Water Resources Systems

Modeling Techniques and Analysis

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Lecture No. # 36

Fuzzy Optimization for Water Quality Control and Reservoir Operation

Good morning, and welcome to this the lecture number 36 of the course water resource systems modeling technique and analysis. We have been discussing now the fuzzy optimization techniques, recall that the fuzzy optimization is especially useful, when you want to impart some latitude in the decision making, flexibility in the decision making, and also in situations when you want to address the conflict in decision making. For example, when there are a number of stakeholders, each one specifying his or her own preferences of solutions. How to what degree is the solution acceptable to the particular stakeholder, and these preferences are all often conflicting with each other. In such situations, we use the fuzzy optimization techniques.

And in the last lecture, I specifically introduced the water quality control problem, where, we are dealing with the optimal fraction removal levels or the optimal treatment levels to be provided at individual discharges, by discharges I mean either the industrial discharges or the municipal and municipal discharges, who provide, who discharge the effluents into the stream to make use of the assimilative capacity of the river.

So, the conflicting objective there in such a problem are the once related with the discharge of themselves, who prefer to minimize the treatment level or minimize the cost of the treatment and those due to the pollution control agencies, who would like to maximize the water quality at several check points, which means that they would like to maximize the treatment level. This is over and above the standards that we are talking about; that means, even when the standards are met, whether the water quality at a particular location is acceptable are not is the condition, is the situation that we are looking into.

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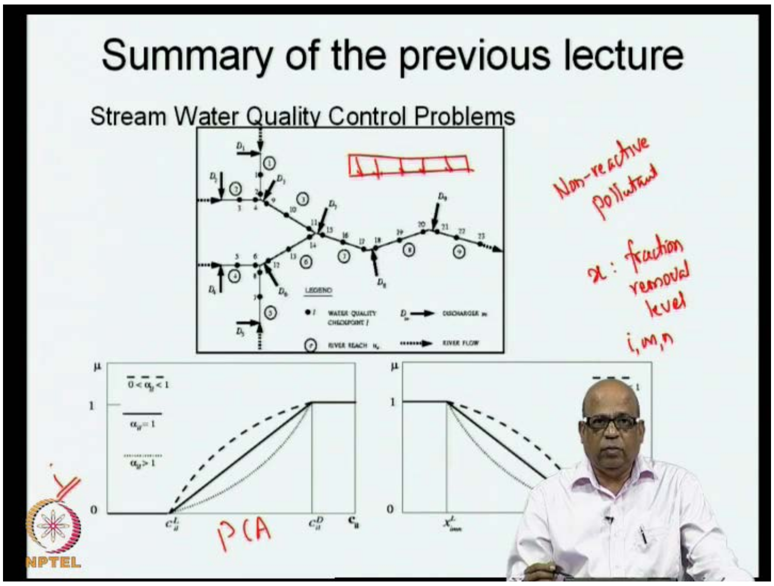

Now, in the last lecture, we introduce this problem. We quickly go through the problem statement. These are the streams, there is the tributary here, so this this dotted arrow indicates a river flow. So, the dotted arrow is a river flow, and then the dark arrows indicate the point sources of the pollutants, the discharges, and then, in addition you may also have a non point source pollution, which may be contributing to the stream all along the length of the stream. And recall that this non point source pollution can be, because of the over land run off that is contributing to the stream all through the stream length. Then, you have the check points; so these are the check points, water quality check points at which you would like to maintain the water quality at acceptable levels.

The water quality, the desirable water quality at one point may be different from desirable water quality at another point, depending on the type of use that you (( )) research, the location of the check point itself and so on. We divide this into several river reaches 1, 2, 3 etcetera; depending on the type of water quality simulation model that you are using, you need to discretize the stream system into several reaches. And in general a particular reach will have one discharger at the begin of the particular reach.

For example, you look at reach number one d 1 is the discharger, and reach number 2 d 2 is the discharger, etcetera. So, you will have one discharger at the beginning of that particular reach. Now, you have the goals of the pollution control agency as well as the goals of the dischargers. Now, the goals of the pollution control agency or the PCA is related to the water quality indicator, and we are in general are denoting the water quality indicator by the index I.

So, this can be DO or it can be DO deficit and so on. If it is DO, we will say if it is an indicator such as DO which indicate which will be reflected which would be reflecting, the higher the water quality the better it is, higher the water quality index value the better it is. Now, that is what is reflected by such membership functions. And we have on the other hand, the preferences of the dischargers themselves. So, these are the membership functions for the dischargers. And these membership functions are related with the treatment levels. So, x here indicates the fraction removal level or the treatment level.

Now, I is a water quality index. So, we have relating the treatment to be given to the discharger given at discharger m for the pollutant n to enhance the water quality indicates index I, water quality index value I. So, that is why we indicate this as x i, m, n; i is the water quality indicator, m is the particular discharger. So, in the particular case d 1, d 2, d 3 etcetera d m, and n is the particular pollutant. Remember the type of pollutants that we are talking about in this example, are non reactive pollutants. Which means one pollutant the effect of these different pollutants at a particular point location for the water quality indicator i or additive, the effects are all additive.

So, they do not react with each other. Now, in this broad frame work of the problem then, what are we looking at? We are looking at the optimal fraction removal levels x i, m, n at all these locations. Such that, the water quality at several of these locations given here, indicated by the water quality check point l is maximized. And therefore, for the water quality indicator, we will have membership functions such as these which reflect the premise that, the higher the water quality indicator value the better it is.

So, in this particular case, we may have a linear membership function like this or a non-linear membership functions like this. And on the y axis is the membership function value, it goes from 0 to 1. Similarly, for the dischargers the lower the discharge, the lower the fraction removal level the better it is, that is reflected by this kind of membership functions. Again the beta i, m, n here; we will will determine the shape of the membership function, when it is linear this indicator this index will be 1.

Now, as you can see there will be large number of such membership functions, because we are dealing with several of the water quality indicators i, several dischargers m and several pollutants n, and associated with each combination of i, m and n, at each location l, you will have a membership function defined. Therefore, there will be set of membership functions, which indicate the preferences or the acceptability of the pollution control agency as well as the acceptability of the dischargers. For example, if the water quality indicator value is here, then the level of acceptability will be to this existence. So, the pollution control agencies are saying, that at location l for the indicator i, this is the type of acceptability or the desirability of a solution is what is indicated for the PCA? And similarly for the dischargers, this is the kind of acceptability is what is indicated?

Now, in such a problem with large number of membership functions like this, what is it that we are looking for, we are looking for a best compromise solution or because we are indicating all the goals and the constraints are fuzzy sets, we are looking at the maximized value of the minimum membership function value, resulting from the particular solution. Remember why we are talking about the minimize minimum membership function value among all these things, because we are defining the fuzzy decision there. So, the fuzzy decision is in fact, determine by the intersection of all such membership function values, and that is what is obtain by minimum of all the membership function values for a given solution.

There is another aspect here, we are actually making decisions on the fraction removal levels x. The x at any particular value location here; for example, at the decision number discharger number 1, we may determine x 1 for a given i and n. This will determine, the water quality at a particular location or this will have this decision will have an influence on the water quality at any given location. Similarly, the decision at this point will have a influence on the water quality at this location.

So, the C il here, which is the water quality at a particular location, will be influence by in general the decisions that we are taking upstream of that particular point. And therefore, you must have models or means or mechanism by which you will relate the water quality at this particular location, with the decisions that you are making upstream of that particular location. And these relationships do exist in elegant mathematical form, we just make use of these relationships and then put them in the optimization problem.

The non point source pollution, that is joining the stream or all along the length is an uncontrollable source, it influence is the water quality at particular at all the check points; however, you cannot do anything about this, but you will have to account for this. What I mean by that is, that the mathematical models that relate the water quality at a particular check point with the decisions that we are making upstream of that, must also make allowance are must also consider; taking to account the non point source pollution file determining the water quality at a particular point, with respect to the decisions that we are making upstream of that particular point.

And account for the all of these, we formulate this as which is particular problem, as a fuzzy optimization problem and determine what is the, what is called as the best compromise solution? So, this is what I introduced in the last class, we will see the details of this in the today's class.

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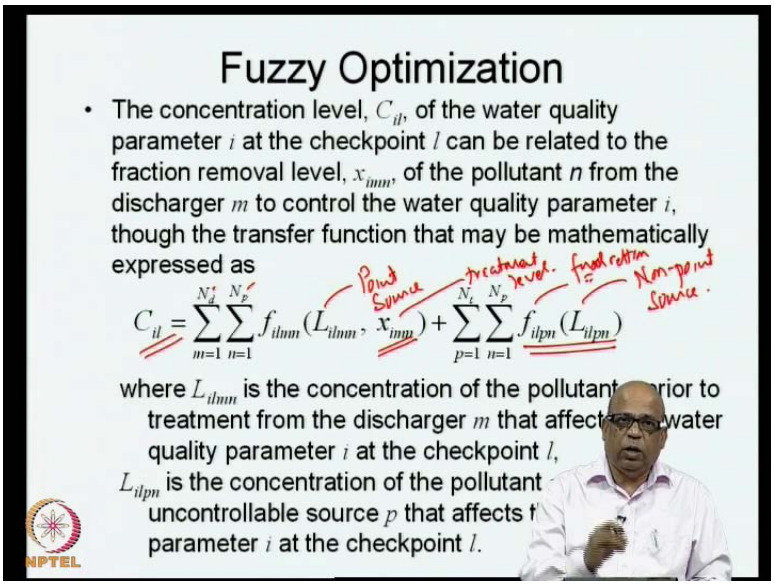

So, as I said, the if you look at the water quality indicators C il, this is the concentration of the water quality indicator i at location l. Now, this is the additive effect arising out of the non point source pollution, this is L ilmn, look at the definition L ilmn is the concentration of the pollutant n, trial to treatment from the discharger m, this is the control level source. So, this is the point source. And this is the non point source.

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Trail to treatment, which means we are making the decision X imn, this is the treatment that we are given. And this treatment we are giving on this; L ilmn, i is the water quality indicator, l is the location or the check point, m is the particular discharger, n is the particular pollutants.

So, it can be use for multiple pollutants. The way it influences C il is given by a functional relationship, which has the arguments as L ilmn and X imn, this is the treatment level. Similarly, from the non point source pollution, the influence that the non point source pollution has on the water quality index or the concentration of water quality indicator i at location l is given by a functional relationship, this is the functional relationship.

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So, what this indicates, that it is a sum total of the effect of the point sources after treatment, and the non point source is on the water quality indicator concentration C il, at location l for the water quality indicator i. Now, these are the number of dischargers; and these are the number of pollutants. For example, you may have BOD as one of the pollutants, you may have nitrate as another pollutant and so on. So, these are different pollutants. Similarly, here you may have uncontrollable sources several of them. So, p is equal to 1 to n t is uncontrollable source, and n is again the number of pollutant, n p is the number of pollutant.

So, what this indicates is a general formulation? Where, we are relating the concentration at a particular location l, for the water quality indicator i, as a aggregate effect of what is happening to the point source is? And the influence of the non point source as much in as much as the concentration C il is concerned. When we deal with realistic problems, we may be dealing with only 1 or 2 pollutants, 1 or 2 water quality indicators and and so on. So, it will be quiet simple, when we are dealing with actual situations, in in terms of the mathematical modelling. But the general formulation will address this kind of a model.

Now, typically these are water quality transport models, which means if you have a mathematical model, which relates the water quality indicator at a particular location, as a function of the dischargers that are coming upstream of that, such mathematical models can be used into the optimization model. To generate, C il as a function of X imn, which is the decision that we are making.

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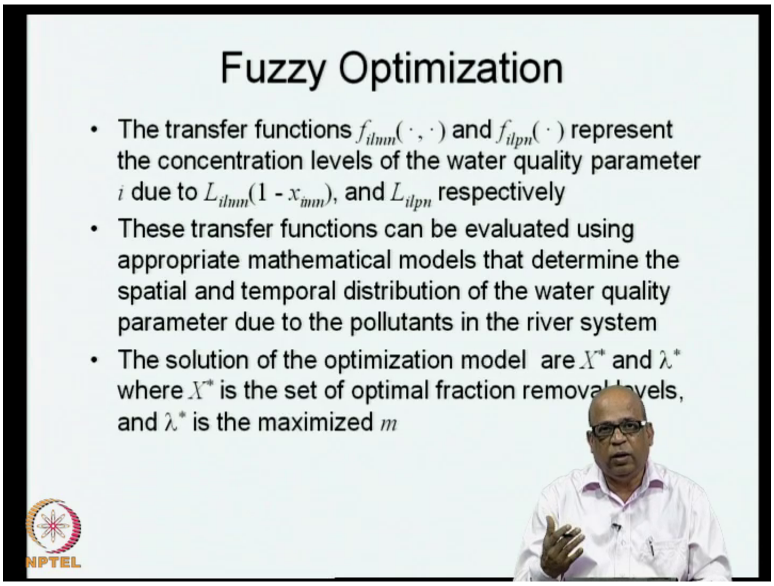

So, this is the general frame work. Now, within this general framework them, now this gives you all the details of exactly what I just now told. Remember here, L ilmn is what I am saying as the pollutant n trail to treatment. Now, trail to treatment, this is what is coming you are giving the fraction removal level of a X imn. So, the actual load, that is coming is 1 minus X imn and into a L ilmn will give you the effect of that on the C il. So, remember we are talking about let say, a load of unit load is coming, we are taking out x fraction of that; so, 1 minus x will act to the stream. So, the effect will be (( )) based on the 1 minus x, that is the idea there.

And the solution of the optimization optimization model is x star, which is the vector of the decision making vector of, I will repeat vector of the fraction removal levels at each of the dischargers. There are m dischargers x 1, x 2, x 3 etcetera x m that is the vector of decisions that we are talking about. And we formulate this typically, as a fuzzy optimization problem, where we are looking at the maximized value of the minimum membership function, typically interpreter as the maximization of the satisfaction level in the system.

So, we are looking at a best at the best compromise solution, in the phase of the conflict that exist in the system, in terms of the acceptability, degree of acceptability of solutions or various stakeholders; by various stakeholders, I mean here, there may be pollution control agency goals different at different locations, different for different pollutants, different for different water quality indicators and so on. So, these all of them from a, set of membership functions; similarly, at the dischargers you have membership functions, which indicate the degree of acceptability to the pollutant discharger m for the pollutant n at the, for the water quality indicator i, because the treatment level is taking place only at one location.

But when the discharger m at that particular location has to decide his or her preferences would be based on the water quality indicator i and the pollutant n, because that is what is the treatment that has to be given at the particular point. And this decision X imn is related to the water quality indicator, concentration C il at location l for the water quality indicator i. All of these membership functions now, we put together and then formulate the fuzzy optimization problem and look at the best compromise solution.

Let us look at a example now. So, if you recall, the general form of the fuzzy optimization problem can be expressed, we did in the last class.

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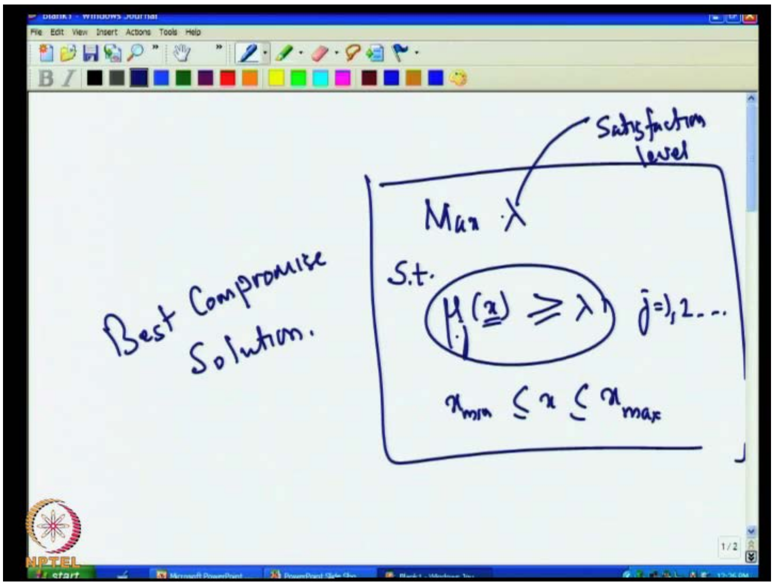

We just recall, we are in general saying maximize lambda, subject to in this particular case you may have mu of x greater than or equal to lambda, and you may have let say mu j of x greater than or equal to lambda, j is equal 1, 2 etcetera, you may have several of such membership functions. So, if you are looking at the the solution x, then you may also have x min less than or equal to x less than or equal to x max.

So, this is the general formulation, where we are defining the membership function associated with solution x, and then this particular set of constraints define lambda as the minimum among all the membership functions; and that minimum value, we are maximized. So, you are solution x will be such that, it maximize the minimum value of lambda. And lambda is interpreted as the satisfaction level in the system. And the solution that arises out of such a model is called as the best compromise solution.

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So, we are actually looking at the best compromise solution, in the presence of the conflict and the preferences x expressed by several players or several stakeholders in the system. We will look at example, and then this should be more clear.

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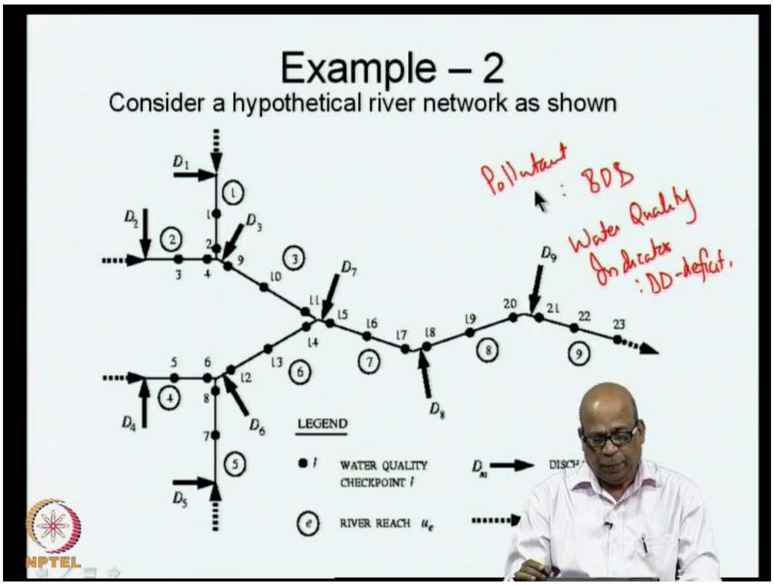

So, we will take a simple example of the hypothetical system, where we have 9 dischargers, 9 effluent dischargers; we have 23 check points 1, 2, 3, 4 etcetera, 23 check points. There are 9 reaches associated with each of the dischargers. So, at every discharges at every… I am sorry at the beginning of every reach, there is a effluent discharger. Look at discharger one that belongs to reach 1, discharger 2 belongs to reach 2, at the begin of reach 2. Similarly, D 8 is at the beginning of 8, D 9 is at the beginning of 9, and so on. The check points shear or decided based on the type of water quality model that we are using.

For example, the discretization of the stream at of your reach into several checkpoints; sub reach is is based on the type of water quality model that we are using. For example, in this particular problem, we will be dealing with the discharger as as the pollutant as the BOD, as BOD, and the water quality indicator...

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We will use it as DO- deficit.

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So, the there are several check point like this. So, we would like to make our preferences, make now, our preferences for the optimization at all these check points, and these are the dischargers D1, D2, D3 etcetera of the dischargers; we will for the purpose of this example, ignore the non point source pollution. So, we will assume that there is no non point source pollution. So, now, within in this problem, within this framework now, we are looking at the optimal treatment levels or the fraction removal levels, that have to be given at all of the location such that the water quality at each of these locations is maintained to the desirable level.

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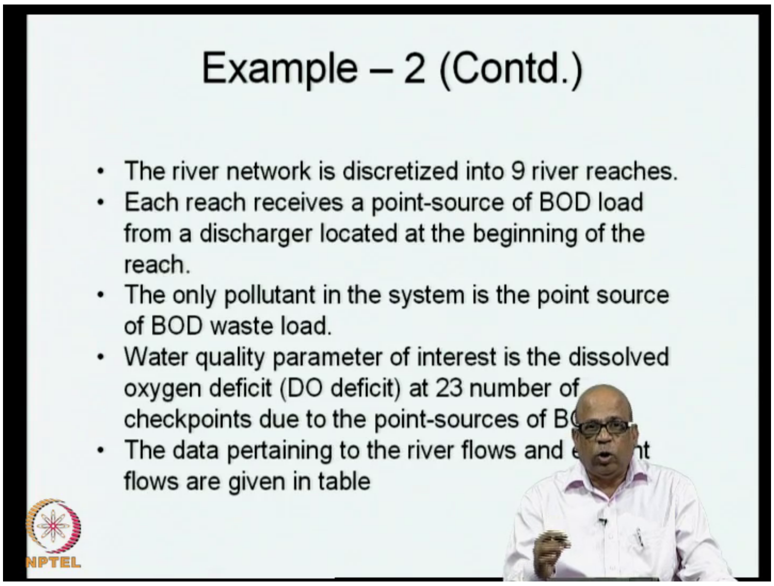

In this problem then now, these are the details that I just mention, we will also have the data patterning to the river flows, effluent flows, etcetera are available. Then you will have for example, you are relating BOD and DO. So, you may use the classical sstreeter phelps equations to relate the BOD and DO at a particular location; that is DO at a particular location with respect to the BOD loads at upstream locations. You will relate using the streeter phelps equations. And the parameters for the streeter phelps equations which typically change from one reach to another reach in this problem are also given. So, these are all part of the data.

So, from this now, we will progress to define the membership functions. The fuzzy membership functions for the pollution control agency, as well as for the dischargers. The water quality indicator that we are using is the DO deficit. So, what will be the preference for the pollution control agency, it would be to maintain the water quality as high as possible at a particular location. And therefore, the DO deficit should be as low as possible. So, your solution should be said that the DO deficit at a particular location must be as low as possible. Therefore, the lower the better is the goal for the pollution control agency.

Similarly, for the treatment levels, the dischargers would specify the goals as the lower the treatment level the better it is. And therefore, the dischargers will also have their own preference is on lower the better. How are these conflicting now? Because both of them are saying with respect to their argument, the lower the value of that particular argument the better it is; however, when you look at the particular problem. The discharger will say, the lower the treatment level the better it is. The pollution control agency will say the lower the DO deficit the better it is.

To make the DO deficit lower, the treatment has to be higher. And therefore, these two are conflicting with each other. Therefore, the students often make this mistake; do not go by the shape of the membership function. Look at the arguments of the membership functions, and then see how the arguments are conflicting with each other; if they are conflicting with each other. And therefore, in this particular problem, because the higher the treatment level will indicate the lower DO deficit, these 2 aims, these 2 goals are conflicting with each other; and therefore, we need to address is conflict in the fuzzy optimization problem, and then obtain the best compromise solution.

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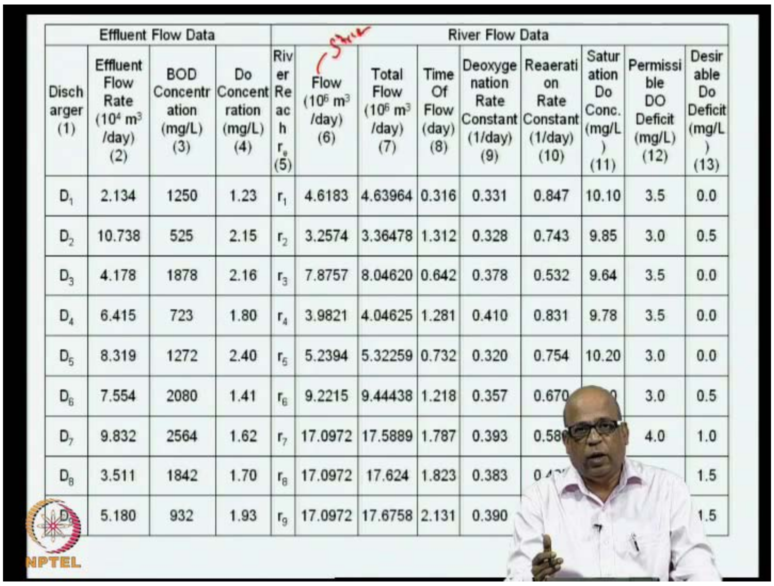

	Effluent Flow Data			River Flow Data								
Discharger (1)	Effluent Flow Rate (10^4 m^3/day) (2)	BOD Concentration (mg/L) (3)	Do Concentration (mg/L) (4)	River Reach r_e (5)	Flow (10^6 m^3/day) (6)	Total Flow (10^6 m^3/day) (7)	Time Of Flow (day) (8)	Deoxygenation Rate Constant (1/day) (9)	Reaeration Rate Constant (1/day) (10)	Saturation Do Conc. (mg/L) (11)	Permissible DO Deficit (mg/L) (12)	Desirable Do Deficit (mg/L) (13)
D_1	2.134	1250	1.23	r_1	4.6183	4.63964	0.316	0.331	0.847	10.10	3.5	0.0
D_2	10.738	525	2.15	r_2	3.2574	3.36478	1.312	0.328	0.743	9.85	3.0	0.5
D_3	4.178	1878	2.16	r_3	7.8757	8.04620	0.642	0.378	0.532	9.64	3.5	0.0
D_4	6.415	723	1.80	r_4	3.9821	4.04625	1.281	0.410	0.831	9.78	3.5	0.0
D_5	8.319	1272	2.40	r_5	5.2394	5.32259	0.732	0.320	0.754	10.20	3.0	0.0
D_6	7.554	2080	1.41	r_6	9.2215	9.44438	1.218	0.357	0.670	[illegible]	3.0	0.5
D_7	9.832	2564	1.62	r_7	17.0972	17.5889	1.787	0.393	[illegible]	[illegible]	4.0	1.0
D_8	3.511	1842	1.70	r_8	17.0972	17.624	1.823	0.383	[illegible]	[illegible]	[illegible]	1.5
D_9	5.180	932	1.93	r_9	17.0972	17.6758	2.131	0.390	[illegible]	[illegible]	[illegible]	1.5

The data that we need; so for, this particular example, you will understand if you want to apply to realistic case study, you should have all the specified data. First is these are the dischargers, in this particular case we have 9 dischargers. The effluent flow rate itself is one of the data that means you have so much of volume of the effluent flow is coming per day. This is the data. This a effluent we will have the BOD concentration, in terms of mg - milligrams milligrams per litre and this is the type of data. Typically, these are industrial effluents; for example, high BOD of 1250 mg per litre, 525 mg per litre and so on. So, these are typically, either the municipal sources or the industrial sources.

So, these may be for example, textile industry, leather industry, etcetera with very high BOD concentrations, they are effluents they are effluents are reaching the river. Then you also have the DO concentrations, which are related to the BOD concentrations. Then you discretize this into river reaches as I just said, you discretize these into river reaches r 1, r 2, r 3 etcetera r 9. So, D 1 is at the upstream of r 1, D 2 is the at upstream of r 2 and so on. Then, you have the stream flow, this flow is stream flow. That is in million cubic meters per day; then you add the stream flow plus the effluent flow, this is per day, this is per day, but remember this is the 10 to the power 4 here, 10 to the power 6 here, and then you get the total flow.

Now, these these minutes digits are written, because it is 10 to the power of 6, and then you are also adding to the 10 to the power 4 here. So, this is the total flow that is coming.

And then what we are saying is at particular reach here. We know the total flow - one is due to the stream flow and another added to that is a effluent flow. So, total volume of flow that is the occurring, in this reach will be this flow plus this flow, that what we are saying? And then from this point, it takes the time of flow for this total flow to reach the end of the reach is governed be the hydraulic of the system, including the cross sectional area, the aslope the roughness, and so on.

So, this is the time of flow from, this point to this point is what is determined and given here. So, this is the time of flow and that is in days. This time of flow I repeat is for the flow, for the total flow to flow from the beginning of the reach to the end of the particular reach, that is in days here, 0.316 day, and for the BOD DO problem we need deoxygenating rate and reaeration on rate. These are also given, this can be determined from the data, but we assume that these are given, their units are 1 1 by day, there is day minus 1, day minus 1 here. Then you have the DO concentration, which will also depend on the temperature which will which is given here, typically it will be of order of 9 mg per litre.

Then we will have the preferences; that means, we will say that anything above 3.5 mg per litre of DO deficit is not acceptable. And we would prefer preferably have a DO deficit of 0, which means as close to the DO level, as close to the saturation level as possible. Similarly, here we may want to have DO deficit of 0.5 as desirable and so on. When as desirable and so on, 1.5 as desirable not less than 3.5 is what is a acceptably here, what is the permissible? So, like this we specify our preferences for each of the reaches. So, at each of the reach, we are saying that these are the membership functions. It means that any check point within that reach 1, 2, 3,4 for these 3,4 for reach number 2, 1, 2 for reach number for 1, 9 10, 11 for reach number 3 etcetera, any of these check points within that reach will have the same membership functions, that is the assumption for the particular problem.

So, we have all the data in place, we have the membership functions for the pollution control agencies with respect to each of the location points or each of the check points are specified. Similarly, we now, specify the discharger's preferences or dischargers membership function.

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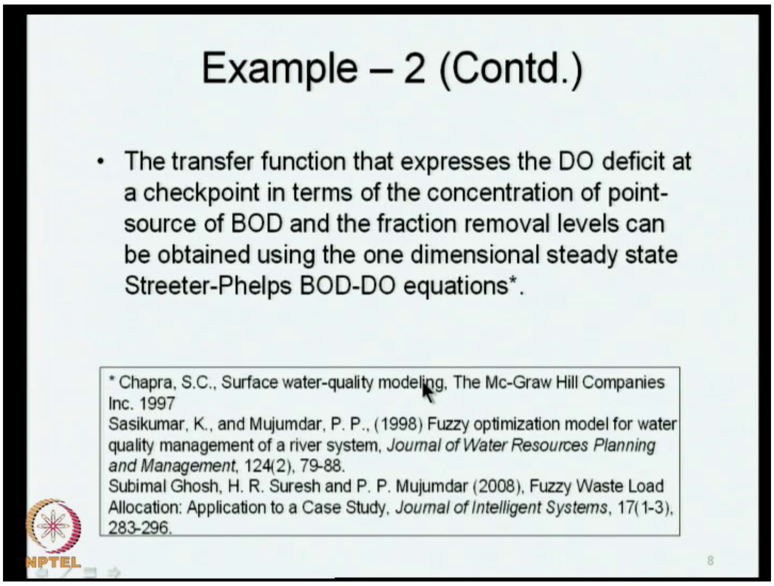

Now, as I said the treatment level that you provide at a particular location for the particular discharger, determines the water quality indicator concentration at a location l; and these is modelled by streeter phelps equation in this particular example. Because we are talking about only one pollutants namely BOD, only one water quality indicator namely DO. So, the dissolved oxygen at a particular location can be determined based on the BOD loads that are coming up stream of that particular location.

And this is determined in the example, by using the classical streeter phelps equation. I I would encourage to go through it is streeter phelps equation, to understand this problem, and typically the chapra book gives the basic of the streeter phelps and these two papers here will provide you, how to use the streeter phelps equation in the optimization problem, in the fuzzy optimization problem? In fact, these two papers also provide you with the recursive relationship, of usage of the streeter phelps equation for multiple loads alright.

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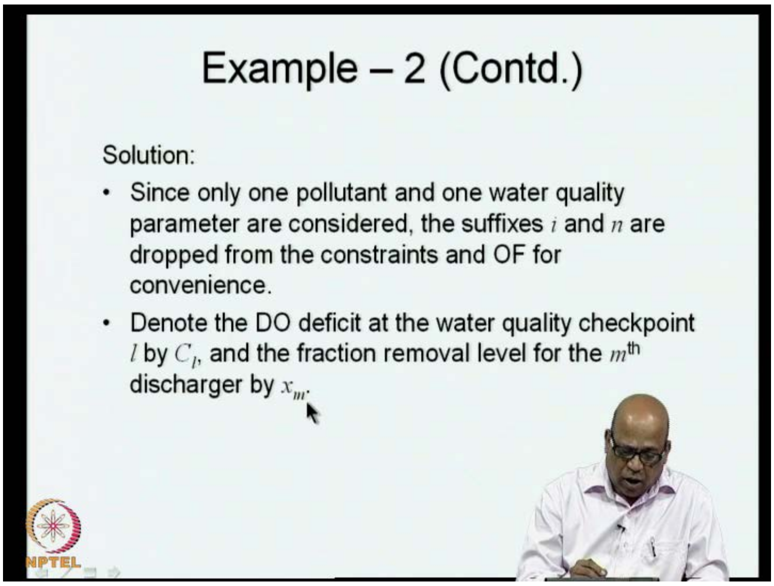

Now, since only once pollutants and one water quality indicator is there. We will simplify the notation by removing the indices i and n, because i is 1 here, only DO deficit, and then only one pollutants namely BOD therefore will remove i and m. So, we will denote the DOD deficit at the water quality check point l by C l, and the fraction removal level for the m th discharger by x m. So, we have only 2 indicator sound m and l. We have removed i and n. x m are the decisions, that we are making and C l is the result of this decision, but we have membership functions as functions of C l for the pollution control agencies, and we have membership functions as functions of x m for the discharger m.

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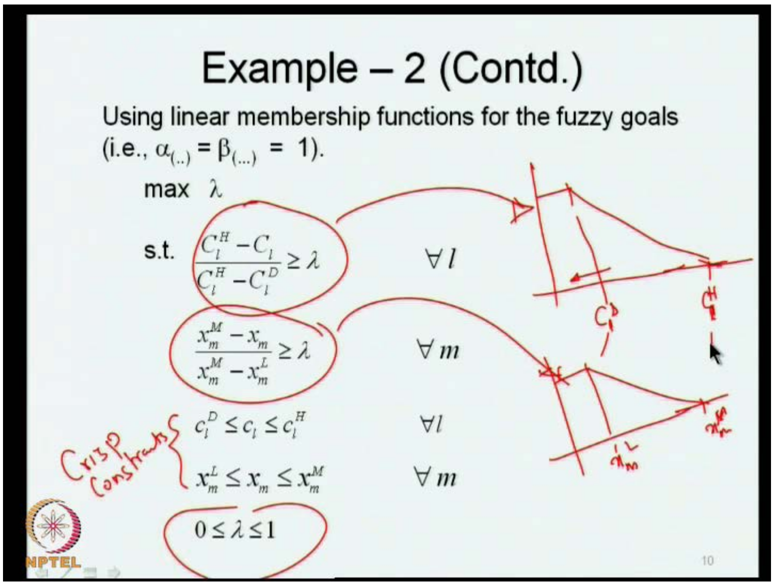

So, the membership functions are, we are actually taking linear membership function. So, the type of membership function, we are taking are like this I will come to that. So, the membership function at were dealing with are like this, as I said, because we are talking about DO deficit. We may say, the lower, the better and we will put it desirable level for the DO deficit as C l D and a permissible level of C l H. So, I will say, this is C l D and this is C l H, which means we are talking about DO deficit, we are saying that we would prefer to have it lower than C l D here, but anything higher than C l H is not acceptable. Therefore, this is we call it has permissible level, and this is the desirable level. So, that is what is reflected by this membership function.

Similarly, for the dischargers, we will have a membership functiona similar to this. We will call this as X m L and this will call it as X m M, which means the dischargers will say that we would prefer to have the treatment levels lower than X m L, but anything higher than X m M is not acceptable to us, so this is the membership function associated with the dischargers. Then we will have crisp constraints, these are the crisp constraints.

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Where we will indicate that C l has to be in fact, (( )) within in the range and X m has to be in fact within this range; because we are looking at the optimal solutions, in the presences of conflict. Then of course, lambda has to be it is, membership function finally, and therefore, it has to be between 0 and 1. So, this is the problem that we deal with now; it is a phelp problem, it is because the decisions X m and the C l can be related in a linear form. So, we will relate C l with X m using streeter phelp equation, and make it as a linear function and therefore, we will able to put this relationship in a linear form, and then solve it using the linear programming problem. So, from the general formulation that I give earlier, we are using alpha as well as beta as one therefore, we are using only the linear membership function in this particular case.

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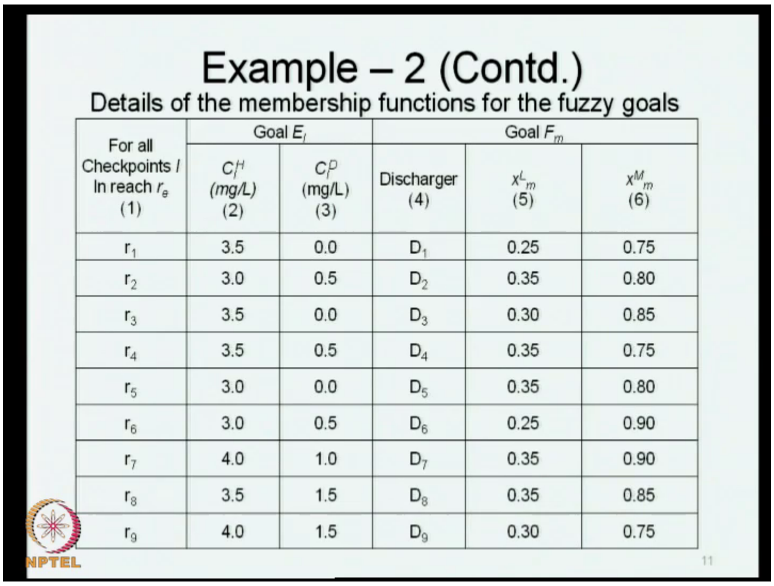
Example – 2 (Contd.)

Details of the membership functions for the fuzzy goals

For all Checkpoints l In reach r_e (1)	Goal E_l		Goal F_m		
	C_l^H (mg/L) (2)	C_l^D (mg/L) (3)	Discharger (4)	x^L_m (5)	x^M_m (6)
r_1	3.5	0.0	D_1	0.25	0.75
r_2	3.0	0.5	D_2	0.35	0.80
r_3	3.5	0.0	D_3	0.30	0.85
r_4	3.5	0.5	D_4	0.35	0.75
r_5	3.0	0.0	D_5	0.35	0.80
r_6	3.0	0.5	D_6	0.25	0.90
r_7	4.0	1.0	D_7	0.35	0.90
r_8	3.5	1.5	D_8	0.35	0.85
r_9	4.0	1.5	D_9	0.30	0.75

Now, we need then the membership function values for each of the dischargers as well as each of the reaches. Do not loose site of what we are doing here? Remember we are talking about this problem now. So, for each of the reach, we need the membership function associated with the pollution control agency. And for each of the discharger, we need the membership function associated with the treatment level. Now, that is what we are writing now. So, for each of the reach, then we need to define l at l need to given C l D, which is the desirable level and we need to give, for each of the location l the permissible level of DO deficit. Similarly, for each of the discharger m, we need to provide what is the lower limit and what is the higher limit?

So, that is what to given here. At each reach r 1 to r 9, we have C l H as well as C l D 3.5, 0, 3.0, and 5 0.5 and so on. Similarly, at each of the discharger D 1 to D 9, we have the lower limit as well as the higher limit of the treatment levels. Now, this is goal FM, if we recall what we did in the last class? And this is the goal E l for the location l. The location l in a particular reach, all the l will have the same membership functions. For example, reach 1 may have check points 1 and 2, both of them will have the same membership function and so on. And therefore, we are define for goal here.

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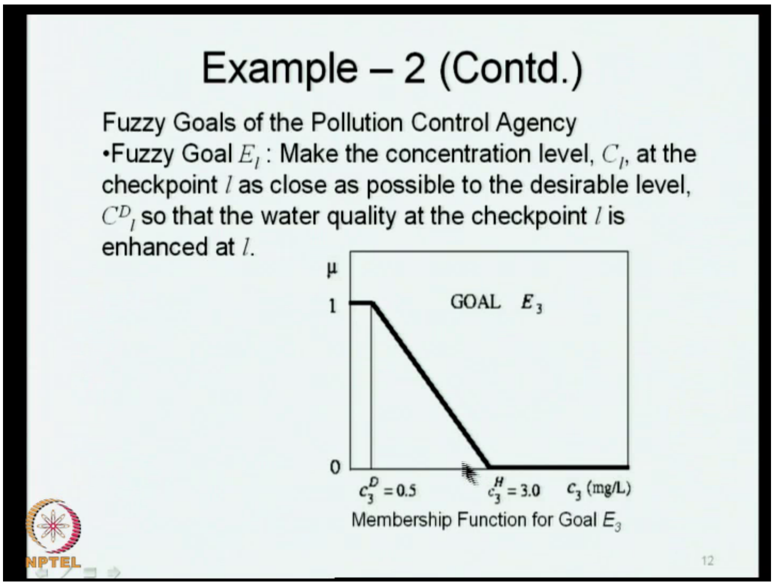

So, what is the fuzzy goal here, it is make the concentration level C l, this is we were talking about the pollution control agency goal, C l at the check point level l as close as possible to the desirable level C l D. So, you want to bring it as close to C l D as possible. So, that the water quality at the checkpoint l is enhanced at the location l. For example, E 3 you look at. So, we are talking about the goal of the pollution control agency. For the location 3, go back to the diagram and see where the location 3 is. Now, the location 3 or the check 3 belongs to the reach 2. And we have defined the membership functions for reach 2. So, for both 3 and 4 the same membership functions as defined for reach 2 are valid.

So, you look at, what we define for reach 2 as membership function, it is 3.0 and 0.5. And therefore, for goal 3 it is 3.0 and 0.5. The permissible level is 3.0 and the desirable level is 0.5, which means that at location 3 or at checkpoint 3, anything higher than 3.0, is not acceptable and we would like to have the DO deficit to be lower than 0.5. So, anything in between is acceptable with variant degrees that is the interpretation.

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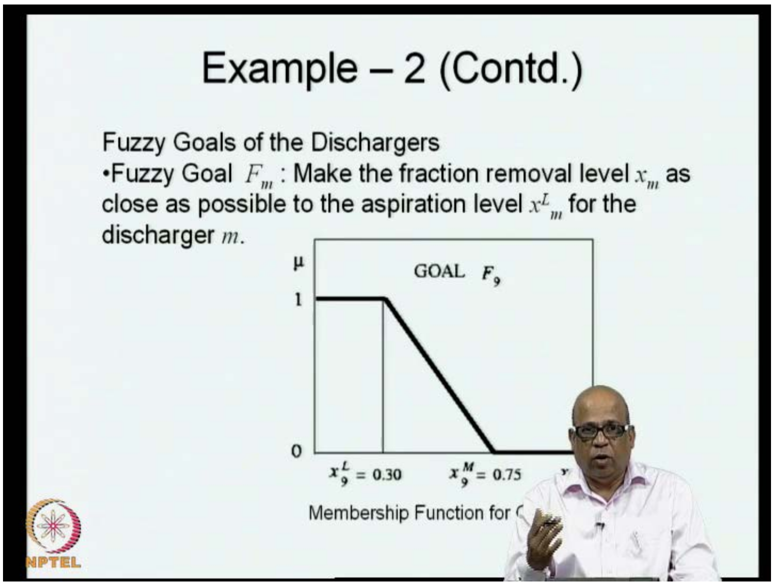

Similarly, now we look at the fuzzy goal of the dischargers. The fuzzy goal can be stated as make the fraction removal level X m as close as possible to the aspiration level X L m for the discharger m, that is the fuzzy goal, which means we are saying, the lower the better. So, for example, at the discharger 9 you look. Go back to the data and see for the discharger 9, we are saying that X M is 0.75 and X L is 0.30. And therefore, this is 0.75 and 0.30. So, like this for each of the dischargers, you define the fuzzy membership functions, similarly for the each of the location, which are 23 number and which are classified into 9 reaches for each of the location, you define the fuzzy goals.

So, you have a set of fuzzy goals associated with the pollution control agencies, which is defined at each of the location. You have a set goal for the dischargers which is defined as each defined at a particular discharger. So, like this you have a hand full of fuzzy membership functions, many of them conflict with each other and within this we want to get the optimal solution or the best compromise solution.

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Example – 2 (Contd.)

A minimal fraction removal level of 0.30 is imposed by the pollution control agency on all the dischargers . Results are as follows

Discharger (1)	Fraction Removal Level (2)	River Reach R_e (3)	Minimum DO Concentration (mg/L) (4)
D_1	0.64	r_1	9.89
D_2	0.70	r_2	8.76
D_3	0.72	r_3	8.50
D_4	0.66	r_4	8.80
D_5	0.70	r_5	9.17
D_6	0.75	r_6	7.65
D_7	0.77	r_7	6.90
D_8	0.74	r_8	6.61
D_9	0.49	r_9	6.07

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Now, the results of this model are exactly what we did we solve this problem now. This is given C l is a unknown determine by the decision X m, the relationship between the X m and C l is model through the streeter phelps equation and therefore, X m is the decision variable.

Now, these are crisp constraints, C l is unknown, but it is related to X m. X m is the decision making, decision variable and lambda is the decision variable. So, we are looking at that particular set of value of the X m, which will maximize the minimum satisfaction level lambda is interpreter as the minimum satisfaction level, we are maximizing that particular lambda variable. So, we use linear programming may be lingo or some such simple software we use, and then solve this problem. You will get these are the solution, D 1 is 0.64 what it says is that the fraction removal level that you need to apply at D 1 is 64 percent, 70 percent, 72 percent and so on.

These are the fraction removal levels that come out as the results. And the minimum DO concentration level, remember this not the DO deficit from the DO deficit, you have to convert into DO concentration, and then obtain the DO concentration. So, r 1 you will have a DO concentration of 9.89, 8.76 and so on. So, as a result of this solution, you will get this particular quality. This is the interpretation of the solution. And this is the best compromise solution in the presence of the conflict, among the pollution control agency as well as among the dischargers.

Now, we will go to a more simpler, more general problem, where we will often using the fuzzy membership functions. In the case of reservoir operation, you will have storage targets as well as the release targets, and typically in the deterministic optimization, when you are doing the reservoir operation what would be you doing is? You would minimize the deficit of the storage from its target, typically the sum of square deficit plus the deficit arising out of the releases. So, this is how you would have determine the reservoir operating policy.

Now, this was the crisp optimization, what will you now do is, will start relaxing that crisp conditions, and then state in terms of a fuzzy membership functions, the deviations from the target themselves; that means, the higher the deviation, the worse it is. These kind of linguistic statements will try to build it into reservoir operation problems, and then see how the solutions look at, the solutions appear.

Essentially, you must keep in mind that we are introducing more and more latitude, more and more flexibility into the solutions by introducing by by treating the crisp optimization problems as fussy optimization problems.

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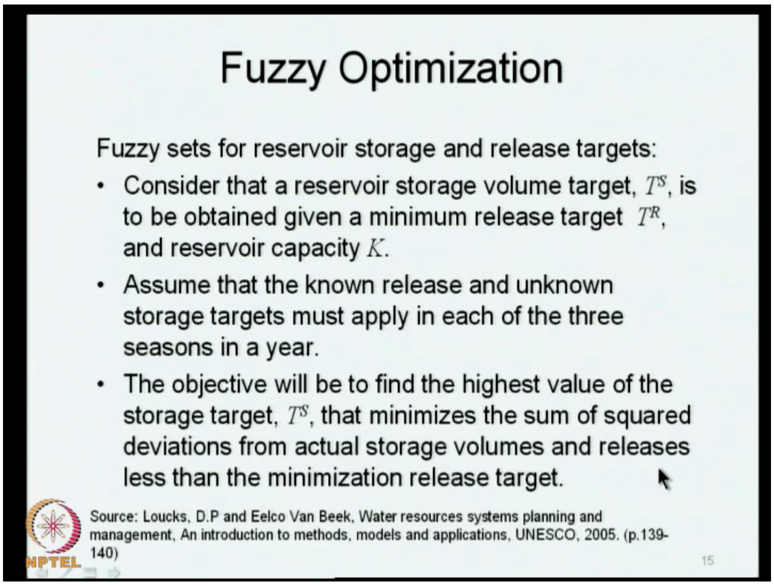

So, first let us look at the crisp problem now. This is taken directly from the classical text book by DP locks and others. For all water resource system students I would encourage them to go through this particular text book, which is extremely compressive, it is almost in imitable style of writing and presenting. So, all of you must have access to this, I understand that this is also available on the net you may just look for this particular text book. So, I am taking this discussion directly from this particular text book.

So, we have a storage target and we also have a release target. The reservoir capacity is known. Now, we are looking at maximum storage, maximizing the storage target itself, that means we do not (( )) specify storage target, but we will formulate the problem such that the storage target, that can be met most of the time is itself maximized, which means on an average we would like to maintain the storage as high as possible. But the release target will specify. And then recon the deficit as only that particular value of deficit, whenever release is smaller not the other side of the deficit, that is we recon the deficit to be non-zero only when the release is less than the target.

We specify the target, and compute the deficit as release minus, that particular target and only whenever it is less than 0; we put it as that particular value. Therefore, we take the mod of that particular amount. So, the objective will be to find the highest value of the storage target T s, we are trying to maximize the storage target, that minimizes the sum of squared deviations from actual storage volumes and releases less than the minimum minimization release target. That is what we mean is, that we set the targets target for release and then look at the minimum value minimum of the deficits for the release from the targets, and then look at the maximization of the storage itself so, that is the idea here.

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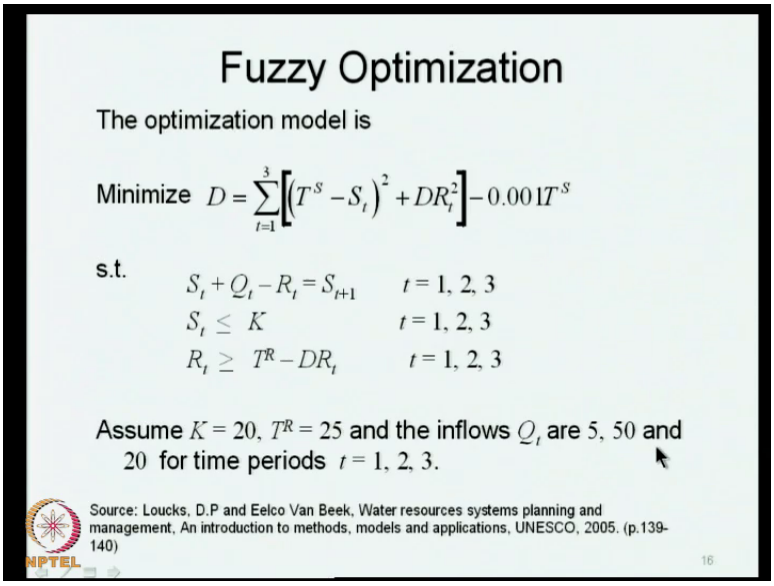

So, the way we write it in a mathematical form is for this particular problem, we take as 3 time periods; so, we are summing the deficits of storage and the in releases, this is the deficit in release, the squares of that minus we are looking at the storage target T s, this is the decision variable, we want to maximise the decision variable T s and therefore, we finalize, because this is the minimization problem. We finalize T s through a negative value here; what does this ensure, because we are using a negative value for the penalty, it ensures that the T s is as high as possible, because we are looking at minimum value, as T s increases, this value decreases and because we are looking at minimum value that would be preferred.

This is subject to the storage continuity equation S t plus Q t minus R t is equal to S t plus 1. This is the continuity that have been talking about all through, the capacity is known, the storage should be less than or equal to capacity and the D R t here is defined such that R t is greater than or equal to T R minus DR t.

DR t is specified in this particular example, we put it as 25 and DR t is the decision variable, which is governed by R t. So, when you solve this what what is it that you should get, we should get, because K is specified, we should get S t, we should get T s, which is the storage target and we should get DR t; these are all the decision variables and there are 3 number of time periods. The inflows are given, Q t is known and similarly, the capacity is given as 20 and there are 3 time periods.

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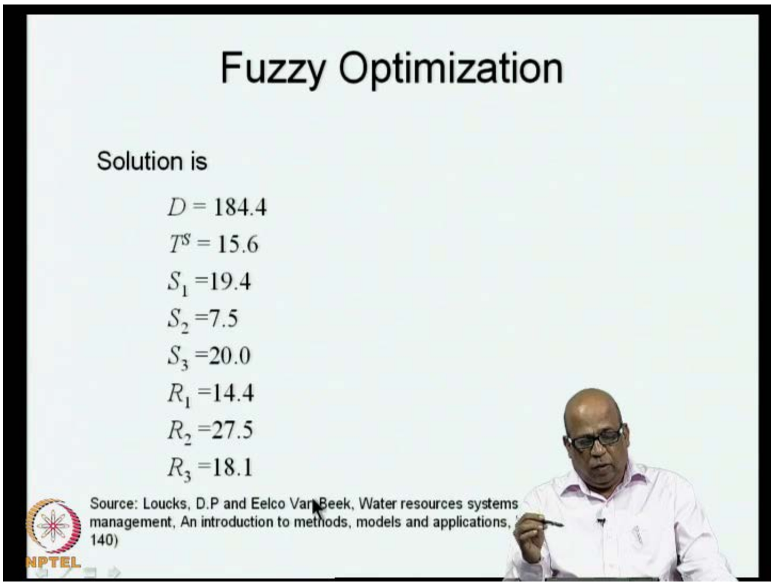

So, we will solve this example and then get the solution as this. So, you get a T s of 15.6, S 1, S 2, S 3 are given and R 1, R 2, R 3 are given 14.4, 27.5 and etcetera. And this is the DR t that you get as 184.4; this is the total value of the objective function, which we denoted it as D. Now, this is the solution for the crisp problem.

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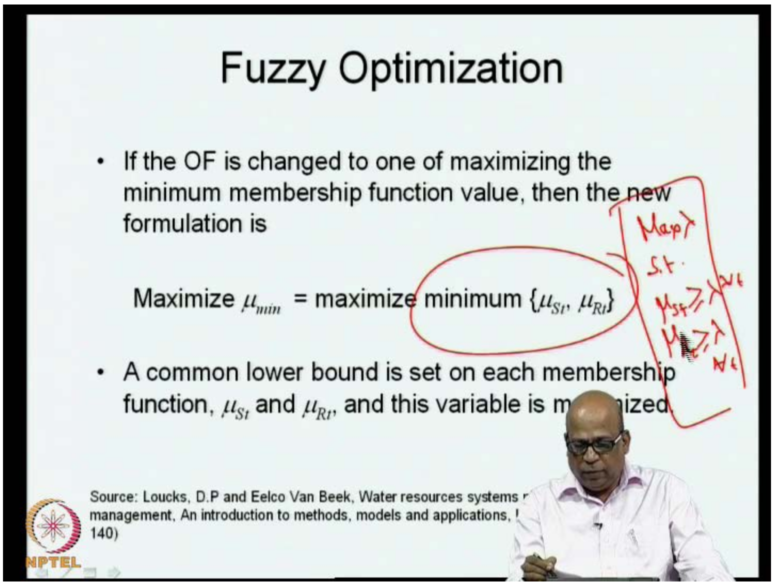

What will now do is, will convert this into a fuzzy problem, and then look at the membership functions rather than looking at the actual values of the target storage, what we will now do is? We start imparting some latitude into the problem relax the constraints, and then look at what is the best compromise solution, that we are getting. Why we call it as best compromise now? There is a conflict between the release and the storage, the more you release, the less will be the storage. You would like to maintain the storage also high, you would like to maintain the release deficit also low. And therefore, there is a conflict that exists; this conflict now, we will model through the fuzzy membership functions, and then look at the best compromise solution.

So, this is the general form of the fuzzy optimization problem. We are looking at maximization of the minimum value of the membership functions. These minimum values together, will define the satisfaction level in the system and then we are maximizing that particular satisfaction level. So, typically we write this as maximize lambda subject to mu S t greater than or equal to lambda for all t, and mu R t greater than or equal to lambda for all t. So, this is the problem now. So, if you know the membership functions, you solve this problem using this particular operating function and these as constraints, you will obtain the value of lambda as well as S t as well as R t.

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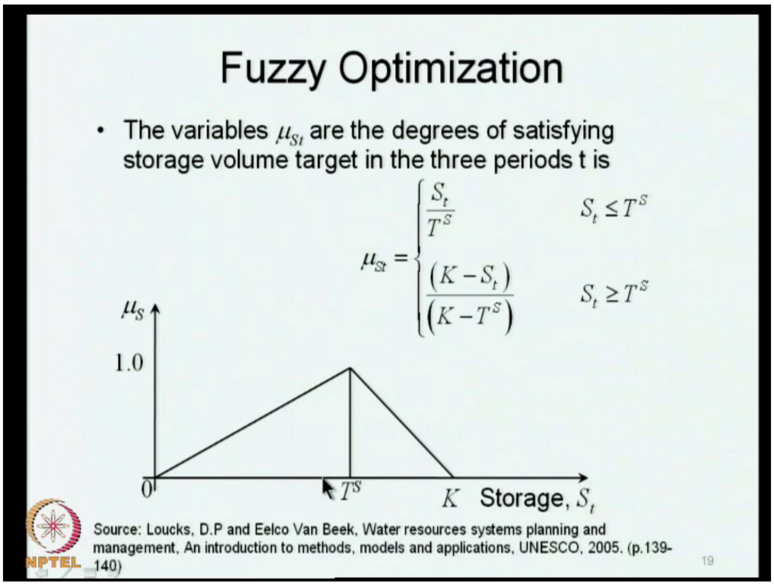

Now, for the S t look at this membership function now. We are saying that you would like to bring the storage as close to the target as possible. So, we may define a linear membership functions something like this. So, from the left when we are approaching, you would like to come as close to S t S as possible. So, we will say, S t by T s whenever S t is lesser than or equal to T s, and as it is starts going up, you do not still want it to be very far away from T s and therefore, you write this as the membership function when you have S t greater than or equal to T s.

So, this defines the membership functions for the storage. What does this indicate? This indicates that the storage, the actual storage value should be as close to T s as possible; that is what is it indicated here.

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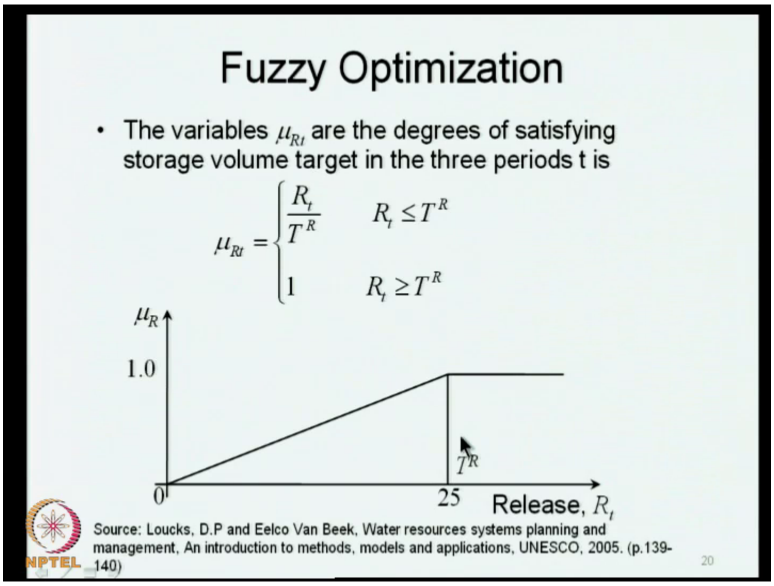

Similarly, for the release your target is 25, but you would like, you would do not mind the release to be greater than 25, because recall that we define the deficit only when release was less than 25, less than the target. Therefore, the fuzzy membership function for this is, that you would like to have the release as close to 25 as possible from the left side. So, R t by T r, that is this line and when R t is greater than T r, we will put the membership function to be 1, which means that anything greater than R t is fully acceptable. So, this is the way, we define the membership function for the storage and the release.

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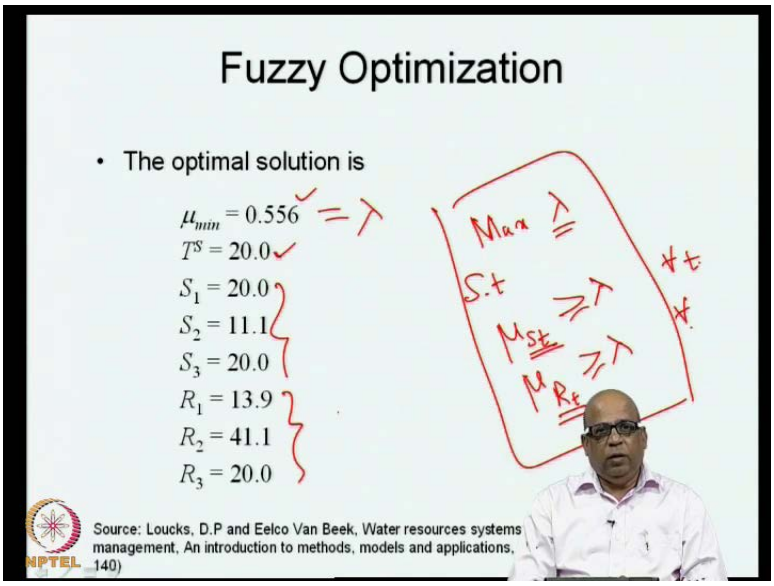

And then solve the problem. When we solve this problem problem, we will get the minimum value, which is lambda actually in our notation, this is lambda. So, what we are saying is we are maximizing lambda subject to mu S t greater than or equal to T s for all t and mu R t greater than or equal to T r for all t. I am sorry, this is lambda. So, we will write this as this greater than or equal to lambda and this greater than or equal to all lambda. Now, that is the solution that we are talking about.

As the result of which, we get S t for all T s solution, R t for all T s solutions. And the associated value of lambda. So, this is the associated value of lambda T s, where maximizing T s is also a solution that we get to the as 21 and then S 3, S 1, S 2, S 3 and R 1, R 2, R 3 this is the solution. When you are comparing the crisp solution with the fuzzy solution, you must keep in mind, the value of the objective function or the values that you actually get here, is not so much important.

What you should compare - What you should what you should keep in mind, is that you have been able to impart flexibility into the solution; and therefore, the solutions are more realistic. We are able to incorporate the preference is from various stake holders, and these preference is you have modelled in the fuzzy membership functions; and therefore, the fuzzy optimization problem provides you more realistic, more flexible solutions rather than the crisp optimization problems.

So, this is what you should keep in mind, when you do the fuzzy optimization. So, essentially in today lecture, we have continued our earlier discussion on the fuzzy optimization. We dealt with the water quality control problem and took a hypothetical example, through which we demonstrated the use of the fuzzy membership functions, which indicate in fact, the conflict that exist in the system, and then we arrive arrive at best compromise solution, when there are several stake holders or several players in the system each with these are around preferences for these solutions.

And these preferences are often conflict with each other, and then we formulate these problems as fuzzy member fuzzy optimization problems, look at maximized minimum value of the acceptability level or the satisfaction level in the system. And therefore, we call this as best compromise solution.

Then towards the end I also demonstrated a simple problem, where we are looking at the reservoir operation problem. You would like to maintain the storage to be as high as possible without increasing the release deficit. So, we formulated linear membership functions for the storage, and then looked at maximum value of that particular storage target, and also we formulated linear membership function for the release, and the then we use the fuzzy optimization to obtain solutions, I I mention in the course of the lecture. That the fuzzy optimization imparts more flexibility, more latitude in decision making and therefore, it is more realistic for actual applications, compare to the crisp optimization.

So, in the next 3 lectures, is the next 4 lectures I will take up some case studies, which apply all the methods or many of the methods that we have dealt with in the lecture so far. So, we will continue the discussion in the next class, thank you for your attention.