

**Water Resource Systems**  
**Modeling Techniques and Analysis**  
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**Lecture No # 37**  
**Conjunctive Use of Ground and Surface Water**

Good morning, and welcome to this the lecture number 37 of the course Water Resource Systems Modeling Techniques and Analysis. Over the last few lectures, we have discussed the fuzzy optimization technique, which is especially useful, when there is a conflict in the water resource systems, which typically exists, because there are several stakeholders in any water resource systems problem.

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**Summary of the previous lecture**

Minimize  $D = \sum_{t=1}^3 \left\{ (T^S - S_t)^2 + DR_t^2 \right\} - 0.001T^S$

s.t.  $S_t + Q_t - R_t = S_{t+1} \quad t = 1, 2, 3$   
 $S_t \leq K \quad t = 1, 2, 3$   
 $R_t \geq T^R - DR_t \quad t = 1, 2, 3$

Maximize  $\mu_{min} = \text{maximum minimum } \{ \mu_{S_t}, \mu_{R_t} \}$

Specifically, what we dealt with in the previous lecture that is the last lecture was to deal with a situation, where we have a storage target and a demand, and the release target; and these two are conflicting in general, because if you make more release, then the storage will be depleted. So if you want to maintain both the storage target as well as the release target, one way of doing that was through our deterministic crisp optimization, where you will take the minimized square deficit like this, and then panelize the T S itself; here

$T_S$  is the storage target, and you were maximizing the storage target. And that is how you **get you** obtain the maximized storage target, and the associated storage is during each other time period for given inflow sequence; and this also decides on the releases for a given  $K$ , that is the storage capacity. And then you are looking at the deviations of the release from the target, that is a **only** whenever there is a deficit release from a target. So this was your crisp problem. This you converted into a fuzzy optimization problem by defining fuzzy membership function for the storage as well as for the release.

Now for the storage, what did we do? We said as it approaches, as the storage  $S_t$  during time period  $t$ , approach is the storage target  $T_S$ , which is again a decision variable, then the membership function becomes close up to 1; as it deviates away from  $T_S$ , the membership function will be close up to 0 on either side of  $T_S$ . Whereas on the release, what we said is that if the release is greater than the  $T_R$ , our membership function is 1. Now we constructed these membership functions, and then looked at maximize... Looked at the objective function, where we are maximizing the minimum of these membership functions  $\mu_{S_t}$  and  $\mu_{R_t}$ . So, the decisions are  $S_t$  and  $R_t$ , and the associated membership functions are  $\mu_{S_t}$  and  $\mu_{R_t}$ ; we are looking at the set of  $S_t$  and  $R_t$ , which satisfies all these conditions which will in fact, maximize the minimum value associated with those particular membership functions  $\mu_{S_t} \mu_{R_t}$ .

So, this is the way we formulated the fuzzy optimization problem. We looked also at the comparison of solutions that we obtained from fuzzy optimization problem with crisp optimization problem for another example problem. So with that now, we more or less complete the techniques that we have to learn for water resources systems analysis. So, right from beginning if you recall, we have been focusing on the techniques of problem solutions; for example, we started with constrained and unconstrained optimization using the methods of calculus. Then we went on to the linear programming, and then the dynamic programming, simulation recover, and then after that we went into probabilistic optimization problems or the stochastic optimization problems, where we dealt with chance constrained linear programming as well as stochastic dynamic programming. Then towards the end, I have introduced the fuzzy optimization problem.

Now, these are essentially the techniques or the tools with which we address a problems, and as we have gone along, we are also dealt with reservoir operation problems, multi reservoir operation problems, and water quality problem in the fuzzy optimization and so

on so. So some of the applications we have dealt with, but the main focus was all through on the techniques themselves.

So, from this lecture onwards what we will do is, over the next four lectures including today's, we will take up some specific model formulations for specific problems dealing with water resources systems. In the process, I will also discuss some case studies, but the emphasis again is on model formulations using the techniques that we have gone through in this particular course.

So, in today's lecture, we will introduce the important model formulation for conjunctive use of surface and ground water. Now this is we are talking about specifically large river basin or large water resource systems, where there is a source of surface water available, there is a source of ground water available, you need to manage both these systems. So, ground water system as well as surface water system together or conjunctively, such that you are able meet the demands, that are necessary; yet at the same time, the system is sustainable; in the sense that, the ground waters do not deplete beyond certain level; at the same time, the ground water does not enter into the root zone or **the** it does not come nearer to the ground surface causing salinity problem, water logging problems etcetera. And in the deficit situation of surface water, you are able to use the ground water to meet the demands.

So, ground water and surface water together, you need to plan; you utilization of ground water as well as surface water over a period of time. Now, this is the extremely relevant in countries like ours, where there are large number of regions which receive deficit rainfall in many years. And then we need to use the ground water resources during the deficit rainfall period; and yet at the same time rich as the ground water wherever there is a whenever there is a excess rainfall. So, these two together that is ground water system as well as surface water system, we need to look at a joined or conjunctive use of both these resources together.

Now, the type of model that I will introduce is extremely simple, and the first cut type of model just to understand how we formulate the models or such problems. It can in fact, be as complex as the details that you would like to input into the model, which all presently discuss, but because this is a first time, I am talking about conjunctive use; let


us look at some physical features of the conjunctive problems, and then see what are the advantages, what are types of constraints that we need to meet and so on.

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## Conjunctive Use

Conjunctive use of surface and ground water resources:

- Impounding stream water in a surface reservoir – transferred at optimum rate to ground water storage.
- Periods of above normal precipitation
  - Use of surface water
  - Artificial recharge of ground water
- Drought periods
  - Pumping of ground water
  - Lowers ground water levels

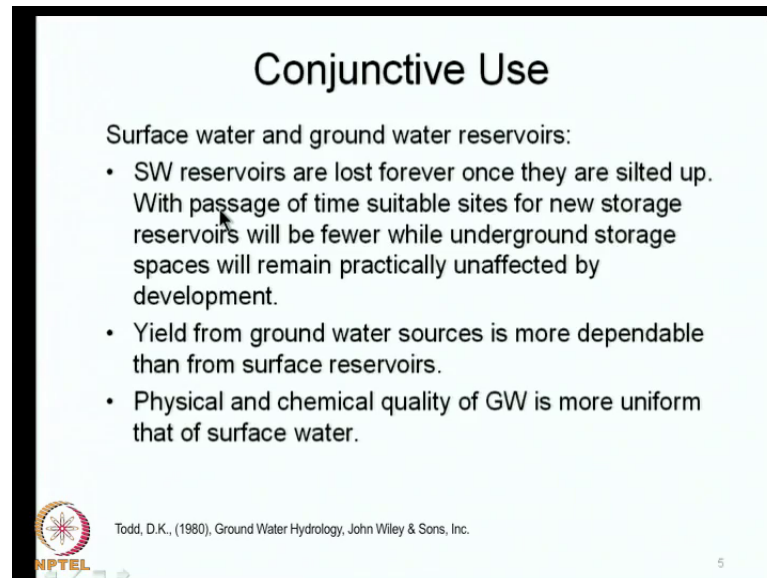
 Todd, D.K., (1980), Ground Water Hydrology, John Wiley & Sons, Inc. 4

So, what does the conjunctive use imply? It implies first of all, we impound the stream water in a surface reservoir, and then we transfer this surface water at some optimum rate to the ground water reservoir. So, we are meeting the demand, yet at the same time, we are also recharging the ground water. Then whenever there is a above normal precipitation, we use the surface water to the extent possible to meet the demands; and then the excess water we use to recharge the ground water; we may have artificial recharge structure, through which we recharge the ground water. And then during the drought periods, where there is deficit rainfall, you have a large amount of ground water storage available, which has been recharged during the excess rainfall period; and we start using the ground water, during the deficit period.

So, ground water actually is a sort of insurance against the droughts; and therefore, we need to look at the ground water as a result storage, in which we keep on putting additional water that is available from the surface sources, whenever there is a excess rainfall or excess surface water available. Now, during the drought periods what happens? Because you do not have surface water sources, you start using more of ground water resources and therefore, the ground water starts depleting.

Now, when we are operating the ground water system, in conjunction with the surface water system, we need to make sure that the ground water levels will fluctuate within an acceptable band; that means, they should not cross an upper limit, because that will cause water logging; they should not cross a lower limit that becomes unsustainable, because the ground water levels will be lower to extends beyond possible usage.


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**Conjunctive Use**

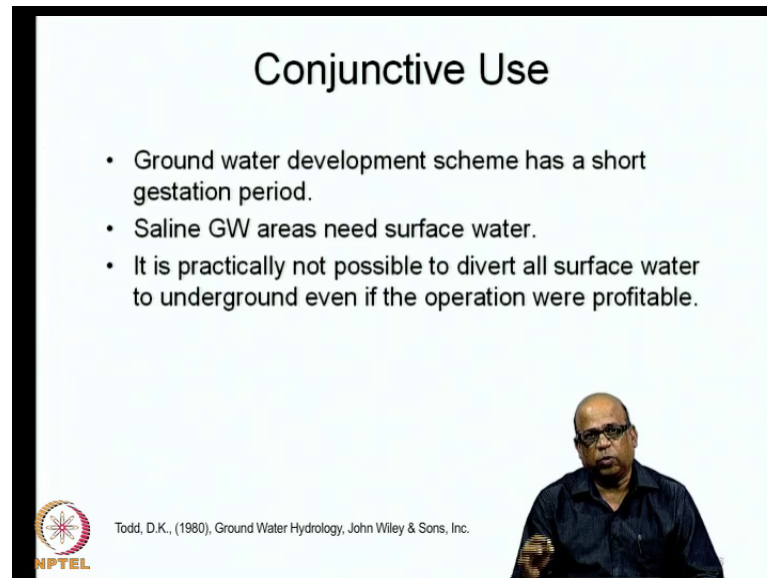
Surface water and ground water reservoirs:

- SW reservoirs are lost forever once they are silted up. With passage of time suitable sites for new storage reservoirs will be fewer while underground storage spaces will remain practically unaffected by development.
- Yield from ground water sources is more dependable than from surface reservoirs.
- Physical and chemical quality of GW is more uniform that of surface water.

 Todd, D.K., (1980), Ground Water Hydrology, John Wiley & Sons, Inc. 5

Then you must also realize here that the surface water reservoirs once they are silted up, typically in the period of about 100 years or 150 years in some cases, the surfaces water reservoir gets silted up. Then you need to look for alternate surface water sources; whereas the ground water storage by enlarge is the stable; it is not dependent on how the development is taking place on the surface of the ground; the ground water storage is more or less fix for a particular region. So, yield from ground water sources is there for more dependable than that from the surface reservoirs; if you are able to manage the ground water resources well, you can depend more on the, that is the reliability of ground water supply can be increased. Also the physical and chemical quality of ground water is more uniform than that of surface water.

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## Conjunctive Use

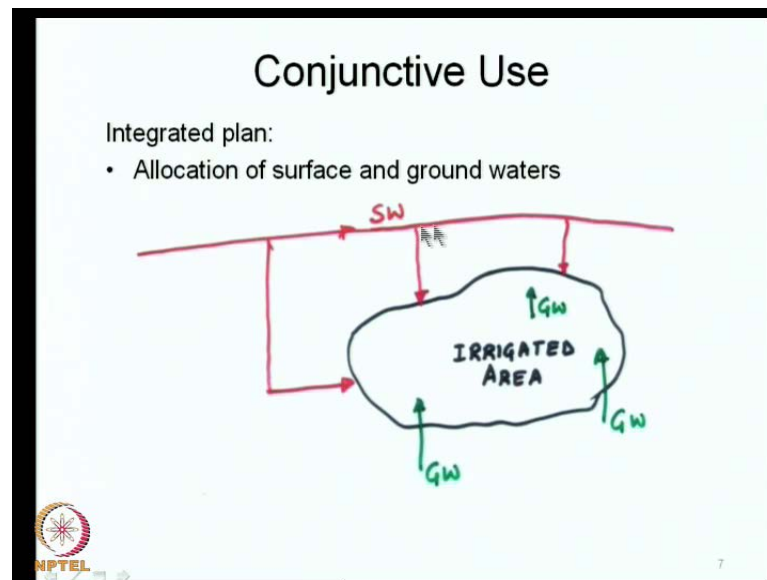
- Ground water development scheme has a short gestation period.
- Saline GW areas need surface water.
- It is practically not possible to divert all surface water to underground even if the operation were profitable.

Todd, D.K., (1980), Ground Water Hydrology, John Wiley & Sons, Inc.

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Now these are some of the advantages; however, in the coastal regions typically, if there is saline ground water that is salinity injects takes place and therefore, the ground water become saline, in which case you need to inject more of fresh water or more of surface water, so that the salinity gets reduced. Now of course, it is not practically possible to divert all of surface water to underground, even if the operation were profitable; what I mean by that is that you have a surface water source, and then you may have certain means or mechanism by which you want to put the fresh water into the ground water, but all of the surface water cannot obviously, be put into ground water, because of several constraints.

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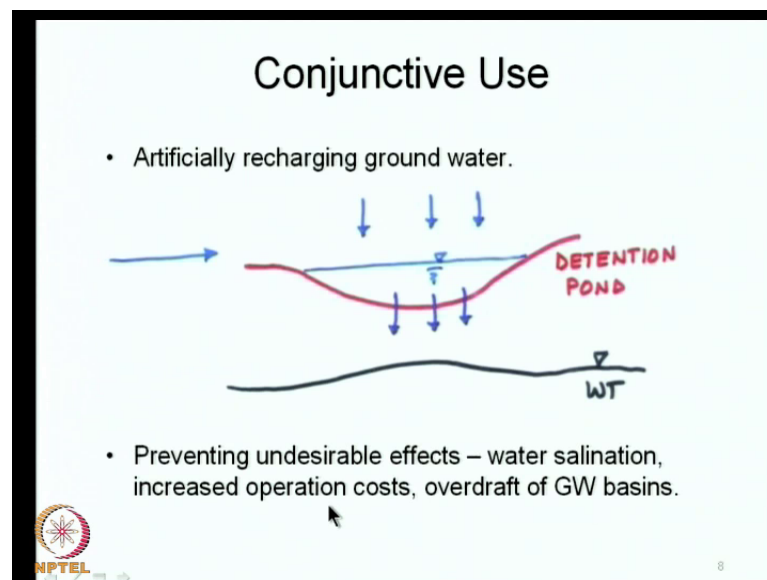
Now, you look at a irrigated area typically, when we are talking about conjunctive use of surface and ground water especially, in our country we talk about irrigation, use of water for agriculture, which is the irrigated agriculture. But this needs to be looked at in a slightly broader context although; now with water being used more in the urban areas especially, the ground water being used more in the urban areas, it is important for us to look at conjunctive use of surface as well as ground water sources in urban context also. But as far as this course is concerned, this lecture is concerned, I will focus mostly on the use of ground as well as surface water for irrigated agriculture.

So typically, we have an irrigated area, you may have the bore wells or tube wells from which you are using the ground water; in addition you may be supplying the water through surface source for example, this is the canal, and from the canal, you have branch canals and field canals etcetera. So, you have a canal network also supplying water from the surface sources. You need to decide, how many such wells are necessary in this irrigated area, and what is the level of pumping that you need to do from each of these wells? Across each time period within a year such that along with the surface water source, you are able to meet the demands to the best extent possible. So, that is the decision that you need to make.

So, the question is how much of surface water, and how much of ground water we need to apply, such that this system becomes sustainable in the longer. Now what do you

mean by sustainability etcetera, we will slightly defer the discussion; but essentially, it means that we are able to maintain this level of irrigation by using surface water and ground water optimally over a long period of time. So, that is the idea. So, for a long period of time, we should be able to meet these demands satisfactorily.

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Now, during the excess period I just mentioned that you may divert the excess rainfall or excess surface water into detention ponds; so you may create some artificial detention ponds and divert this water, so that the water seeps through, and then adds through the water table. In the conjunctive use, we must also be alert to the undesirable effect, undesirable effects of excess use of ground water or lowered use of ground water, which will **which will** bring the ground water to near surface.

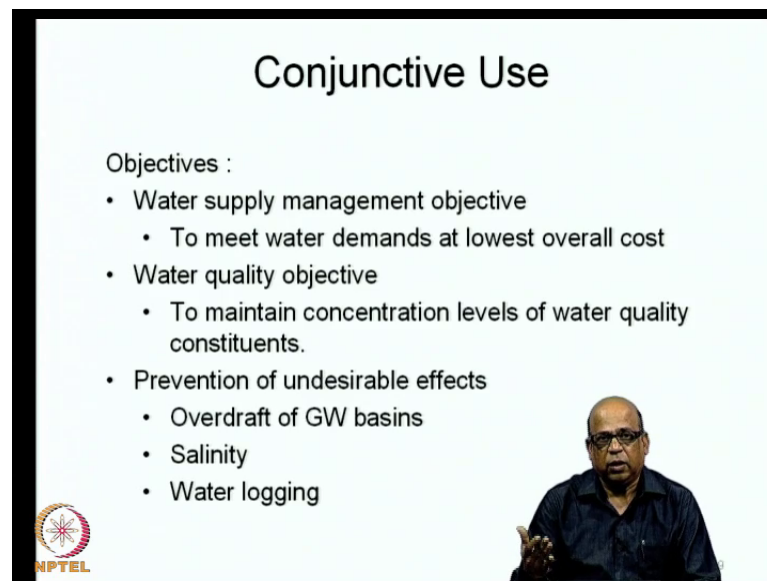
So, there are two issues that you need to keep in mind especially, when you are dealing with large irrigated areas. If you do not at all use the ground water, and simply you are using the surface water, from the surface water irrigation, there is a recharge that is taking place to the ground water, as the water flows through the canals, there is seepage, and part of the seepage and in fact, most of the seepage may contribute to the ground water. As the result of which, the ground water levels will be continuously increasing, if you do not use the ground water at all. And then the ground water may enter into the root zone of the crops, because we are talking about the agriculture irrigated that is the irrigated agriculture. Once a ground water enters a root zone, it causes salinity problems,



water logging problems, which affects the irrigation; and therefore, you need to manage the ground water as well as the surface water systems, such that the ill effects are nullified or ill effects are taken care of.

So, salivation is one of the effects that you need to be alert; then increased operation cause, what do we mean by this, is that if we were only putting the surface water systems, then there is a initial capital cost for creating the infrastructure. But the operational cost would be minimal, because the water flow by gravity, and once the infrastructure is in place, you do not have to do much about the surface water resistance; whereas if you want to develop the ground water, you need to have energy cost, associated with the ground water usage. So, there is a increased operational cost, if you start using ground water. Then there is a danger of overdraft of ground water basics. So, we need to look at all of these, when we are looking at the conjunctive use of surface water and ground water.



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**Conjunctive Use**

Objectives :

- Water supply management objective
  - To meet water demands at lowest overall cost
- Water quality objective
  - To maintain concentration levels of water quality constituents.
- Prevention of undesirable effects
  - Overdraft of GW basins
  - Salinity
  - Water logging

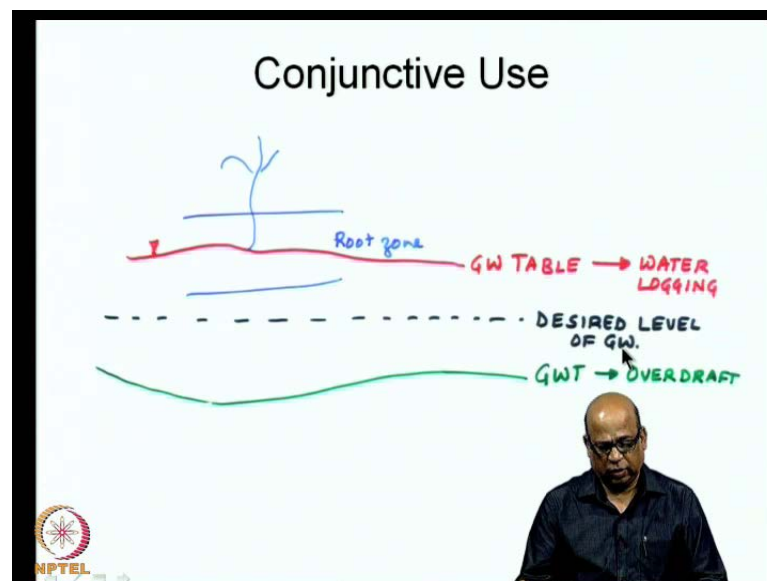
 

Now, for the systems models, when we are looking at the conjunctive use of surface and ground waters, we may have several objectives; for example, the first level of objective that we may want to fabulate is to meet the water demands at lowest overall cost. So, you want to create a surface water infrastructure, and you want to create a ground water infrastructure, to meet a certain demand pattern; and this infrastructure that we create

along with the operational cost must be minimum; yet at the same time, you should be able to meet the demands; that is the first level of objective.

Then you may have several other objectives associated with the conjunctive use problem; for example, you may have a water quality objective, where you may want to maintain the concentration levels of water quality constituents, both in the surface water as well as the ground water. So, you may have quality objectives, which we can be made through, which can be studied through the system's models. Then as I just mentioned, there may be undesirable effects; over draft of ground water levels should be discouraged or should be avoided; then salinity should be avoided or salinity should be minimized by using the surface water, which is the fresh water into the ground water; that means you dilute with the ground water with the surface water, so that the salinity levels decreased. Then water logging, where the ground water level rises to near surface, near ground surface, this should be avoided. So, these are some of the objectives that we need to look at.

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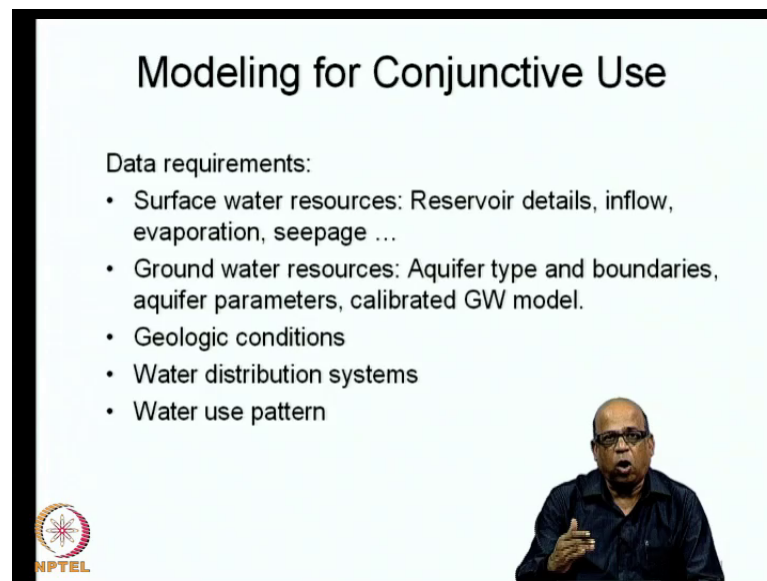


Now, whatever I just mentioned, this picture represents that. Let us say, this is the agriculture area, in which we are growing certain types of crops, let say wheat or paddy or some other crops like this maize and things that. Now, this crops here will have a root zone like this; the effective root zone. By root zone, I mean the zone of soil below the crop, from which the crops can extract water. So, in general, the root zone will be, the

effective root zone will be more than the physical root length that the crops have, because the crops can suck water from this particular soil, which may be more than the physical length of the root itself.

Now, if you do not use the ground water correctly, what may happen is the ground water table may keep on rising, and then enter the root zone. So, this is the ground water table. So, it may enter the root zone. Now this causes the problem of water logging. However, if it will keep on lowering the ground water table that means, if you keep on exploiting the ground water without properly recharging it; then what happens is the ground water table goes so below the ground level that it becomes unsustainable or unusable. So, this causes over drought of the ground water. So, we may want to have the water level of the ground water table to fluctuate within certain band, it has to be below the root zone, and at the same time it should not go below certain specified water levels. So, we may specify desired level of ground water or desired range within which the ground water can fluctuate.

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**Modeling for Conjunctive Use**

Data requirements:

- Surface water resources: Reservoir details, inflow, evaporation, seepage ...
- Ground water resources: Aquifer type and boundaries, aquifer parameters, calibrated GW model.
- Geologic conditions
- Water distribution systems
- Water use pattern

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The slide features a circular logo with a star-like pattern and the text 'NPTEL' below it. In the bottom right corner, there is a small inset image of a man with glasses, wearing a dark shirt, gesturing with his hands as if speaking.

With that background now, we will see how we formulate simple systems models for conjunctive use of surface and ground water. So typically, you will have a surface source, you may have a several surface sources, but the surface source may consist of a surface reservoir, there may be a dam with a canal network and so on. So, you have a surface reservoir; so you will recover the surface water resource data, these will include reservoir details for example, all this salient features of the reservoir, the area capacity

relationship, and height of the dam, the type of the dam, the type of canal network and so on. Then you deal the hydrology at that point including the inflow, evaporation, seepage losses and so on; the complete hydrology is what is required. Similarly, at the ground water, you may need the type of aquifer, you may need the boundaries of aquifer, then aquifer parameters typically, the storage coefficient time, the transmissibility, and you may need a calibrated ground water model, because we are now talking about conjunctive use of surface as well as ground water, and there is a continuous interaction, so ground water has its own flow mechanism, you need to be able to model the ground water flow, and the surface water has its own hydrology, and that hydrology needs to be model.

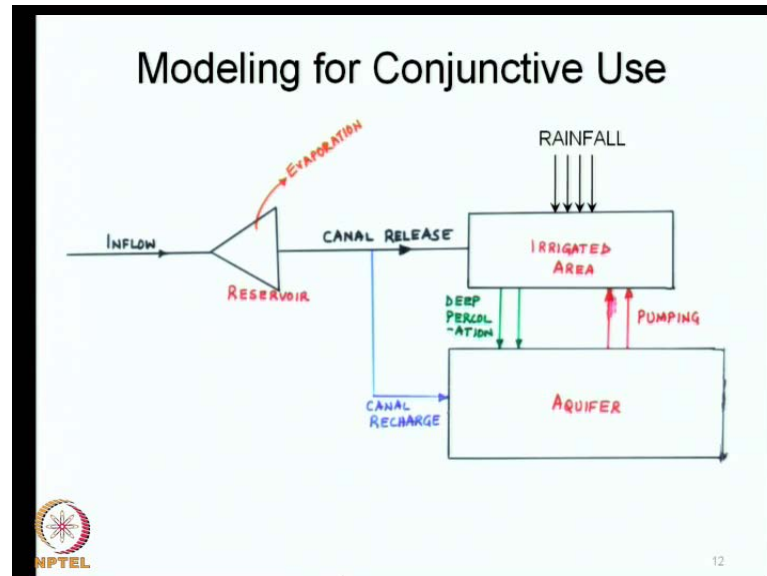
Then you need to know the geologic condition, because we are also talking about the recharge that adds to the ground water and therefore, the geologic conditions are important. Then water distribution system both for surface water as well as for the ground water, how many well, how many wells are there, what type of wells, what kind, what location of wells and so on. And then of course, a water use pattern; this should depends on the cropping pattern, and what type of crop is grown, what is the crop calendar, and what is the evaporative demands, evapotransportive demands of the crops and so on. So, all of these information would be necessary for getting a good model on surface and ground water conjunctive use.

Now, assuming that all of these information is available, what we will now do is, we will take a simple case of one surface reservoir, and one ground water reservoir, together we will see, how there is a interaction, and then how we model this? I say it is a simple model, because we are not considering the continuity of ground water storage with respect to the type of flow that takes place, where looking at it as a box, in which the ground water storage is fluctuating. And similarly, we put levels, within which the ground water levels can fluctuate and so on. So, it is a lumped model, its its in a very large scale, it is a very large scale lump model, you can replace it with any of the distributed models for example, you can take Morflow and then (( )), whatever I am saying now within the Morflow.

So, you can include as many details as you desire, and such modules are readily available, we should able to plug in those models and then address the problems of conjunctive use. However, the focus here now is, how to formulate the problems, given

that there is a surface water model, given that there is a ground water model, you need to put them in a system's model that is a focus of this lecture now.

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So, we will take a system like this, where you have a surface reservoir; at the surface reservoir, there is a inflow, which is governed by the catchment of the reservoir, the inflow in the catchment of the reservoir, and then there is an evaporation taking place, there may be also direct seepage from the reservoir itself; you have a canal release, you may have a canal network here, and this is the just a conceptual diagram, in which I am showing just one canal, but it is the actually a network of canals; and the canal release is made to an irrigated area, there is a seepage that takes place as the water flows from the reservoir to the irrigated area; this is through the canals, and in conjunctive use modeling, perhaps you will allow for some seepage rather than making everything perfectly lined and preventing the seepage, because the idea is also partially recharge the ground water.

So, typically the field channels etcetera, you leaved and lined, so that there is seepage that is taking place. So, canal seepage goes as canal recharge, part of it at least goes as canal recharge, then at the irrigated area, you have the rainfall, and then you have excess water going as depercolation that is the water above the field capacity of the soil goes down as the depercolation; and part of this depercolation will add as recharge. You also have pumping at the ground at the aquifer to supply water to the irrigated area; and part

of this pumped water also comes down as deep depercolation, and then it joins the aquifer. At the aquifer you may have a boundary like this; you may have a net inflow and a net outflow. So, the aquifer will take it as one box, and look at the continuity here; irrigated area we take it at one box, and then look at soil moisture continuity if desired; then the surface water reservoir is another box, in which we look at the surface water continuity.

So, simply put all of these different components together, and then look at an optimization problem. What is the problem here? The problem is to decide how much to be released from this reservoir here; and how much to be pumped from the aquifer, during each intra season time periods. Now, these time periods can be typically a month, it can be 10 day period or it can be a seasonal, seasonal time periods. So, you decide on the time period of planning, and then you look at the optimal use of surface as well as the ground water. So, the question that you are asking is, during each of this time periods, you have so decided, how much should be released from the surface water, and how much should be released from the ground water, such that over a period of time, the system becomes sustainable; in the sense that, you are able to meet all of these demand; yet at the same time, you are able to maintain the ground water levels within those prescribed limit, such that it does not come nearer the surface nor is it overdrafted that is the idea here.

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The slide is titled "Modeling for Conjunctive Use". It contains the following text:

Model formulation:

- Decision variables:
  - Reservoir release during period  $t$ :  $R_t$
  - Ground water pumping during period  $t$ :  $GW_t$
- $t = 1, 2, \dots, 12$  (monthly periods)
- Objective:
  - To maximize the net benefits in a year

Handwritten notes in red ink include a box labeled "Monthly Time Periods" with an arrow pointing to the time period list, and checkmarks next to the decision variable terms. The NPTEL logo is in the bottom left corner, and a presenter is visible in the bottom right corner.

So, the decision variables that I am taking about are the reservoir release during each of the time periods. Now I am formulating this model, this particular example for monthly time periods. So, we will do the operation for monthly time periods. Remember, this is the planning model, where you will plan how much to use from surface water, and how much to use from ground water; the actual operational model, which is the real time operation will be slightly different, where you will take the input from the planning model, and then incorporate in real time. So, right now, you look at these as a planning model that means, how much to be used, how much do we plan to use form surface as well as from ground water that is the idea here.

So,  $R_t$  is one of the decision variables, that is reservoir release during time period  $t$ , and ground pumping during time period  $t$ ,  $GW_t$ , so I will denote this as  $GW_t$ . Now we will say, as the simplest first cut of objective function, we will say that we want to maximize the net benefits in a year. Now the net benefits need not be always economic benefits, you may have intangible benefits also included in the model. But right now for simplicity, we will simply take it as economic benefits, occurring out of the water resources system operation. Which means what? The amount of water that we applied to the irrigated area in every time period  $t$ , both from surface water as well as from ground water has a benefit associated with it. And this we will assume that we are able to express in terms of the economic benefits. So, that is why, we say that we would like to maximize the net benefits in a year, for its conjunctive use system.

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**Modeling for Conjunctive Use**

- Constraints:
  - Ground water balance ✓
  - Surface water balance ✓
  - Minimum and maximum drawdown ✓
  - Reservoir storage limits ✓
  - Meeting irrigation requirements ✓
  - Total GW pumping in a year ✓

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Then on this, we will have to put the constraints. So, we define the objective function, we define this decision variables; now we will there has a set of constraints that we need to look at. First we have to look at the ground water balance that is from time period to time period, we are taking monthly time periods now. So, starting with first time period, which is the first month, we need to look at ground water balance; how the ground water levels are fluctuating, which means we need to look at the initial storage, ground water storage to which how much is recharged, how much recharge is added, how much of it is taken out as pumping, and how much is loss through other means, which will discuss presently, and how much is at the end of the period storage. So, that is the ground water balances.

Similarly at the surface water reservoir, you have the surface water balance. The inflow comes in, there is a existing storage, and then there is a evaporation taking place, there are other losses through seepage taking place, and then you are releasing certain amount of water that may be spill, that is taking place at surface reservoir etcetera, all of these together will govern the surface water balance. Then we may specify the maximum and minimum draw down; that means, we will specify that the ground water level, water table should be within certain **certain** band that is the maximum and minimum water draw down. Then you may have storage limits at the reservoir that is the surface water reservoir, you may specify a dead storage, and there is a capacity associated with it.

So, the surface storage should be within these limits that is reservoir storage limits. Then you have the irrigation requirements, because you are operating this particular system for a given cropping pattern, and you would like to meet the irrigated agriculture demands; and therefore, you will specify the constraints associated with the demands themselves that means, every time period you need to meet the demands to the best extent possible. Then apart from specifying the limits for the ground water level, for the ground water fluctuation, you may also want to place certain upper limits on total pumping through the year. So, you may specify certain constraints on the ground water usage by specifying the volume of water that is pumped from the ground water all through the year, which means every time period that is the decision variable associated with the ground water pumping. So, you know how much is pumped every time period, and you aggregate that the total pumping that you have decided based on the model should be less than or equal



to certain pre specified amount. So, that is the another constraint, that you may want to put.

So, like this depending on the type of situation that you get, you may specify different constraints. So, we will take these constraints, and then look at how to put it as a optimization model. So, what did we do? We decided the decision variables, we set the decision variables or the release from the reservoir surface reservoir  $R_t$  during time period  $t$ , and the ground water pumping during time period  $t$ ,  $GW_t$ ; these are lumped models, so we are looking at the total pumping from the **ground** aquifer, and the total release from the reservoir, surface water reservoir. And then we decided on the objective function, we set the objective function is to maximize the net benefits occurring out of the water application at the reservoir, at the irrigation area.

So, we are applying the water both from surface source as well as from the ground source; the actual amount of water is that is applied at the irrigated area has the net benefit associated with it; and then that is what we want to maximize. And then we decided on a set of constraints including the ground water level fluctuation, including the storage that fluctuate **fluctuates** between let us say the dead storage and the maximum storage, including the meeting of irrigation demands, so we may want to say that  $R_t$  must be always greater than or equal to  $D_t$ , where  $D_t$  is the demand, and  $R_t$  is the release and so on.

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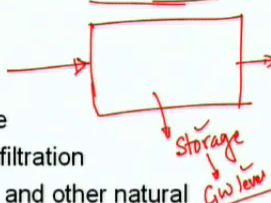
### Modeling for Conjunctive Use

Components of GW balance:  
 GW inflow – GW outflow = Change in GW storage

$$S_{gw} = W_p + W_r + W_c + W_{as} + W_{ag} + GW_i - GW_b - GW_e - GW_o - GW_{ET} \pm GW_n$$

where

- $S_{gw}$  = volume change in GW storage
- $W_p$  = recharge from precipitation infiltration
- $W_r$  = recharge from streams, lakes and other natural water bodies



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So, this now, we will put more formally in a mathematical form. And look at how we formulate the different constraints and the objective function. So, first we look at the ground water balance itself. Remember we are talking about the ground water as one box here, in which there is a net inflow and there is a or there is an inflow and there is a outflow, so we may have a net outflow from this system. And this  $e$  has storage, and we may say that the storage during every anytime period, must be within certain limits, and we assume that we have a model, because of the parameters being known; from the storage, we are able to convert the storage information into ground water level information.

So, we will be able to access both the storage as well as the ground water level at each of the time periods. Now, how to do this is the different and slightly more involved topic altogether, we will not go into this; for the purpose that we are looking at namely, put it in our systems model, you assume that this is like a box, in which the storage is fluctuating much the same way as the surface water reservoir storage fluctuates, and then we will incorporate constraints associated with it. So, what we will do is, we will take this as the change in the storage; there is a net inflow that is coming here, and then there is a out flow that is going in, going out of the system. So, we will take this as change in the storage.

So, we will say volume change in ground water storage, this is  $S_{gw}$ ; we include all the terms that add to the ground water storage. So, all these positive terms are actually adding to the storage, and all the negative terms indicate how much is extracted, for because of various reasons from the storage; we will see one of the one by one all of these terms, and then you may have a plus minus term, depending on whether it is adding or it is being extracted. So, the left hand side here is the volume change in ground water storage. So, change in storage. Now, the first term  $W_p$  is the recharge from precipitation infiltration; as I said from in the irrigated area, there is a precipitation taking place, the precipitation in excess of the field capacity in the soil that is the precipitation, that is going down into the soil, in excess of the field capacity, which is infiltration in excess of the field capacity goes down as recharge.

We will assume that all of that goes down recharge, although you must remember there is a unsaturated zone, water zone in which the flow also takes place horizontally. So, not all of the seepage or not all of the infiltration comes down as the recharge, but for the

time being we will assume that all of that comes as recharge. Then you may have recharge from streams, lakes and other natural water bodies that is  $W_r$ .

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### Modeling for Conjunctive Use

$W_c$  = recharge by storage structures, canals, distributaries and other irrigation works

$GW_i$  = ground water inflow

$W_{as}$  = recharge from surface water applied for irrigation

$W_{ag}$  = recharge from return circulation of GW applied for irrigation


$GW_b$  = GW discharge to streams and springs

$GW_e$  = GW extraction by pumping and flowing wells

$GW_o$  = GW outflow

$GW_{ET}$  = ET loss of GW from phreatophytic vegetation

$GW_n$  = other items, if any (e.g., artificial recharge through injection well)

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Then we have recharge by storage structures, canals, distributaries and other irrigation works. So, these should be because of the seepage through the canals, and the storage, and so on. Then you may have the ground water inflow, if you are looking at the aquifer boundaries like this, typically what we do in the conjunctive use models, we look at the irrigated area, which is on the surface, and then we demarcated the boundaries of the irrigated area, and then look at the aquifer boundary below the irrigated area. Physically the aquifer boundaries and the irrigated area boundaries will not match with each other, but for the purpose of conjunctive use we take only that much area of the aquifer or that boundary of the aquifer, which is just below the irrigated area. And therefore, anything outside of that, this is now looked at as a control volume, and then anything outside **outside** of that you account for through inflow into that system as well as outflow from the system.

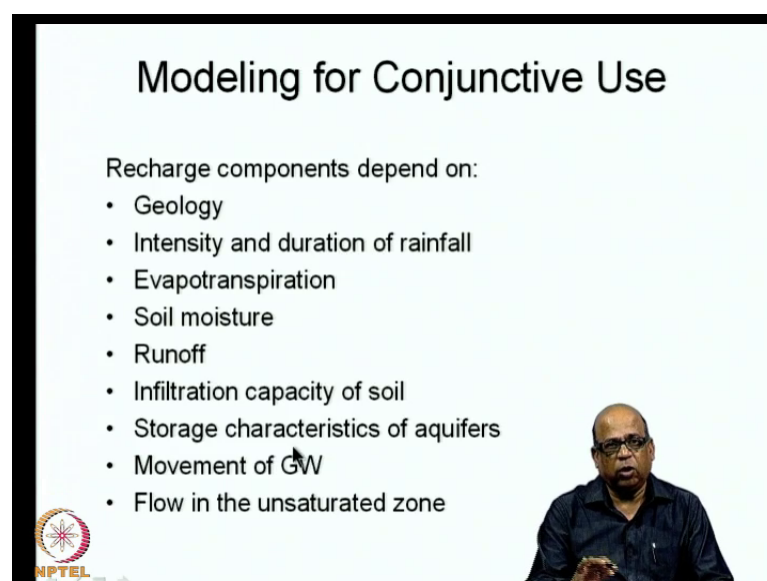
So,  $GW_i$  is this, what is coming into the control volume or the aquifer, then you have recharge from surface water applied for irrigation. So, this is  $W_{as}$ . Then  $W_{ag}$  is recharge from return circulation of ground water applied for irrigation; what we meant here is that as you see from the surface water, there is a part of thing that is coming down, and adding to the aquifer, which is the recharge due to the surface water

application. From the ground water, there is also a recharge that is taking place, and that is return circulation, from the pumping that is what is meant by these two terms. Then you have GW b, which is the ground water discharge to streams and springs. So, from the, within the aquifer, you are also taking out some water or it is contributing some into the streams as well as springs.

Then you have GW e, which is the ground water extraction by pumping and flowing wells. So, you **you** have actual pumping or some wells, which are just directly flowing into well irrigated area, and then you have the ground water outflow. So, these are various components; if you want to go into further details, you can also add the ET losses of phreatophytic vegetation that means there are vegetations, which are directly dependant on the water table itself; they take the water directly from the water table. So, they are called as phreatophytic vegetation; typically, these are large depths of roots, that means large lengths of roots, which extend into the water table itself.

Then you may have other terms included depending on the situation for example, you may have artificial recharge through injection well, directly it may be recharging into the ground water, in which case it becomes a positive term. So, like this you identify various terms in the ground water balance; this is simply the net change in the storage is equal to what has been input minus what has been taken out that is all.



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**Modeling for Conjunctive Use**

Recharge components depend on:

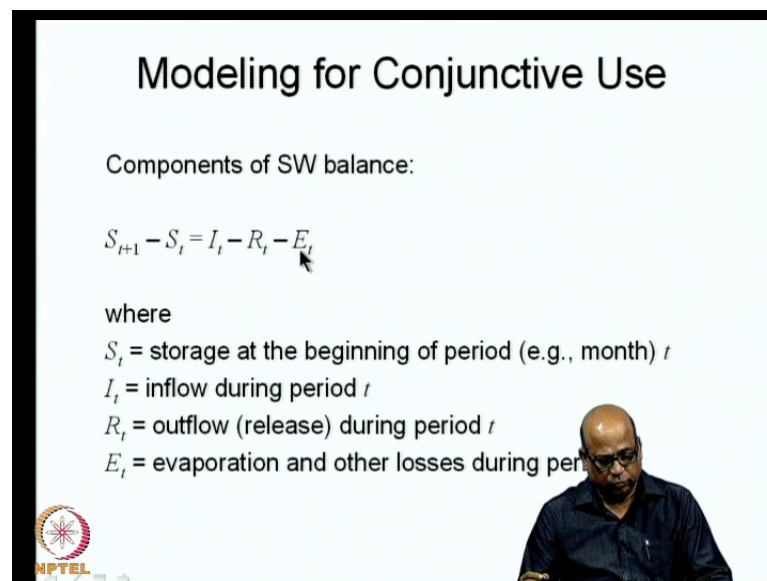
- Geology
- Intensity and duration of rainfall
- Evapotranspiration
- Soil moisture
- Runoff
- Infiltration capacity of soil
- Storage characteristics of aquifers
- Movement of GW
- Flow in the unsaturated zone

Now, the recharge components that we are talking about two different recharges now, several recharge in fact; one is directly through the rainfall, another is what is applied on the irrigated area. This comes as infiltration, and whenever the moisture content below the crop is more than the field capacity, the moisture content in excess of the field capacity comes from as depercolation, and we are assuming that all of the depercolation goes as recharge. So, the recharge component depends on first of all the geology, what type of aquifer that you have; it may be hard rock or it may be alluvial and so on. So, it depends on the geology itself.

Then the intensity and duration of rainfall, evapotranspiration, soil moisture, runoff, infiltration capacity, storage characteristics, movement of ground water in the aquifer, and then the flow in the unsaturated zone that is the (( )) that I just talked about, and so on. So, that are ways of computing the recharge, which generally is based on all of these factors. We now going to these details, we will simply assume that we know how to compute the recharge, for a given data, which pertains to all of user match.

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**Modeling for Conjunctive Use**

Components of SW balance:

$$S_{t+1} - S_t = I_t - R_t - E_t$$

where

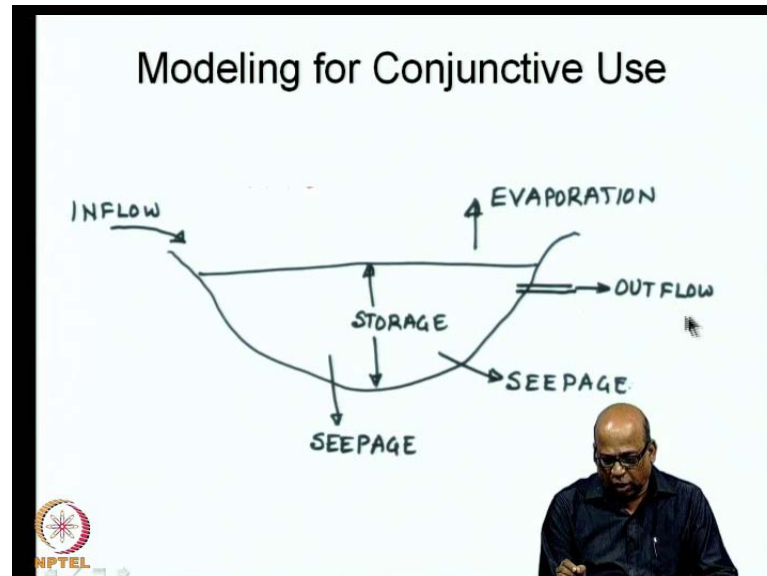
- $S_t$  = storage at the beginning of period (e.g., month)  $t$
- $I_t$  = inflow during period  $t$
- $R_t$  = outflow (release) during period  $t$
- $E_t$  = evaporation and other losses during period  $t$

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Then at the surface water reservoir, you have the surface water continuity, this is the same continuity that we have been talking about in the reservoir systems. So, will write the change in storage  $S_{t+1} - S_t$  is equal to, this is the inflow minus  $R_t$ , which is the decision variable minus  $E_t$ , which is a evaporation and all other losses together.

So, we will write a simple continuity equation like this. Remember  $R_t$  is the decision variable here,  $S_t$  also becomes decision variable, whereas  $I_t$  and  $E_t$  are the data.

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So, simply what we are doing is for the surface water storage, as I mentioned the surface water source need not be always a dam and associated reservoir, it may be simply a tank or several tanks together, irrigation tanks. So, we are looking at irrigation tanks and ground water usage simultaneously or conjunctively. So essentially, we are looking at the inflow that is added to the storage that is the seepage that is taken place, you have an out flow, and this  $R_t$  is the decision variable, and then from this evaporation is taken place. So, this decides the continuity or the storage continuity.

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### Modeling for Conjunctive Use

Example : From drawdown conditions,

- Total GW that can be pumped in a year is  $Q_p$
- Canal recharge: 30% of release
- Recharge from irrigation: 10% of water applied
- Inflow in period  $t$  :  $\{I_t\}$
- Net benefits for each unit of water applied in period  $t$  :  $B_t$
- Ground water storage in period  $t$  :  $GS_t$
- $G_{min} < GS_t < G_{max}$
- Reservoir capacity :  $S_{max}$
- Irrigated demand in period  $t$  :  $\{D_t\}$



Now, in this broad framework now, we will take a simple example, I keep repeating simple, because the conjunctive use problems are in fact very complex. You need to look at the two dimensional ground water module, you **you** may have two incorporate or include two-dimensional ground water model into the optimization problem itself. We will come to all those complexities, if the time permits at the end of this lecture. So, from the draw down conditions, this for an example, we will say that the total ground water that is pumped in the year is  $Q_p$ . So, we will specify the volume of ground water as  $Q_p$ , then whatever is released from the reservoir as it goes through the canal network, 30 percent of that goes as canal recharge; that means it goes out as seepage through the canals, and then adds to the canal recharge. So, 30 percent of the release goes as recharge.

Then whatever is applied at the irrigated area, 10 percent of the water applied comes down as recharge; now whatever is applied at the irrigated area may come from surface water source as well as or from ground water source or from the both these sources together. But whatever is applied, 10 percent of comes of that water also comes as recharge. Then we have the inflow at the reservoir as  $I_t$ ; and we will say that net benefits for each unit of water applied in period  $t$  as  $B_t$ ; we will indicate this as  $B_t$ ; we will assume that monitory returns are known, per unit of water applied in every time period.

We will denote the ground water storage in period  $t$  by  $GS_t$ ; so this is ground water storage. And we will specify that the  $GS_t$  should be within certain limits, minimum to maximum. Similarly at the reservoir, we have the reservoir capacity, and the irrigation demands are denoted as  $D_t$ . So, all of these are data, we know  $I_t$  is known,  $D_t$  is known, and  $Q_p$  is fixed. So, all of these is data; this is known, this is known, this becomes the decision that is  $GS_t$  becomes the decision variable, as indeed is  $S_t$ , which is the storage at the reservoir.

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### Modeling for Conjunctive Use

$$\text{Max } \sum_{t=1}^{12} B_t (GW_t + 0.7R_t)$$

Benefit per unit water applied at the field      Amount of GW applied      Amount of SW applied = 70% of release

s.t.

- Reservoir storage continuity

$$S_{t+1} = S_t + I_t - R_t - E_t \quad t = 1, 2, \dots, 12$$

- Ground water balance

$$GS_{t+1} = GS_t + 0.3(R_t) + 0.1(GW_t + 0.7R_t) - GW_t \quad t = 1, 2, \dots, 12$$

Recharge due to : Canal seepage      Deep percolation

We will write the now the optimization problem. The objective **from objective** is to maximize the net benefits, occurring out of the conjunctive use of surface as well as ground water. We know that the benefit per unit water applied at the field is  $B_t$  per unit water; and how much we have actually applied? We have applied  $GW_t$ , which is the through the ground water source, and  $0.7 R_t$  which is from the surface water source. Remember we are talking about the actual applications at the field level. So, this is the irrigated area, and you are bringing  $R_t$  here, 30 percent goes off as recharge. So, what is the actually applied here is  $0.7 R_t$ ; and at the same time you have also applied  $GW_t$ , which is the ground water pumping.

So, this term here that is  $GW_t$  as well as  $0.7 R_t$ ,  $GW_t$  plus  $0.7 R_t$  will indicate the total amount of water. And for every unit of water that you apply, you have a benefit of  $B_t$  therefore, this gives you the total benefit occurring out of the water application. And this



we sum over  $t$  equal to 1 to 12, there are time periods, and therefore this gives you the total benefit across the year. Then we write the reservoir storage continuity, and ground water balance; in the ground water balance now, we have discarded several terms, we will simply take it as  $GS_{t+1}$ , which is the ground water storage at the beginning of time period  $t+1$  is equal to the ground water storage at the beginning of time period  $t$  plus whatever as come from the canals seepage as recharge, we released  $R_t$  from the reservoir 0.3 of  $R_t$  that is the 30 percent of  $R_t$  comes as canal seepage, and that is the recharge.

Then whatever is applied at the irrigated area, we applied ground  $GW_t$ , which the pumping from the ground water plus 70 percent of the release  $R_t$ ; and this is the total amount that is applied; 10 percent of that comes as a recharge, so that adds, and minus  $GW_t$ ; now this is the decision variable,  $GW_t$  is the pumping. So, these are the two major constraints that we need to look at; one is the storage continuity, another is the, that is the reservoir storage as well as the ground water balance.

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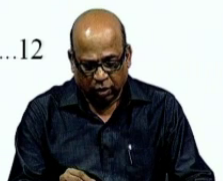
### Modeling for Conjunctive Use

- Minimum and maximum drawdown
 
$$G_{min} \leq GS_t \leq G_{max} \quad t = 1, 2, \dots, 12$$
- Reservoir storage limits
 
$$S_t \leq S_{max} \quad t = 1, 2, \dots, 12$$
- Irrigation requirement
 
$$GW_t + 0.7R_t \geq D_t \quad t = 1, 2, \dots, 12$$

$\nwarrow$   
 GW  
 application

$\nwarrow$   
 SW  
 application

$\nwarrow$   
 Demand

Then we put all other conditions that ever ground water storage must be within this two limits; the surface water storage must be less than or equal to  $S_{max}$ . And then we put the condition that whatever you apply at the irrigated area, namely the ground water pumping, ground water application plus the surface water application must be greater than or equal to the demand itself  $D_t$ .

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### Modeling for Conjunctive Use

- Constraint on total GW pumping
 
$$\sum_{t=1}^{12} GW_t \leq Q_p$$

$t=1, 2, \dots, 12$   
 Total pumping in a year      Maximum permissible pumping in a year
- End of the year storage
 
$$S_{13} = S_1$$

End of the year storage = beginning of year storage for next year

$$GS_{13} = GS_1$$
- Non-negativity
 
$$\left. \begin{array}{l} R_t \geq 0; \quad GW_t \geq 0 \\ S_t \geq 0; \quad GS_t \geq 0 \end{array} \right\} t=1, 2, \dots, 12$$




And we have also specified that the total pumping in the year must be less or than or equal to  $Q_p$ , we have said the  $Q_p$  is the maximum pumping that is allowed in terms of volume. So, every time period in time period  $t$ , you have  $GW_t$  as the pumping. So, the total in the time period, in the year is  $t$  is equal to 1 to 12  $GW_t$ , this should be less than or equal to  $Q_p$ ; for this is not there. So, this is the maximum permissible pumping in a year. And then because you are doing it in a deterministic sense, you will put  $S_{13}$  is equal  $S_1$ , and  $GS_{13}$  is equal to  $GS_1$ , to indicate that end of the period storage is equal to beginning of the storage for next year. And then you also add the non negativity conditions. So, this is the complete model, although a very simple model, I keep repeating simple, because you it is all lumped, ground water aquifer is lumped, the irrigated area is lumped, surface water reservoir including the canal networks everything is lumped.

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### Modeling for Conjunctive Use

- Data

Period t (month)	Inflow $Q_t$ ( $Mm^3$ )	Demand $D_t$ ( $Mm^3$ )	Evaporation $E_t$ ( $Mm^3$ )
1	70.62	245	10
2	412.75	308	8
3	348.40	308	8
4	142.29	308	8
5	103.78	285	6
6	45.00	190	6
7	19.06	190	5
8	14.27	78	5
9	10.77	65	6
10	8.69	0	8
11	9.48	0	
12	18.19	0	



So, this is the model that we write; and then we will take the example, where the data is given like this. You have the inflows in million cubic meters, you have the demands in million cubic meters, you have the evaporation in million cubic meters.



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### Modeling for Conjunctive Use

- The economic returns per unit of water applied at the irrigation field for the 12 periods are as follows :

*B<sub>t</sub>*: 60, 50, 50, 50, 50, 90, 90, 90, 90, 0, 0, 0.

- Reservoir capacity : 350  $Mm^3$  ✓
- Volume of water that can be pumped from the aquifer over the year = 1000  $Mm^3$  ✓
- Maximum volume of ground water that is allowed to be pumped in a period = 200  $Mm^3$  ✓
- Canal seepage adding to groundwater = 30% (i.e., all water lost due to seepage adds as recharge to groundwater) ✓
- Recharge due to irrigation applied = 10% ✓



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And then you also have the net economic returns. So, these are the  $B_t$  values; now the  $B_t$  values are for several for 12 time periods, these are like this; 60, 50 etcetera, remember  $B_t$  is the net benefit that you get per unit water applied at the irrigated area. Then reservoir capacity is known; then we also specify volume of water that can be pumped

from the aquifer over the year, which is  $Q_p$ ,  $Q_p$  is known; and maximum volume of ground water that is allowed to be pumped in a period. Now, these we may put it as  $G_{max}$  that means,  $G_w$  is our decision on the pumping, but we will say that  $G_w(t)$ , which is any period  $t$ , the pumping in any period  $t$  must be less than or equal to 200 million cubic meters; you should not pump more than this. Then we have taken canal seepage as 30 percent, and recharge due to irrigation as 10 percent. So, all of these data are known.

So, we put this data into this model now. So, this is known, this  $G_w(t)$  is being it comes out as decision variable,  $R(t)$  comes out as decision variables, and in this,  $S(t)$  comes out as decision variables,  $R(t)$  comes as decision variables,  $G_w(t)$  is decision variables and so on. So, all of the terms that are required to run this model are given now; through this data, we use this data, and then make the run using a lingo model, it is a linear programming problem. So, we use lingo, and then obtain the solution.


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### Modeling for Conjunctive Use

• Solution: (All values in  $Mm^3$ )

t	$G_w(t)$	$R(t)$	$0.7R(t)+G_w(t)$	$D(t)$	$RECH(t)$
1	230.65	20.50	245.00	245.00	30.65
2	16.19	416.88	308.00	308.00	155.86
3	0.00	28.74	20.12	308.00	10.63
4	0.00	134.29	94.00	308.00	49.69
5	40.20	97.78	108.64	285.00	40.20
6	140.60	70.57	190.00	190.00	40.17
7	0.00	271.43	190.00	190.00	100.43
8	30.87	67.32	78.00	78.00	28.00
9	0.00	7.78	5.44	65.00	2.88
10	0.00	0.00	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.00
12	0.00	0.00	0.00	0.00	0.00

*Water Applied* (handwritten red text pointing to the 3rd column)  
*Water Demand* (handwritten red text pointing to the 4th column)



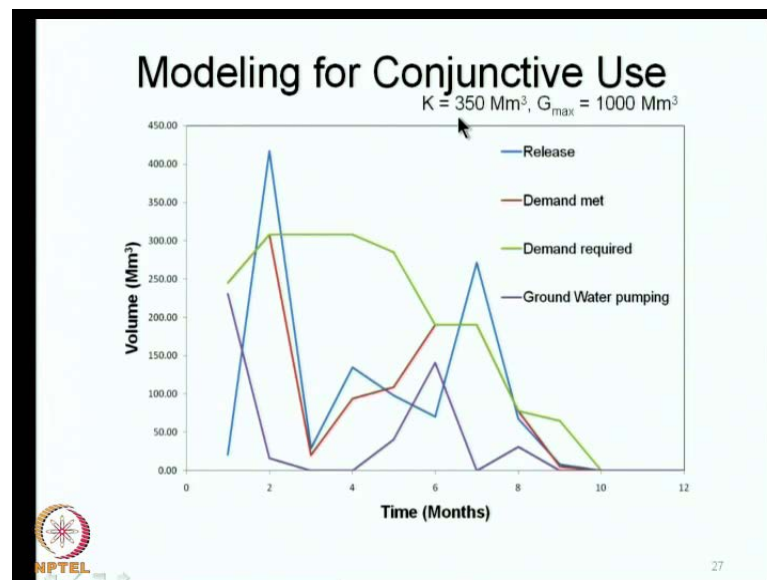
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Now, the solution here is, it provides how much of ground water to be used, and how much of release to be given from the reservoir. these are all in volume minutes, and for comparison purpose of seeing, for seeing how much of the demand is met, we put this term now; this term here is the water applied, actual water applied at the irrigated area. And this is the water demand. And we also look at the recharge that is taking place during different time periods. So, the demand here is not met for several time periods,

and there is recharge that is taking place, so that the demand can be met in the other time periods to the extent possible.

Now you can do several analyses with this. So, this is the type of model, several types of analysis with this. First thing is you see that there are certain periods, in which the demands are not met; for example, this period, there is a big deficit; this period, there is a big deficit, and this period, there is a big deficit and so on; which means that there is a crunch recourse both with ground water as well as surface water together, there is still a deficit of resource. So, you can start doing a sensitivity analysis, start increasing the reservoir storage, and then see how the solution will be; start increasing the ground water availability, and then see how the storage, how the solution changes and so on.

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So, typically from such solution, you get a idea of what kind of demands that are met; for example, you look at this red line here, this is the demand pattern; and anywhere else, which is not seen, it coincides with the green level; green is demand required. So, wherever red is not seen, it is coinciding with the green. And there are several time periods, using this typically between 2 and 6 months as well as from 8 months to 12 months, the demands are not met to a great extent. Then you will also look at how much of ground water that is pumped, how much is released from the reservoir, and the demand that is required and so on. So, you look at all of these in fact, you can also put the recharge, what kind of recharge that is taken place and so on.

Now on this, you can do several sensitivity analysis to see how sensitive is the solution with the reservoir capacity K, you try increasing the reservoir capacity K, but you will very soon realize that this solution may become insensitivity, insensitive to reservoir capacity itself, because it may be limited by the inflows. If your inflows themselves are small, then no matter how large a dam you build, you may still be not able to meet the demand. And therefore, you may have to start increasing the ground water that is available, but the ground water will be limited by the actual aquifer conditions therefore, you may not be physically able increase the storage beyond a certain point. So, this is how you model the conjunctive use of surface as well as ground water, and this is the simple simplest model.

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### Conjunctive Use

Ground Water Balance Equation

Two dimensional, unsteady flow in an isotropic, homogeneous, unconfined aquifer is given by (Willis and Yeh, 1992),

$$\frac{\partial}{\partial x} \left( T \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( T \frac{\partial h}{\partial y} \right) = S_y \frac{\partial h}{\partial t} + Q_p - Q_R$$

$h$	Ground water level (m)
$T$	Transmissivity m <sup>2</sup> /day
$S_y$	Specific yield
$Q_p$	pumping rate per unit area m <sup>3</sup> /day/m <sup>2</sup>
$Q_R$	Recharge rate per unit area m <sup>3</sup> /day/m <sup>2</sup>
$x$ and $y$	Cartesian coordinates in plan
$t$	time in days


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You can also include a two-dimensional unsteady flow for the ground water flow itself. In this example that I just discussed what we did was that we took **took** it as a lumped model. However, you can go into two-dimensional unsteady flow model, and write a finite element code for this convert this differential equation into a finite element numerical method; and then at each element, you write the continuity equation. So, this type of complex models are available in literature, you can just go through it, but the broad philosophy of the system's model remains to be broadly what I have discussed here namely, that you identify a objective function, write constraints associated with the ground water, write constraints associated with the surface water and so on.

Now, if you **if you** want to put a finite element model for the ground water, then all of these constraints that I have return must be written for each of the element. Typically we may write element to be choose the element of size 2 kilometer by 2 kilometers; and then write the constraints for each of the 2 kilometer grid; and it will also depend on the soil moisture balance; it may depend also on the soil measure balance at each of these nodes the elements that you have written, 2 kilometer by 2 kilometer; and then the mesh type that you choose for the finite element and so on. But such complex models are in fact, available.

And then on the top of that you can also include the uncertainties associated with the inflow, rainfall, the evapotranspiration and so on; you can build stochastic models for conjunctive use of surface and ground water, but broadly the philosophy remains the same. So, in today lecture then we have just discussed the conjunctive use model; we will continue the discussion in the next class, where we introduce another type of model formulation, not for conjunctive use, we will look at certain case studies and so on. Thank you for your attention.