Water Resource Systems

Modeling Techniques and Analysis

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Lecture No # 32

Stochastic Dynamic Programming for Reservoir Operation (2)

Good morning and welcome to this the lecture number 32 of the course Water Resource Systems Modeling Techniques and Analysis. Now we are discussing the explicit stochastic optimization techniques and specifically for reservoir operation; over the last two lectures, I have been talking about the chance constrained optimization problem and the stochastic dynamic programming. In fact, in the last lecture, we introduced the stochastic dynamic programming for reservoir operation. Let us recollect what we did in the last class.

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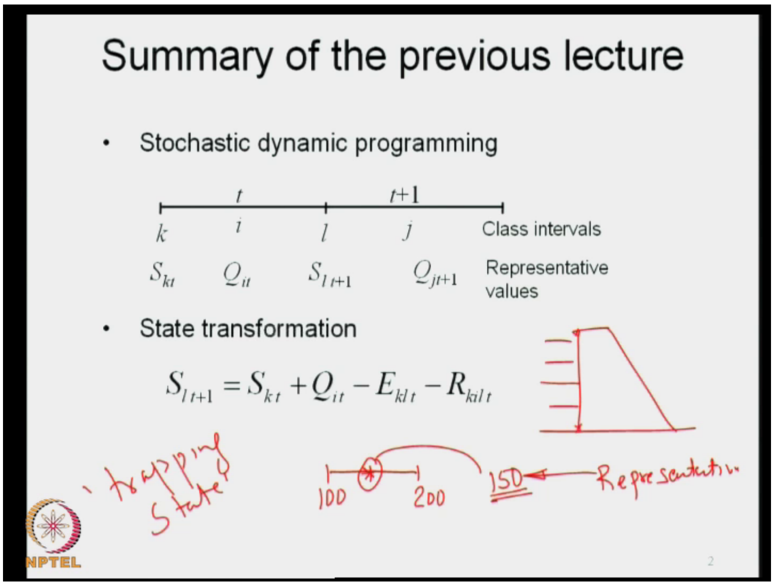

So, if we recall, in the last lecture, I discussed about the discretization of the state variables; recall that we discretize the storage, reservoir storage into a number of class intervals like this; let say from 0 to maximum capacity, we divide it into a number of class intervals 10, 15 and so on; and then each class interval is represented by a representative value, for example the storage class interval k that is the storage at the beginning of the time period t is represented by k, and that storage class interval is characterized by a interval; for example, storage between 100 and 200, we may say, it belongs to a particular class interval k; and the represented value, which is typically taken as the midpoint, although there are several better ways of doing this, depending on the actual distribution of the storage within this range, we may device different methods of taking the representative value, but for the time being, we will assume that we take the mid midpoint of the particular class interval as the representative value.

Similarly, the inflow is also discretized into several class intervals, and each class interval of the inflow is also represented by a representative value i, that is Q it. So, this is the notation that we use for two adjacent time periods t and t plus 1, the storage class interval that is the class interval for the storage at the beginning of the time period t is represented by k, and the associated representative values S kt, the class interval for the inflow during the time period t is represented by i and the associated representative values Q it, class interval for the storage at the end of the time period t, which is also the storage at the beginning of the time period t plus 1 is represented by l and its representative value is S l t plus 1, and the class interval for the inflow during the time period t plus 1 is represented as j and its representative value is Q j t plus 1.

So, we are looking at discrete values of storage and the inflow, in the stochastic dynamic programming, this is in fact, thus discrete states stochastic dynamic programming; and we write the state transformation, which is in fact, just the continuity equation as S l t plus 1 is equal to S k t, which is the storage at the beginning of the time period t plus the inflow during the time period t minus the evaporation losses minus the releases in the time period t; now the release is the function of the initial storage k, the inflow i and the end of the period storage l in the time period t; now when you discretize the storage and the inflow, we must make sure that the discretization or is such that there are no what are called as the trapping states.

In fact, when you take up any realistic case study, you will realize that the discretization of the state variable itself is a important exercise, what we mean by that trapping state is, let us say that you have certain pattern of discretization for the inflow, certain scheme of discretization for the inflow, and a certain scheme for the storage, let us say storage you have discretized into 15 class intervals, which means k is equal to 1 to 15 and inflow you may have discretized into 5 or 6 class intervals i is equal to 1 to 6 or (( )).

Now when you apply this continuity equation over a period of time, it may so happen that the storage gets trapped into one of the class intervals for several time periods, and it is unable to come out of that class interval, because of the way you have discretized, and therefore the discretization must be such that we avoid trapping states, which means the storage should not be trapped in a particular class interval for a long period of time, it should keep on moving from one class interval to another class interval associated with the particular class intervals that we have defined for the inflow.

So, remember that the discretization of the inflows as well as the storage must be done a hand in hand with each other. So that you avoid the trapping states; there is enough literature available on this, you can just refer to the available literature or how to how do we discretize this. In fact, with the advent of fast computers, it is possible for you to make both the storage as well as the inflow discretization to be very fine, quite fine, so that, you will avoid the trapping states.

Now, we will progress for there now, which means essentially, we are now talking about discrete state stochastic dynamic programming problems, in which all the state variables and in this particular case, we are taking only two state variables namely, the storage at the beginning of the time period and the inflow during the time period as the two state variables; but as I have been mentioning in actual realistic problems, you may want to add more and more state variables for example, you may add soil moisture as one more one more state variable, you may also add the rainfall in the command area is another state variable and so on.

All the state variables that we are considering in this particular framework of the dynamic programming that I am talking about are all discrete state variables; we can take on only discrete values, and each of the state variables is discretized into a number of class intervals with a represented value associated with each of the class intervals; with that background now, we will start writing the recursive relationships; much as we did in the dynamic programming for reservoir operation, recall in the discrete state dynamic programming, I am sorry in the deterministic dynamic programming, what did we do? We started with some time period for into the future, and then progressed backwards; in that case the inflow was deterministic, we will do the same thing now, we will start with a time period for into the future, and then progresses backwards, progressed in the backward direction from that time period in the future, until we hit a steady state policy.

The difference being that instead of taking the inflows as deterministic, now we are accounting for the random nature of the inflow in deriving the reservoir operating policy; how do we account for that? Recall that, we are treating the inflows as a one step Markov chain, the inflow to constitute a one step Markov chain, and they are represented by the Markov chain is completely defined by the associated transition probability P ij t and how did we define the P ijt? For completeness sake, I will just review what we said for P ij ts.

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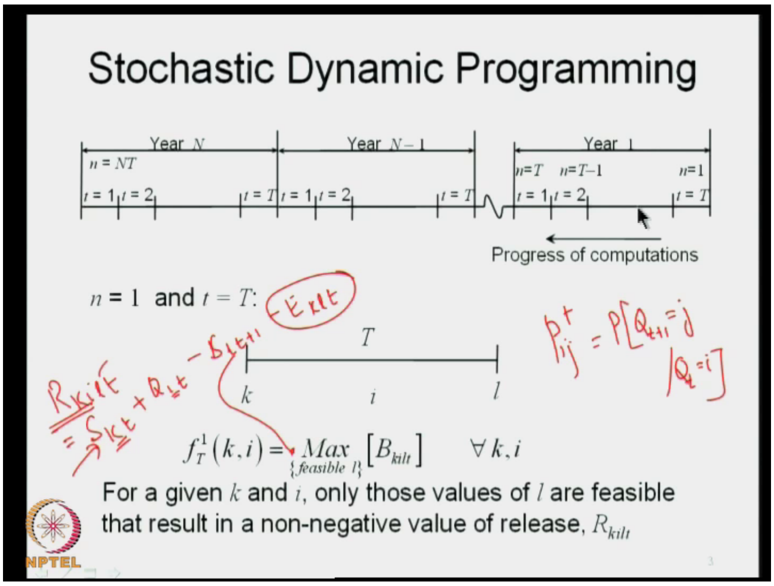

We said P ij t, we are referring to that inflow transition probabilities; as probability of Q t plus 1 belonging to the class interval j, given that the inflow in time period t belongs to class interval i, and this equality sign, I am using it to indicate that it belongs to the particular class interval; what does it say? Starting with a class interval i in period t, the inflow, the probability that the inflow goes to class interval j in period t plus 1 is P ij t. So, t we are denoting it here, to indicate a transition from period t to period t plus 1; now this is the notation that we are using. So, when we start in a particular period for into the future, and we progress backwards like this, in the backward direction; so we say year number 1, year number 2, year number 3 etcetera as the computation progress; the last time period in a year is indicated by capital T.

So, we start with the first stage of the dynamic programming n is equal to 1, n is equal to 2, n is equal to 3 etcetera; at the end of the first year, we will come to n is capital T, which is number of periods in the time period; let us say you are talking about the monthly operation of the reservoir, in which case, the capital T will be 12 and therefore, we start with n is equal to 1, n is equal to 2, n is equal to 3 etcetera, at the end of the year number 1, we are at n is equal to 12; the next time period will be again n is equal to t is equal to 12, t is equal to 11 etcetera for monthly time period; whereas, n keeps an increasing n is equal to t plus 1, n is equal to capital T plus 2 and so on, it keeps on increasing, until we reach the steady state in a some year n, where n will be equal to N into T.

So, the index n here, keeps track of the position in the computation in terms of the stage. So, the stage is continuously increasing, whereas the small t here keeps track of the period within the year; for example, if it is the monthly operation, it will be a only between 1 to 12, always between 1 to 12, like this; whereas, n will keep on progressively increasing. So, with this now, we formulate the recursive relationship for the stochastic dynamic programming; we start with the last period, which corresponds to stage number 1. So, n is equal to 1and t is equal to capital T, this is the last stage.

We are looking for a system performance measure, so for n is equal to 1 and t is equal to T, we write this as f 1 of T (k,i) that means, for a given k and i, these are the state variables, storage at the beginning of the time period and the inflow during the time period; for a given k and i, we may want to maximize the system performance measure B kilt; from the last lecture, recall that, this is the return from the system or the system performance measure, which depends on the storage at the beginning of the time period t k, the inflow during the time period i and the end of the period storage l in the time period t.

For example, if you are looking at the hydropower optimization, you may want to maximize the expected value of the hydropower that is generated from the reservoir system, then your system performance measure B kilt will perhaps be a function of the actual hydropower generated, now the actual hydropower that is generated will be a function of the release that is made to the turbine made made through the penstocks to the turbines, and that release itself is a function of k as well as l; therefore, we write the release as R kilt it is also a function of i.

And the head, that is the power that is generated will be a function of both the release as well as head, and the head depends on the storage; and therefore, the retard or the system performance measure will depend on k il in period t, that is why we write B kilt; and we are looking for the maximization of this system performance measure for a given (k,i), and we are making the search over all possible l values, which are feasible. So, for a given k and i, we are asking the question what is the optimal l; now this optimal l, we are making search over all possible values of l and picking up that particular value of l or those particular values of l, which will maximize the values of value of the system performance measure.

Now when we are making this search, let us say l is equal to 1, 2, 3 etcetera up to 15; 15 class intervals; you want to make search over all the 15 class intervals; you will pick up, you will make this search only over those particular values of l, which are feasible; what do we mean by that term feasible? Remember that we wrote R kilt as S k t plus Q it minus S l t plus 1 minus E klt, which is the evaporation. Now, for the time being, you do not worry about evaporation. So, essentially S kt plus Q it minus S lt plus 1 will give you the release; we are making search for a given and given i in period t, we are searching over the possible values of l; now, when we put a particular value of l, it is possible that the R kilt may turn out to be negative and that l becomes infeasible. So, if your R kilt as defined by this equation becomes negative, that particular value of l becomes infeasible.

So, we are making the search over only those those values of l, which result in a non negative value of the release R kilt as defined by this. So, that is what way I mean by feasible l. So, in the last period capital T, which corresponds to stage number one here as we are starting the computations; we write the equations for the system performance measure as simply f 1 T (k,i) is equal to maximize overall feasible values of l B kilt for all k i; so, we are defining this function for all k i, that is a idea.

Now, because there is nothing to look beyond, because this is the last time period, this itself becomes a system performance measure. I encourage you to go to the dynamic programming lecture where we have derived the stationary policy; it is a exactly similar to that except that we are accounting for the randomness of the inflows through the inflow transition probabilities. So, let us go to stage number 2 now; in stage number 2, we are moving one time step in this direction now. So, t will be equal to T minus 1.

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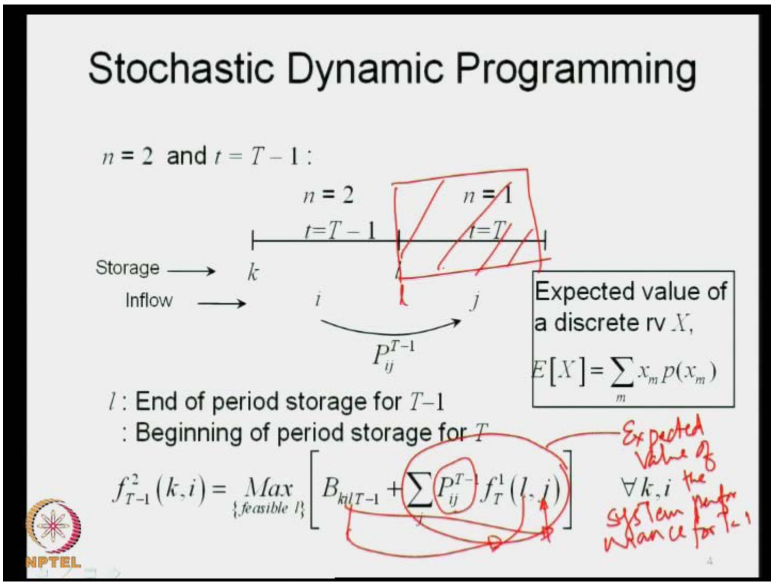

So, I write for the stage number 2, the time period t corresponds to T minus 1, that is the stage 2 corresponds to time t is equal to T minus 1, so we are here now, n is equal to 2; and n is equal to 1, we have already solved, this part is already solved; and the solution of this has defined f 1 capital T (k,i). So, for all given values of combinations of (k, i), we already have the optimal values of f 1 T (k,i), we will use that in solving for the stage number 2. So, stage number 2 computations, if you understand correctly, then all other stages are straight forward; at stage number 2, we are in period t is equal to capital T minus 1, if we are solving for monthly time periods, this would have been capital capital T will be 12, therefore 12 eth month, we would have solved we are going to 11 eth month now.

We are solving it for a given storage k, and a given inflow i; we are making search over all possible values of l, and picking up that particular value of l, which will maximize the system performance over these two period together. In fact, which will maximize the expected value of the system performance measure over these two periods together, that is a problem that we are posing at stage number 2; I repeat that for a given k value, for a given i value, we are looking for that particular value of l, among those possible values of l, such that the expected value of the system performance over these two periods together is maximized; starting with a given k and for a given i, the continuity equation will determine the release, because we are making a search for a given l. So, the storage transformation from k to l is known; what do we need for getting the expected value for the time period t capital T, which is the stage number 1,we need both these values k and i for that particular time.

So, l is what defines the beginning of the storage, beginning of the storage, beginning of the period storage for this particular time; we need the inflow, which is j when we are reckoning time period T minus 1, and the transition from inflow i to inflow j is governed by the transition probabilities P ij T minus 1, why I write T minus 1? Recall that we define the transition probabilities for time period t as transition from period t to period t plus 1, and we are looking for transition form period T minus 1 to period t here - capital T; and therefore, this transition probability is written as P ij t minus 1; so you understand this recursive relationship in the light of the definitions here; we will write for stage n is equal to 2 and for the time period T minus 1, so we are defining f 2 T minus 1 of (k,i).

So, for a given value of k and for a given value of i, I am defining this as equal to maximization over those feasible values of l as I just defined of what is resulting from period T minus 1, because k and i are known, we know the system performance measure for a particular l, you are making the search over a particular l for a given value of k and i; therefore, the immediate return or the immediate value of the system performance measure for this will be B k il T minus 1; why there are no probabilities here, because k is fixed, i is fixed, you are saying that for a given k and for a given i, and for that particular value of l, over which I am making the search what is the return from this particular time period T minus 1, that will be simply B k il T minus 1; but associated with this l, we have to pick up that particular system performance, which we have already optimized for the next time period; how do we pick up? The next time period capital T, which is the previous stage n is equal to 1; we have defined the system performance for a given initial storage k and the inflow i.

Now when we come to the next time period, the given initial storage for the next time period t will be l here. So, the particular value of l, over which we are making the search, defines the storage for the next time period capital T. So, this is the same l that we are picking up; and we also need the inflow i, what happens to the inflow i, when we are talking at, we are looking at the next time period; we are given the inflow I, and this inflow goes to a class interval j in the next time period, which is capital T, with a probability of P ij T minus 1. So, we are looking at this class interval j, and the transition between i and j is governed by P ij T minus 1.

Remember, the expected value of x any random variable x, when x can take on a only the discrete discrete values is given by the sum of x m into P of x m, where P of x m is a probability of the random variable taking on a value of x m, and the x m is a associated value itself. So, we apply this, and then write the expected value of the system performance measure for t is equal to capital T, n is equal to 1; i goes from i to j with the probability of P ij T minus 1, if it goes to j, for this given l, the system performance measure is f 1 T (l,j), which we have already solve for this time period. So, this becomes the value of the random variable x m, and this is the associated probability and therefore, this term together gives the expected value of the system performance measure, of the system performance measure (No audio from 24:26 to 24:34) for the time period T minus 1. So, up to this point; I again repeat, because this is the one crucial thing that you must understand, that we are summing over all possible values of j here, may be I will erase all these; so that I can explain it better.

So, we are looking at a given value of k and i what is a expected system performance measure we first look at what is the immediate return from T minus 1, capital T minus 1, period T minus 1, k is known, i is known, we are making a search over a possible value of l; therefore, we can determine B k il in time period T minus 1 there is nothing random about this because k and i are fixed and we are making search over a given l; when we come to the next stage next time period capital T or the previous stage, the i here goes to j with a probability of P ij T minus 1, which is the transition probability; and the l here determines the storage at the beginning of the time period for that time that time period t, and we pick up this f 1 T (l, j) from the previous stage computations; we define k here as l resulting from here, and i here as j that particular value of j resulting in this summation, and the associated probabilities for this, for example, if you are looking at j is equal to 2, I starting with i is equal to 1, I will use probability 1, 2 in time period T minus 1, and l may be, that particular value of l may be 3, so this will be 3, 2, when j is 2.

So, like this we vary j and write the summation terms, and then associated values of f 1 T (l,j), we pick up from the previous level, when we solve the example, it will be more clear. So, understand that, for this time period T minus 1, we get the deterministic value of B k i l T minus 1, because there is no uncertainty associated with it, as you have fixed the storage state k, and you are solving it for a given value of inflow i, but when we go look at the next time period capital T, there is the transition probability associated with the transition of inflow from i to j, and therefore, we multiply it by the transition probability, because I looking at the expected value of the system performance. So, this is how we write the recursive relationship for the next time period t.

So, we define for the time period T minus 1 stage 2, we define the expected system performance measure; we use this when we go to the next time period, so we write it for the next time period, which is T minus 2 - capital T minus 2, which corresponds to n is equal to 3 and so on. So, like this we keep on writing the recursive relationship, connecting with the previous stage always; when we are solving for n is equal to 2, I am connecting it to n is equal to 1; solving for n is equal to 3, I always connect it to n is equal to 2, n is equal to 2 solution would have already contained the solutions for all the time periods up to the end of the time horizon, which means up to the stage number 1; when I go to n is equal to 4, I connect it to n is equal to 3, because the solution for n is equal to 3, would have contained solution up to n is equal to 1 and so on. So, like this, we progressed in the backward direction, keep writing the recursive relationship at every stage, and then solve the recursive relationship.

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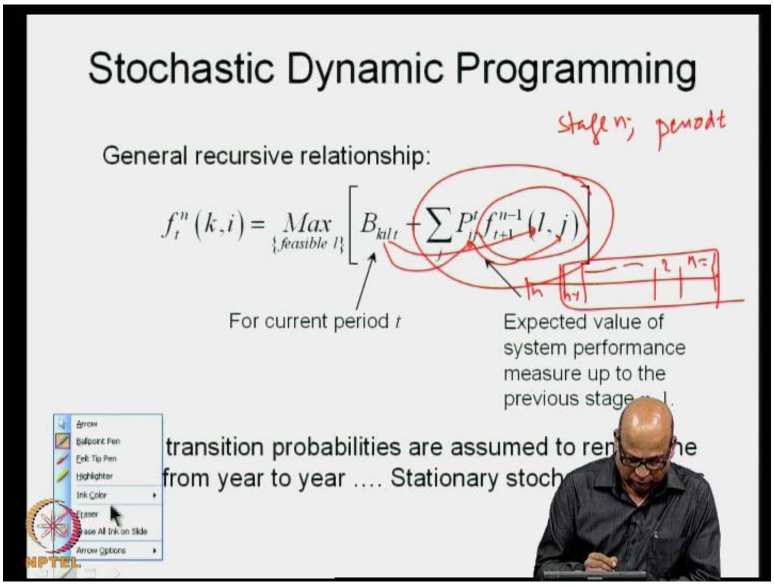

So, any general stage n, which corresponds to the time period t. So, stage n and period t, we write the recursive relationship as f n t, now I use both these indices to indicate that the time period the stage n corresponds to the time period t within the year f n T (k,i) is equal to maximize over feasible l. So, which means our search is over the feasible l, the deterministic value B kilt, because you have define k and i in time period t, and you are looking for that particular value of l; therefore, B kilt gets fixed for the current time period t plus the i here goes from i to j with a transition probability of P ij t, if it goes to j, then we get a value of system performance f n minus 1 t plus 1 associated with this j, and this particular l is the same l for which we are solving this.

So, this defines the expected value of the system performance measure up to the previous stage n minus 1, what do I mean by; up to the previous stage, we are moving in the backward direction, so n is equal to 1 n is equal to 2 etcetera, we have come up to n minus 1, and we are solving it for n. So, up to this point, we have solved everything, and then this term here, defines the expected values of system performance measure up to the previous stage that is the idea there. So, this is the general recursive relationship now, for this particular time period, f t n (k,i); now, look at these terms P ij t, these are the transition probabilities.

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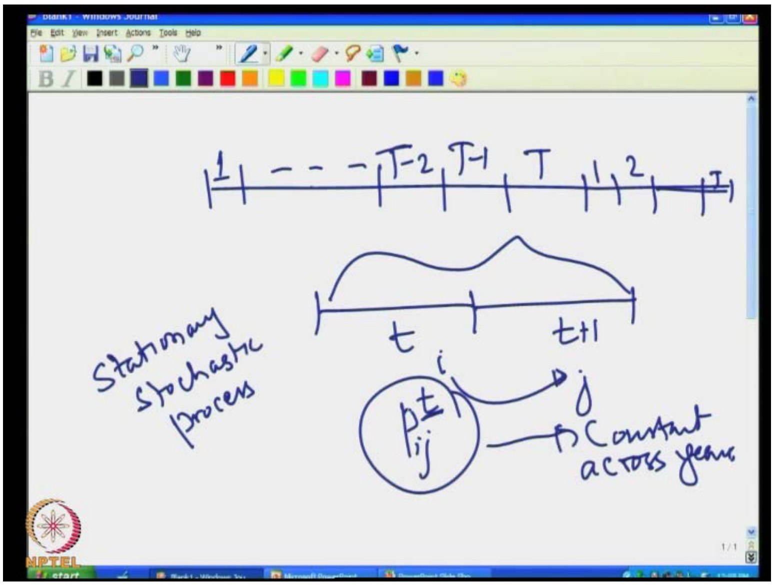

So, we are saying that in a year, let me explain that more clearly, in a year, there are T time periods T minus 1, T minus 2 etcetera, we have up to 1; if it is a monthly time monthly operation, you have 12 number of such periods; now, any two adjacent time periods you take, t and t plus 1; the associated transition probabilities, there are P ij t. So, P i j t here, are in fact, in this particular class, I am also using capital P here in the slides; however, generally it is used as small, because we are indicating probabilities, individual probabilities, and capital P is generally used for matrices - notation of matrices; however, here I am using interchangeably.

Now P ij t is the transition probability of the inflow going from class interval i to j; now when we come to in the next year, we start with t is equal to 1, t is equal to 2 etcetera capital T; for any given combination of t and t plus 1 that is the two adjacent time periods t and t plus 1, the transition probabilities remain the same across years, let us say from period number 2 to 3, we have defined the transition probabilities 2 to 3, the transition probabilities remain the same next year also, next year also and so on.

So, across the years the transition probabilities remain constant, often the students mistake this, remember I am saying it is changing from period t to t within the year; however, across the years, for the same time periods t these will remain constant. So, that is what you must remember, and such stochastic processes, where the transition problem where the probability structure does not change from year to year are called as the stationary stochastic processes. So, we are assuming that the inflow constitutes a stationary stochastic process, in as much as the transition probabilities do not change from one year to another year, it is because of this assumption that the recursive relationship as we solve will converge to a steady state policy.

So, the P ij t, the assumption in the P ij ts is that the inflow transition probabilities will remain the same from year to year, and that is what constitutes a stationary stochastic process, and because of this assumption, as you solve this recursive relationship over time, we achieve a steady state policy, I will define what is a steady state policy presently. Now we started with the last time period in the in some distant year, and then started progressing backward, in the backward direction, and every time we added one stage, and then solve the problem for that particular stage using the recursive relationship. So, you keep on doing this, year number one, you completed; let us say you have 12 time periods, you completed 12 time periods; keep going 13 time period, 13 time period, 14 time period, then you go up to 24 eth time period that completes year number 2, progress further 25 eth time period, 26 eth time period etcetera, like this, you keep on solving this recursively from one period to another period, progressively increasing, the the stage being progressively increasing and every time you are associating with the particular time period within the year.

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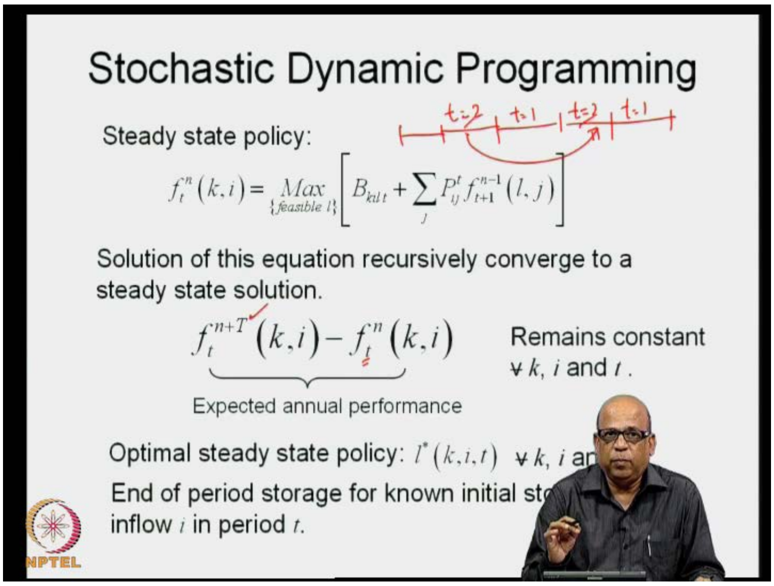

Like this, when you solve this over a period of time, a stage will come in the computations, when the annual expected system performance measure will remain constant; what do I mean by the annual expected system performance measure? For a given combination of (k, i), in the time period t, the system performance measure that you get associated with the stage n plus capital T, where capital T is the number of time periods in the year minus for the same combination (k, i), for the time period t in stage n. So, this n to n plus capital T is the elapsed time for 1 year, let us say you had two time periods, so t is equal to 1, t is equal to 2, we came here; then again it will be t is equal to 1, t is equal to 2 and so on, like this. So, for the same time period t, when you are looking at in the previous, the previous year you have the difference of capital T and in this particular case two time periods. So, the same time period t, corresponds to t period T stages apart from the current stage.

So, we take the and therefore, this difference here now, f t n plus T minus f t n will provide the expected annual system performance, why annual? If we are looking at two time periods the difference in the system performance here between this stage and this stage will give you the system performance measure, for that particular year, for a given

t. So, and therefore, this indicates the expected annual system performance measure, expected annual system performance; when this annual system performance measure, the expected value of annual system performance remains constant, for all given k and i, and also for all time periods t. So, it reaches a it reaches a value that will not change further within certain limits, in such a situation, we say that the steady state has been reached.

So, we keep solving this recursively in the backward direction and keep checking for this value here, that is the annual system performance, actually the expected annual system performance, when it reaches a constant value, nearly a constant value, then we say that the steady state policy has been reached, and then we terminate the combination, we terminate the computation I am sorry; what is the steady state policy then? Remember, we are solving for l, we are making the search over l; so, our purpose is to get for a particular combination of k and i, which is the initial storage k and the inflow I, we want to get an optimal end of the period storage l; so, the policy is defined by l, for a given (k,i).

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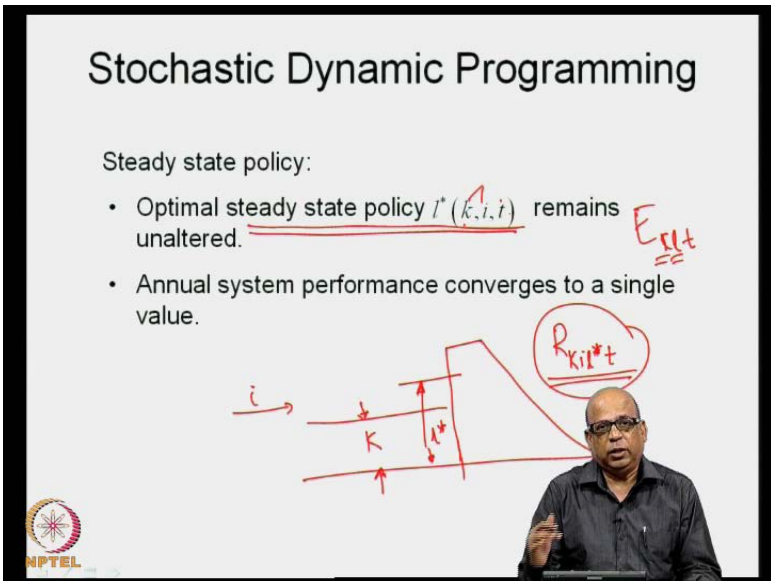

And this policy, we denote it as l star k, i for f time period t. So, this in fact, is the steady state policy that is for a given k, i in time period t, we define what should be l star; now what is the implication of this? In time period t, you are let us say you are in storage state k, and the inflow state i is known, then this will define, what should be my end of the period storage l star. So, given this l star, you know how to calculate R, because R is given by R k i l star into t. So, you know how to get the release from the reservoir. So, this is how we define the policy; for a given initial storage k and for a given inflow i, you know, what is the end of the period storage, and that is what defines the release to be made from the reservoir; of course, it also includes E klt, that is a evaporation loss associated with the initial storage and the end of the time period storage in time period t.

So, this is how we denote or we specify the policy, reservoir operating policy; you specify initial storage k, you specify the inflow, the policy will give you l star, which is the optimal end of the period storage that needs to be maintained. Now this optimal end of the period storage l star is in fact, the steady state policy, in the sense that we have solve the recursive relationship accounting for the random nature of the inflows until the steady state has been reached, and finally when the steady state is reached, we capture the those values of l star, and then tabulate them; for a given value of k, i, for each of the time periods t, we tabulate the l star values; and that is what we give the steady state policy; how to implicate the steady state policy in real time; I will discussed slight later, but this is how we derived the steady state policy.

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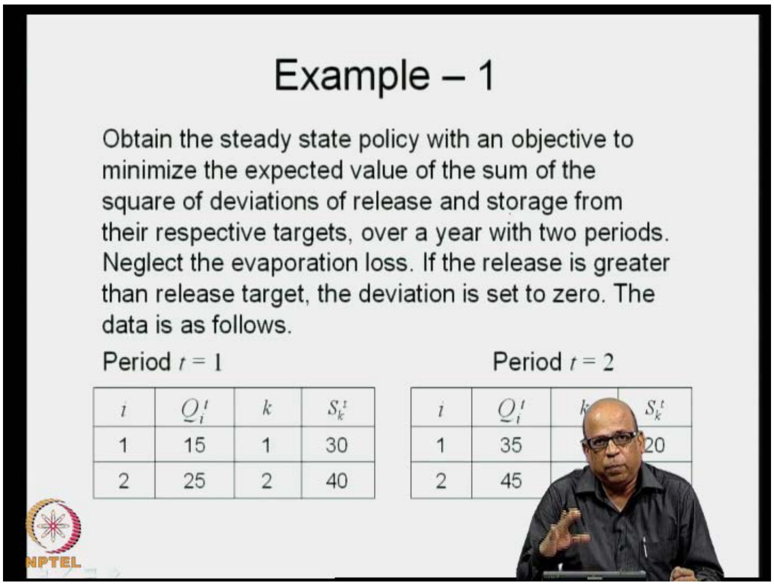

We will now look at in numerical examples; so that all the certainty of the problem are understood correctly. First of all you know, for the students or the practitioners want to apply the stochastic dynamic programming, you must know what kind of data that is necessary, for the what kind of discretization you do, what kind of inputs that are necessary for the stochastic dynamic programming and so on. So, this numerical example will clarify the type of data that you need to… You need to procure before you start applying the stochastic dynamic programming.

You are solving the stochastic dynamic programming for a given reservoir, therefore, the maximum storage is fixed; and you are looking at only the live storage of the reservoir, and the live storage of the reservoir, you are discretizing into several class intervals; let us say the live storage of reservoir is some 800 million cubic meters or some (( )); and you are looking at the discretization between 0 to 800, this you may divide into 10, 15, 20 number of classes; either 10 or 15 or 20 number of classes; associated with each of these class intervals of storage, you will have a storage representative value.

So, you will need first that discretization scheme for the storage, by discretization scheme I mean, how what are the intervals of each of the class intervals and what are the associated representative values; then similar to that, you also look at the inflows, and you need the inflow discretization class intervals, inflow discretization scheme; in realistic cases, the inflow discretization is done based on the historical data, let us say you have fifty years of historical data, and you are doing this for a monthly time period, t is equal to 1, t is equal to 2 etcetera t is equal to 12; for each these months, then if you have fifty years of data, each of these months will have fifty years of values.

So, look at actual values that I have been realized in the historical data, and then discretize these inflows into several class intervals; each of the class intervals, in a time period t, is represented by a particular representative value of the inflow. So, this is the first level of data that you need to generate, storage discretization scheme and the inflow discretization scheme; now these are done of an iteratively by trial and error that is you do the storage discretization, then you do the inflow discretization may be run yours standard operating policy to make sure that the inflow and the storage discretization or compatible with each other in the sense that, they do not resulting a trapping state, a storage trapping state; all of these, you will learn through a experience as you start applying to realistic cases; then the third important data that you need is the transition probabilities. So, between two consecutive time periods t is equal to let say 1 to t is equal to 2, how the inflow transition probabilities are computed is what you must know.

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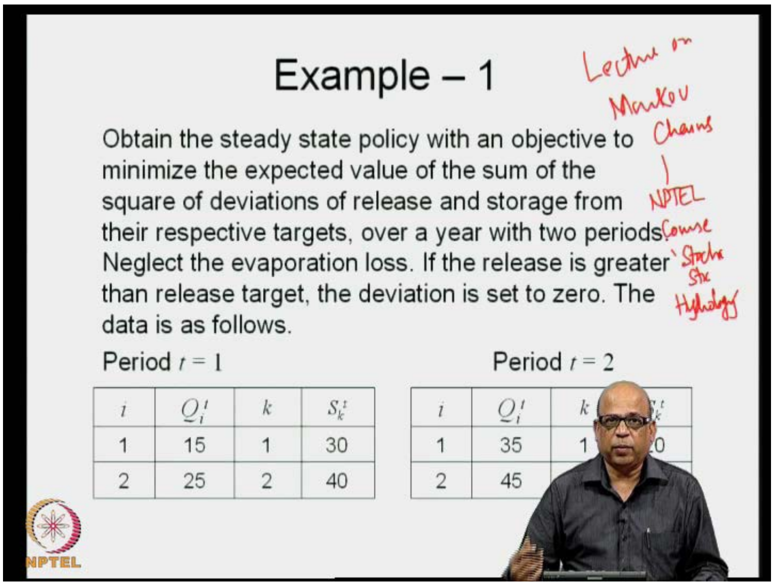

I suggest that you go to the NPTEL course on stochastic hydrology again, and look at the Markov chain, lecture on Markov chain; Markov chains, there are two lectures there; you can look up this course - NPTEL course on stochastic hydrology. (No audio from 45:11 to 45:18) So, there we have discussed how to obtain the transition probabilities for a one step Markov chain using their relative frequency approach. So, you need the inflow transition probabilities form one time period to another time period; then you will need the system performance measure itself, that is what is the measure with which you are optimizing the system, for example, your objective may be hydropower optimization, your objective may be irrigation optimization, your objective may be flood control optimization and so on.

So, associated with each of these objectives are the main objective with which you want to optimize the reservoir, you need the system performance measure; the system performance measure as I have been repeatedly saying is in general, a function of the initial storage k and the inflow i in time period t and the final storage l in time period t. So, you must know, how to compute this system performance measure for a given value of k, given value of i and given value of l in time period t, so you must have a mathematical statement of the system performance measure. So with all of this data, then we enter the recursive relationship; so this example that I am demonstrating will make it clear on the type of data that you need, how you compute compute the B kilt for a given objective function, for a given objective, and how we apply the recursive relationship. So, let us go through the example now.

So, the purpose here is to obtain the steady state policy with an objective to minimize the expected value of the sum, of the square of deviations of release and storage from their respective targets. So, this is the objective; we are setting a target release, we are setting a target storage for each of the time periods, and in this example, we are considering two time periods; and then we are saying that the sum of the squares of the deviations from the target source set for the storage as well as the reservoir as well as the release; the sum of the deviations must be minimized that is the objective; we will neglect for this exercise, we will neglect the evaporation loss; further if the release is more than the target, we will say that the deviation 0. Let say that your target release in each of the time period is 5, but we are able to released more more than 5, then we will assume that the deviation is 0; that means, we we do not penalize for releasing more than what is the target, but this can also be accounted for, whenever you want the release to be just close to the target and not more than that; then you can also apply penalty functions for that.

Now we will have the descritization, as I said you need the descritization for that. So, for t is equal to 1, we are defining Q i t; now for convenience, we may use i and t as subscript and superscript respectively like I had on here, as long as you are clear that you refer to it the time period t, when you are looking at i; it is ok. So, for time period t is equal to 1, we have representative values associated with the class interval 1 as 15, here for the inflow and as 25 here for the inflow class interval 2; similarly for storage class interval 1 as I represent it value 30 denoted as k t, corresponding to storage class interval 2, I have the represented value for t, associated with class interval 2 for S k t, similarly for period two we have define this. So, you need their representative values as the first set of data.

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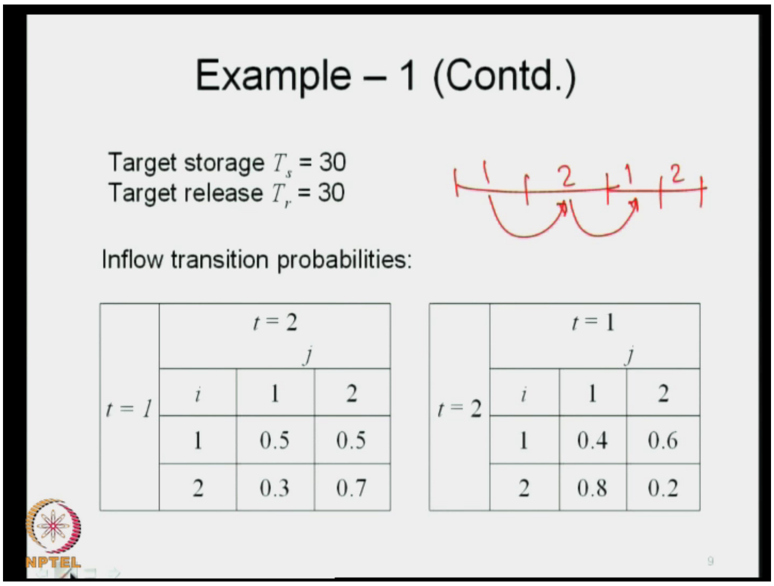

Then for this example, we are putting a storage target of 30 and the release target as 30, both of them are 30 on its each. The most important requirements for the SDP are the inflow transition probabilities. So, we need to define the inflow transition probabilities for the inflow going form time period t is equal to 1 to 2 as well as from t is equal to 2 to 1. So, we define this as, for given i, which corresponds to t is equal to 1, i can be either 1 or 2, because we have only two state possible for the inflow; remember here, we are looking at a two time period, two inflow state, two storage state problem. So, there inflows are discritized in to two class intervals, in each of the time periods; the storage is descritized in to two class interval, in each of the time periods; and there are two number of time periods.

So, i is equal to 1 and 2; from i is equal to 1, it goes to j is equal to 1 in time period t is equal to 2, with the probability of 0.5; starting with i is equal to 1 it goes in to class interval j is equal to 2 in time period 2 with the probability of 0.5; remember the transition probabilities being stochastic matrices, the rows must add up to 1, so 0.5, 0.5; that is its says that starting with class interval one, the inflow has to go in to one of the class intervals and therefore, the sum of the probabilities must be equal to 1; similarly starting with class interval 2 in time period 1, the probability that it goes in to class interval 1 in time period t is equal to 2 is 0.3; starting with class interval 2 in time period t is equal to 1, the probability that it goes in to class interval 2 in time period t is equal to 2 is 0.7; similarly from t is equal to 2 to 1.

Remember we need to define the transition probabilities for each of the time periods. So, if you have two time period problem, so you will have t is equal to 2 here, t is equal to 1 here, t is equal to 2, t is equal to1. So, from 1 to 2, it goes and that 2 to 1 its goes. So, the transition probabilities from t is equal to 2, t is equal to 1 also have to be different and these are the transition probabilities.

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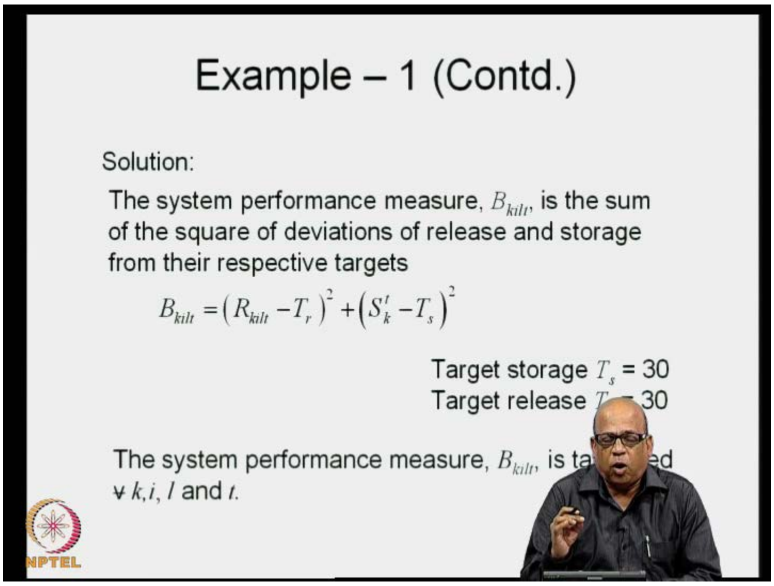

Then we start with the system performance measure. So, the first step in the stochastic dynamic programming after the data is ready is to compute or enumerate or tabulate in fact, the system performance measure values for all the combinations possible; let us say, you have 12 time periods, and you have define the system performance measure in a mathematical expression, if you are looking at the particular type of objective that we are considering; that means, every time period you have a target release, every time period you have a storage target, and you are looking at the some minimization of the some of the square deviations.

So, for each combination of K i l for each of the time periods, you must first enumerate or tabulate the system performance measure that is what we will do now for this. So, we have define B kilt as the sum of the squared deviations, this is the system performance measure, sum of the squared deviations of the release form its target and the storage from its target. So, T s is the storage target, which is 30 units, T r is the release target, which is

30 units; for a given combination of k, i and l in time period T, I must be able to enumerate this, and this will enumerate across all the time periods.

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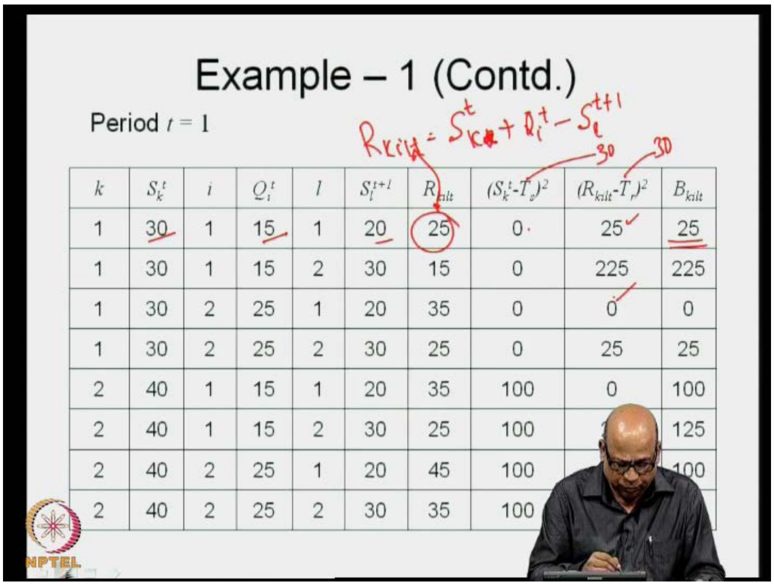

So, we as a first step, we tabulate the system performance measure B kilt, already we calculate the B kilt; look at this, we start with k is equal to 1, i is equal to 1, l is equal to 1, in time period t is equal to 1; when k is equal to 1, we know that S k t, this is k is equal to 1, S k t is 30 units, so, you put 30 units; similarly k is equal to 1 in i is equal to k is equal to 1 and i is equal to 1, in period t is equal to 1; the inflow is 15, Q i t in time period t is equal to 1 is 15 units. So, like this, we tabulate and for l is equal to 1 a set t plus 1 that is in t plus 1, you are looking at t plus 1 will be 2 will be 2 time period 2 and you are looking at class interval 1 time period 2 class interval 1 the storage is 20. So, we write this as 20.

And then we calculate R kilt, how do we calculate R kilt? R kilt is equal to S k t, we can write it either here or apply this plus Q i t minus S l t plus1; so, this is 30 plus 15 45 minus 20, which will be equal to 25, so, this is 25; and then you calculate these two terms separately S k t minus t S t s is 13 and T r is 13. So, S k t, which is 30, 30 minus 30 whole square is 0; R kilt is 25, 25 minus 30 that is minus 5 whole square that is 25, and B kilt is the sum of these two. So, this plus this will give you B kilt, which is 25; then you change k is equal to 1, i is equal to 1, l becomes 2 now.

So, for this you calculate again; remember, whenever the R kilt is more than 30, you recall the you reckon that particular deficit as 0; for example, here R kilt is 35 and the target is 30, because R kilt is more than that, you can set, you set to 0 in this particular case; like this, you enumerate for all combinations of k i and l, the B kilt values for both the time periods t is equal to 2 as well as t is equal to 1; remember, when you go to t is equal to 2, the t plus 1 that you refer here will become t is equal to1. So, in this particular case, t plus 1 will be equal to1; that means, you will have to look at time period 1 for these values. So, the first step that we have finished is, compute the possible values of B kilt for both the time periods.

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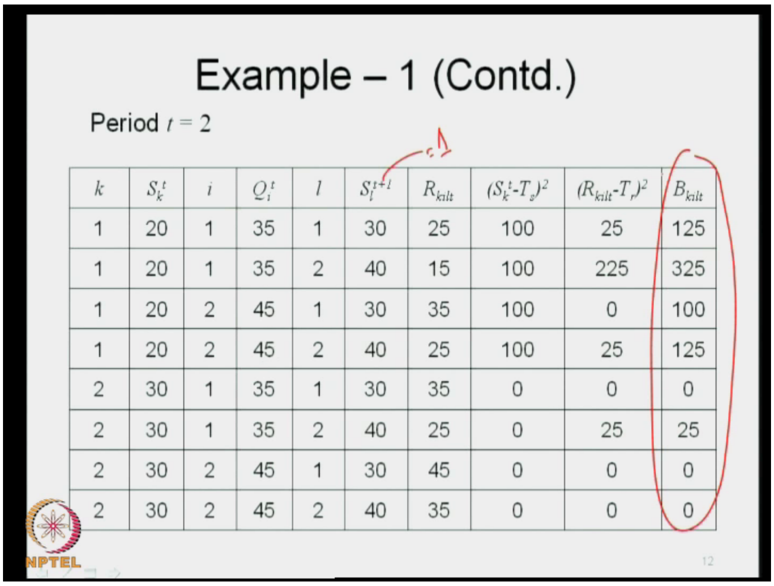

k	S_k^t	i	Q_i^t	l	S_l^{t+1}	R_{kilt}	$(S_k^t\text{-}T_s)^2$	$(R_{kilt}\text{-}T_r)^2$	B_{kilt}
1	20	1	35	1	30	25	100	25	125
1	20	1	35	2	40	15	100	225	325
1	20	2	45	1	30	35	100	0	100
1	20	2	45	2	40	25	100	25	125
2	30	1	35	1	30	35	0	0	0
2	30	1	35	2	40	25	0	25	25
2	30	2	45	1	30	45	0	0	0
2	30	2	45	2	40	35	0	0	0

So, this is the first step, we will use these B kilt values in actually solving the stochastic dynamic programming recursive relationship. So, that we will continues example in the next class, so essentially, what we did in today's class is to write the general recursive relationship for the stochastic dynamic programming. We start at a time period farther into their future, the last time period in the year and progress in the backward direction; each time writing the expression for the system performance measure, which connects the immediate system performance measure for the current time period with the solution that you have obtained for the previous stage, and which includes the solution until until the last stage, last time period where the first stage. So, like this, we related from one time period to another time period.

Remember that the difference between what we did in stochastic dynamic what we did in stochastic dynamic programming today, and what we are done earlier in the deterministic dynamic programming for the steady state policy is that, we are talking about the expected value of the system performance measure, because there is the probability associated with the inflows, and this probability, we are… With this random nature of the inflow, we are accounting for through the assumption of the single step Markov chain; and therefore, we put their inflow transition probability, which essentially give the probability of the inflows going from one particular given state i in time period t to the state j in time period t plus 1, and these are the transition probabilities, which we define the expected value of the system performance measure.

When we solve this problem recursively, when we solve this equation recursively over the period of time, the annual system performance measure becomes constant after certain point, certain point in the computations and that is when we say that the steady state policy is reached, we are just started discussing a numerical example, in which the first step is to calculate the B kilt values, the data that you need are the inflow transition probabilities, the storage descritization scheme as well as the inflow descritization scheme for a given objective. So, we will continue the the numerical the discussion on the numerical example in the next class, thank you for your attention.