Water Resources Systems

Modeling Techniques and Analysis

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Lecture No. # 34

Fuzzy Optimization (1)

Good morning and welcome to this, the lecture number 34 of the course water resources systems - modeling techniques and analysis. Now, in the last lecture, we concluded our discussion on the stochastic dynamic programming for reservoir operation. Essentially in the explicit stochastic optimization techniques that we covered, namely the chance constraint linear programming and the stochastic dynamic programming, the type of uncertainty that we are addressing is one due to randomness. So, specifically we consider the randomness of inflows to the reservoir in both the cases.

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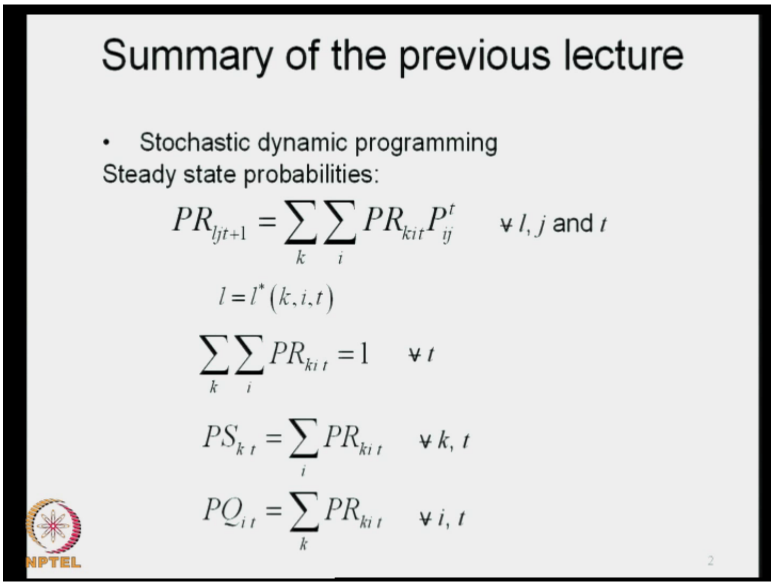

In the last lecture, we also talked about the steady state probabilities of the releases and the storage as well as the inflow. Once we obtain the steady state policy which is given by l is equal to l star for a given k, i and in a period t, we use this specification of the steady state policy to obtain the steady state probabilities of the release as well as steady state probabilities of the storage which are in fact the marginal probabilities, these if we re-call will give the join probabilities of the storage being in state k and the inflow being in state i which defines a unique l is equal to l star, and therefore, from the join probabilities you can get the marginal probabilities of the storage as well as the inflow. Now, this is what we did in the last lecture and also we solved a numerical example to show how these marginal probabilities of storage as well as inflows are obtained once you get the steady state policy.

Now, we go to a different topic today, this is the topic of fuzzy optimization. As I have just mentioning in the explicit stochastic optimization, the type of uncertainty that we considered was one due to randomness and specifically the randomness in inflows - the reservoir inflows is what we considered in the earlier two techniques that we covered. In water resources systems, as indeed in any engineering systems, there are other types of uncertainties which are which are the uncertainty due to subjectiveness, which are due to vagueness, imprecision and so on and specifically in water resources systems, because of a large number stakeholders that are involved, large number of conflicting objectives that are involved, large number of subjective judgments that are involved, we come across uncertainties due to vagueness, imprecision and fuzziness and so on. So, this is the type of uncertainty that we start introducing now. And specifically cover one technique call the fuzzy optimization where we address uncertainty due to imprecision.

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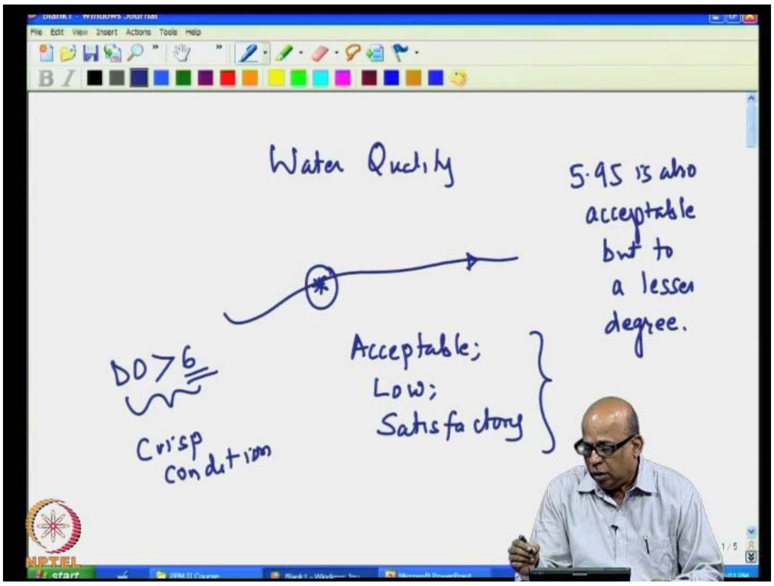

So, we will talk about fuzzy optimization today in today lecture and perhaps cover a specific application in the next lecture. Now, we need to understand a little bit about the fuzzy concept - concepts of the fuzzy sets and so on. For example, you look at water quality; let us say we are looking at water quality at a particular location. And we are looking at a stream and then saying that we are interested in water quality at this particular location. Now, we may say the water quality is acceptable or we may say water quality is low or we may say it is satisfactory etcetera. So, these are the linguistic statements of our preference to the water quality here. So, a certain level of water quality may be low, certain water quality level may be acceptable and so on. Whereas the crisp definition of water quality may be that I may say that my D O level being greater than equal to 6, I may say as acceptable or it is a good water quality or we may say this is satisfactory and so on.

So, in the crisp definition, we put crisp conditions like this. So, this is the crisp condition, if we are having a crisp condition like this, that means the dissolved oxygen at this particular location, if it is greater than 6 then the water quality is good, if we say this. A water quality level very close to 6 let us say 5.995 m g per liter, this is in milligram per liter is not acceptable. So, anything lower than this is just not acceptable and that is why we call it as a crisp condition or crisp constraint; whereas what we may say is divide prefer it to be greater than 6, however, I am not awares to a solution likely less than 6. I may say that 5.95 is also acceptable, but to a lesser degree. I am sorry, this is we will say 5.95 is also acceptable, but I prefer it to be greater than 6, but to a lesser degree. So, this is when we start introducing the concept of degree of acceptability of solutions.

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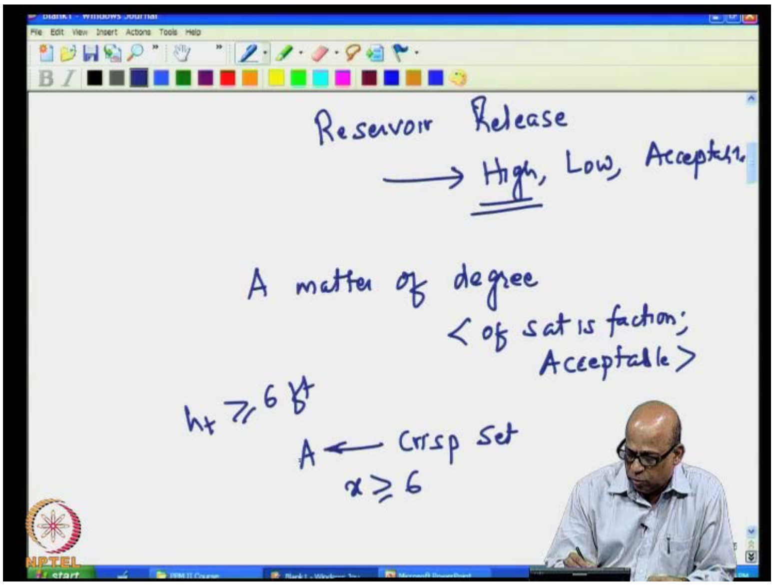

Look at another example where we are saying the reservoir demands are high or let us say the reservoir release, we are talking about with respect to demands as being high, low, acceptable and such such other quality at its statements. We may say reservoir release to be high in the crisp sense whenever it is more than the demand and low in a crisp sense when when it is, let us say less than the demand and acceptable again a setting up some reasonable limits and so on. Whereas in the fuzzy sense or in the subjective sense, we may say that any reservoir release is high to certain degree, is high is low to certain other degree, is acceptable to certain other degree and so on.

So, the solutions now start becoming a matter of degree and indeed whenever talking about (( )) design solutions etcetera, these are degree of satisfaction or of acceptability and so on; that is what is the degree to which a solution is acceptable and so on. In a more (( )) example, you can talk about the height of a person being, let us say a person being tall, in certain situations you may say a person who has a height of more than 6 feet is tall. Now, that may be a crisp condition in which we may say the height should be greater than or equal to 6 feet for A set which is a crisp set. So, if we define A to be a crisp set, we may say x greater than or equal to 6, we belong to the set A in the crisp sense.

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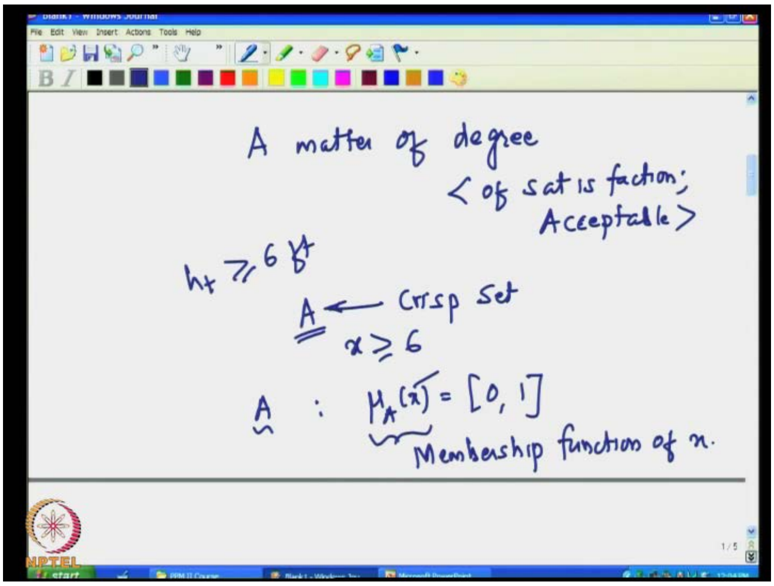

Whereas, in the fuzzy set, we may write this as the fuzzy set is denoted as A tilde here, we may say that mu A (x) is equal to 0 to 1 which means given any x it belongs to the set A with a degree of membership between 0 and 1. So, this is called as the membership function function of x, when you have the fuzzy set A. So, this is the membership function of A. So, the element is x. So, this is the membership function of A. Understand this concept correctly, let us say that you are talking about water quality and we are saying that - the water quality which is denoted by the dissolved oxygen at a particular location, and that water quality level is x and we are saying A is a fuzzy set consisting of or denoting the low water quality. Let us say A is a fuzzy set indicating low water quality.

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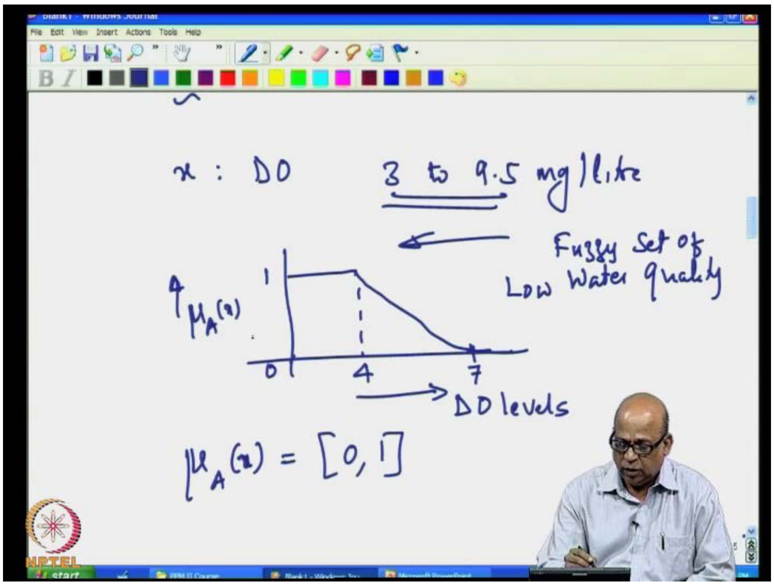

Now, for any given x, let us say x is a D O in the range from let us say 0 to… 0 will avoid, because it is anaerobic conditions will say between 3 to 9.5 m g per liter. So, these are all the possible values of D O levels. Now, for any given D O level - dissolved oxygen level the water quality at that particular location is low to a certain degree. For example, if you take 9.5 it is low to a degree of 0, 8 low to a degree of 0, 7.5 low to a degree of let us say 0.9 0.1 and so on. So, as you approach lower and lower values of the dissolved oxygen, the degree of belongingness of the particular value of x to the set of low water quality starts increasing in this direction. So, this we may write as a membership function for the set A indicating that the lower the dissolved oxygen the higher is the degree to which it becomes low. So, this is the low water quality. That is the fuzzy set of lower water quality.

And here I may write a D O levels and this is the membership function, and the membership function goes from 0 to 1, always the fuzzy membership functions mu A (x) is a closed interval between 0 to 1. And this we may put some very high water quality level, let us say a saturation level. But typically we may put some levels below the saturation. Let us say we put 7 m g per liter and then we say that anything below 4 is low to the degree of 1. So, this is the pictorial representation of the membership functions. So, mu A (x) indicates the membership of x in the fuzzy set A. And sometimes even for membership function we use this tilde to indicate that A is in fact a fuzzy set.

So, a major distinction between a crisp set and a fuzzy set is that - the crisp set has binary value 0 or 1, whereas the fuzzy sets which are completely described by the membership functions. The membership functions of the fuzzy sets are closed intervals between 0 and 1 which means that any value between 0 and 1 is it denotes the membership function in that particular fuzzy set. Now, the fuzzy membership functions which indicate the degree of belongingness of a particular element to the fuzzy set being considered, completely describe the fuzzy sets. In fact, the fuzzy sets are suppose to be functions which map on to which map the discourse on to a particular interval 0 and 1 - closed interval 0 and 1. The x the exercise that we do on crisp sets, can the operation is specifically of union, the intersection and so on are also defined for for this sets in a slightly different context. We will see that when we looked at fuzzy decisions.

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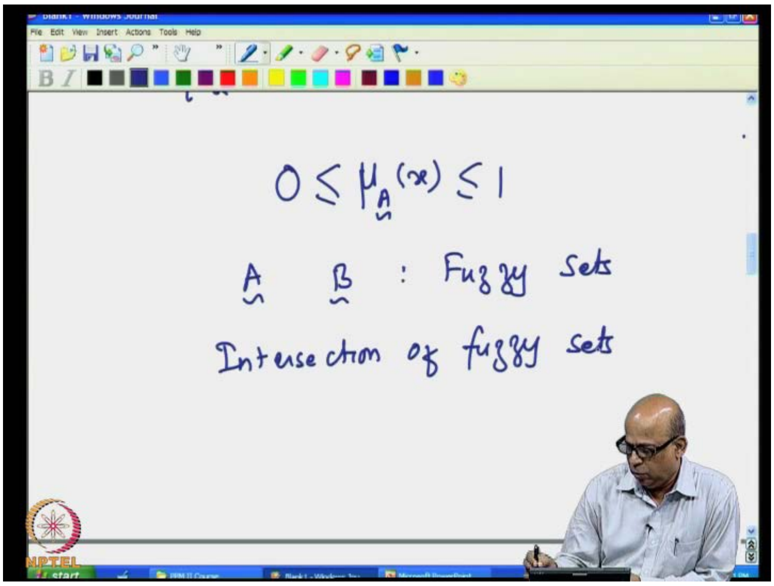

Now, in water resources, there are several occasions or several situations, where we need to look at variables as fuzzy variables or the objectives as fuzzy fuzzy objectives, constraints as fuzzy constraints and so on. And therefore, incorporating the fuzziness in the optimization problems, in the systems problems becomes important. We will also look at the confluence of two fuzzy sets. Let us say that we we have now define the fuzzy membership function as of x going between 0 and 1 and this is the fuzzy set. We may have two fuzzy sets, let us say A and B, you have two fuzzy sets. The most important operation that we do is the one of intersection of two fuzzy sets and this is what we do what we use in the fuzzy decision.

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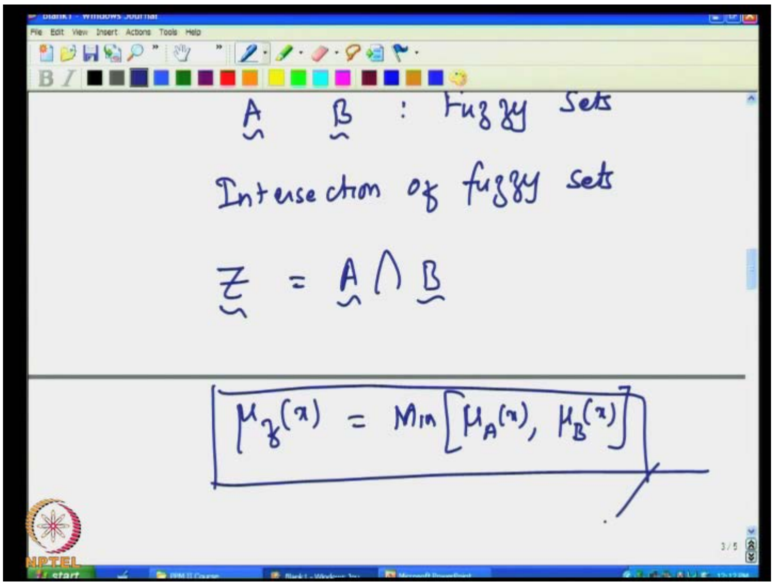

So, let us understand this, what we mean by that is Z which is a fuzzy set, A intersection B which are the two fuzzy sets will give the membership function as a set the fuzzy sets are completely defined by their membership functions. So, whenever you want to define the fuzzy set you have to associate them with the membership functions. So, we define the fuzzy membership function for the intersection Z as minimum of the membership in A and the membership in B of the particular value of x. So, this is an important result which will be using in most of the applications, especially in fuzzy optimizations, this is what we use. So, these are the some of the preliminary concepts that we use in the fuzzy optimization. Remember, fuzzy logic, fuzzy influence systems etcetera are also commonly use in water resources. But I am not touching those in this particular lecture. I am just focusing on fuzzy optimization, starting off with the crisp optimization, how we allow for fuzziness, vagueness etcetera in water resources systems is the concern of today lecture.

Now, with this background now we will look at how starting with the crisp optimization, we formulate the fuzzy optimization techniques and how we interpret the results and so on. As I just mention the notion of fuzziness arises whenever there is a subjectivity, there is a vagueness, in fact there is a conflict, because of several stakeholders preferring their own sets of solutions and so on. So, whenever we have large number of players in a particular system, each with his or her own preferences for let us say a constraint or an objective and so on and they specify that preferences in the form of acceptability or otherwise of certain solutions. These statements or these linguistic statements are converted into fuzzy sets fuzzy membership functions, and then all the fuzzy membership functions are put together, and then we are looking at in some sense the best compromise solution. In most engineering decision problems or engineering design operational problems and so on, what is that we are looking for? There are a large number of possible problems, large number of feasible problems out of which we want to pick up the best are the most optimum solution.

Now, when we are looking at the best solution, we have to address certain uncertainty associated with the variables that lead to this particular optimal solution. Randomness is one form of uncertainty where you are looking at the natural fluctuations or natural variations of the stream flow, of the rainfall, of the soil moisture, etcetera through probabilistic concepts. But there is also an important source of uncertainty that comes, because of the qualitative statements, the vague statements, the vague nature of the objectives and so on. That are not unable to be solve by the probabilistic concepts. That is where we introduce the fuzzy concepts. In fact in most recently literature, you will also see fuzzy stochastic optimization techniques. For example, fuzzy stochastic dynamic programming, starting with what we did in the stochastic dynamic programming in the last few lectures, you start looking at the state variables being fuzzy in addition to the stochastic. And then you start talking about fuzzy stochastic dynamic programming. This is as a matter of information; you can just look up the most recent literature. In fact, (( )) from Iran, they have address this problem very efficiently in the recent years. So, we will start talking about a simple introduction to fuzzy optimization now.

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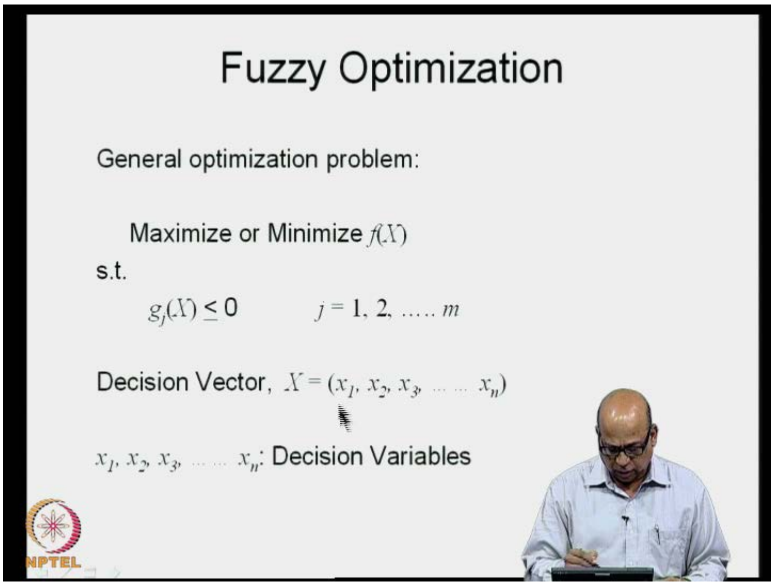

Re-call that we defined our general optimization problem as one consisting of an objective function, maximization or minimization of f(X) subject to m number of constraints g j (X) less than or equal to 0, and this is the decision vector X x 1, x 2, etcetera, x n these are the decision variables. There are m number of constraints, there are n number of decision variables.

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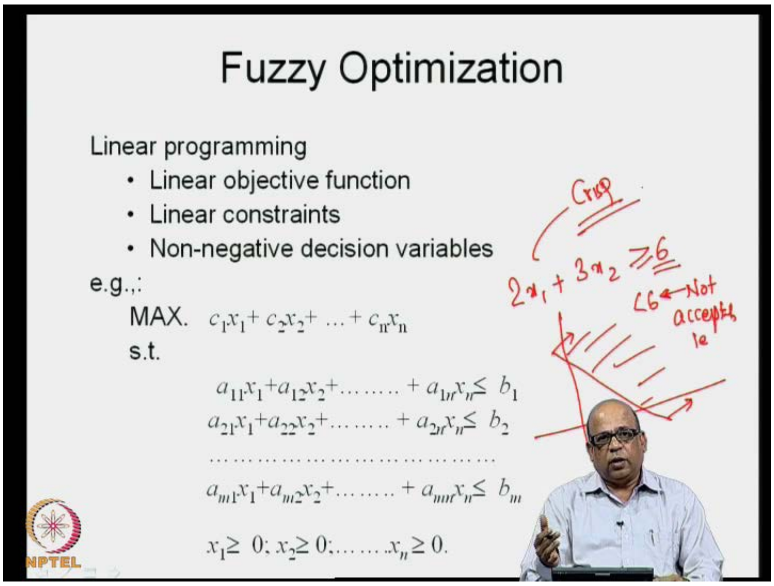

In the linear programming, what we did? We introduced three major requirements for a linear programming problem. The objective function must be linear, the constraints must be linear and the decision variables are non negative. And we write the linear programming problem in this form c 1 x 1 plus c 2 x 2 etcetera x n, it can be either maximization or minimization, and then there are set of constraints as this b 1 to b m are the right hand side of the constraints, a 11, a 12, etcetera are the coefficients associated with the variables x 1, x 2, etcetera in the first constraint, second constraint and so on. So, this is the general form in which we wrote the linear programming problem.

Now, look at any of these constraints. Let us say that - you are writing a constraint in the linear programming of the form 2x 1 plus 3x 2 is greater than or equal to 6. And then when we wrote, when we solve this problem using the graphical method, we would have plotted this particular constraint, and set your feasible space must be such that you are looking at region which will be to the right of 2x 1 plus 3x 2 - the line 2x 1 plus 3x 2. So, you may draw this line and then say that my objective my solution must always lying beyond this particular line, and that is how you define this particular constraint. It means that 2x 1 plus 3x 2 has to be 6 greater than or equal to 6 in a final solution. Anything less than 6 while make the solution infeasible and therefore, anything less than 6 is not acceptable. This is the way we handle in the crisp optimization techniques.

Now, what we start doing is that we start allowing for a latitude in the constraints which means that I would prefer the solution to be greater than 6 in this particular case. But I am not awares to solutions which are slightly lower than 6. And what is this slightly, I define through membership function. That means I will say that I my desirable value for this constraint is 6 and above, but anything less than 6 is also acceptable, but to a lesser degree. So, we start associating degrees of acceptability of solutions. In this constraint, I will say that - I while I will prefer my solution such that 2x 1 plus 3x 2 is greater than 6, but I will also accept solutions which are less than 6, but to a lesser degree. Now, this matter of degree is what we introduce in the fuzzy optimization technique.

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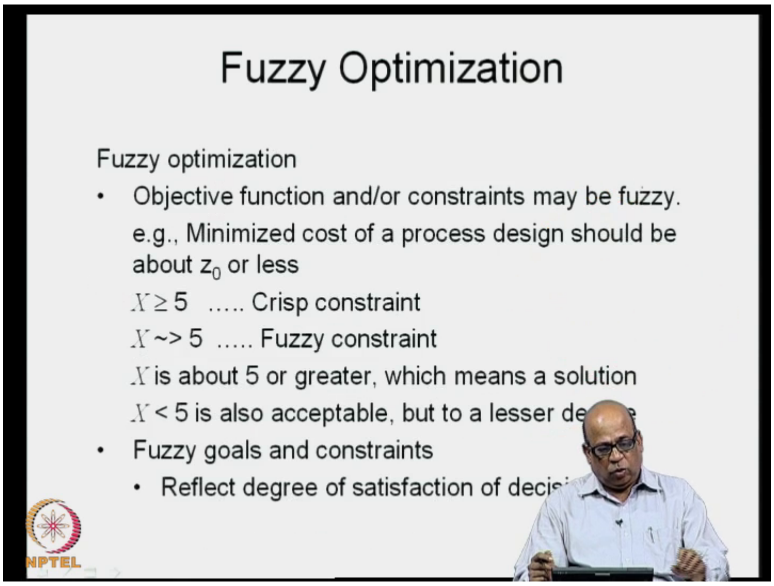

So, in the crisp constraint, we use to say that x greater than or equal to 5, this is the crisp crisp constraint. We write this as x is about greater than 5, now this notation here we will use for indicating about 5 or greater when we are looking at 5 here. So, the crisp constraint x greater than or equal to 5, we may say as x about greater than 5 which means that x less than 5 is also acceptable, but to a lesser degree. This is what we mean by a fuzzy constraint. Now similarly, minimized cost of a process design should be about z 0 or less. Instead of looking at the absolute minimum value of z 0, instead of saying that minimum minimized z 0 subject to several constraints, we will now say that I would prefer z 0 to be about some given value or less. So, we may say that my acceptability of the solution is highest when z 0 is so much, and it keeps on decreasing as z 0 keeps on increasing in a minimization problem.

So, we associate degrees of acceptability for objective functions as well as for the constraints. And this degree of acceptability we reflect through fuzzy membership functions. Now, the goals which are the objectives as well as the constraints, they all become fuzzy sets, and therefore, they are all completely define by the respective membership functions. And therefore, we do not distinguish between an objective and a constraint in in the fuzzy optimization problem, everything is fuzzy membership functions. So, through the fuzzy membership function we address the uncertainties or we address the latitude that we are providing in the decision making.

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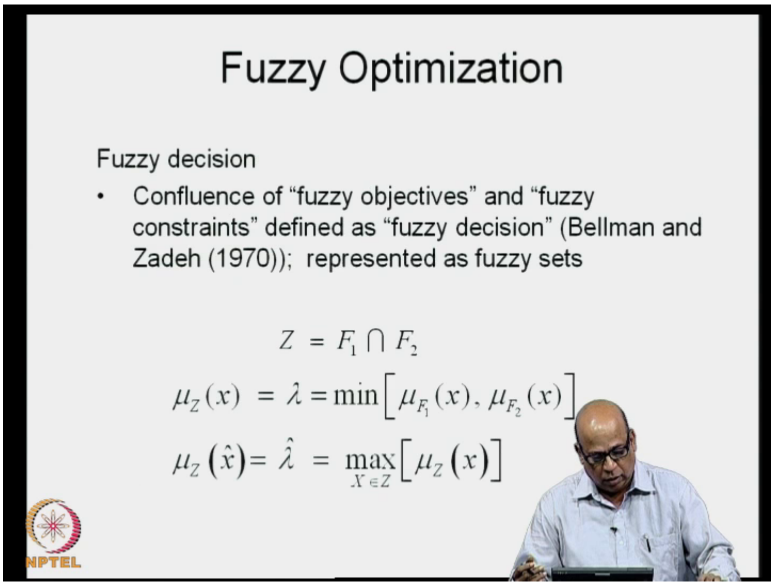

These these are some concepts that we need to be clear. This is the Bellmen and Zadeh have defined the fuzzy decision. Now, fuzzy decision is what is the important for fuzzy optimization. If you have two fuzzy sets F 1 and F 2, the intersection of the fuzzy sets F

1 and F 2 defines the fuzzy decision, and the intersection as I just mention is defined by its membership function mu z (x) is equal to minimum of mu F 1 (x), mu F 2 (x). So, we take the minimum value of the membership functions of the element x in the fuzzy set F 1 and the fuzzy set F 2 and that is what we define the membership function for z. Within this z then we are looking for that particular value of x bar x cap which will maximize the value of lambda. So, we will look for the maximum value of the membership function as defined by this. So, we are looking for that particular value of x which is denoted as x cap which will maximize the membership function in z. That is what we write here maximize mu z (x), mu z (x) is defined like this. So, we are among all possible values of x, we are looking for that particular value which maximizes the membership function x membership function z. Now, this is only for two fuzzy membership functions that I have indicated, but in reality there will be large number of fuzzy membership functions.

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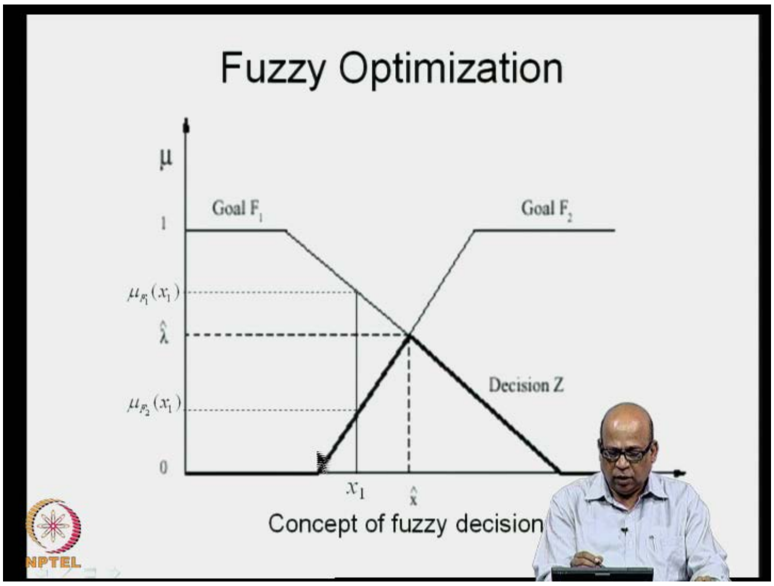

Let us look at this concept more clearly; what we are saying here is that we may have a goal F 1 that is we are looking at two goals or two constraints or one goal and one constraint and so on. As I said it does not matter which is the goal, which is the constraint, as long as we have the associated membership functions there. Let us say my goal may say that as x increases my acceptability decreases - acceptability of the solutions decreases. So, I will have a membership functions something like this in a linear form. And then my constraint we say that as x increases my acceptability increases. So, you have a conflicting statement for two goals; goal F 1 and goal F 2, and they are completely defined by the membership functions. For example, you may look at goal F 1 as the 1 dealing with irrigation. It may say that the higher the release for irrigation, the better it is for me; whereas, you may look at the hydropower for example, hydropower which depends on the head. So, if you make more release for the irrigation the less is available as head for hydropower and therefore, it is less acceptable. So, like that you may have conflicting goals.

You look at water quality situations; the higher the treatment level the better for the pollution control agencies, because you will have a water quality; the higher the water treatment level the worst for the discharges or the industrial inflow discharges, because it involves cost. So, like this as you involve more and more stakeholders, you will have more and more conflict. And the conflicts are typically indicated like this in the through the fuzzy membership functions. Now, the fuzzy goal F 1 is represented by its membership function; fuzzy goal F 2 is represented by its membership function. The confluence are the intersection of this which is defined as the minimum value between the membership function F 1 and F 2 is denoted like this. So, at every point as you progress you take the minimum value between F 1 as well as F 2. So, this darken line here indicates the membership function for the decision Z. So, the decision Z is simply indicated by the minimum value of membership function in F 1 and the membership function in F 2 for a given value of x and this is what defines the fuzzy decision.

Within this fuzzy decision then once you define the fuzzy decision, you are looking for that particular value of x cap which maximizes the membership function of decision Z. So, lambda cap is the maximum value of the fuzzy decision membership function of fuzzy decision and the associated value of x is x cap. What does x cap indicate now? x cap indicates the best compromise solution, because this was saying do not far away from this point and this goal is saying go as far away as possible from this this particular point, and therefore, x cap happens to be the best compromise solution that you can get in such a situation.

However, as I said this is the simplistic representation, first of all the goals are only two and you do not have any other constraints and also there are linear. In general, you may have non-linear response functions or non-linear membership functions; you may also have a large number of constraints and goals which are all fuzzy. And also in the fuzzy optimization, you may also have certain crisp constraints. For example, technologically feasible solutions, you cannot have them as fuzzy constraints; there will be certain limits that you have to observe; those defined the crisp constraints. The fuzzy constraints, the fuzzy goals etcetera are completely defined by their membership functions and in addition you may also have crisp constraints associated with the physical conditions that have to be met in a particular situation.

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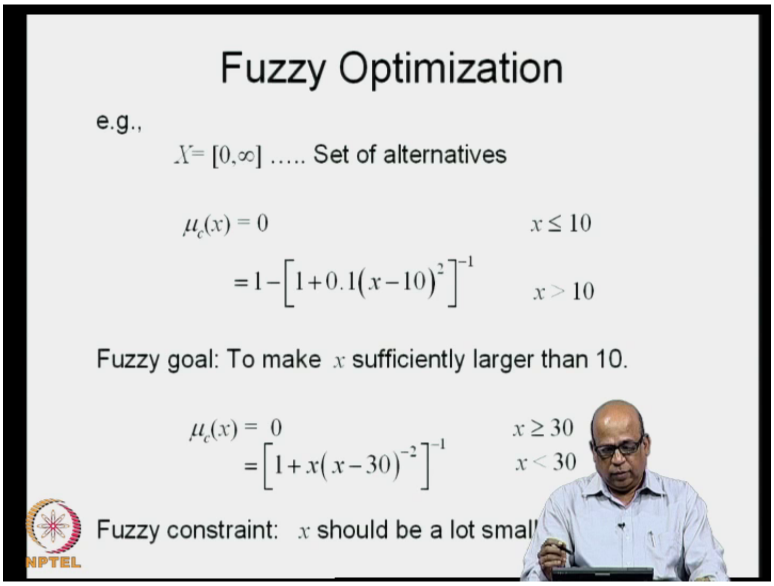

So, let us look at an example where we want to make x sufficiently larger than 10. Let us say this is our fuzzy goal, and this statement to make x sufficiently larger than 10. I will define through a membership function, saying that the membership function is 0, if x is less than or equal to 10, and a certain value which increases in a non-linear form as x greater than as x is greater than 10. I will show pictorially presently. But I am defining this as a non-linearly increasing value as x moves farther away from 10 to the right side. There may be another constraint which says that x should be a lots smaller than 30. So, one goal is to make x greater than 10 as for great a larger than 10 as possible. And the other one says - the constraint or goals says that - my x should be a lots smaller than 30.

Now, these are conflicting objectives, conflicting fuzzy sets and we define the membership functions. So, for x should be a lots smaller than 30, I will say my membership function is 0 for x greater than or equal to 30, and this particular function which is a non-linear function as I will show for x less than 30; what we are doing here is - that as x becomes lower and lower, and approach is as goes as it goes to the left of 30 the value keeps on increasing, because this on the denominator here. This is how we define. Now, remember the interpretation of the membership function in the in the context of optimization is in fact the acceptability of the solutions. So, what does this mean? This says that I want to make x sufficiently larger than 10. And here it is says, if it is less than or equal to 10 my acceptability is 0, and as it starts increasing form 10 my acceptability degree of acceptability keeps on increasing, as I will show in the diagram here.

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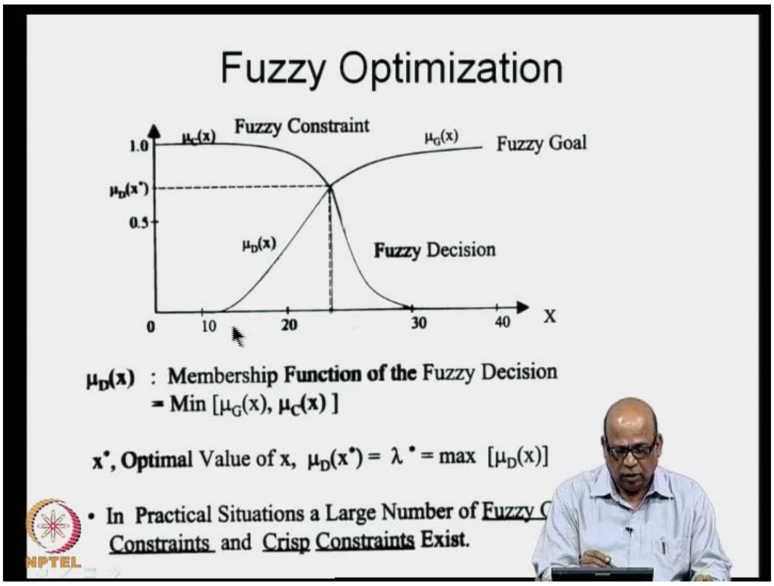

So, these statements are now plotted. So, this says that as you go away form 10, my degree of acceptability of solution keeps on increasing in this particular fashion. And the other other constraint says that it should be lots smaller than 30. So, anything greater than 30, I am not accepting it; anything smaller than 30 as it moves away from 30 to the left side, my degree of acceptability keeps on increasing in this particular fashion, the maximum value is 1 in both the cases. Now, this defines the fuzzy decision. That is we take the intersection between these two fuzzy goals and fuzzy constraints fuzzy goal and fuzzy constraint and then this defines the fuzzy decision, and that membership function we define it as mu D (x) where D is the decision. And in this fuzzy decision, we want to look at that particular value which maximizes the membership function of x. So, we are looking for that particular value of x, I will call it as x star which maximizes the membership function here. So, this is what we do in the fuzzy optimization.

We will collect all the membership functions and then look for that particular value of membership function in the decision space defined all these membership functions. How do we define the decision space? We define it by taking the intersections of all the membership functions, what I define for two membership functions is also valid for n number of membership functions. So, if we have n number of fuzzy sets F 1, F 2, F 3, etcetera, F n the fuzzy decision is defined by z is equal to F 1 intersection F 2 intersection F 3 etcetera F n. And the associated membership function is simply mu z (x) will be the minimum value of the membership functions of all this fuzzy sets. So, the optimal value in this particular case, we will look at that particular value of x star which maximizes the minimum value of the membership functions of C as well as G. Why minimum, because we are defining the decision z as the minimum of these two. So, mu D (x) star we will write it as lambda star as maximum of mu D (x) and mu D (x) is minimum, mu G (x), mu C (x) which is the membership function for x in G and membership function of x in C.

So, we are actually looking at maximization of a minimum value. So, these are called as the max min type of optimizations. That means we are looking to maximize the minimum value of the of the membership function. In practical situations, as I said there may be a large number of fuzzy sets, large number of fuzzy I am sorry large number of fuzzy goals, large number of fuzzy constraints in addition you may also have several crisp constraints.

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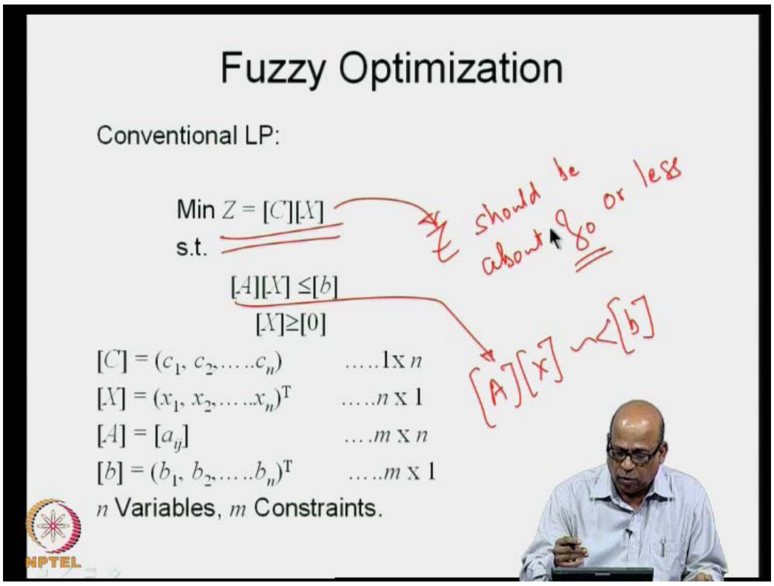

So, let us go on to formulate the fuzzy linear programming starting with the crisp linear programming. In a matrix form, we write the crisp linear programming as minimize Z is equal to C comma C X where C is the matrix of C is the vector 1 by n and X is the vector x 1, x 2, x 3,..,x n transpose. So, this is n by n n by 1. We write the coefficient matrix as A, and A has a size of m by n, and X again is n by 1 transpose, and the right hand side b is m by 1 vector. So, this is the way we wrote the crisp linear programming. I will write it as conventional linear programming. Now, from this now what we will say is, instead of saying I want minimum value of Z, I will now say that my Z should be about z 0 or less. So, instead of saying I want the absolute minimum that is resulting from this problem. I will say that I would prefer the solution to be around z 0 or less, and even if you slightly greater than z 0, I will accept the solution; however to a lesser degree. That is what we convert this statement into.

Similarly, these constraints we will say instead of saying A X should be less than b less than or equal to b, we will write this as A X should be about less than or equal to b. That is we will say that the solution which is greater than b in a particular constraint b i is also acceptable, but to a lesser degree. So, we are now converting the objective function into a fuzzy set or fuzzy objective, and the set of constraints into a fuzzy set of constraints. What did we do in the process? We have allowed for latitude in the constraints and we have allowed for some flexibility in the objective function instead of looking for 0 one type of solution, we are saying that we are unable or we accept solutions which violate to certain degree the original constraints and we also set certain limits for the objective function, and say that on either side of this we are we accept the solution to deferring degrees. So, in fuzziness the concept is one of degree. So, everything is a matter of degree is the basic premise in the fuzzy logic.

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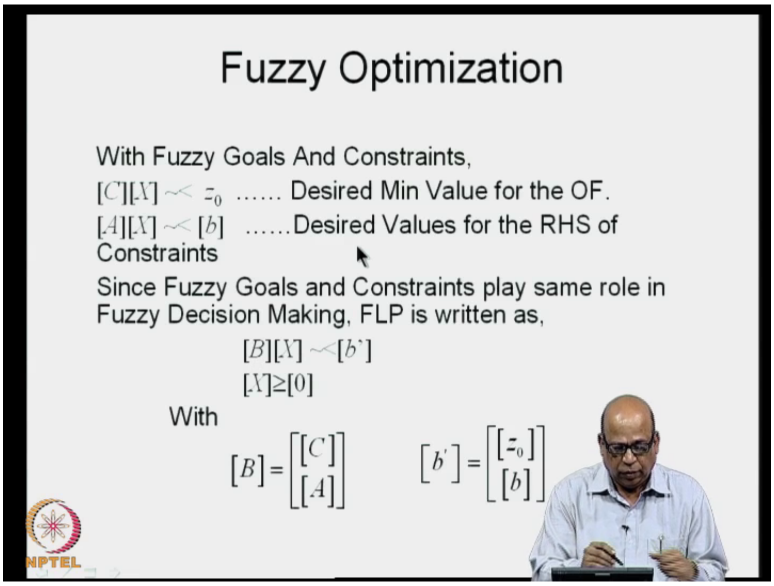

When once we start formulating the crisp problem into fuzzy optimization problem, what will we do? As a set, we will say instead of minimize C X, we will say C X should be about z 0. That means we are saying that it is a desired minimum value of the objective function. So, I will write this as C X is about less than or equal to z 0. Similarly, A X is less than or equal to b, I will write it as A X is about less than b, which is b is the desired value of the RHS of the constraints. Now, all of these are fuzzy and therefore, what we do is - look at the dimensions here, we will write C and A can be combine, because C is 1 by n and A is m by n. So, I can combine these two and I write another matrix b as combining C and A here. Similarly, z 0 is a scalar. So, 1 by n and b is m by 1. So, I will combine these two and write this as b dash.

So, essentially what I did is - the objective functions and the constraints have last there identity now, they become fuzzy sets with fuzzy membership functions. So, I will write this as this entire problem as B X is about less than b dash where b dash is another matrix another vector z 0 and b, z 0 is the scalar and b is m by 1 vector. So, I am I am able to do this. So, finally, we are written for these two sets together we are written only B X is about less than b dash. Then, so for the i th constraint here, we know there are m number of constraints here. So, including this now we may have m plus 1 number of constraints including the objective function. So, you take any particular constraint of this form b x less than b dash. Because it is a fuzzy constraint, this is the fuzzy constraint now; we need to define an associated membership function. Now, this has for the i th term, we will define the membership function - a general membership function

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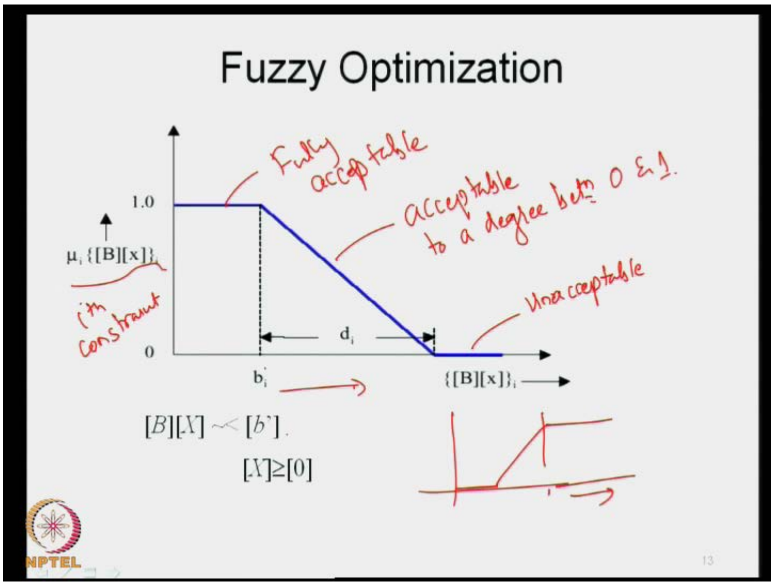

Let us say, you look at this particular membership function. So, this is for the i th constraint. So, mu i of B X i, B X is a left hand side of the constraint. So, for the i th constraint I am writing, and this is the value of the B X i and this is the membership function for the B X i which means this is the left hand side value. We would prefer it to be less than b dash and therefore, below b dash I will say that the acceptability is 1 or the membership function for that particular constraint is 1. As it is starts increasing farther away from b i b dash as it is starts going farther away from b dash for the i th constraints. So, which is b i dash I am writing. As it is starts moving away from b i, my degree of acceptability starts decreasing and beyond the certain latitude that we have provided, beyond the certain distance b i, this is totally unacceptable. So, this is unacceptable range and this is fully acceptable and in between we are saying that acceptable to lower degree between 0 and 1. So, this is what the membership function indicates.

So, for the i th membership function, we may have a distance or a latitude we provide and then say that - in this region my solution is still acceptable while I prefer the solution to be less than b i. Now… Or this is one way of membership function. Now, there may be certain other requirements where we may say that it may be the other way round. That means as you move from… It may increase and then go up to this point. That means as the higher the better is what is reflected here, the lower the better is what is reflected here. So, depending on situations you may formulate the membership functions for the i th constraint.

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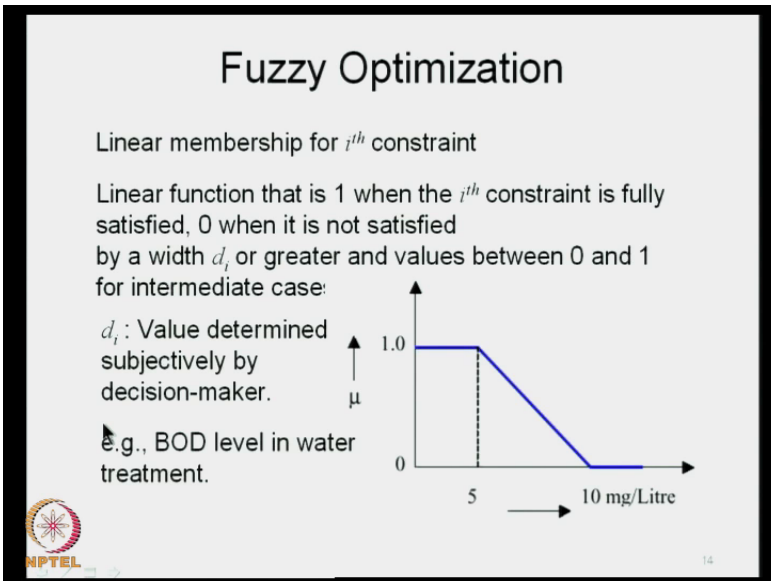

Now, starting with this now what we will do is. An example of this this particular type of membership function is the one dealing with water quality. Let us say that we are looking at the objective for water treatment and we may say that I would prefer my BOD level after treatment to be less than 5, but I am not awares to accepting solutions up to about 10 m g per liter. And therefore, I would prefer my solution in this region, but I would accept the solution to lesser degree between 5 and 10 m g per liter, but beyond 10 m g per liter I do not accept the solutions. So, this is the flexibility that we have building in into the objective function value.

Now, the d i there which is the distance or the latitude that you have providing for the i th constraint that becomes 5 here - 5 m g per liter. Now, the d i values as as I demonstrate here is actually determines subjectively by decision makers. In fact, you can do some sensitivity analysis on this and the parameters of these membership functions, for example, 10 here, 5 here etcetera are in fact design decisions. And then you can also address through sensitivity analysis or otherwise through gray systems and so on, through interval analysis etcetera; the sensitivity sensitivity of the solutions for specified values of these parameters; so, these are the membership parameters.

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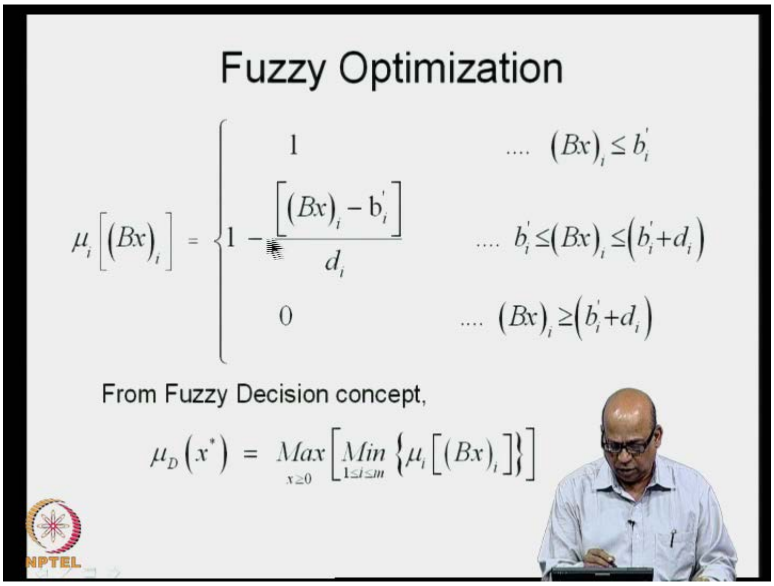

Now, you look at this, now B X about less than b dash and this as completely define our original problem. So, we will start with that now. For this type of membership function, I can write now b i dash is here and this is b i dash plus d i and therefore, I will write this membership function as mu i B X i for the i th membership function as 1 when B x i less than or equal to b dash that is here. So, as long as the B x i is less than this value here, the membership function value is 1 and defining the membership function. And in this region, it goes from 1 to 0 in the distance b i dash to b i dash plus d i. So, that is what I write here; 1 minus B x i minus b i dash divided by d i for this region for this region b i dash to b i dash plus d i. So, that is what I have written here. And as it goes beyond b i dash plus d i my membership function value is 0. So, essentially I have depicted this particular membership function through these expressions here. Or remember if we have the function which is of this type, then it will not be 1 minus, it will be simply this term here, and b i dash will be here in that particular location. So, this will… In such a situation, this term will not be there.

Now, there are m number of constraints here and we are looking at the decision Z which is the intersection of the fuzzy sets - m number of fuzzy sets each defined by its membership function like this. So, we take the minimum of the membership functions, we constitute the fuzzy membership fuzzy membership function for the decision Z. And in that we look for that particular value of x which will maximize the membership function. So, this becomes a problem now; mu D (x) star that is the fuzzy decision concept, we are saying that x star is such that the membership function of x star in the decision D is maximum for over all these constraints - all the membership functions. So, we are looking at the maximized value of the minimum membership function value.

So, we are looking for that particular value of x star which maximizes the membership function in the decision fuzzy set D. So, that is the i d whole i d of fuzzy optimization. We convert the objective function and the set of constraints as fuzzy sets, we define the membership functions associated with both the objective function as well as the set of constraints, and then combine these, because they are all membership functions, you combine all of these and then (( )) the problem as one of fuzzy optimization in which you define the fuzzy decision. The fuzzy decision itself is defined based on the intersection of the membership functions of each of the each of the fuzzy sets that we are consider. And within this fuzzy decision, you look for that particular value of x star which maximizes the membership function for the decision set. That is what we do here and that is what needs to max min type of problem - maximization of a minimum value is what we are looking at.

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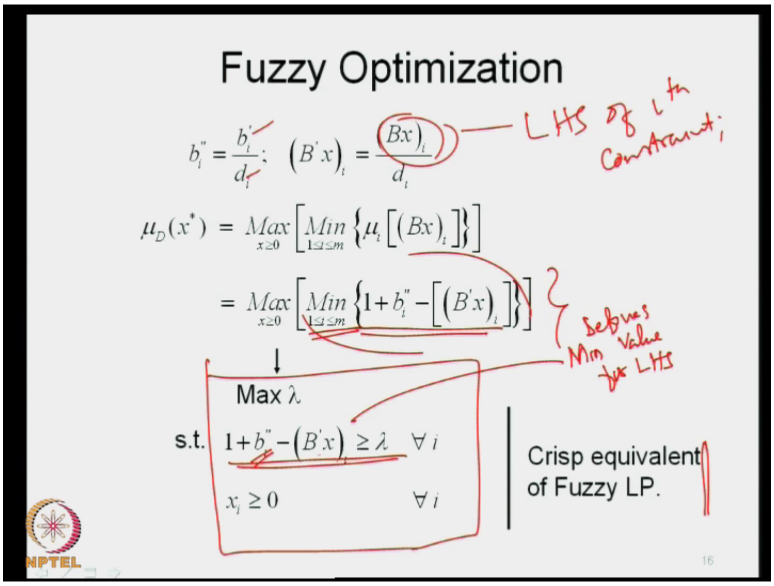

Then we simplify this a little further what we will now do is, you look at this region now, B x i minus b i dash by d i. So, I will write this as b i dash by d i as b i double dash and B dash x i as B x i by d i. Remember, this is not transpose, this is another matrix B x B dash x i for convenience I write it as B dash x i as B x i by d i. And then we simplify this. So, essentially what we are then getting is 1 plus b i double dash, looking at this looking at this b i dash by d i dash I will write it as b i double dash and B x i by d i dash I write it as B dash x i. So, we will get this as max min 1 plus b i double dash where b i double dash is defined like this minus B x B dash x i where B dash x i is defined as B x i by d i. This B x i is simply the left hand side of the constraint i and d i is associated latitude or the distance that you have provided for the i th constraint.

So, finally the problem reduces to this now. That is we are looking at minimum over all the constraints of this term 1 plus b i double dash minus B dash x i where b i double dash and b i B dash x i are defined like this. You know b i dash, you know d i, you know B x i dash, B x i this is the left hand side of i th constraint. Remember, I keep repeating by constraint we mean including the constraint due to the objective function, including the fuzzy set due to the objective function. So, we no longer distinguish between the original objective function and the constraints.

Now, this is now written as in a more elegant form as follows. We are looking for maximum value of this objective function that is minimum value of these. Therefore, I will write this as the term within the bracket here 1 plus b i double dash minus B dash x i greater than or equal to lambda, this will define the minimum value for the left hand side. Why we are saying that my lambda should be such that any value for this any acceptable value for this must be greater than or equal to that, which means that you are actually defining the minimum value for this. And this minimum value you are maximizing. So, maximize lambda subject to this constraint being greater than or equal to lambda. So, this is actually the max min formulation. This now terms out as the crisp equivalent of fuzzy LP. Remember this, these are known you can determine this B x i dash are known, because B x i is known and d i is known, therefore, you can determine this, and this is for all i, i is equal to 1 to m, and therefore, this turns out to be the crisp equivalent of the fuzzy linear programming. And we can solve this using any of our linear programming techniques to obtain the associated values of x and lambda.

So, in today lecture then, we have just introduced the concept of fuzzy optimization. We started with the uncertainties due to we started with the concept of uncertainties which are not necessarily due to randomness, which are not due to randomness. But they are also due to they are due to subjectivity, vagueness, imprecision etcetera which are very common in the decision making engineering decision making problems. Particularly, so, so in water resources systems problems where a number of stakeholders are involved and each one has his or her own preferred solutions, and then the stakeholders or the players in the game will indicate the preferences for the solution and these preferences are indicated or or quantified through the membership functions. So, fuzzy membership functions indicate the degree of acceptability of a particular solution. And like this depending on the number of constraints, number of objectives and so on you may have a large number of fuzzy membership functions.

Fuzzy membership functions completely define the particular fuzzy set or a fuzzy set is completely described by its its membership function. We collect all these fuzzy membership functions and form the fuzzy decision; the fuzzy decision is formed by the intersection of all the fuzzy membership function. Once you get the fuzzy decision that is which is also a fuzzy membership function, which is also a fuzzy set I am sorry and the membership function for the fuzzy decision is defined by the intersection of the membership functions for all the associated fuzzy sets which which define the constraints as well as the objective functions. Essentially what we are doing is from the crisp optimization, we start allowing for latitude in the decisions, instead of saying that my x should be greater than or equal to a certain value 5 for example, we say that I prefer the solution to be greater than 5, but I am not awares to solutions which are also less than 5, but I will accept these solutions which are less than 5 in that particular case with lesser degree of acceptability.

So, everything is a matter of degree is the basic premise of the fuzzy concepts. We introduce this in the fuzzy LP problem and then we formulate the crisp equivalent of the fuzzy LP. And then we can solve the LP problem in a crisp form. So, we will continue this discussion in the next lecture and I will consider a numerical example to make these points cleared. Thank you for your attention.