65, these are the inflow that we obtain; that means, essentially we would have fit a CDF. Now, in the absence of any regress methods, you simply use the (( )) formula to fit the CDF, that is arrange them in the decreasing order, and then associated probability is calculated etcetera; you please refer to the stochastic logic course, in which some regress methods are given for estimating the CDF for a given sequence of inflows. For this value of 0.65, these are values which correspond to F I t inverse 1 minus P, so these are the values. So, you get one sequence associated with specified value of P, and these are the demand values, which are specified; there are independent of the reliability; remember, now these are the irrigation demand values known.

So, when we solve this you get a certain maximum hydropower that is generated; then you increase P, let us say P is equal to 0.65, you make it P is equal to 0.7, this inflows sequence will change, as you change this, the inflow sequence will change, because that will depend on the probability level, and for each of the month, there is different CDF and corresponding to that particular probability level, in fact 1 minus probability of that, you will get a different inflow, and as you change P, the inflow sequence will be different; therefore, you rerun the model; keeping the irrigation demands same here, and then you will get another maximum value of power that is generated; remember every time that you solve the problem, you will get maximum EB t, that is some of EB t which is the total power that is generated over the year; and thus you will get a trade of between the reliability of meeting the demand P verses the maximum power that you can generate associated with that reliability of meeting power demand; and that is what we get here.

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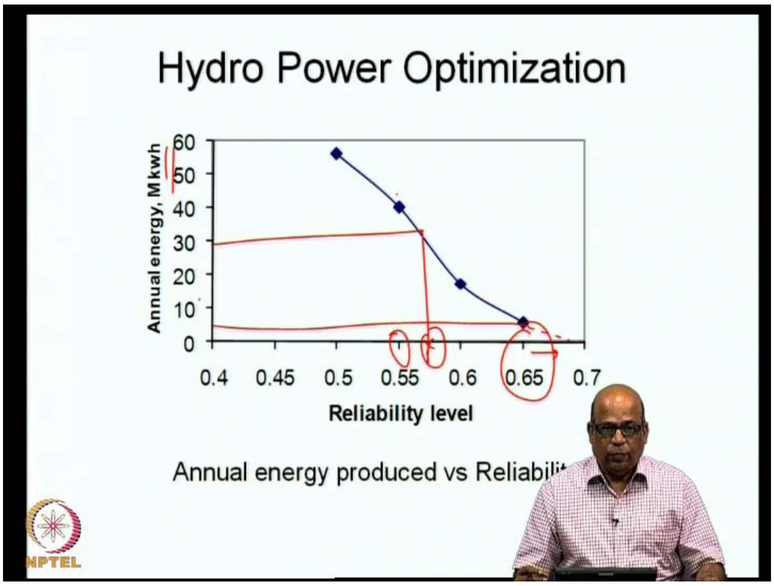

So here, you will get the reliability level, you fixed 0.65, for the 0.65 it is comes to somewhere around 6, this is billion kilo watt hour; so, the unit is kilo watt hour here and million kilo watt hours; then as you increase reliability, this will be smaller and smaller beyond certain point, you cannot increase further. So, you cannot go beyond let us say 0.7, 0.8 and so on; you can examine that if you increase the reliability further, the problem becomes infeasible; and therefore, you will start looking at, how much can I increase the annual energy? If I want minimum of 30 million kilo watt hours, what is the reliability with which I can meet the irrigation demand. So, I may have to sacrifice the irrigation reliability to about 0.58 let say, to achieve a minimum achieve a energy generation of million kilo watt hours, 30 million kilo watt hours.

So, this figure that you are seeing now, provides you a good tool with which to make decisions on how to operate the reservoir; do I operate the reservoir at 55 percent reliability of meeting the irrigation demands, in which case I will get higher power or do I sacrifice on a power and meet the irrigation demand at a much higher level, let us say I want meet at 65 percent; then I will be able to generate only 5 million kilo watt hours. So, this gives you a trade of between irrigation reliability and the power that can be generated associated with every solution here, remember there is a operating policy;

what do I mean by operating policy? You will get the b t, which are the decision parameters as the as the decision variables out of the optimization problem; the b t is in fact, will decide on the operating policy, because S t plus 1 is equal to b t.

And therefore, b t which comes as the decision variable, will dictate the storage that that is to be maintained at every time period, and that becomes an operating policy. So, the operating policy here will be decided based on a trade of between the reliability of meeting the irrigation demand here, and the associated maximum hydropower that you can generate; remember this is the maximum hydropower that you can generate associated with this particular level and how this gets so decided? It gets decided, because associated with this reliability level, there is a particular in flow sequence that you need to use, and that inflow sequence which is governed by the probability distribution of the inflow, which has been determined based on the historical data, that will dictate how much power, how much water is available for power generation after meeting the irrigation demand at a particular reliability level.

So, this is how you optimize the hydropower generated, this is one of the methods where we have use the chance constraint method; you can also use the stochastic dynamic programming, you can use the basin stochastic programming and so on. So, there are several methods available for hydropower generation, hydropower optimization this is one of the simpler way of doing it by considering the inflow uncertainty.

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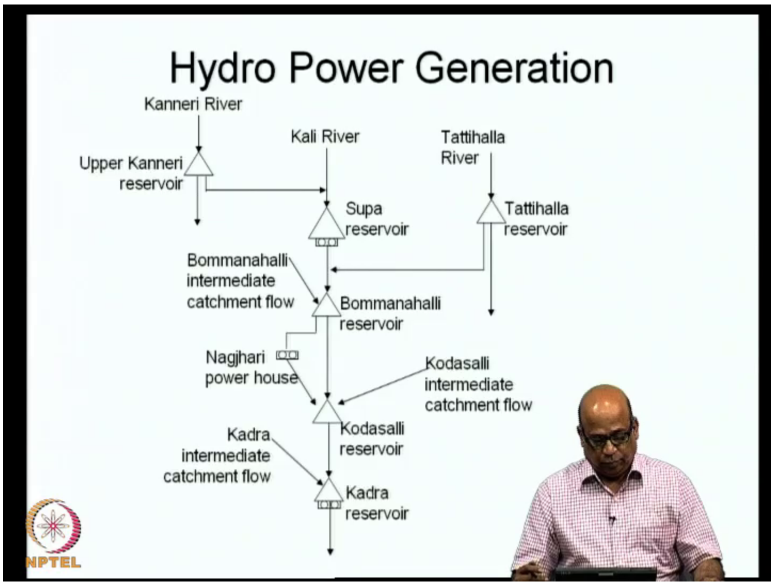

Now before I close the hydropower discussion, I will just explain how we do multi reservoir operation for hydropower generation, within just one or two minutes, I will just explain this. Now this is the KHP stages to, that is Karnataka hydropower project, electric hydropower project stages to, where you have the Supa reservoir here, and you have Nagjhari power house, there is a power house here, there is a power house here, and at Kadar there is a power house here; now this is the multi reservoir system operated only for power generation.

I have discuss in the earlier lecture, when I discussed hydropower, how to simulate for single reservoir; use the same procedure for multiple reservoirs, except that at every reservoir you look at the total contribution from, coming from upstream reservoirs, for example, this upper cannery reservoir is a balancing reservoir, which will supply water as and when it is required; similarly Tattihalla is a balancing reservoir, it will supply as and when it is required; Bommanahalli supplies to Nagjhari power house. So, the main power house here is the nagjhari power house, there is the another power house here, an Supa Supa dam; there is another power house here at the Kadar; suppose for example, you are looking at the nagjhari power house simulation; then what you need to look at?

You see all the intermediate catchment flow that comes here, plus the release contribution from the Supa reservoir, now that release contribution comes out of the power house therefore, it would have generated power house. So, look at the total power that is generated at Supa reservoir as well as at the Nagjhari reservoir. At every every reservoir, you just look at the continuity of that particular reservoir, how much is release that is going out, how much that has been coming from the intermediate catchment as well as from the upstream reservoir and so on. So, you have for example here, you have Bommanahalli contributing here, something comes from the Nagjhari power house and there is also an intermediate catchment flow that comes here, and that feeds into the Kadra reservoir. So, you write the continuity equations at each of these reservoirs, by accounting for what is coming from outside upstream reservoirs plus what is coming from the intermediate catchment flow, and then generate the power; using the same single reservoir power simulation single reservoir simulation that we have studied earlier.

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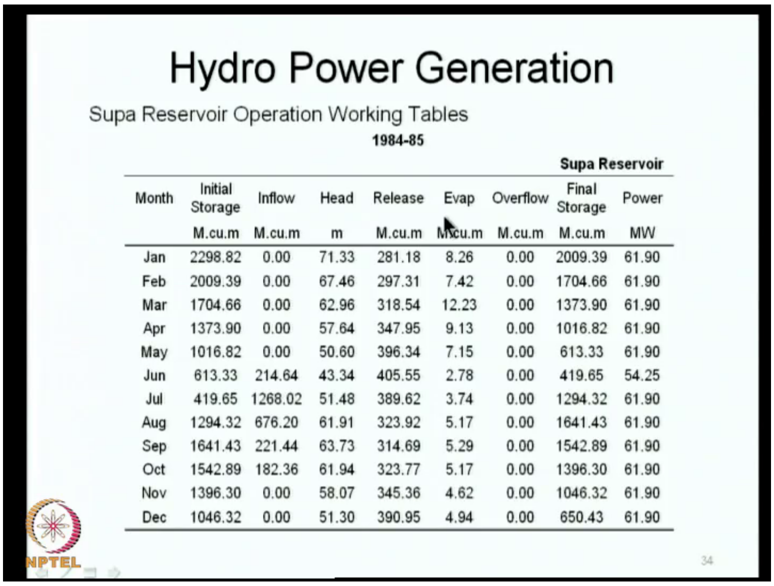

Hydro Power Generation

Supa Reservoir Operation Working Tables

1984-85

Supa Reservoir

Month	Initial Storage	Inflow	Head	Release	Evap	Overflow	Final Storage	Power
	M.cu.m	M.cu.m	m	M.cu.m	M.cu.m	M.cu.m	M.cu.m	MW
Jan	2298.82	0.00	71.33	281.18	8.26	0.00	2009.39	61.90
Feb	2009.39	0.00	67.46	297.31	7.42	0.00	1704.66	61.90
Mar	1704.66	0.00	62.96	318.54	12.23	0.00	1373.90	61.90
Apr	1373.90	0.00	57.64	347.95	9.13	0.00	1016.82	61.90
May	1016.82	0.00	50.60	396.34	7.15	0.00	613.33	61.90
Jun	613.33	214.64	43.34	405.55	2.78	0.00	419.65	54.25
Jul	419.65	1268.02	51.48	389.62	3.74	0.00	1294.32	61.90
Aug	1294.32	676.20	61.91	323.92	5.17	0.00	1641.43	61.90
Sep	1641.43	221.44	63.73	314.69	5.29	0.00	1542.89	61.90
Oct	1542.89	182.36	61.94	323.77	5.17	0.00	1396.30	61.90
Nov	1396.30	0.00	58.07	345.36	4.62	0.00	1046.32	61.90
Dec	1046.32	0.00	51.30	390.95	4.94	0.00	650.43	61.90

So, typically then what you get in such a situation is, the working tables; for example, for Supa reservoir, you will look at the working tables; now this working table is a is similar to what we have discussed earlier, except that here we have considered the multi reservoir operation. So, we take account, take in to account the multi reservoir operation, and then simulate the hydropower at each of this location. So, essentially then we have discussed in today's class the hydropower optimization specifically dealing with the using the chance constraint linear programming for a single reservoir system, and then I have just explain towards the end of the class, how we use the multi reservoir simulation for hydropower generation; these are available as handouts, so because a multi reservoir simulation, I could not cover in detail; but it is just the extension of what we do for single reservoir systems; thank you for your attention we will continue the discussion.

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