Water Resources Systems

Modeling Techniques and Analysis

Prof. P.P. Mujumdar

Department of Civil Engineering

Indian Institute of Science, Bangalore

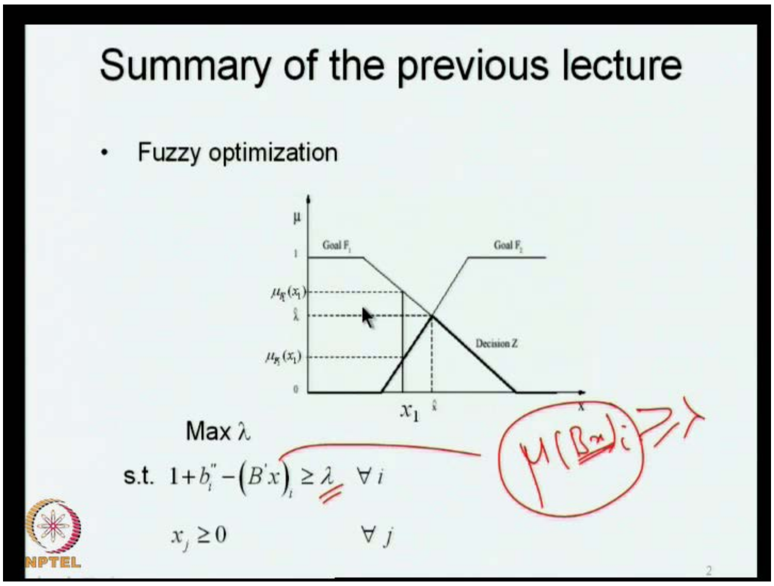
Lecture No. # 35

Fuzzy optimization (2)

Good morning and welcome to this, the lecture number 35 of the course water resources systems - modeling techniques and analysis. So, in the last lecture we discussed the fuzzy optimization, in fact we just introduced the fuzzy optimization. Re-call that the fuzzy optimization essentially gives a latitude in the constraints as well as in the objective function. What the crisp objective crisp optimization problem provided as a crisp solution, we start relaxing that solution and then start looking at the constraints, where we introduce certain latitude or relaxation or flexibility in the constraints.

Now, these types of problems are essentially useful, when you have, let us say the resource constraints. For example, we may say x 1 is less than or equal to 4 where x 1 is one of the resources. Then, we want to say that x 1 is not exactly less than or equal to 4, we may we are also accepting solutions which are slightly greater than 4 also. That means we start relaxing the right hand side values of the constraints, and then start looking at flexible solutions. And these types of optimization problems are essentially useful when you have large number of players, stake holders or large number of objectives, each conflictivity the other one. And then you are looking at acceptability of solutions. That means stake holders provide their level of acceptability of solution and so on. So, whenever there is a subjective judgment that is involved, subjective level of acceptability that is involved, we convert these crisp optimization problems into fuzzy optimization problems, and then address it using the techniques that we are dealing with now.

(Refer Slide Time: 02:15)



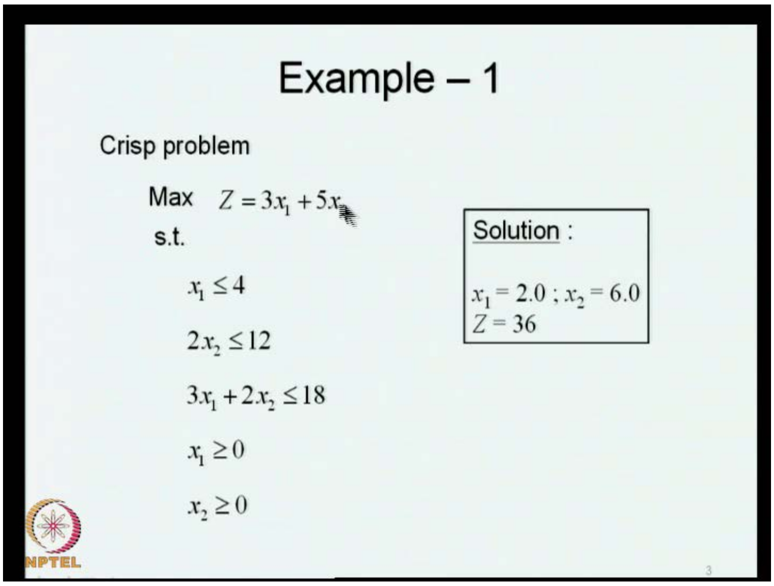
So, essentially the fuzzy decision - the concept of the fuzzy decision if you re-call. The fuzzy goals and the constraints of the fuzzy objectives and the constraints are all represented by their membership functions - their respective membership functions. Say for example, the goal F 1 which is a function of x, states here in this particular example - that the lower the value of x 1 the better it is, the lower the value of x the better it is for this particular goal. And this goal here, goal F 2 says that the higher the value of x 1 the better it is.

So, these two are conflicting and they are represented by their membership functions. The intersection of the two goals forms the decision Z and these in fact is the membership function for the decision Z which is given by the intersection of these two, and remember the intersection of two fuzzy sets is simply the minimum value of the membership function at each of the x 1 values. For example, we start taking the minimum value between F 1 and F 2, and then start going here, because there is no F 2 here, F 2 is 0 here, we take this value and then the minimum value at all this points will lead to this particular set or this particular membership function for the decision Z. It is within this decision Z that we are looking at the maximum membership function value.

So, in the case of conflicting goals like this, we define the fuzzy membership function for the decision Z, and then look at that particular value of x which will maximum which will provide as the maximum value of the membership function on the decision Z. And that is the problem that we wrote like this, maximize lambda subject to, these are essentially the left hand sides of the membership functions greater than or equal to lambda. So, I can… In fact write this simply as the membership function for the constraint B x i which is greater than or equal to lambda. What does this constraint ensure? This constraint ensures that the left hand sides which are the membership function values associated with the particular value of x that defines the left hand side value of the particular constraint is a minimum value.

So, this is the minimum of the membership functions. That in fact defines the space Z here, the membership function for the set Z here, and then through maximizing lambda we are looking at that particular value of the membership function so defined, which is the minimum value of the membership function; we are looking at the maximum value of lambda there. So, these are in fact called as a max-min type of problems, where we are looking at the minimum value of the membership function through this constraint - through this set of constraints, and then such minimum values we are maximizing. So, that is the max-min type of problems. Alright, what will we do is? Now, will take a simple crisp linear programming problem, the type of problem set where addressing now or in fact to the fuzzy linear programming problems, because we are saying that all these are linear membership functions and also we are introducing the non-negativity concept non negativity condition and therefore, these are the fuzzy linear programming problems.

(Refer Slide Time: 05:56)

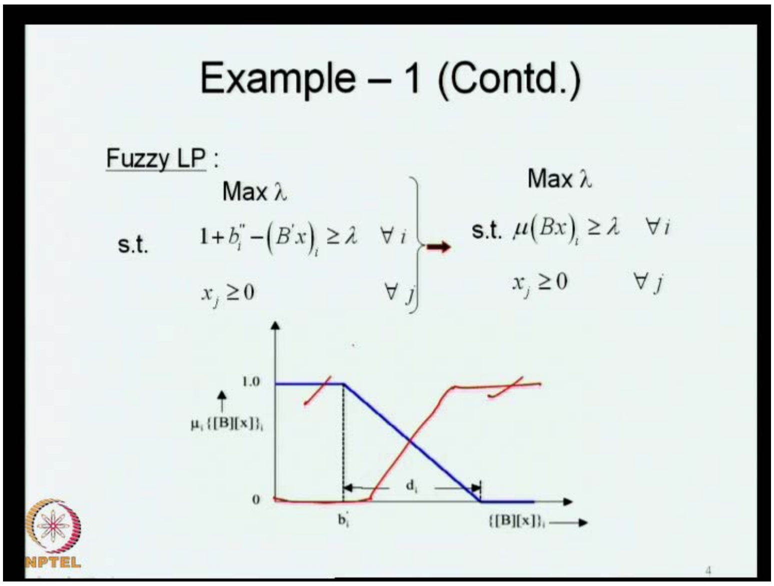


So, we will just take a crisp problem that we solved earlier on in the linear programming case. When I introduce the linear programming, I put this particular simple problem. Just a two variable problem with simple constraints here, you can also solve it is in the graphical method. So, maximize Z is equal to 3x 1 plus 5x 2, x 1 less than or equal to 4, 2x 2 less than or equal to 6, 3x 1 plus 2x 2 less than or equal to 18. This is the simple problem with non-negativity conditions. The solution for this is x 1 is equal to 2, x 2 is equal to 6 and Z is equal to 36. What this problem states is that - the solution should be such that x 1 has to be always less than or equal to 4, and 2x 2 has to be always less than or equal to 12 12 or x 2 is less than or equal to 6, and 3x 1 plus x 2 is less than or equal to 18. This is what this problem states. And we obtain the value of Z as 36 for such a crisp problem. We use the solution of the crisp problem in general, as a guide to look at the type of objective function values that you can expect.

For example, in this case, you got an objective function value of 36. So, we know that the objective function value can be around 36, 38 or it may be 34 and so on. So, if you are looking at maximization problem, you may say that I would like my objective function value to be around 37, 38 and so on. And then look at the constraints x 1 is less than or equal to 4, because you are saying my objective function can be larger than this particular objective function. You may want to relax the constraints a little bit, say that x 1 I would prefer to be less than or equal to 4, but I am not awares to taking solutions which are slightly more than 4, for x 1 more than 4. And therefore, we relax this constraint and put it as a fuzzy constraint with its associated fuzzy membership function. And that is what we do for all the constraints. So, we said the objective function as a fuzzy objective function, we said all the constraints as fuzzy constraints, and then formulate this crisp problem as a fuzzy problem - fuzzy linear programming problem.

So, re-call that my general form of the fuzzy optimization is this; maximize lambda subject to the membership function for the i th constraint greater than or equal to lambda. Now, when I say membership function for the i th constraint, it also includes the objective function of the original problem. Because this also we convert it into a fuzzy constraint and then handle everything as fuzzy constraints.

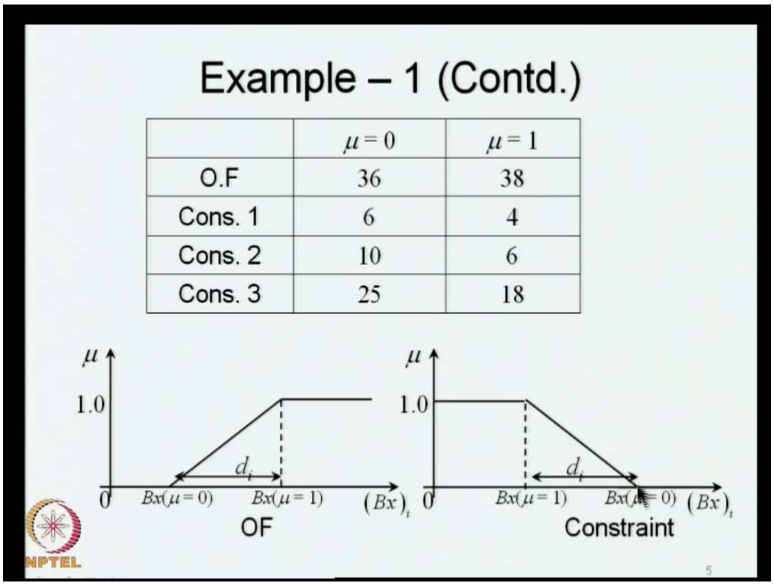
(Refer Slide Time: 08:45)



So, in general, we write this as maximize lambda subject to set of constraints associated with each of the constraint in the original problem including the objective function and the non-negativity condition. So, we can simply write this instead of going through all this b i double dash etcetera. I will simply say this is the membership function for the i th constraint, (B x)i greater than or equal to lambda. So remember, what we are saying here is that including the objective function, we said the membership functions to be greater than or equal to lambda, and then that value of lambda will maximize. So, this ensures the minimum value of the membership functions for the i constraints, and then that minimum value we are maximizing. That is a idea there.

And this is a type of membership function that we consider, it can be either this way or you can also have the membership function the as a wave which means the higher the better kind of membership function. So, you can either have this or can have this. So, we will look at this problem now again. Z is equal to 3x 1 plus 5x 2, x 1 less than or equal to 4 and so on. For the Z, we will form the membership function based on the objective function value that we obtain in the crisp solution. And then we will relax these constraints and then gives some latitude, and then define the membership functions accordingly. Because it is the maximization problem; what we are referring to is, the higher the objective function value the better it is. So, I expect a solution somewhere around 36 and then I will say that the higher the objective function value the better it is.

(Refer Slide Time: 10:43)

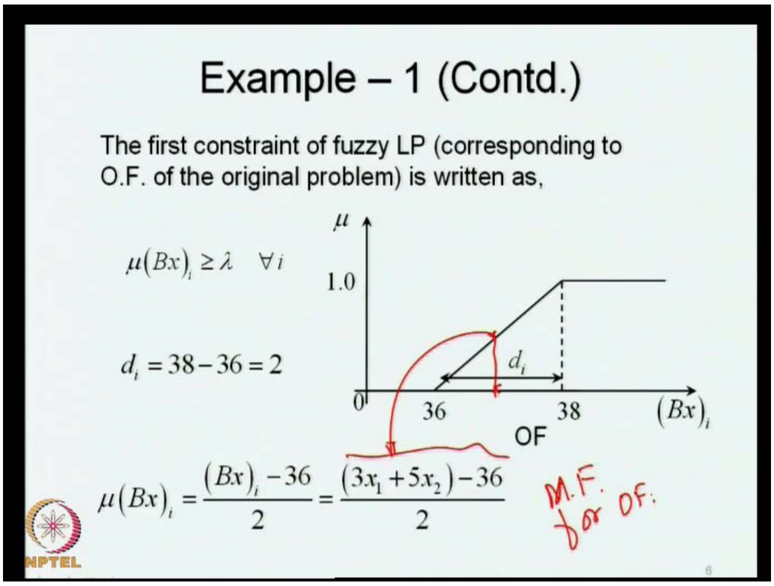


So, what we will do is? That I will put my mu is equal to 0 for the objective function. So, I am formulating now the membership functions for the objective function. I will say my mu is equal to 0 at Z is equal to 36. That means the objective function value is 36. And I would prefer the solution to be more than 38. So, at 38 I put a membership function value of 1. Similarly, you go to the constraints, x 1 is less than or equal to 4. So, I will say that my mu is equal to 1 for the membership function corresponds to the left hand side to be equal to 4. That means x 1 should be less than or equal to 4 in that particular case. But I am not awares to the solutions up to 6. So, I can go up to 6 for that particular constraint, which means essentially the left the right hand side we are relaxing a bit here. Instead of saying 4, I am saying up to 6 it is a acceptable, but as it crosses 4 my degree of acceptability becomes smaller and smaller. So, that is what it is reflected here.

So, at mu is equal to 1 my value is 4 and anything less than 4 is preferred, because it has mu is equal to 1; anything more than 4 is acceptable up to a distance of 2 which is which leads to the constraint value being equal to 6, and beyond 6 it is mu is equal to 0 again. Similarly, constraint 2, it is says x 2 is less than equal to 6 and that we will put it as x 2 is less than or equal to 6 is preferred, but we are not awares to going up to 10 and with a lesser degree of acceptability. As x 2 moves away from 6 here, my acceptability of the solution becomes smaller and smaller, and that is what is reflected by this membership function, and that is how we formulate the constraint. Similarly, that last constraint 3x 1 plus 2x 2 we will say that 18 is acceptable that is here, 18 is equal to mu is equal to 1. So, anything is less than or equal to 18 is preferred, but we can go up to 25 with reducing degree of acceptability.

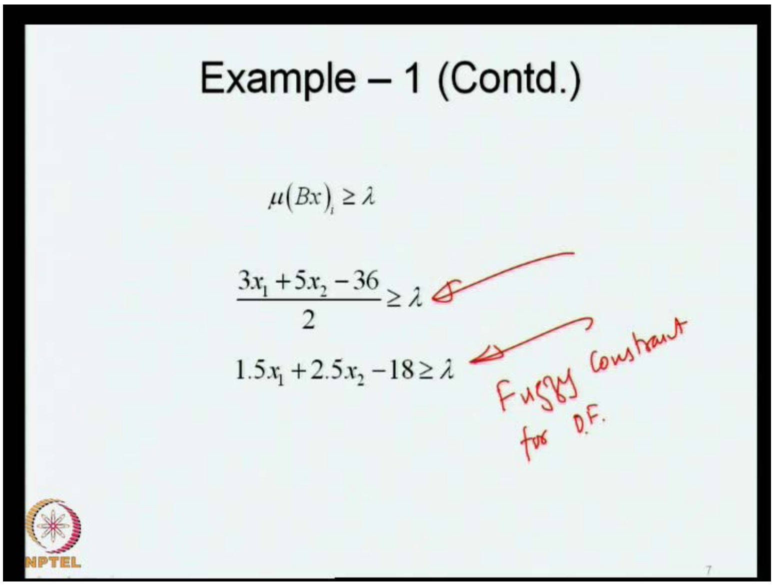
So, this is how we formulate the fuzzy membership functions associated with the objective function and the three constraints in this example. Now, remember some of the constraints may be of greater than or equal to type. In which case, you have to formulate the constraints accordingly. So, in this particular case I have put all of them as less than or equal to, if someone some constraint was greater than or equal to type, then you may get constraint of a membership function of this type.

(Refer Slide Time: 13:43)



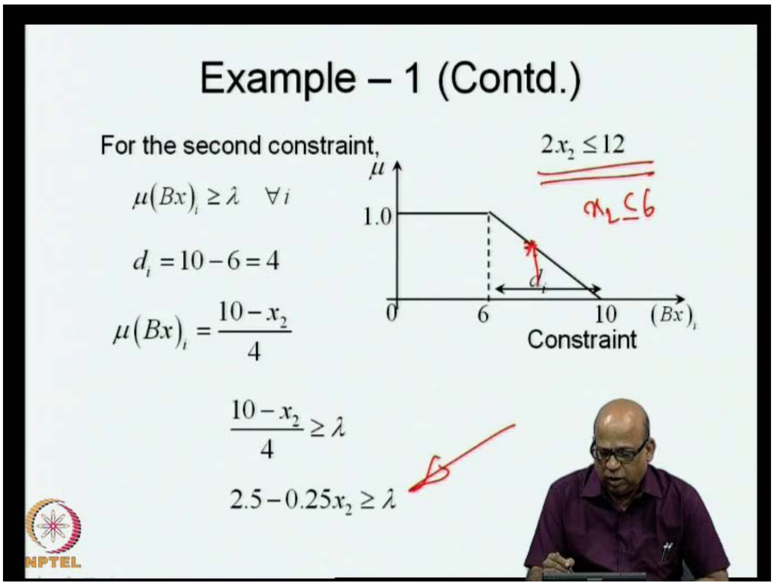
Now, we will formulate the crisp equivalent of that. You look at the objective function constraint - this particular constraint. So, 36 to 38 we are saying that my mu is between 36 to 38. So, I will put 3x 1 plus 5x 2 which is a objective function value here, 3x 1 plus 5x 2 that is (Bx)i minus 36. So, I am just writing the mu x mu Bx for this region here, 36 to 38. So, that will write it as 3x 1 plus 5x 2 minus 36 divided by 2, this 2 is this distance here, going by the earlier definitions that we have used. So, essentially what I am writing is any general value of (Bx)i in between, I will put the objective function value in terms of this. So, this is the objective function value membership function. So, this is membership function for the objective function value.

(Refer Slide Time: 14:52)



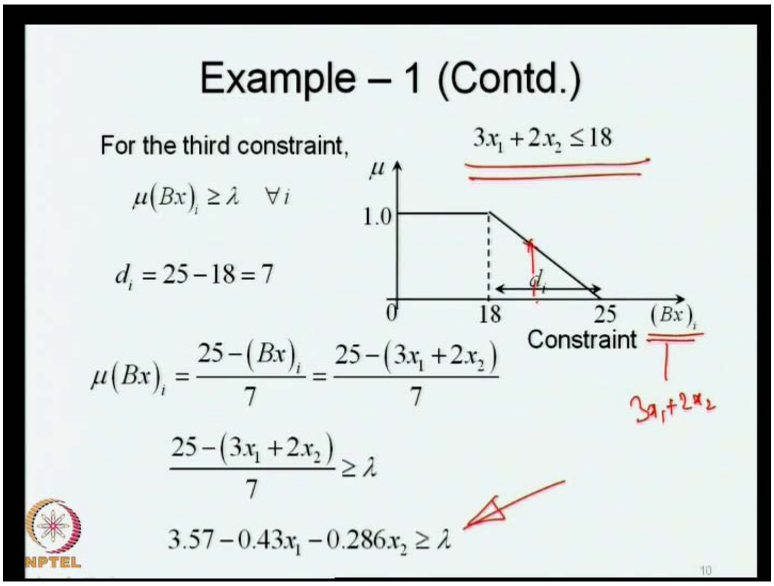
Similarly, then will simplify this and then write this as the constraint. This becomes the constraint corresponding to the objective function value. That is, we will say this as the fuzzy constraint for the O F. Remember, in doing this what we achieved is, that we are saying that I would prefer the solution to be higher than 36 and then preferably more than 38. So, between 38 and 36 my degree of acceptability comes down and the objective function is 3x 1 plus 5x 2, and therefore, we write that particular statement as 1.5x 1 plus 2.5x 2 minus 18.

(Refer Slide Time: 15:48)



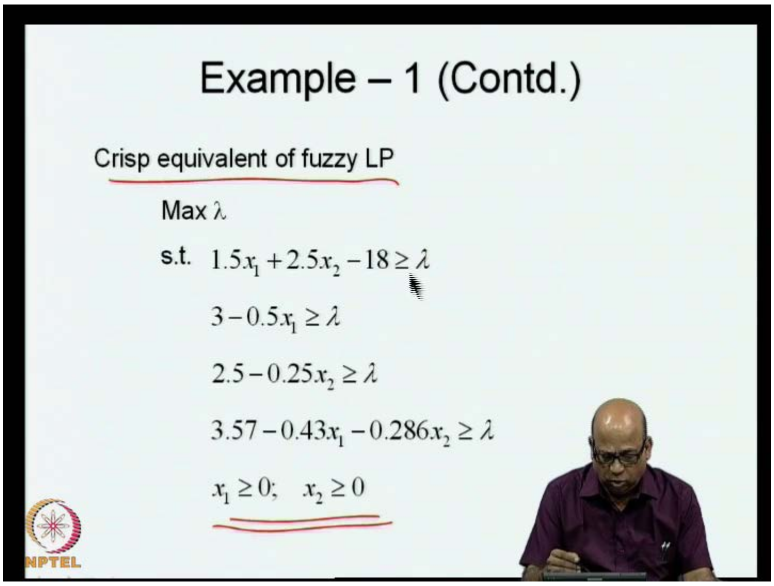
Similarly, for each of the constraints we do. For for example, we take x 1 is less than or equal to 4 as the constraint, and then we use this membership function, and then write this as the crisp equivalent. So, we are writing for any level in between here. Similarly, for the next constraint 2x 2 is less than or equal to 12. So, I am writing this as x 2 is less than or equal to 6 and that is a constraint that is defined F the membership function for that particular constraint, and then we get this as the crisp equivalent. We are writing the expression again for any middle value, any given value here of x 2 between 6 and 10.

(Refer Slide Time: 16:34)



Similarly, the third constraint 3x 1 plus 2x 2 less than or equal to 18. So, this is the constraint left hand side values. So, this is actually 3x 1 plus 2x 2 along this direction. And then, we put that use the same same method of converting and write the expression for any value in between 18 and 25, and that is a value that you get here. So, what we have done is, we have converted the objective function using the membership function for the objective function into one constraint. We have converted the first constraint into another constraint using the membership functions; we have converted similarly all the constraints using the membership function.

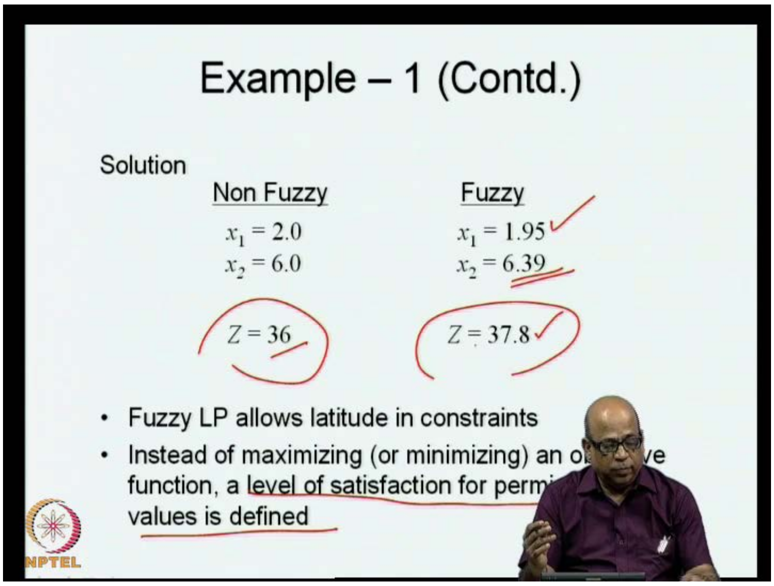
(Refer Slide Time: 17:25)



So, the crisp equivalent of the fuzzy LP becomes this now. Remember, we started with the crisp problem, then we relax the constraints and formulated also the objective function as the fuzzy objective function, and looked and formulated a fuzzy linear programming, where we are maximizing lambda subject to each of the membership functions being greater than or equal to lambda. And that is how we are ensuring that we are maximizing the minimum value of the membership function of the decision space Z. And then we convert this fuzzy LP into a crisp form. So, that we can solve the linear programming problems and this is the crisp form. So, this is maximizing lambda subject to each each of these constraints and this is the non-negativity constraint. Remember all of these constraints, we have just derived based on the membership functions that we have derived; we have defined earlier.

So, this is the crisp equivalent. We can solve this using the linear programming software or including even you can use the perhaps graphical method will not be easy, because you have lambda also as one of the decision variables here. So, there are three decision variable; x 1, x 2 and lambda. So, you use any of the methods; use the simplex method or use the simple software (( )) and then you can solve.

(Refer Slide Time: 19:07)



When you solve this you compare it with what you had obtained earlier. So, we got the non fuzzy solution as x 1 is equal to 2, x 2 is equal to 6 and Z is equal to 36. This is the crisp solution before we converted into fuzzy optimization. And then we relax the constraints, and then also formulated the objective function as the fuzzy objective function, and then solve the fuzzy linear programming x 1 becomes 1.95. So, I am showing the solution for this now. x 1 becomes 1.95 x 2 as 6 earlier that becomes 6.39 and z is 36 here, this becomes 37.8. What did it achieve now? You look at the first constraint x 1 was less than or equal to 4. So, it has reduced x 1 further, and x 2 was less than or equal to 6. But we have set that we are not always to going up to 10.

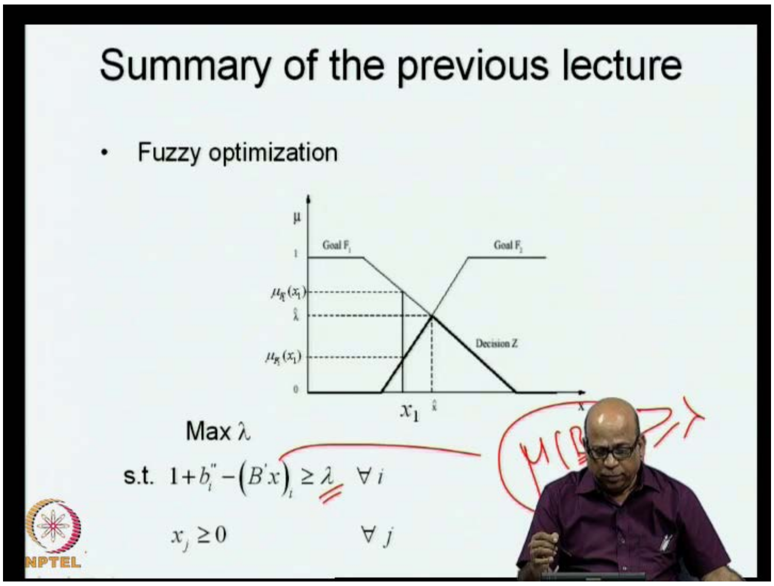
If you look at the membership function for the second constraint, 2x 2 is less than or equal to 12 or x 2 is less than or equal to 6. We said that while we would prefer the solution is less than 6, we do not mind going up to 10 with reduction in our acceptability. In the solution for the crisp optimization problem, x 2 took a value of 6. Look at this, x 2 took a value of 6. So, that was the binding constraint. And then when you relax this, it had a more flexibility to play around with solution. So, instead of saying that x 2 is exactly 6, we are saying that we are not always to going slightly higher than 6. And therefore, it immediately jumped on to this and put a value of 6.39.

Whereas, the first constraint which was x 1 less than or equal to 4 was not a binding constraint in the solution, because x 1 is 2 in the solution. And therefore, it did not the solution did not matter much, it just went into 1.95, it just reduce the value of x 1. But it could achieve a value of 37.8. Why could it not go to further solutions? When my membership function value for Z is in fact the higher greater than 38 you will have 1. So, it has come very close to 38, but it could not reach 38 quite, because of the last constraint. We also had another constraint 2 3x 1 plus 2x 2, and because of this all the three constraints it could not achieve a solution more than 37.0 whatever it could get, 37.8.

But essentially, it is not the actual values that are important here in this particular case. That we could go from 36 to 37.8 was not the idea. The idea is to provide latitude in decision making or flexibility in decision making. We are essentially saying that - if our resource constraints are slightly relaxed and then we become more flexible in our optimization, then these are the solutions that we get. So, essentially what we are saying is that instead of saying x 1 has to be less than or equal to 4 or for example, the second constraint 2x 2 has to be less than or equal to 6, if we say that we would preferred 2x 2 to be I am I am sorry 2x 2 to be less than or equal to 12. But we are not awares to slightly relaxing this. That means x 2 can be greater than or equal to 6, and that is the type of solution that we get.

So, the fuzzy linear programming, essentially allows some latitude in the decision making instead of maximizing or minimizing an objective function, what we are now doing is, look at the objective function value of the fuzzy optimization, we are saying maximize lambda. The identity of the initial objective function namely maximize Z is equal to 3x 1 plus 5x 2 has been lost in the final objective fuzzy optimization problem - objective function. But it has been accounted for in one of the constraints as fuzzy constraint. So, instead of maximizing or minimizing an objective function what we do in the fuzzy optimization is that a level of satisfaction for permissible values is defined. At we use this level of satisfaction as a major, and then optimize the objective function value in the fuzzy optimization. The fuzzy optimization is in fact more useful when there are conflicting objectives, when there are number of stake holders, all of whom express their level of acceptability to different solutions. In fact, the interpretation of the membership function, you should not lose sight.

(Refer Slide Time: 24:36)



The membership functions for the goals here, as shown here, is what is important, what it shows is that these two are conflicting; goal F 1 and goal F 2 are conflicting. Similarly, you may have several such goals and not necessarily linear goals, you linear membership functions, you may have several non-linear membership functions and so on, and define a several variables. And therefore, it becomes a space rather than just intersection of lines. Among such conflicting objective functions, the degrees of acceptability of solutions for different players or different stake holders will be different, and then you are looking at the best compromise solution. So, one goal says this is the type of solution that is preferred, the other goal says this is the type of solution that is preferred. They are both conflicting and you are achieving the best compromise solution.

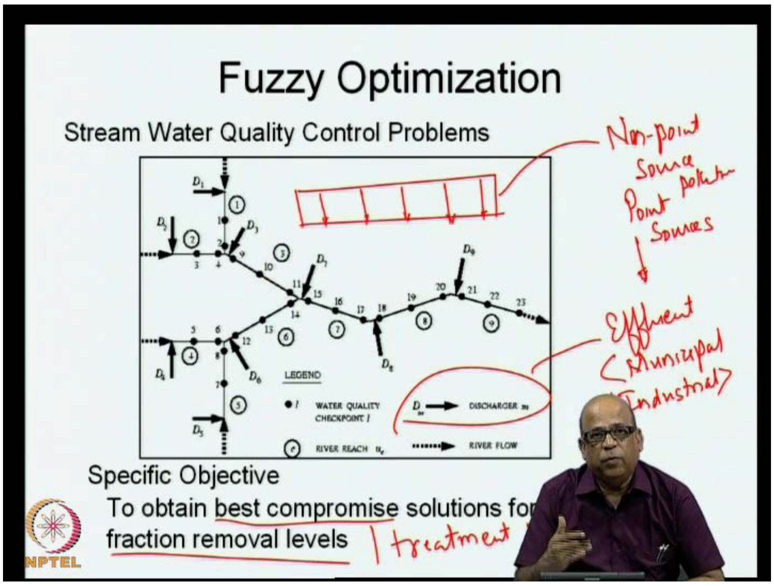
So, the fuzzy optimization is ideally suited, whenever there is a large conflict in the system, there are large number of linguistically stated goals which can be converted into associated membership functions. You also want to introduce latitude in the decision making, you want to provide or imparts some flexibility in the decision making, in such situations the fuzzy optimization becomes extremely handy. And the solutions are quite simple, as I just demonstrated or you have to do is convert the fuzzy linear programming into crisp linear programming using the membership functions, and then maximize the level of acceptance lambda is in fact the level of satisfaction in the system. And this is also called as the best compromise solution. So, with that brief the ground now we will look at some applications.

Now, the typical application that I will deal with for the fuzzy optimization is a general water quality control problem, which has large number of stake holders and it also has degree of conflict. For example, you are looking at the water quality in a stream, when there is an effluent discharge that is taking place at a particular location. So, straight away you would look at this problem from two different perspectives; one is to use the assimilative capacity of the river to the best extend possible. So, that the waste can be discharge into the river, the industrial or municipal effluence etcetera can be discharge into the river. So, the rivers are receptacle receptacles of the effluent discharges. And therefore, you would like to make the best use of the assimilative capacity of the river. So that, the municipal industrial waste etcetera may be perhaps after certain treatment can be discharge into the water bodies like rivers.

The other perspective of looking at which is also equally important is that I would like to maintain the water quality above a certain threshold value. So, we would not want to (( )) the water quality beyond a certain point. So, on one side we are saying that - the water quality should be as high as possible; on the other side we are saying that - the assimilative capacity should be used to the best extends possible, and therefore, it should be we should be able to discharge the effluence to as high a level as possible, both in terms of the volume as well as in terms of the concentrations of the pollutants themselves. So, these are two straight away conflicting goals; one is to maintain the water quality as high as possible, the other one is to discharge effluence to as large the quantity as possible. So that the assimilative capacity of the river is utilize to the best extend possible. Now, these are typical conflicting goals that we come across in most of the development and environmental situations; development verses environmental.

So, it this particular problem will demonstrate that it should not be development verses environmental, it will be development and environmental together where there are there are conflicts and then we arrive at there are methodologies which are the fuzzy optimization methodologies through which we can arrive at best compromise solutions. So that the environment is not degraded yet at the same time you are able to maintain the development at certain level.

(Refer Slide Time: 26:37)



So, will look at this particular example now; this is the stream water quality control problem, this is the stream and there is another stream that joints at this location, this is the general figure - general stream. There are effluent discharges, the D 1, D 2, D 3, etcetera that I am showing here are the effluent discharges. It is shown here Di m is the discharger, this is the effluent discharger. Now, these are typically either municipal effluence or the industrial effluence. And these are the point sources - that is point sources of pollutions.

(No audio from 30:35 to 30:50)

Now, we are interested in maintaining the water quality at several locations such as these shown by dots here. So, these are the water quality check points. That means we would like to maintain the water quality at several locations at some pre-specified levels. In fact, we would like to make the water quality as high as possible at these locations. Then we may also have a non point source pollution which is either uniformly adding the to the stream or it may be in a adding to the stream in a non-uniformly manner. So, this is may be a non point source pollution.

Now, in this problem, what is it that we want to achieve? Let us say that we are looking at a particular check point 19, the water quality at this particular location is determined by what is happening upstream of that location all of these. So, there is a there is a dilution effect taking place, because of the stream flows. There is also re-erosion and other processes that are taking place, because of the immediate atmosphere in which the water is contact, and the effluence that are being discharge into the stream. Given the effluent discharges at various locations, it is possible through mathematical models to obtain the water quality at this location. So, for the discussion purpose, you just assume that if I specify, let us say the D O D loads bio-chemical oxygen demand loads at this various locations in the stream, it is possible for us to obtain the dissolved oxygen at this particular location, and similarly for any other locations. So, given the upstream conditions, it should be possible for you to get the water quality at these locations.

Remember the water quality at this location will also depend on the hydraulics of the system. For example, what is the time it takes for the flow to come from this point to this point? What is the cross section, what is the (( )) coefficient and all the open channel characteristics and so on, they will all determine how the flow takes place in all the streams and then reaches this particular point. The effluent discharge or the pollutant let us say you put a D O D, the effect of D O D on the D O at this location is governed by certain transport processers and the transport processers for non reactive pollutants can also be formulated using mathematical models. So, for the discussion of these optimization problem, you assume that the water quality at particular location can be determined through the mathematical models for given loadings of the effluence discharges, for given hydraulics of the system; that means we specify the discharge, we specify the cross section at various locations, we also specify the temperature, relative humidity etcetera, so that the reaction rates can be determined and so on. So, all the physical process is that govern are all known and the parameters thereof are cannot be determined.

Now, the question is let us say for a given level of discharges D 1, D 2, D 3, etcetera. These are the effluent discharges for given levels, the water quality at this point is very bad; that means the dissolved oxygen level may have reach 3 mg per liter or some those things. So, we are looking at the water quality be indicator as dissolved oxygen. So, you would like to increase the dissolved oxygen level form the existing 3 to 6 let us say. That at this point, we want to have the water quality such that the dissolved oxygen is at least 6 mg per liter. Whereas, if you do not have any treatments here it would be 3 mg per liter, then you start looking at the treatments that need to be given; that means there was certain effluent load that was coming, we say that this is not acceptable, you start treating now, so that the water quality at particular location increases.

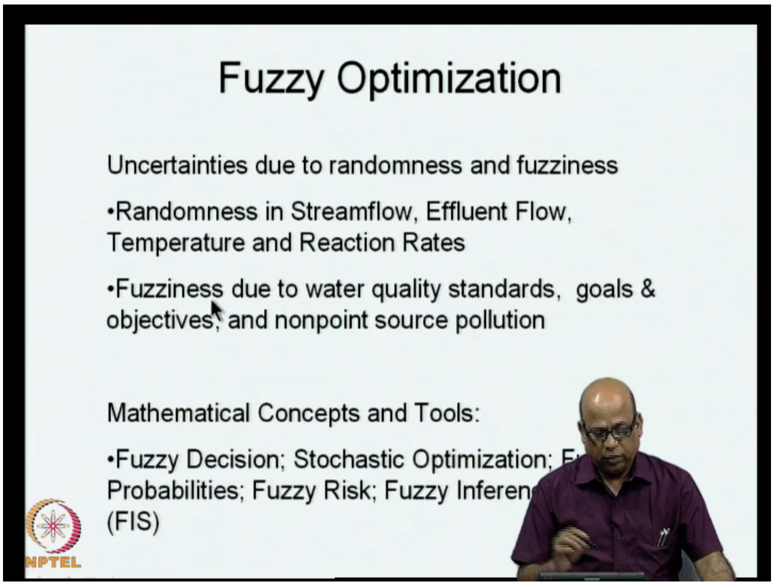
The decision then becomes how much to be treated, how much of the discharge is to be treated, effluent discharge to be treated at this location, at this location etcetera. So, at each of the given point sources of discharges, what is the level of treatment that you need to achieve? That is the question. It is easy to say that you do not discharge anything; that means 100 percent treatment. That is even if you are generating some pollutants here or the effluence here, waste here, you make sure that 100 percent of that is treated and let in to the river. But that is practically not possible. Because the effluent waste has to be dischargeable somewhere and the rivers have been good receptacles of the waste and therefore, the question that is more pertinent is - that you arrive at optimal treatments at all of these locations such that the water quality is not (( )) beyond a certain point or the water quality is maintained at least up to a certain level at several check check points. So, the question that we posses to we want to obtain the best compromise solutions for effluent fraction removal levels.

The fraction removal levels are in fact the treatment levels - these are the treatment levels. Why we say best compromise solution, because there is a degree of conflict associated in this associated with this problem. The degree of conflict arises, because of the sets of objectives that are conflicting with each other namely, one corresponding to the pollution control agencies who would like to maintain the water quality to be as high as possible. The other corresponding to the discharges themselves or the developmental goals, where you would like to discharge as much as possible into the stream. The way that is generated after treatment after certain treatment, you would like to discharges into the stream.

Now, the question is what is the optimal treatment? So that the water quality is maintained yet at the same time the cost of treatment or not very high or the cost are minimized. For one side you would like to maintain, the cost of treatment to be low and the other side you would like to maintain a high water quality, these two are conflicting. And these two are conflicting with respect to several of the sources here. For example, we may look at different check points and at each of the check points you may want to have a objective. At each of the discharger you may want to have a objective objective in terms of its acceptability - the solutions acceptability. For each of the pollutant you may have degree of acceptability. So, there are a large number of goals associated with the number of the pollutants, associated with the number of locations at which would like to achieve the solutions that is the check points, and also associated with the water quality indicators themselves. For example, you may form one membership function associated with dissolved oxygen, another with nitrate, for example, another with p h, another with turbidity and so on. So, with respect to different water quality indicators you may have different responses, with respect to different discharges you may have different responses. For example, D 1 you may have its own acceptability, D 2 you may have its own acceptability and so on.

Then from the pollution control angle you may have at different locations, you may have different responses. For example, my response to site 17 may be much different from the response to response at site 22, where I may want to use the water for gardening purpose, for irrigation purposes and so on. And therefore, the water quality requirements there will be much different from if I am simply using the water for some other purposes, any other non portable purposes. And therefore, your degree of acceptability of solutions will be different also at different locations. And therefore, you will have a large number of membership functions, associated with this problem. So, this problem can be post as a general fuzzy optimization problem, and then form objective form the membership functions and then look at the optimal solutions.

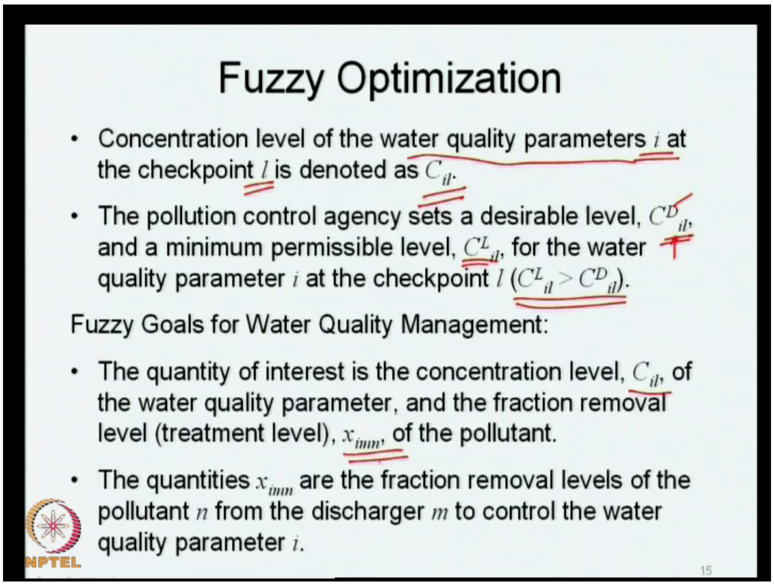
(Refer Slide Time: 40:35)



So, these are some uncertainties that we introduce here. For example, the randomness in stream flow can be addressed in this and then effluent flow temperature, and reaction rates, the fuzziness is what is our focus in this, we want to address the fuzziness due to water quality standards; that means how much can be let down here. And even if the water quality standards are met, can we if further optimized the solution.

Let us say that the water quality standards say that at a particular location, your D O D is should not be more than 30 mg per liter or something for the effluent. So, all of them maintain 30 mg per liter. Yet at the same even with this, the water quality at particular location is not acceptable then we look at the optimal solutions. And that we form using the fuzzy membership functions.

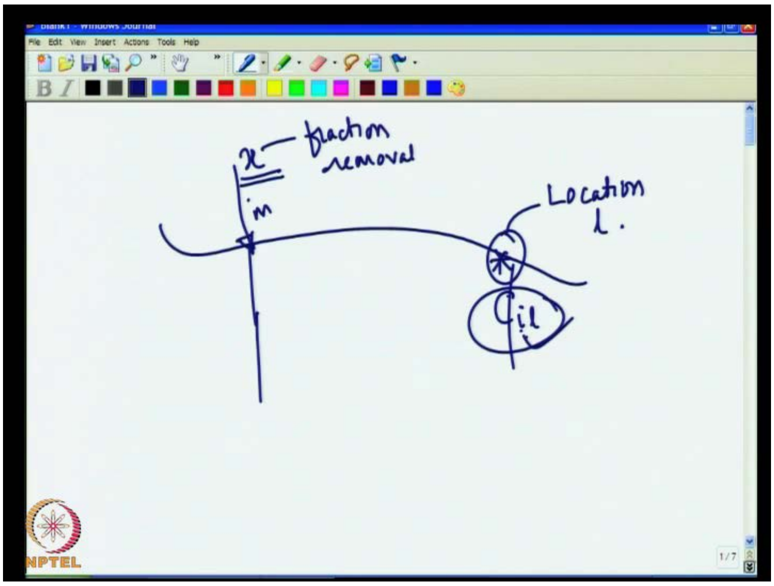
(Refer Slide Time: 41:28)



So, we will introduce this problem now, what we will do is a check point l, we denote the concentration level for water quality parameter i. Now, these are some general notations. As I said this water quality parameters can be D O at a particular level or you may talk about dissolved oxygen deficit instead of talking about the absolute values of D O, nitrate levels, Ph level, turbidity level and so on. So, those are the water quality parameter. And the concentration level at check point l we call it as C i l. Now, we may have a desirable level for the particular pollutant i or the water quality parameter i at location l. So, this we put it as C i l D.

Now, I am starting to formulate the fuzzy membership functions. So, we say that there are two players now here - two major players; one is the pollution control agency, another is the discharges - set of discharges. So, we say that the PCA or the pollution control agencies sets a desirable level C i l D, this is a desirable level. And also a minimum permissible level C i l L which means that you cannot go below C i l L, but I would like to have as high as C i l D, and in this particular case, we may say that C i l L is greater than C i l D, for example, if you are looking at D O deficit. The D O deficit the minimum permissible level will be higher than the desirable level. So, depending on the type of water quality parameter i that we are looking at, this condition can be populated.

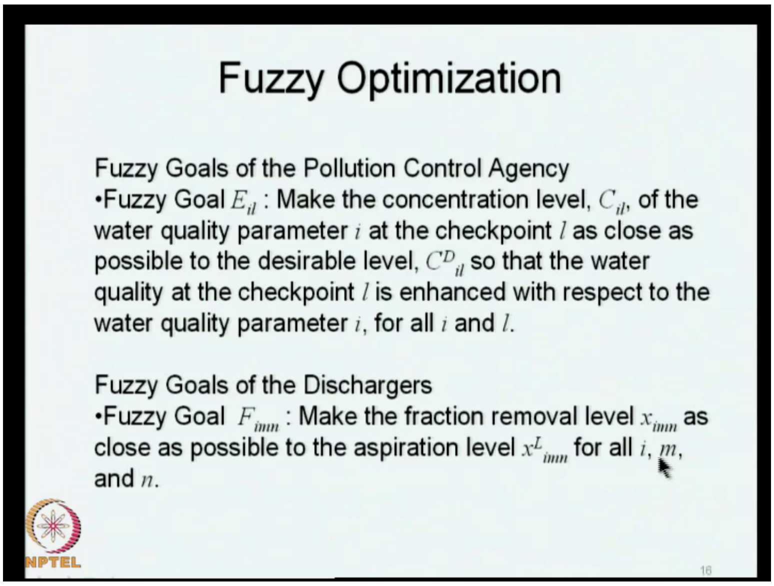
(Refer Slide Time: 44:10)



Then we formulate the fuzzy goals for the water quality management. For example, we are saying that C i l is the concentration level of the water quality parameter i and it is also a function of the fraction removal level x i m n. How, if we look at that transport process, let us say that at a particular location in the stream, you have the effluent coming at a particular location. Let us say this is x and you are interested in the concentration C i l and this is the location l. As you treat this that is this is a fraction removal level. So, as you change x your C i l will different will be different. The relationship between what is coming here and what is the result here can be written as crisp mathematical problems. So, you can determine C i l which is the concentration of the water quality parameter i at the location l, as a function of the fraction removal level at the discharger m, let us say this is the discharger m. So, at the discharger m you are treating certain effluent and you can determine the C i l corresponding to that. That is the idea there.

So, with these functions in place we say that x i m n and C i l are dependent on that is C i l is dependent on x imn, where x imn is the fraction removal level or the treatment level of the pollutant n from the discharger m for the control of water quality parameter i. Why I am stressing this so much is that - the responses can be much different for different pollutants, different dischargers, different locations as well as for different water quality parameters. And all this responses together will form a set of membership functions and from the membership functions you want to look at the optimal solutions. That is the idea there.

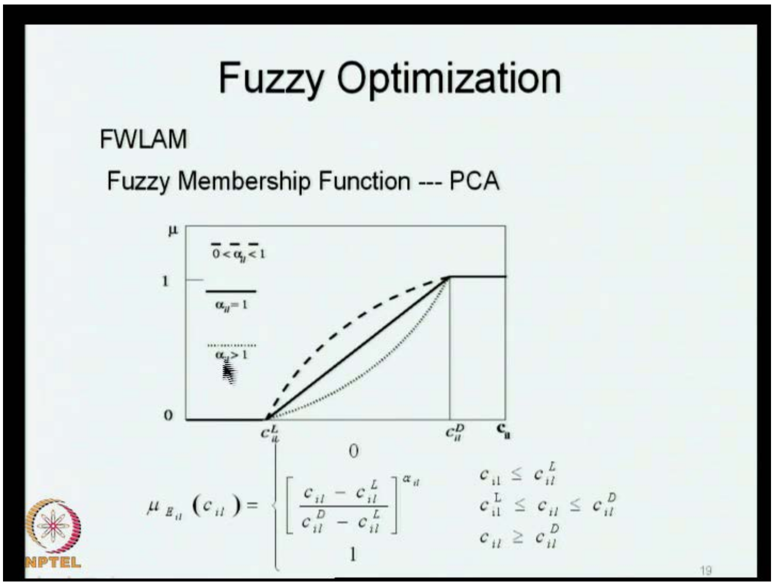
(Refer Slide Time: 46:20)



So, then we define the fuzzy goals; that means now we are coming to the objective functions. Look at one of the goals of the pollution control agency, we will state in investic terms first and then convert it into a fuzzy membership function. So, we say that our goal is to make the concentration level C i l of the water quality parameter i at the check point l as close as possible to the desirable level C i l D. So that the water quality at the check point l is enhanced with respect to the water quality parameter i for all i and l. This is the most general statement of the pollution control agencies objective or goal.

But you look at the goal of the dischargers, they say that make the fraction removal level x imn as close as possible to the aspiration level, each of the dischargers may say that I I would like to have my treatment level at x L imn; that means the lower the better is the dischargers goal here - fuzzy goal for all i, m and n. Now, these goals can be stated as fuzzy goals and then we formulate the fuzzy membership functions. There are all these details here available. But we will go to how we formulate the fuzzy membership functions now.

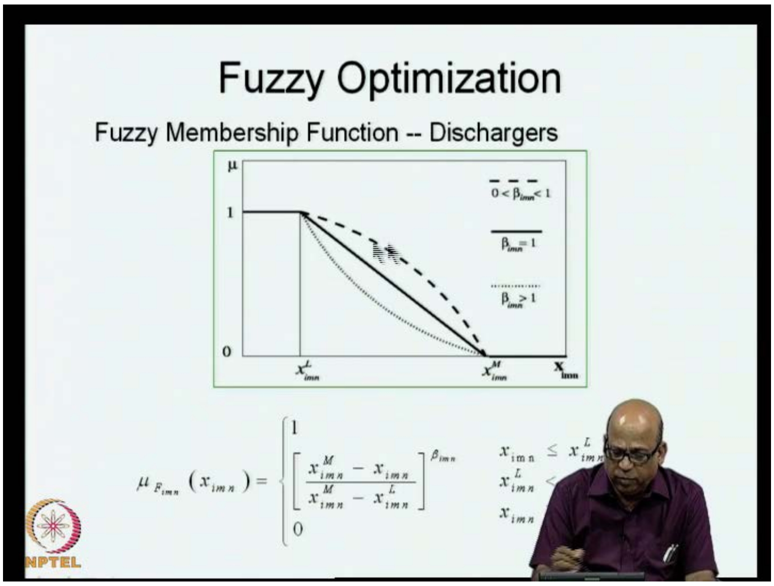
(Refer Slide Time: 47:45)



Remember, if we are looking at a water quality parameters such as the dissolved oxygen, we will be saying the higher the better. So, this is on the x axis is the concentration of the dissolved oxygen for example, then we are saying the higher the better. So, in general we may have a non-linear membership function such as is or a linear membership function such as is, and this is the desired level, and this is the lower level; that means we will say that anything less than this is not acceptable, and we would prefer the oxygen concentration to be higher than this. This we will define for all l and this can be defined for all similar i, similar i in the sense that - the higher the better. There may be dissolved oxygen, the higher the better, these kind of water quality parameters we may use such membership functions. So, this can be generally written as in a general sense it can be written as a non-linear function like this and the alpha i l which is a index here will determine the exact shape of this. For example, alpha i l is equal to 1 will give you a linear shape and so on. If alpha i l is less than 1 you will get this shape.

Now, the shape of the membership function at a particular location will decide the response or the acceptability of the solution. For example, if my D O level is here, this is my degree of acceptability, if you follow this membership function; this would be my degree of acceptability, if you follow the linear membership function and so on.

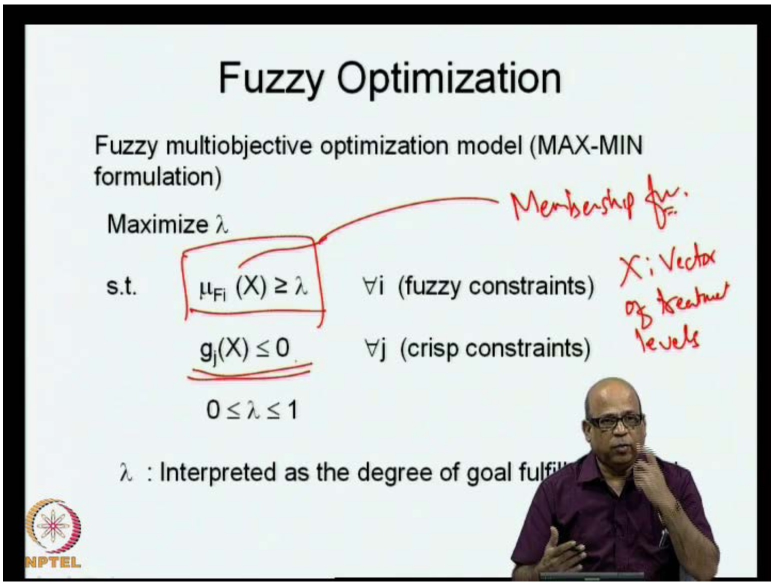
(Refer Slide Time: 49:42)



So, like this we also formulate the membership function for the discharges. So, for the discharges we will say that the lower the treatment levels the better it is. So, we may formulate non-linear membership function like this or linear membership function like this and then put in a general form where the beta imn for the water quality parameter i, for the discharger m, for the pollutant n. So, like this we may formulate membership functions associated with each of the i, m and n depending on the responses that you get from the particular dischargers for the the given water quality parameter and n. Essentially, this membership functions shows that the lower the treatment level the better, that is a that is a implication of the membership function. Whereas, for the pollution control agency, this is the higher the water quality indicator the better it is; that is a pollution control agencies membership function.

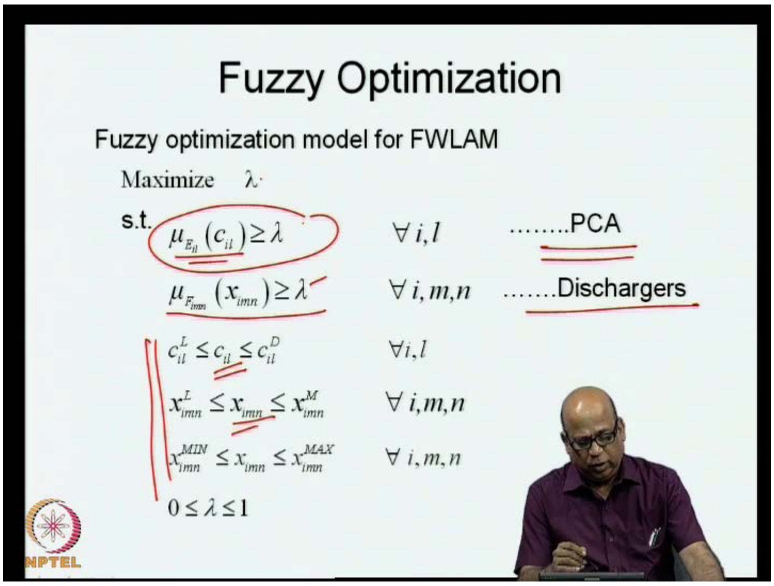
So, you have a large number of sets of membership functions now; associated with the goals of the pollution control agency and the dischargers. All of this will pollute together and then look at the best compromise solution that is comes out of this. How do formulate that? We go go to the fuzzy optimization, use the general fuzzy optimization technique, where I will write mu F i of x which is the membership function. These are the membership functions associated with that is the left hand side - left hand side are the membership function, which include the goals of both the discharges as well as pollution control agencies.

(Refer Slide Time: 50:58)



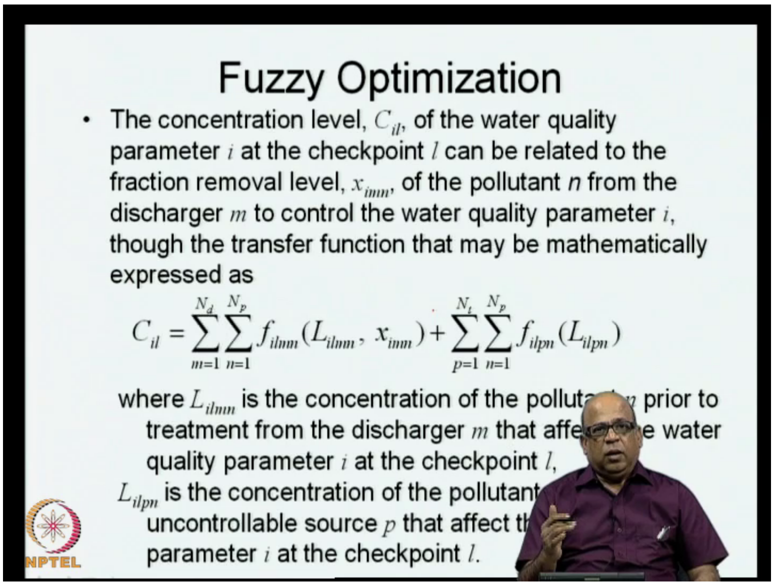
Now, x is a vector of treatment levels in this case. What I mean by that is that x will be x 1, x 2, x 3 etcetera at various discharges what is the level of treatment that you want to give. Now, this will determine the concentration C i l, and therefore, it will define the membership function for the pollution control agency. So, your decisions that you are making will be on x, what is the optimal treatment level? From the x you transfer the membership function values into the pollution control agency goals, which are associated with C i l, which is the concentration of the water quality indicator i at the location l. How do we do this? We use water quality simulators to obtain the concentration at a particular location for a given discharge given treatment level x at a particular point upstream of that. And then formulate the general fuzzy optimization using the fuzzy constraints and the crisp constraints, you may also have some crisp constraints, for example, the technological limits on treatment levels that are possible, they will define a set of crisp constraints and so on. So, you may also have physically based crisp constraints, you use all of that and formulate the fuzzy optimization technique problem.

(Refer Slide Time: 52:58)



So, in the particular case, what we will do is that for the pollution control agency we have these sets of goals, these sets of fuzzy constraints, for the dischargers you have these sets of constraints. Remember in fuzzy optimization, you pick up the fuzzy membership function and put it greater than or equal to lambda, and you are maximizing the particular value of lambda. That is that is the idea here. And these are the crisp constraints, what we may we are saying here is - that is the C i l which is the concentration of water quality parameter i at location l must be within a certain range. Similarly, my treatment level should be within a certain range. Now, these are the maximum limits or the minimum acceptable levels.

(Refer Slide Time: 53:54)

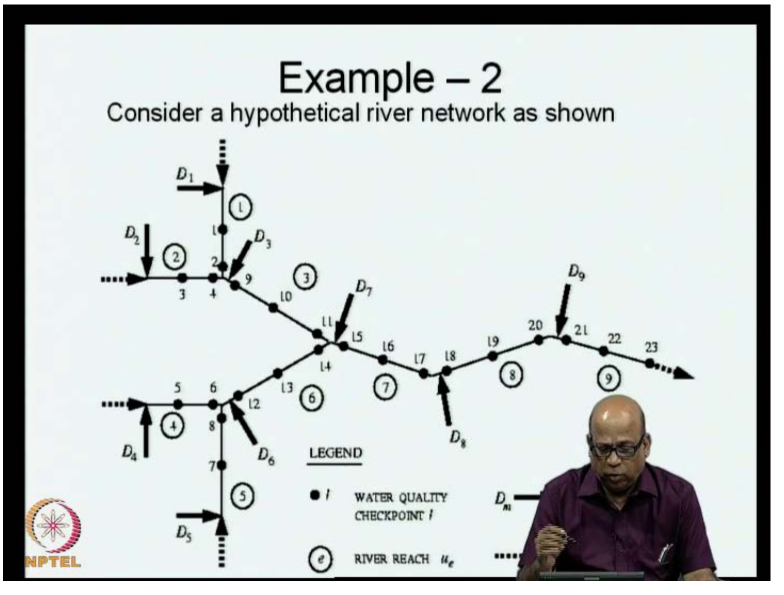


So, like this we formulate the general fuzzy optimization problem, and then solve the problem. Now, there is one small technical detail here - is that as I said the fraction removal levels at a particular location in the river will determine the concentration of the water quality parameter, for example, the dissolved oxygen is a water quality index or water quality parameter at a particular location. If you apply certain treatment level upstream of that then this water quality parameter will water quality at this particular location will enhance. And the relationship between the water quality at a particular location here and the upstream treatment levels that you are giving can be determine through some mathematical formulations. In this course will not worry too much about it, it is all available in the literature. So, if if you are doing a course on water quality modeling, you will know all these techniques; right now, what will do is? We will assume that is such relationships are available and typically they look this.

So, this water quality concentration at a particular location l can be written as a function of what has happened upstream of that with respect to the treatment and the functional nature here. So, we will look at the concentration C i l of the water quality parameter i at the check point l is related to the fraction removal level x imn and also L ipn. Now, L ipn is the concentration of the pollutant n from the uncontrolled level source which means that you have also apart from the pollute control level source which are the point sources, you may also account for non-point sources.

So, in general the water quality parameter here is an aggregate effect of the point sources as well as the non-point sources. Now, this is how you transfer the decisions that you take x imn onto the water quality indicators. Once you know the water quality indicator you know what are the associated membership function values. And therefore, you have to relate the x imn with C i l and then pick up the associated membership function values and then put it in the objective fuzzy optimization problem.

(Refer Slide Time: 56:39)



Now, that is what we do. We will look at this particular example and look at the numerical values that we obtain when we solve a problem for optimal solutions for the treatment levels, when there is a conflict between the dischargers and the pollution control agencies.

So, in the next lecture I will continuing this example and demonstrate and show how the fuzzy membership functions can be quantified for such a problem, and then we use this fuzzy membership functions into fuzzy optimization and then look at the solutions. Remember, it is not just a technique that is important here it is the interpretation of the results that are also important that is also important. And how we formulate the membership functions; to represent certain goals, certain constraints and so on are more important or extremely important here. For example, if you are looking at the purpose of irrigation, what should be your fuzzy membership function? If you are looking at reservoir operation for a general purpose meeting that demands and so on, what should be your membership function, for the water quality what should be our membership function and so on.

So, the way you formulate the responses from different users, for a general water resource problem is what is important. So, they will discuss this (( )) in the next lecture, and look at the solutions and interpretation of the solutions. So, thank you for your attention, we will continue the discussion in the next class.