Water Resources Systems

Modeling Techniques and Analysis

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Lecture No. # 33

Stochastic Dynamic programming for Reservoir Operation (3)

Good morning and welcome to this lecture number 33, of the course, Water Resource Systems - Modeling Techniques and Analysis. In the last lecture, I discussed the stochastic dynamic programming formulation, and then we just started towards the end of the last lecture. A example An example of stochastic dynamic programming for reservoir operation. We considered a two state, two info state, two storage state and two time period problem. We will continue the discussion on the problem today.

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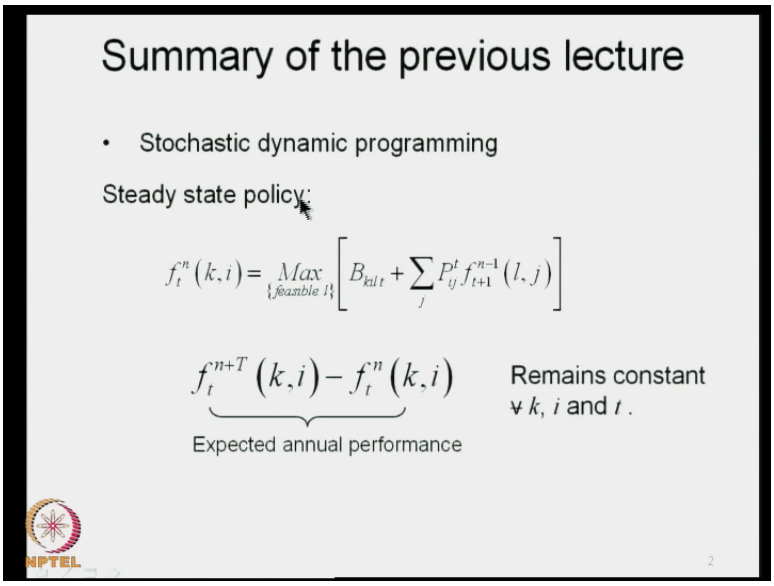

So, re-call that for the steady state policy, we will do stochastic dynamic programming, we are talking about stochastic dynamic programming for a reservoir operation problem, and specifically we are interested in obtaining the steady state operating policy. We wrote the recursive relationship as given here. For a given initial storage state k and the inflow state i in time period t which corresponds to the state n, we are searching over all possible feasible final state storage - final storage state l in time period t, which will maximize the expected value of the system performance measure. Now, this term here gives the system performance expected value of a system performance measure, because in time period t, k is known, i is known and we are making search for a specific l, B kilt is completely known. There are there is no uncertainty associated with it, because we are fixing k i and l.

However, when we are re-writing it with the next time period which is the previous stage, the i here goes to j with a probability of P ij t which is the transition probability. And we are making the search or l, so this gets fixed l, and this term here - the summation term gives the expected value of the system performance measure for all the time periods until the end of the time arisen. And that is how we define the expected value of the system performance measure. We say the steady state is reached; in fact the steady state is reached when the annual expected system performance measure converges to a certain value. So, it remains constant of a certain number of iterations, it remains constant for all k, i and t. This is the annual system performance measure. So, this is what we discussed in the last lecture. In today’s lecture, we will look see completely the problem that I started in the last towards the end of the last lecture. As I mention, the data that you will require for such a steady obtaining steady state operating policy is the discretized values of the stream flows, discretized value of the storage and the performance measure - B kilt values.

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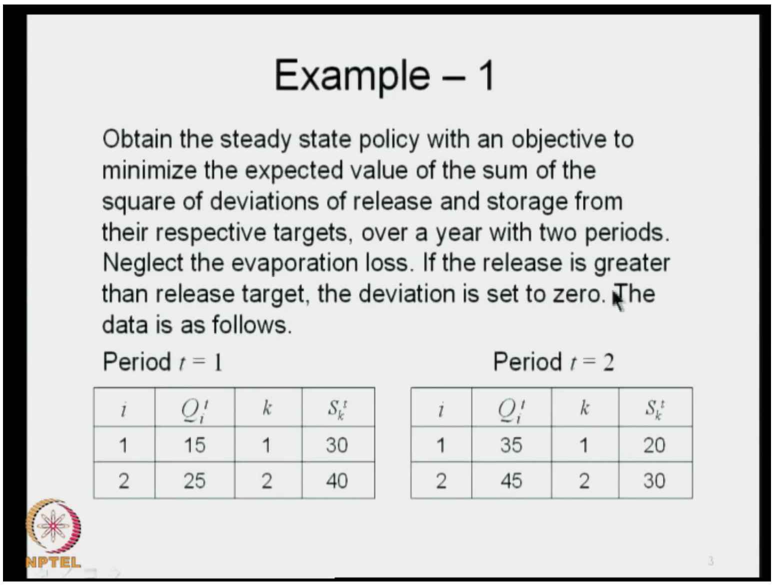

So, the first step is to obtain the performance measure. Re-call that this is the data that we considered. You have the transition probabilities as well as the storage and inflow state discretized discretized values. So, this is Q i t for i is equal to 1 2, t is equal to 1. Similarly, this is S k t for k is equal to 1 and 2, t is equal to 1, similarly for t is equal to 2. Now, this is the discretization that is available with us.

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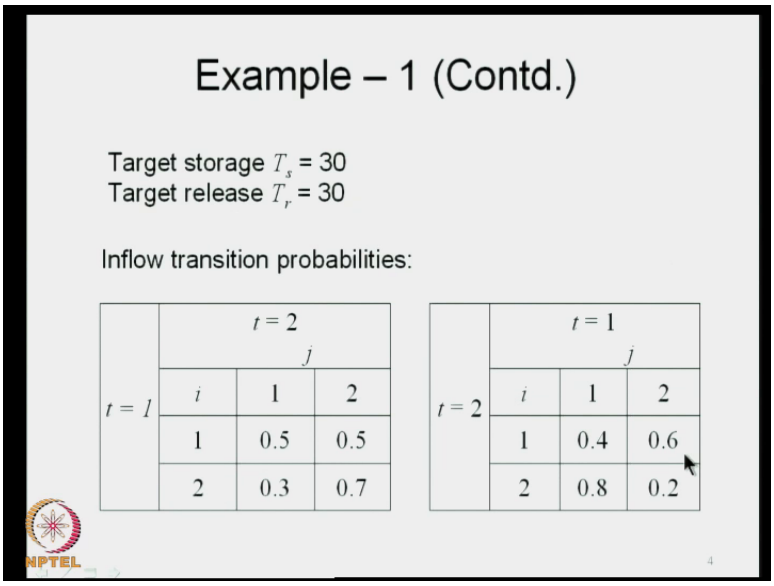

We also have the transition probabilities, starting with i is equal to 1 in period t is equal to 1. The probability that the inflow goes to j is equal to 1 in period t is equal to 2 is 0.5 and so on. So, these transition probabilities we have for both both t is equal to 1 to 2 as well as t is equal to 2 to 1. We have the target storage as 30 units and target release as 30 units.

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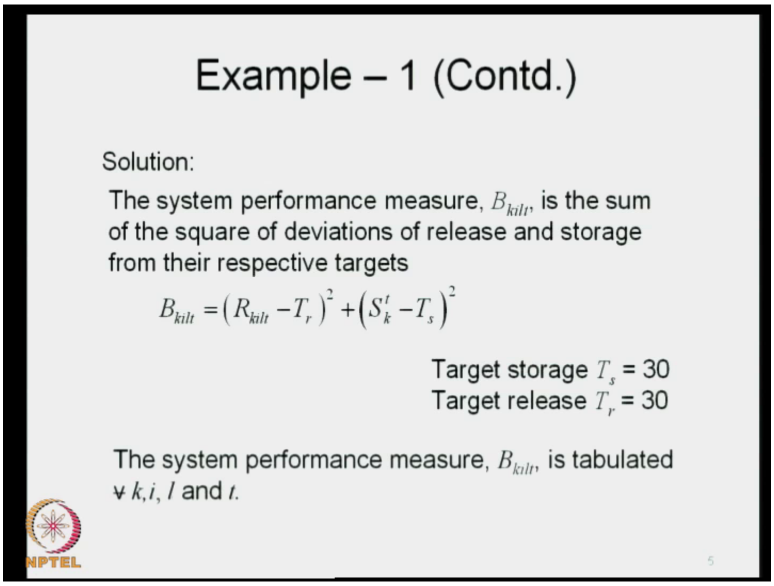

And we have defined the system performance measure as the squared sum - the sum of the square as I repeat the sum of the squares of deficit release from its target and the deficit storage from its target, and therefore, for a given kil in period t the B kilt is defined by this term.

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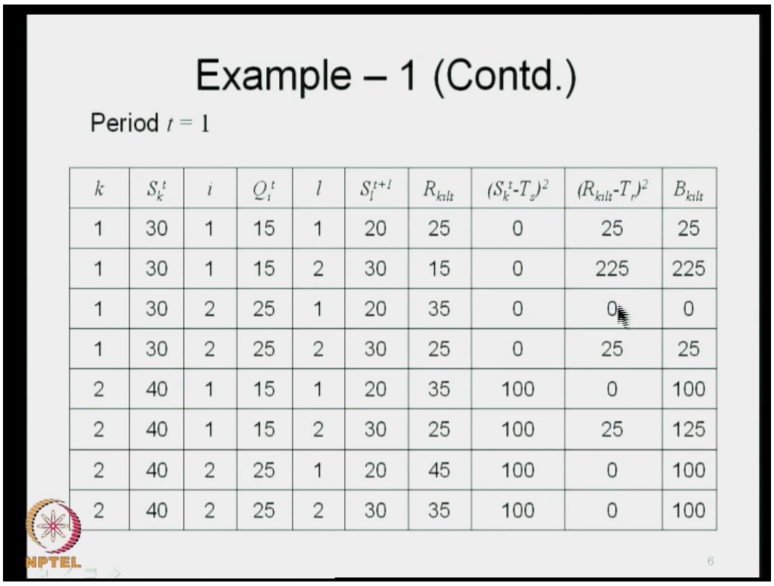

So, the first step is to obtain the B kilt. In the last class, I have explained how you obtain this. So, for a given k i and l in time period t, you obtain R kilt and knowing the S kilt, you obtain this deficit. So, remember that for this particular exercise, you have said if R kilt is more than the target then we said the deficit to 0 - associate release deficit to 0. For example, here R kilt is 35 and then we said that deficit to 0 here. So, we obtain like this, the B kilt values for all combination of k, i and l, l is equal to 1, 2, 1, 2, etcetera, it changes; when i changes from 1 and 2; similarly, k changes from 1 and 2; and for both the periods t is equal to 1 and t is equal to 2. So, we obtain the B kilt.

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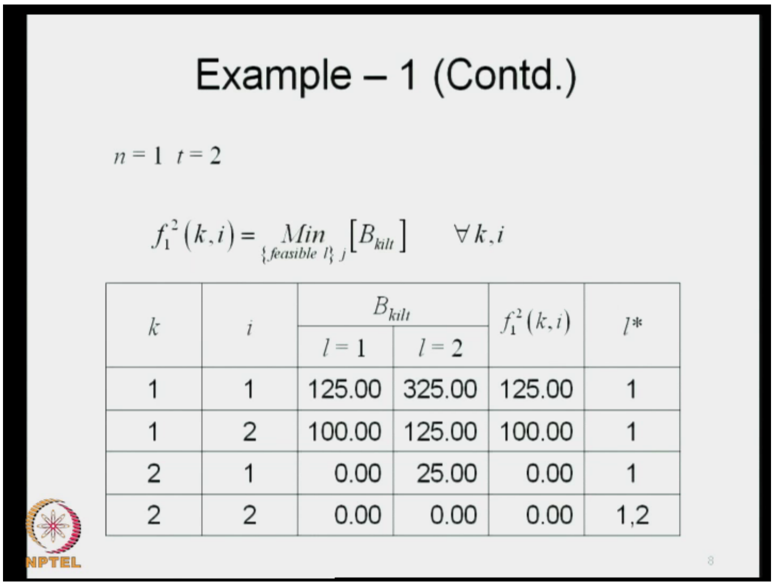

k	i	B_{kilt}		$f_1^2(k,i)$	l^*
		$l = 1$	$l = 2$		
1	1	125.00	325.00	125.00	1
1	2	100.00	125.00	100.00	1
2	1	0.00	25.00	0.00	1
2	2	0.00	0.00	0.00	1,2

Now, we will start applying the recursive relationship, what we are doing now is that we are starting with the last time period in a year. So, t is equal to 2 is what we are starting with t is equal to 1, then t is equal to 2, t is equal to 1, like this we are progressive in the backward direction starting with some year in future, and n keeps track of the progress of computation. So, n is equal to 4, n is equal to 5, etcetera. It is keeps on increasing, whereas the time period t will be only 1 and 2 within the year in this particular case. So, we start with this time period, t is equal to 2 and n is equal to 1, and define f 1 2 for a given value of (k, i) has minimum value of B kilt, because we do not have anything to look beyond this; this is the last time period and which corresponds to this stage equal to 1. We take different combinations of k and i, and obtain the B kilt values for all feasible l values, and pick that particular l value which results in the minimum value of B kilt for a given combination of k and i. That is what we are doing now. So, this is time period t is equal to 2, we start with k is equal to 1, i is equal to 1. And make a search your l is equal to 1 and l is equal to 2. So, the question that we are asking is in time period t is equal to 2 if my storage state is k and the inflow state is i whether my end of the period storage state must be equal to 1 or it should be equal to 2 such that the B kilt value that will be minimum. So, that is the question that we are asking. So, for 1 1- we pick up the B kilt value from this table. So, we are referring to t is equal to 2, 1, 1, l is equal to 1. So, B kilt is 125. So, I put 125 here; then 1, 1, 2 in time period 2; in time period 2, 1, 1, 2 that is 325. So, I put 325. And pick up the minimum resulting from these two values which is 125, and I write l star as 1 which is the l value corresponding to the minimum value of B kilt. That is the l star t. Similarly, 1 2 in time period t is equal to 2, I look for l is equal to 1, l is equal to 2. So, this is 1 2, l is equal to 100, 1 2 - l is equal to 2 it is 125. So, 100 - 125 then get the minimum as 100 and that 100 results from l is equal to 1 and therefore, I put l star is equal to 1. So, the l star value that I am writing here is in fact, the final storage value 12 which corresponds to the minimum value of f 1 2 k i which corresponds to the value of f 1 2 k i. Similarly in 2 1 in time period 2, t is equal to 2, 2 1, 1 is 0, 2 1, 2 is 25. So, 0 and 25, minimum is 0; I put it as 1. 2 2, 1 is 0, 2 2, 2 is 0. So, 0 and 0; so, both of these are optimal values and I write l star as 1, 2 which means both of them result in the same minimum value. So, this is steady state forward when we are looking at the last time period. We will go to the next stage which is t is equal to 1 and n is equal to 2. So, n is equal to 2 and t is equal to 1, we have to relate with what has being done in the previous stage.

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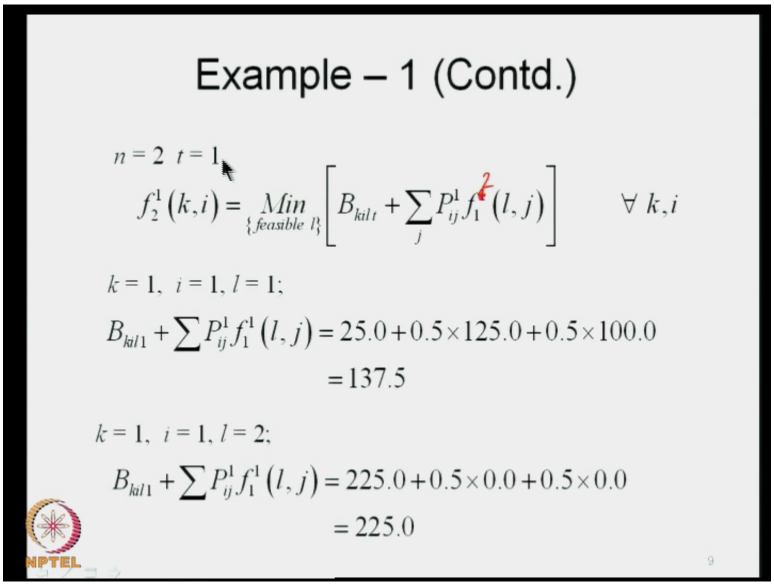

So, we write the recursive relationship starting with the general recursive relationship as f 1 2 that is stage 2 time period 1 for a given (k, i) as minimum over all feasible value of l, B kilt plus sigma over j, j varies from 1 to 2 in this case, P ij 1 this 1 here refers to the transition from time period t is equal to 1 to time period t is equal to 2. We are in time period t is equal to 1. So, what is the probability that it goes into a particular state j in time period t is equal to 2 is given by P ij 1; f 1 1 which is what we have solved earlier. This should be f 1 2 in fact, (l, j). So, we were referring to time period t is equal to 2; so, this is f 1 2 (l, j) for a given value combination of (k, i). So, let us say we look at k is equal to 1, i is equal to 1 first. This have the demonstration of the calculation k is equal to 1, i is equal to 1 and t is equal to 1, l is equal to 1. So, we go to B kilt values of t is equal to 1 first, associated with 1 1 1; we pick up the value 25. So, we are just writing this equation now, B kilt. So, this is 25; that is 1 1 1 in time period t is equal to 1, B kilt is 25. Then we sum over j, the p ij t values multiplied by the associated values of the expected system performance measure that we computed in the previous stage which is stage number 1 corresponding to period number 2. So, first we will take i is equal to 1 and j is equal to 1 and expanding on the summation. So, i is equal to 1, j is equal to 1 in time period t is equal to 1.

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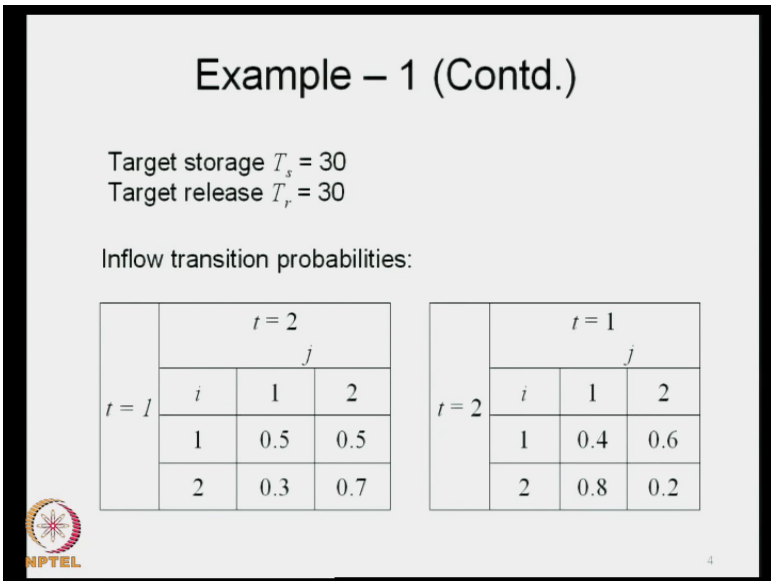

So, looked at the transition probabilities here, t is equal to 1 to 2, i is equal to 1, j is equal to 1, the probability is 0.5 which means starting with a class interval of i is equal to 1, the probability that the inflow goes in to class interval 1 in time period t is equal to 2 is 0.5. So, I will take that as 0.5; this is 0.5 and if it goes in to class interval 1, because l is equal to 1. I will be looking at the previous stage competitions for 1 comma 1 that is l is 1 and j is 1, and I will go to the previous stage competitions and pick up the value f 1 2 of (1, 1); so, f 1 2 of sorry f 1 2 of (1, 1) which is 125. I repeat that my arguments here would be 1 comma 1. So, I will pick up f 1 2 of 1 comma 1 which is 125; that is the 125 that I write here.

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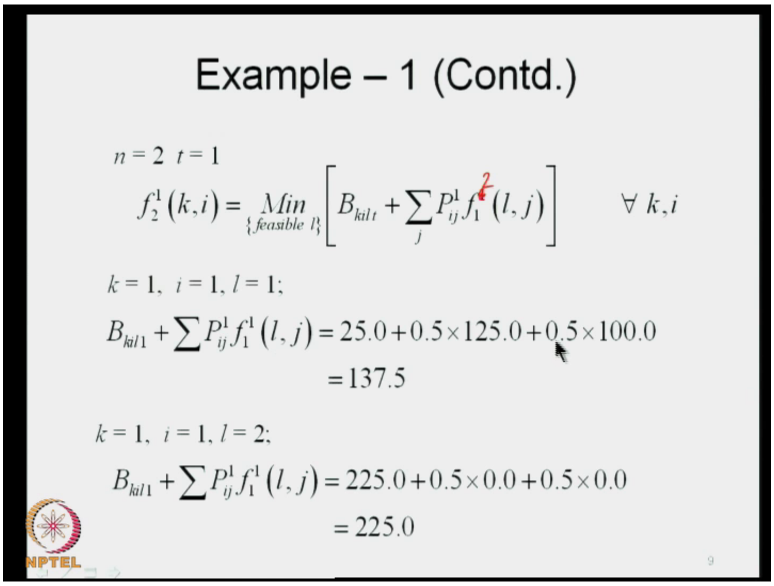

Next, the last term is for j is equal to 2. Now, i is equal to 1, but I am going to j is equal to 2 the probability is again 0.5. Look at this 1 to 2, I go with the probability of 0.5. So, 0.5 and if i go to j is equal to 2 what would be this argument now, l is 1 and j is 2. So, 1 comma 2; so, previous stage combination 1 comma 2; that is 100; so, I will put it as 100. So, that is how I get 137.5. So, for k is equal to 1, i is equal to 1, l is equal to 1 in t is equal to 1, I will get the system performance measure which is f 2 1 of (1, 1) as 137.5. Similarly, k is equal to 1, i is equal to 1, l is equal to 2 that is I am changing the l now. So, this we will get; remember I repeat the same exercise except that I will be putting l is equal to 2 here, and first I will take j is equal to 1 and j is equal to 2. So, these are the values that I will be using - 2 1 and 2 2. So, I get a value of 225. Like this I will get for various combinations of k, i and l, the terms indicated here - that is B kilt plus P i P ij f 1 2 (l, j) summed over r j. So, this is the term that I am representing here. Then out of these all possible values of l, for example, l is equal to 1 here, l is equal to 2 here, out of this I will pick up that which results in the minimum value; for this particular combination k and i. And this is what we do in tabular form.

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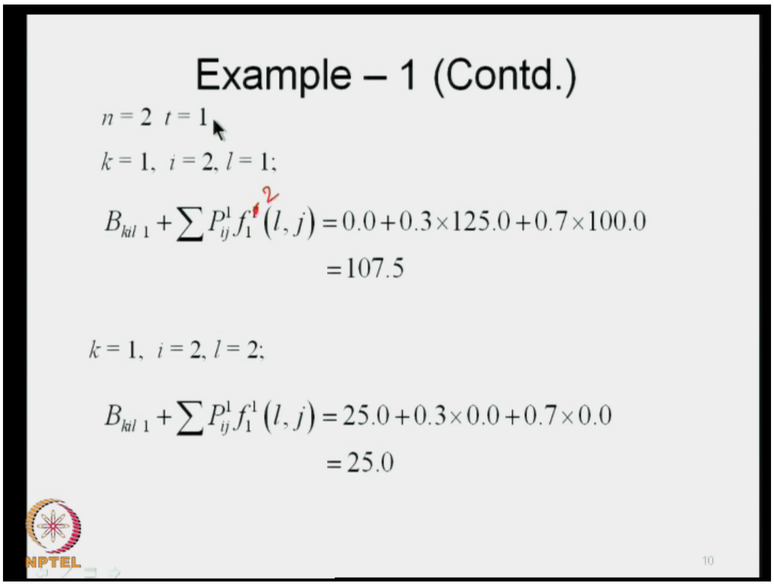

Similarly, I have given for n is equal to 2, t is equal to 1, i is equal to 1, k is equal to 1 and i is equal to 2, for completeness let us discuss this also. So, I am now changing k is equal to 1, i is equal to 2, what did I do earlier? I put k is equal to 1, i is equal to 1, I exhausted all possible values of l. Similarly, I now exhaust all possible value of l which is l is equal to 1 and l is equal to 2; for this particular combination k is equal to 1 and i is equal to 2. So, I write this as B kil 1, this is the 1, sigma P i j f 1 2 this is; this must be 2, (l, j) which is you look at now combination of 1 comma 2 in time period t is equal to 1. So, I will go back to our B kilt values in time period t is equal to 1; I am looking at 1 comma 2 for l is equal to 1, I get a value of 0. 1 comma 2 for l is equal to 2, I get a value of 25. So, this is what I substitute now; 0 for 1 comma that is 1 2 1 in time period t is equal to 1. So, l is equal to 1, I am putting a value of 0 here. Then I am still at l is equal to 1. So, I will pick up the value of P i j 1 now. What is my j? I will set j is equal to 1 first, i is 2, So, P 2 1 of 1; look at the transition probabilities. P 2 1 of 1 which is 0.3. Similarly, P 2, 2 of 1 is 0.7, we will use these transition probabilities; this is 0.3 and 0.7, and the associated values of f 1 2 (1, 1) here, f 1 2 (1, 2) here. That we get from previous combination; f 1 2 1 is 100, f 1 2 2 is 125. So, that is what we put here. Let me extend that again. So, we are going to the previous time period with k is equal to 1, i is equal to 2 and l is equal to 1. So, I will pick up this corresponds to l is equal to 1 here and j is 1 or 2. So, this is 1 comma 1 and this is 1 comma 2 in time period 2 which is the previous stage. So, I will go to the previous stage; 1 comma 1 and 1 comma 2. So, these are the values that I will pick up; 125 and 100. So, this is 1 I am sorry this is 125 and this is 100, and these are the probabilities. So, like this you get 107.5. Similarly, then I change l is equal to 2 from l is equal to 1; re-do the exercise; pick up the associated values; remember, because my i does not change, these probabilities will remain the same, because t is the same. These probabilities will remain the same as the previous calculations, only these two values will change now and B kilt values will change, because l is different now. So, like this, I calculate for all combinations, I calculated this term now within the brackets, and then we go to the tabular form and then solve the example.

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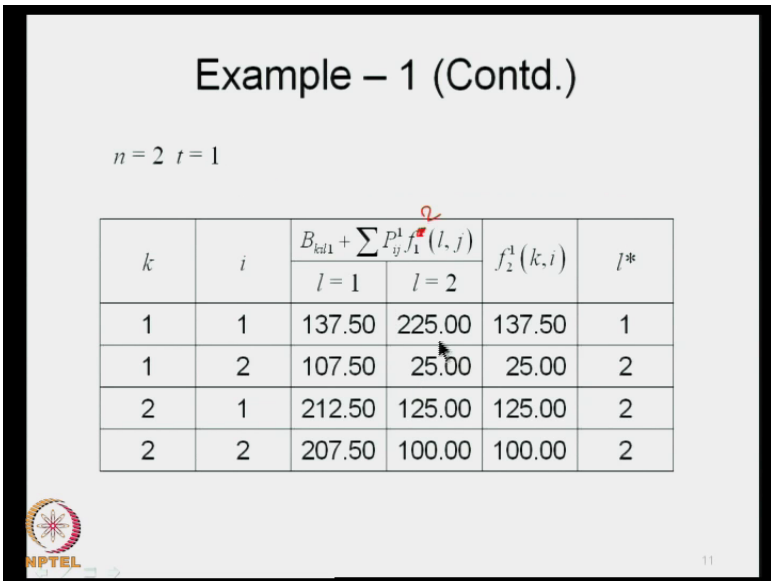

So, this is k is equal to 1 and i is equal to 1 in time period t is equal to 1, I will get these value B kilt plus this bracketed term as 137.5, 225; how do you I get? These are the values 137. 5 and 225. I pick up the minimum of that 137.5 and put this as equal to 1. That is the l star which corresponds to the l value of the minimized expected value of this which is 137.5, therefore, l star is equal to 1. Similarly, 1 comma 2, I get 107.5, 25; these are the values. And 25 is the minimum, therefore, l star is 2. Similarly, you can continue the example, 2 comma 1, I get 212.5, 125.0, 125 2, 2 comma 2, 207.5, 100, 100 and 2. You can just continue this and verify that these values, in fact result. So, essentially what we did here is for t is equal to l which is n is equal to 2 now - the second stage; we have answer the question, if we are in state k - storage state k and inflow state i, whether I should go to l is equal to 1 or l is equal to 2, such that the expected value of the system performance measure until the end of the time horizon is minimum. So, this gives the expected value of the system performance measure until the end of the time horizon, because we are considering both the time periods now - both the stages n is equal to 1 as well as n is equal to 2. So, this was my n is equal to 1, this is now n is equal to 2, this was t is equal to 2 and this is t is equal to 1. So, related what is happening in this particular period with what has decided through optimization, we are applying actually the Bellman's principle of optimality until the end of the time horizon, what is the expected value of the system performance measure. This we continue we go to the next time period which is again t is equal to 2 next which corresponds to the next stage n is equal to 3. So, n is equal to 3, t is equal to 2, this is the recursive relationship. We are relating with the second stage which corresponded to period number 1 and that is what we have solved earlier. Second stage which corresponds to period number 1, for all given combinations of k and I, we have solved and obtain this. That is f 2 1 (k, i) is defined, we use these values and then optimize the next stage values.

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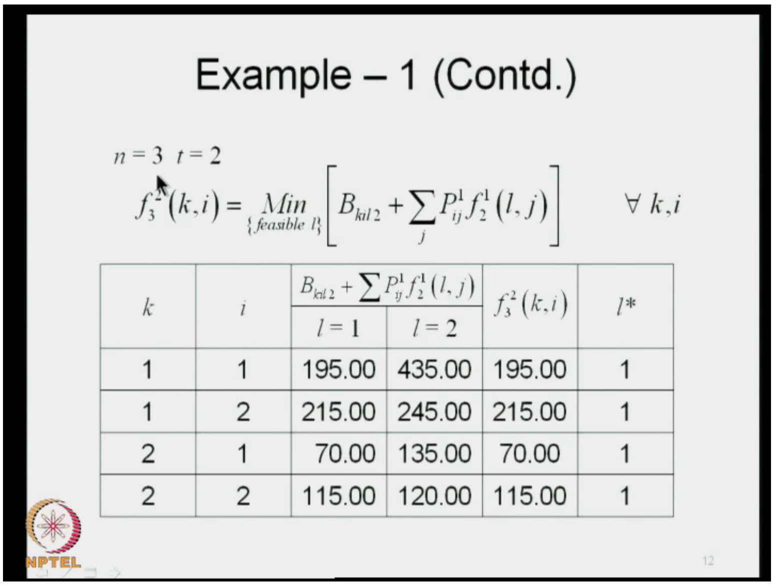

k	i	$B_{kl2} + \sum_j P_{ij}^1 f_2^1(l,j)$		$f_3^2(k,i)$	l^*
		$l = 1$	$l = 2$		
1	1	195.00	435.00	195.00	1
1	2	215.00	245.00	215.00	1
2	1	70.00	135.00	70.00	1
2	2	115.00	120.00	115.00	1

So, l is equal to 1, l is equal to 2, remember we are again going back to t is equal to 2 as we are progressing in time and we solve this again for all combinations of (k, i), we will search your possible values of l and l is equal to 1 and l is equal to 2, obtain the l star values and the associated expected value of system performance measure - minimized expected value of system performance measure. Like this, we keep on continue continuing. So, n is equal to 3, I go to n is equal to 4, I obtain these values; I go to n is equal to 5, I obtain this values; then I go to n is equal to 6 I obtain this values and so on, like this we continue. When we reach around forth cycle or n is equal to 8 in this particular case, we start checking for the attainment of steady state policy. Typically we all have the computational to go on for three cycles, four cycles and etcetera, and then start looking for start making the examination for whether the storage steady state policy has been in fact reached or not yet. So, after the forth cycle, what do you mean by cycle? It is a number of years. So, each cycle in this particular example with consist of 2 years, and like this you keep on continuing. So, n is equal to 8, n is equal to 10 and so on. At that point, you start looking for whether the steady state start examining whether the steady state has been reached or not. So, I will show how we examine this.

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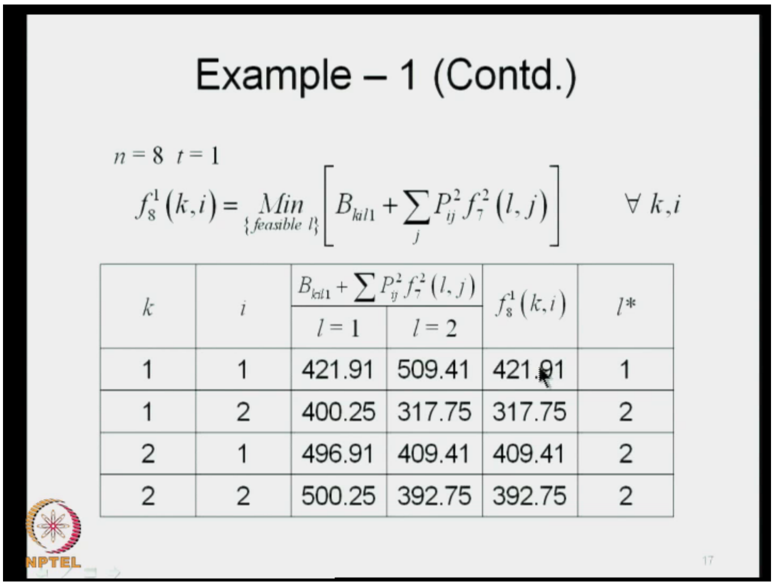

So, I will go to n is equal to 8, we complete the combinations for l is equal to 8 I am sorry n is equal to 8; that is the eighth stage which completes the forth cycle and we obtain values like this. Now, what does this value indicate now? This is the total accumulated expected value of the system performance measure in stage number 8 up to stage number 1. So, this is the total accumulated expected value of system performance measure - minimized expected value of system performance measure. Now, if we looked at the difference between the value that we show obtain here, and the value that was obtained two stages before, that gives you the annual expected value of the system performance measure for this particular time period t is equal to 1, for this particular combination (k, i). Now, that is what we will do now. For a given combination of (k, i), we will look at the difference between the system minimized expected value of the system performance measure as obtained in stage number 8 and the minimum expected value of the system performance measure as obtained in stage 6. So, this is the value for a given combination of (k, i), and that we will examine for all possible values of (k, i), if such difference remains the same or more or less it is the same, not only for this time period t is equal to 1, but also for t is equal to 2. Then we say the steady state is reached. So, essentially what we are doing is? We are looking at the expected annual system performance measure to remain constant across all combinations of initial storage k and the inflow i for both the time periods in this particular case for both the time periods t is equal to 1, t is equal to 2. So, you get the same expected annual system performance measure. That is when we say, the steady state policy has been reach. So, this is the examination that we will do.

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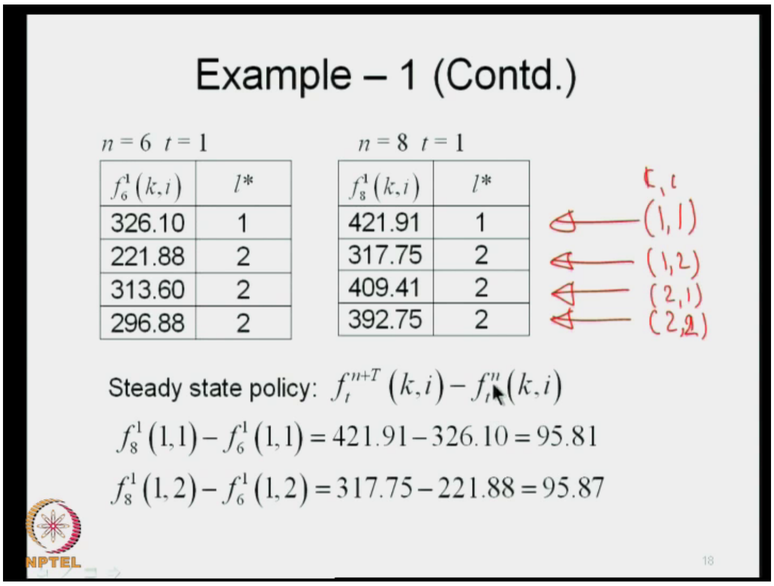

So, I am repeating here n is equal to 8 table and n is equal to 6 table, both of which correspond to the same time period. So, I am just picking up the annual system performance measures here, 421.91, etcetera. So, this column I am picking and then I go to n is equal to 6, I pick up this value, this column and write here. So, this is n is equal to 6, n is equal to 8. Now, this is k is equal to 1, i is equal to 1, 1 2 2 1 2 2, similarly, 1 1 1 2 2 1 2 2. So, we will pick up for 1 1 combination. So, this is for k is equal to (1, 1) and this is for (1, 2), this is for (2, 1) and this is for (2, 2), k and i. So, what I am writing here is the condition for thus steady state policy, n plus capital T, capital T in this particular case is 2 that is the total number of periods in a year; for a given number of (k, i) which corresponds to the time period small t minus f n comma t, f n t for the same combination (k, i). And this gives you the minimum expected value of the system performance measure.

So, for example, if I put n is equal to 6 now; 6 plus 2 is 8. So, I look at n is equal to 8 and n is equal to 6. So, f t f 81(1, 1), I am putting k is equal to 1, 1 is equal to 1 minus f 6 (1, 1) which is 421.91 minus 326.10 which is 95.81. And similarly, I go to (1, 2), 317.75 minus 221.88 this comes to 85.87. So, this is approximately the same. In fact, if you just ignore the last digit, it is the exactly the same. This we will do for other time periods also that is I will go to time period t is equal to 2 now; t is equal to 2, I pick up n is equal to 7 and n is equal to 5; and look at (1, 1) as well as (1, 2); similarly, (2, 2) (2, 1) and (2, 2) also you can verify, 409.41 minus 313.60; similarly, 392.75 296.88 and so on, you get nearly the same values. Similarly, for t is equal to 2 which corresponds to n is equal to 7 and n is equal to 5. So, this is in the fourth cycle, this is in the third cycle. So, I get values of 95.67, 95.96. These are nearly the same as what we got here 95.81, 95.87. In fact, if you want to achieve a much better much better comparison then what you do is? You carry the computations further; I have examine it at the end of the fourth cycle, you can still carry out further, then you will achieve a better performance, but in general this kind of result is acceptable 95.67 95.96 which is 95.81 here and 95.87 at here, we can reasonably be sure that the steady state has been reached and these will no longer change. Remember here, what happens is? Once a steady state is reached and if you still further carry out the computations, let say n is equal to till 8, then you go to n is equal to 10, n is equal to 12, n is equal to 14 and so on, these values here the l star values will no longer change. Although these may be changing, because we are accumulating the values, these would be changing, but the difference does not change, these will also change, but the difference will remain more or less the same. But these l star values which are in fact the steady which indicate in fact the steady state policy, they will remain the same once you reach the steady state.

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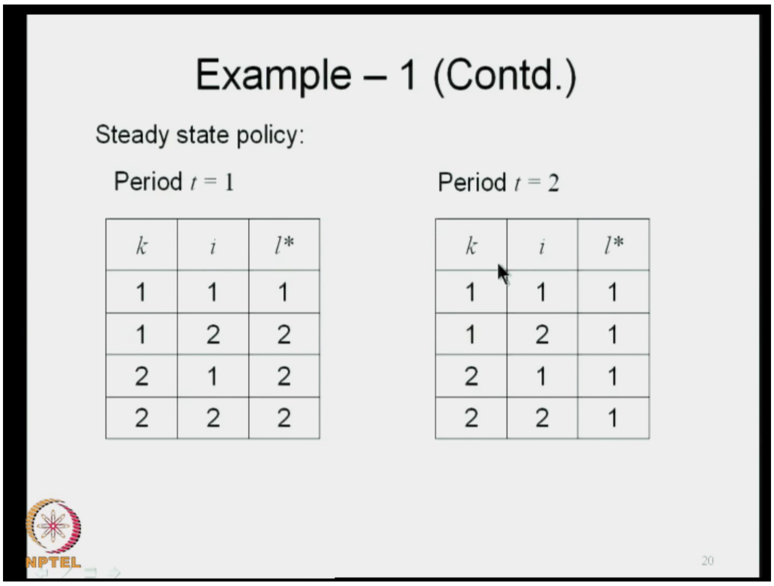

And the steady state policy done is specified by for period t is equal to 1, we specify the steady state policy for a given (k, i), l star should be 1 that is (1, 1) in period t is equal to 1 steady state policy is 1, (1, 2) steady state policy is 2. So, what does this indicate? This says that the steady state policy for the reservoir operation in time period t is equal to 1 is given by this table, if you are in storage state k which is the beginning of the time period storage in time period t is equal to 1 if you are in state 1, and the inflow is in state 1 then your l star must be 1; that is the end of the time period storage should be in state 1 and so on. And this is the steady state policy, in the sense that it should be applied over a long period of time with the same policy; you do not change the policy in between. So, when you apply it to over a long period of time, year after year for a given (k, i), you always go to this table in that particular time period, pick up the l star value and maintain your end of the period storage then over a long period of time when you apply, you will get the best system performance measure. That is the idea for steady state policy.

The application of the steady state policy does not mean that every time period, every year, you will get the best system performance measure. Remember this, because we are looking at the expected value of the system performance measure and this expected value of system performance measure is generally is only achieved over a long period of time. So, the steady state policy is a long term operating policy for the reservoir. Similarly, for period t is equal to 2 we define l star for a given (k, i). Now, what does this mean? This means that for a given reservoir system, you have now derived a steady state operating policy which you will keep on applying period after period, year after year, simply look at the storage state, look at the inflow state, how to obtain the inflow state to the (( )) I will tell you slightly later, and then go to the table, pick up the l star value, and achieve the L star value. Go to the next time period, again look at the k value which is the initial storage and then look at the expected inflow during the time period, therefore, we will look at the inflow state i and go to the table pick up the l star value, apply that. Like this, you keep on doing period after period, you apply the same policy. This means that when the steady state is reached, there are you can then talk about the probability of the storage being in a particular state in time period t, and the inflow being in a particular state in time period t, and the release being in a particular state in a time period t. By because for a given (k, i) there is the fixed l star that you are applying every time period t, now fixed l star that you are applying for a given time period t, and this you are repeating over and over again; which means that you achieve the steady state of the system, and therefore, there are steady state probabilities associated with the storage, the inflow as well as the release. Now, we will determine the steady state probabilities. Now, these have important connotation - important implications for systems studies on a particular reservoir.

You may want to know what is the probability of my release being in such a state, in time period t in a given time period t; when you applied this reservoir using the steady state policy. So, the implications of the steady state policy are studied through the steady state probabilities of the storage, the release as well as the inflow. What we did through the steady state policy is that we defined l star for a given (k, i) in time period t. Now, if there is the unique l star for a given (k, i), you look at this table now for any given combination of (k, i) there is one l star associated with it - one and only one. So, there is the unique l star associated with this. You look at time period t is equal to 2, the l star value corresponding to 1 results from a combination of (1, 1), (1, 2), (2, 1) and (2, 2). The l star equal to 1 in time period t is equal to 1 results from a combination of only (1, 1) whereas l star is equal to 2 results from a combination of (1, 2), (2, 1), (2, 2). So, if we are looking for a given value of l star in the long run, we must look at the combinations from which that particular value of l star is resulting. And we use this information as provided in the steady state policy to derive the steady state probabilities.

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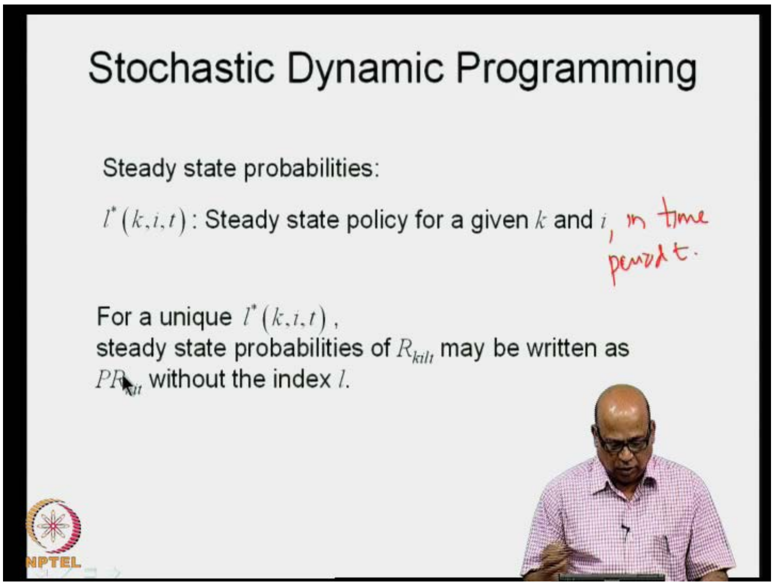

So, we have said now l star (k, i, t) is in fact the steady state policy for a given k and i in a time period t. Now, for a unique l star k i t, what is that we are looking at? We are saying probabilities of R kilt, we can write it as simply probability of R kit, because this combination (k, i) results in a particular value of l star k i t. We delete the index l, because index l is uniquely defined the moment (k, i) are specified for the time period t. So, we will write this as PR kit to indicate the steady state probabilities of the release.

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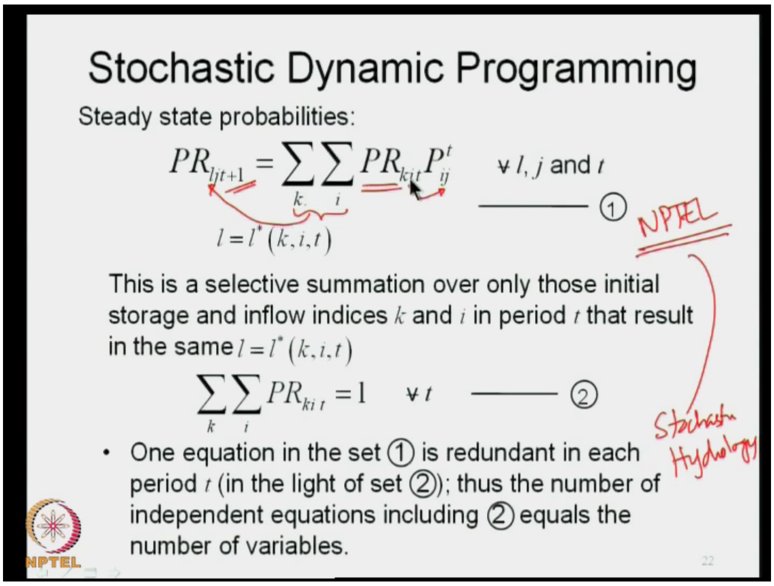

Now, how do we obtain the steady state probabilities? Look at this, we are saying l is equal to l star (k, i, t), and therefore, we are writing it as probability of R kit. Now, the probability of R kit for a given (k, i) multiplied by the transition probability (i, j). Now, this term here indicates that from i you are going to a particular state j, and this combination of (k, i) results in a particular value of l. So, l is equal to l star (k, i, t). So, this PR kit which is the probability of R kit when you multiply it with the transition probability P ijt of the inflow, you will get PR ljt plus 1, because you are now going to the next time period t plus 1, from the time period t you went to next time period t plus 1. I repeat that again, you are summing over those combinations of (k, i) which results in a particular value of l. This is what I have showed earlier; that a particular combination of (k, i) results in a particular value of l. So, you are looking you are picking up all those combinations of (k, i) which result in a particular value of l. For example, l is equal to 2 is resulting from 3 combinations, whereas l is equal to 1 with resulting only from 1 combination in time period t is equal to 1.

So, we sum will pick up those values of (k, i) which resulting this particular value of l, and write this equation for those particular terms. Now, this is the selective summation, this you know, you must understand correctly and in fact when when I continue the example; I will explain this. So, this set of equations here is written for all l, j and t. So, we say l is equal to 1, let say I am writing for t is equal to 1. So, t is equal to 1. So, I will write for l is equal to 1, j is equal to 1, t plus 1 is equal to 2. So, 1 1 2, I will put that and then I will go to the time t is equal to 1, pick up those combinations (k, i) which results in l is equal to 1; that is the idea, and put those many terms here. So, this summation you must understand correctly. So, this is the selective summation over only those initial storage and inflow index is (k, i) in period t that result in the same l is equal to l star kit. So… And then there is another set of equations. So, this is one set of equations, but being probabilities they should all added to 1. For example, PR kit is equal to 1 summed over all possible values of (k, i). So, this has to be satisfied.

Now, when we write these equations for two by two problem that I am considering; one of these sets for each time period will be redundant, because we have to satisfy this particular condition. So, we use only those for independent equations from these two sets and then solve. When you when you looked at the independent equations, you will see that the number of independent equations including 2, the all the equations in 2 have to be included equals the number of variables, and therefore, you should be able to solve for PR kit the idea here is that you are solving for PR kit which is also PR ljt plus 1. These are known transition probabilities are known and you will generate those many equations as you have the number of terms PR kit. For example, in in the numerical example that I am talking about you have k is equal to 2 that is k is equal to 1 or 2, i is equal to 1 or 2 and time period t is equal to 1 or 2. So, 2 into 2 into 2 you must generate 8 equations from this. And you use this equation for both the time periods. So, two equations are available and you use 6 from here and 2 from here and generate 8 equations which are independent equations and then solve for PR kit. So, let us look at how we are obtain the steady state probabilities. Now, once you obtain these release probabilities. Now, these are the probabilities of the release; once you obtain these… Understand it this way that this probability now, PR kit give the probability of the storage k and inflow i occurring together. So, this is the joint probability of the storage k and the inflow i in time period t. So, once you have the joint probabilities of the storage k and the inflow i, you can obtain the marginal probabilities. I again refer you to the NPTEL course on stochastic hydrology where discussions on joint probabilities and marginal probabilities are discussed. So, this is NPTEL stochastic hydrology course you can refer. Once you have the joint probabilities like this PR kit, you can obtain the steady state marginal probabilities of storage k as well as of inflow i.

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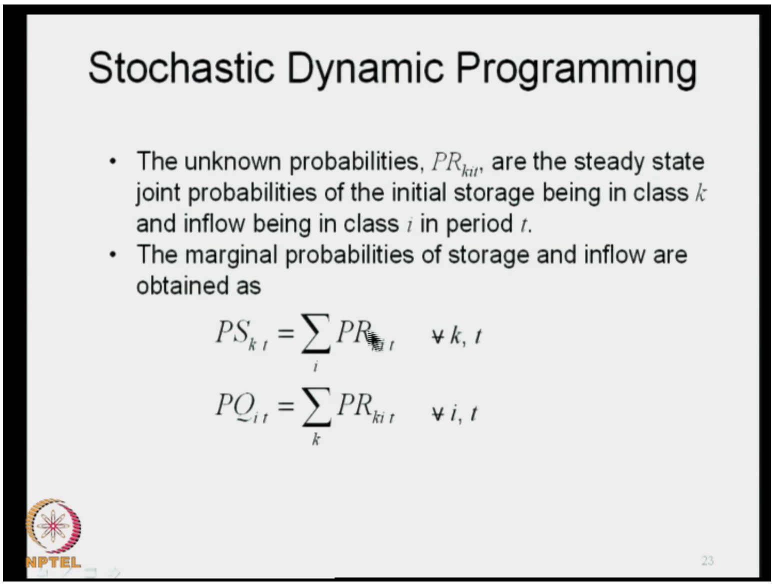

How how we obtain? This is the joint probability of k and i together in time period t. If I sum over all i, I will get the joint probability of storage k. Similarly, if I sum these probabilities over all k, I get the joint marginal probability of inflow i. So, this is how you get the marginal probabilities of storage as well as inflow. Now, what is the physical significance of all these? We are saying that we are operating the reservoir system over a long period of time using the steady state operating policy. If we do this, then the overall long period of time, the storage will be in this particular class k with this probability, for all k you can define this. Similarly, the inflow will be in this class i in time period t. So, this is what it gives. We will see how we obtain these probabilities from the example.

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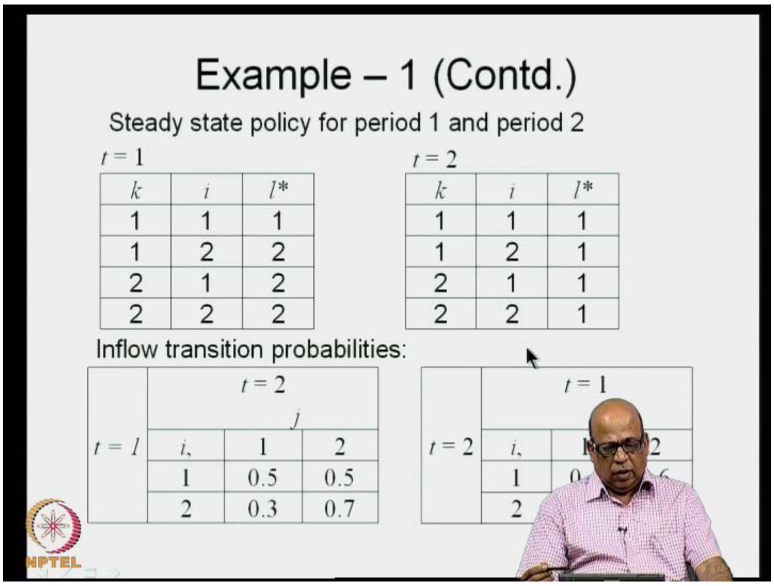

So, in the numerical example that I just solve, for t is equal to 1 we got this policy 1 1 1 1 2 2 2 1 2 2 2 2. Similarly, for t is equal to 2 I got this policy, and the inflow transition probabilities are known. Remember for obtaining the steady state probabilities, what you need is - the l is equal to l star kit which is the steady policy; you need the steady state policy as well as you need the inflow transition probabilities. So, we will look at the steady state policy as well as the inflow transition probabilities and write these equations now. Let us say that I want to write for time period t is equal to 1 and l is equal to 1, j is equal to 1. So, I will write 1 1 2 that is PR 1 1 2, and then look at those combinations of (k, i) in time period t is equal to 1 which result in the same l is equal to 1; we will just write that.

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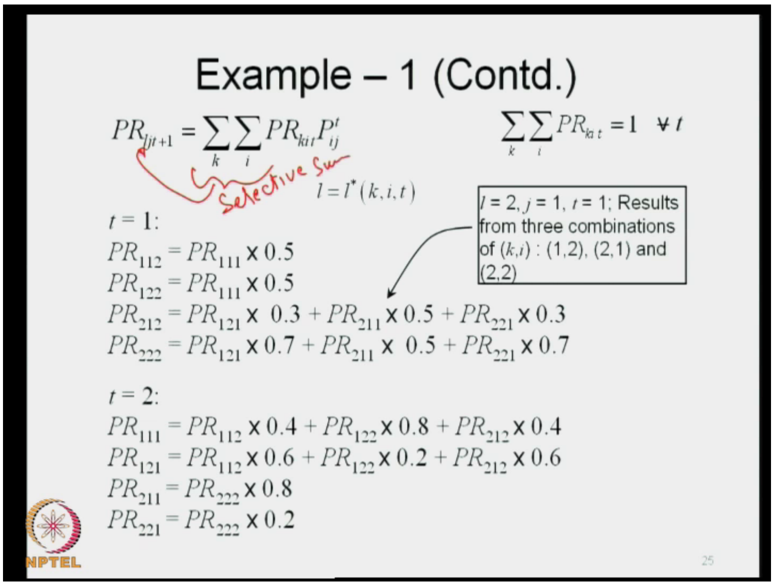

So, I am writing this set of equations now, PR ljt plus 1 is equal to sum over (k, i); this is the selective sum, I will say selective sum it is not for all (k, i). This is selective summation only those value of (k, i) which results in this particular l. So, let us write for t is equal to 1 now. Now, in t is equal to 1, I will first put l is equal to 1, j is equal to 1 and because t is equal to 1, t plus 1 will be equal to 2. l is equal to 1 in t is equal to 1 has resulted from only one combination of (k, i) which is (1, 1).

So, I will have only one term here which corresponds to (1, 1) in time period t, and the associate probability of going from i is equal to 1 2, j is equal to 1 in time from time period t to time period t plus 1. Let me first explain this term and then will come to this. I am writing for t is equal to 1 and I am setting l is equal to 1 here. So, I will start with left hand side, I will write PR 1 comma 1 comma 2. So, 1 l is equal to 1, j is equal to 1, because t is equal to 1, t plus 1 is equal to 2 is equal to I will go to the table and look at the combinations of (k, i) which have resulted in this particular value of l which is 1 in this case, and this particular case we have exactly one combination which is (1, 1), and therefore, I will have only one term here PR 1 1 1, then I will write i is 1 here and j is 1 here. So, i will pick up P 1 1 in time period t; P 1 1 in time period t is 0.5. So, that is how I get like this. Similarly, PR 1 2 2, but we will look at this now. PR 2 1 2 that means, I am setting l is equal to 2, j is equal to 1 and t is equal to 1. So, first I have to look at those combinations of (k, i) in period t is equal to 1 which result is l is equal to 2. So, go to the steady state policy for t is equal to 1, there are three term three combinations here namely

(1, 2), (2, 1) and (2, 2). I will write three terms associated with this (1, 2), (2, 1) and (2, 2) and period t remains the same 1 1 1 here. So, the terms here will be PR 1 2 1 and PR 2 1 1 and PR 2 2 1, then I will pick up the associated probabilities. So, 1 2 1 that is k is equal to 1, i is equal to 2 and I am going form ljt plus 1. So, j is equal to 1. So, 2 2 1 I have to go; this is i; this is j. So, 2 2 1 I have to go in time period t. So, 2 2 we will look at the transition probabilities for 2 2 1, 2 2 1 this is 0.3, 2 2 2 is 0.7. So, this is 0.3 here that is 2 2 1, then similarly 1 2 1 I have to go 1 2 1 as we are seen is 0.3 0.5 here, there is 1 2 1 is 0.5. So, this is 0.3, this is 0.5, then I have to go from 2 2 1 again which is 0.3. So, this is 0.3. Like this, you write the equations from t is equal to 1, t is equal to 2. You just verify this, you use this as the guideline; l is equal to 2, t is equal to 1, t is equal to 1, these results from three combinations of (k, i) which is (1, 2), (2 1) and (2, 2) and that is why you write these expressions. In addition, you have to use these expressions also; that is for all (k, i), you must have the sum of this must be equal to 1. So, use these equations and out of these 4 you delete 1 equation, out of these 4 you delete 1 equation, you will get 2 sets of equations from here, therefore, you will have 3 plus 3 plus 2 - 8 number of equations; solve the 8 number of equations to get the steady state policy steady state probabilities.

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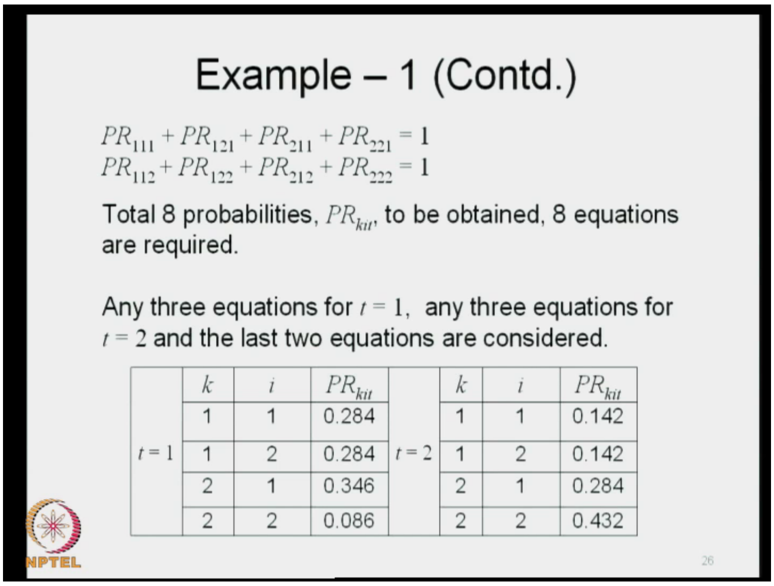

So, when you solve that, you will get the steady state probabilities like this 0.284, 0.284 this is you are getting for PR kit for a given combination of k and i in time period t; when you solve these equations simultaneously you will get these values. Similarly, for t is equal to 2, you will get these values. Using these values now, you should be able to get the storage probabilities as well as the inflow probabilities.

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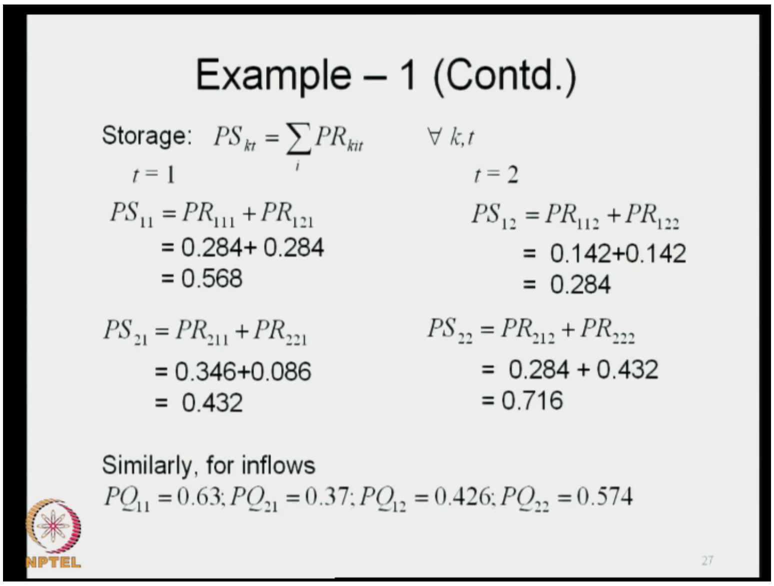

We look at this storage probabilities, t is equal to 1, probability of S kt is given by summation over all i for overall PR kit for (k, t). Let say that I want to write for t is equal to 1 and I am getting it for k is equal to 1, then what does is give? 1 comma 1 comma 1, 1 1 1 plus the I am summing over i. So, 1 comma 2 comma 1 so, i becomes 2 here. So, I will sum PR 1 1 1 and PR 1 2 1. Look at this PR 1 1 1 and 1 2 1. So, I add 0.284 plus 0.284, I get 0.568. Similarly, for 2 comma 1 I will write that is k is equal to 2, I get a probability of 0.432. Remember being probabilities these two should sum to one; 0.568 plus.532; similarly, for t is equal 2 I write, I get 0.284 and 0.2 0.716 which added to one. For storage, we added we summed over inflow the joint probabilities for getting inflow steady state probabilities we sum over the storage states k. So, when you do that for inflows, you can just verify PQ 1 1 is 0.63 and this is 0.37 and 0.426 and 0.574. So, this is how you obtain the steady state policies.

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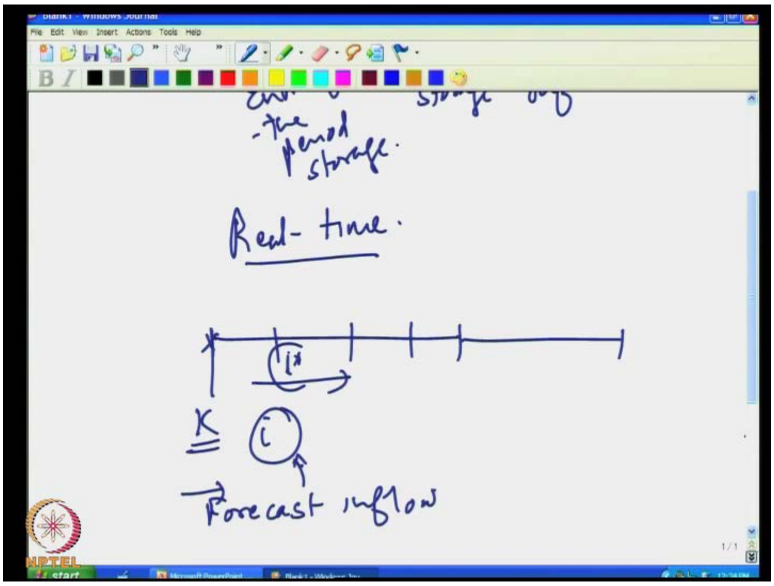

So, in summary for stochastic dynamic programming then that we have obtained the steady state policy for a reservoir system, and then we are saying that we define l star for a given inflow and this is the storage state and the this is the inflow state. For a given combination of storage and inflow in time period t, we know, what is the end of the period storage that has to be maintained. So, this is the steady state policy. Now, how do we apply this in real time that. In real time, we are given the l star (k, i, t). So, that we do is, let say that your progressing like this in real time, you know the storage state k which is measure. So, this is known, inflow is not known. So, what you do is you have a forecast of the inflow, and from the forecast inflow you get i, go to the table and obtain your l star and try to achieve the l star to the best extend possible. Why I say best extend possible is because of the discretization, it may not be always possible for you to obtain exactly the same l star as has been defined as the steady state policy. You try to maintain the l star to the best extend possible and then go to the next time period; now, which means what, from a given k and forecasted inflow you made a particular release which resulted in l star, when you go to the next time period, because the forecasted inflow will not be exactly equal to the actual in flow where l star may have change which means the storage at the beginning of the this period would have been change, again go to this forecast for the next time period use your l star for that time period and come to this point. Look at the storage, forecast for the next time period, apply your steady state policy and come to this period and so on. So, like this you apply the steady state policy in real time during the inflow forecast. How to get the inflow forecast etcetera is slightly different topic altogether; again you can refer to the stochastic hydrology course. So, to get some methods of obtaining the forecast for inflows.

Now, this is how we apply the steady state policy. So, that completes the topic of stochastic dynamic programming. Essentially, we use the stochastic dynamic programming for deriving long term steady state operating policy. In fact, we can apply these two multi reservoir systems also by looking at several number of state variables corresponding to each of the reservoir, you may have to define a state variable; much relate the deterministic stochastic programming - stochastic dynamic programming also suffers from the course of dimensionality, and therefore the number of state variables have to be limited. So, in the explicit stochastic optimization techniques, you have studied now CCLP which is the transcription linear programming as well as stochastic dynamic programming. From the next class onwards, I will start looking at the fuzzy optimization and specifically the fuzzy linear programming for water resource systems. Thank you for your attention. .

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