Water Resources Systems

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Module No. # 06

Lecture No. # 31

Stochastic Dynamic Programming for reservoir operation (1)

Good morning and welcome to this, the lecture number 31 of the course water resource systems modeling techniques and analysis. Over the last few lectures, we have been now discussing the explicit stochastic optimization techniques for specifically determining the reservoir size as well as obtaining the optimal reservoir operating policy. The only random variable we are considering is the inflow with known probability distribution, but because storage and reservoir release, both are functions of inflows, by virtue of inflow being a random variable, the storage as well as the reservoir release become random variables.

So, in the last two lectures, I discussed one stochastic optimization technique, the chance constraint linear programming. Recall that in the chance constraint linear programming problem, the constraints are expressed as reliability constraints, probabilistic constraints or the chance constraints. To make sure that we are able to use the linear programming, as well as to make sure that the inflow probability distribution information that we have is transformed to the release as well as the storage we use, what is called as the linear decision rule.

By using linear decision rule, we achieve two things. One is that, the probability distribution information on the inflows is transformed to release in most of the cases. The complete variability of the inflow is transformed directly to release without affecting the storage. Therefore, the storage is treated as the deterministic variable. All these we discussed in the last two classes and then, we form a deterministic equivalent and then, solve the (()) problem using the deterministic equivalent.

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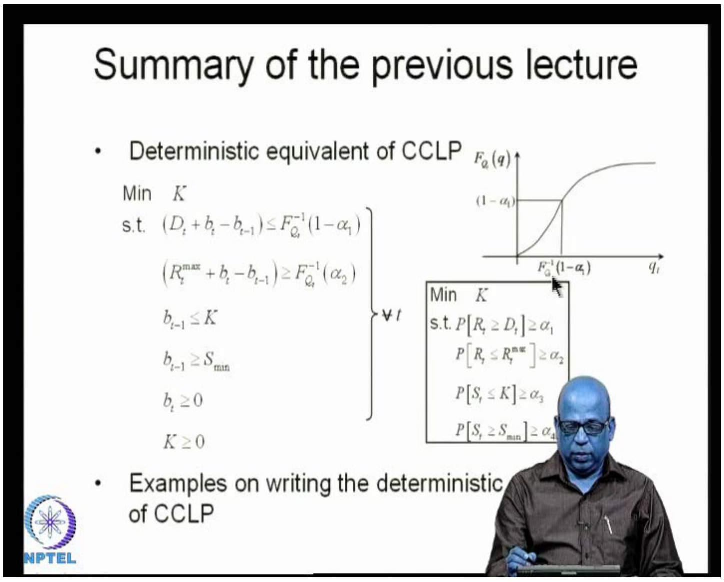

So, this slide shows how the deterministic equivalent that we discussed last time. So, this was your original problem with chance constraints on minimum release, maximum release, maximum storage and minimum storage. These are the chance constraints, and we obtain the deterministic equivalent using the probability distribution of the inflow during time period t. Recall that the inflow distributions will be different during different time periods, either in terms of the parameters or in terms of the complete distribution itself. There, we also examined some examples of writing the deterministic equivalents of chance constraints in the previous lecture.

Now, in today's lecture, we will introduce an important topic of stochastic dynamic programming. In fact, for reservoir operation decisions, indeed for the steady state operating policy, stochastic dynamic programming is ideally suited. It is an ideally suited technique. Recall that in the topic on dynamic programming, we discussed the steady state policies or the stationary policy keeping the inflows as deterministic in that particular case. You may refer to the lectures on dynamic programming, on reservoir operation in fact, and then, see how we obtain the steady state policies.

Now, what we will do is, we build on that dynamic programming except that the inflows are no longer deterministic now. We will treat the inflows as stochastic. In fact, we treat the inflow sequence or the inflow process as the stochastic process and then, incorporate the probability information on the inflows into the optimization model. So, the stochastic optimization or the stochastic dynamic programming technique that we are going to introduce now is an explicit stochastic optimization technique because we incorporate the probability distributions directly into the optimization model itself. So, the optimization model itself incorporates the probability distribution, includes the probability distributions.

In the course of this lecture, we will also introduce an important concept of Markov chains and the assumption of Markov chain is what we use in building the stochastic dynamic programming. Much similar to the deterministic dynamic programming, we will be discussing the discrete states stochastic dynamic programming. That means the state variables will take on only the discrete values as we discussed in the deterministic dynamic programming ok.

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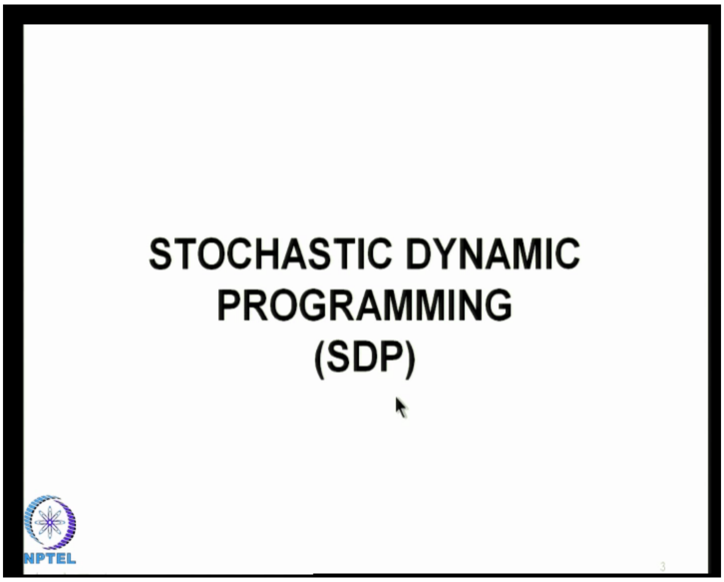

We will start with the basics of stochastic dynamic programming. So, this is an important topic that I will be introducing now. Perhaps over the next 1 or 2 lectures, I will be covering some examples related to this. Today, we will discuss the foundation of the stochastic dynamic programming as well as write the general recursive relationship for SDP. Typically, we are introducing the SDP for a reservoir operation problem, where we are dealing with a reservoir of known capacity and the historical data on inflow is known and given this, we are looking ahead into the future, and deriving what is called as the steady state operating policy for that particular reservoir taking into account the uncertainties associated with the inflows and the uncertainties associated with the inflows, we model through what is called as the Markov chain which I will introduce presently.

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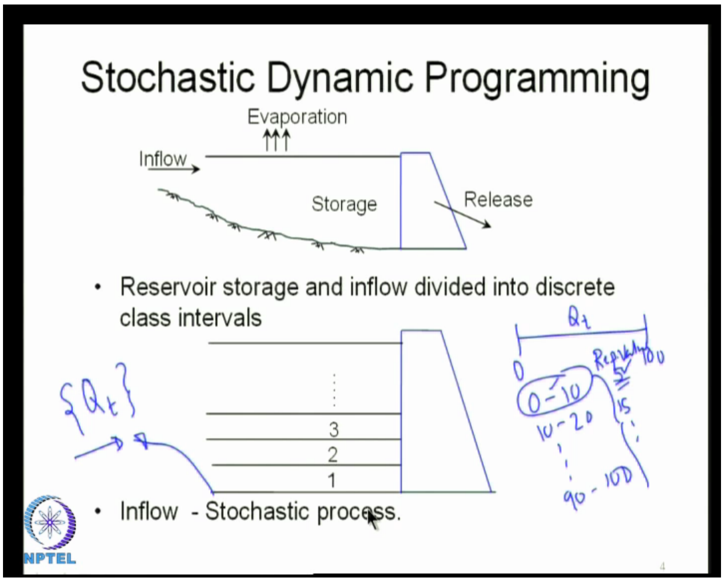

So, if you look at the reservoir, now again we will recall the storage is influenced by the inflow and the release and the evaporation. So, all of these are interrelated. There is an inflow that is taking place and then, corresponding to the area of water spread, there is an evaporation that takes place. Then, you are making release and therefore, the storage also changes. So, all these variables are interrelated. Now, in this, now the inflow process is a random process because it is governed by natural variability of rainfall and other process is such as infiltration and then, also your over land flow and so on.

So, there are large numbers of random process that govern the inflow. So, inflow is a random variable. Because inflow becomes a random variable, the storage and the release will also become random variables as we have discussed in the chance constraint LP. In the stochastic dynamic programming much like what we did in the dynamic programming or deterministic dynamic programming. We treat the storage as well as the inflow as discrete variables which means, they can take on only discrete values and therefore, the inflow values are divided into a number of discrete classes.

For example, you may have an inflow range of let us say, 0 to 100. I mean divide this range of for Q t into 0 to 10, 10 to 20 etcetera, like this 90 to 100 into several, into number of classes, 10 classes, which means what? If my inflow is anywhere between 0 and 10, I will say that the inflow belongs to this particular class interval and so on. Similarly, the reservoir storage itself is also divided into a number of discrete class intervals between the 0 storage, and the maximum storage. We divide the storage into a number of class intervals. If it is less than a certain value, I will say it belongs to the class interval 1. Between this value and this value, I will say it belongs to class interval 2 and so on up to the maximum capacity.

So, both inflow, as well as reservoir storage, we divide them into a number of discrete classes. Further, each of these classes will have a representative values. So, we have the concept of a class interval, and the representative value for that class interval. Typically, we may take the representative value to be the mid-point itself. Say for example, we may have 0 to 10 as a class interval and the class representative value may be 5. 10 to 20 is a class interval and class interval may be 15 and so on.

So, typically we use the mid-points of a particular class as the representative value for that particular class. The representative value indicates or we use the representative value as the value for the inflow belonging to that particular class interval which means, even if your inflow is 2 units here, as long as it belongs to this class interval, we use the inflow value to be 5. Even, if it is 9.5, we use 5. Even, it is 0.5, we use 5. So, the representative value is the value that we use to represent the particular class interval, similarly for the storage. So, we have the storage class intervals and the associated representative values.

Now, these are the two important concepts that we must be aware of before we proceed to formulate the SDP problems. Then, we treat the inflow as stochastic process, which means the inflow process which we denote it as Q t is a stochastic process. What we mean by that is, let us say you have last 50 years of data and the last 50 years of data is, it forms a time series and that process are, it forms the stochastic process and that is the process we modulate using some of the assumptions that we follow, but the inflow process is essentially treated as a stochastic process.

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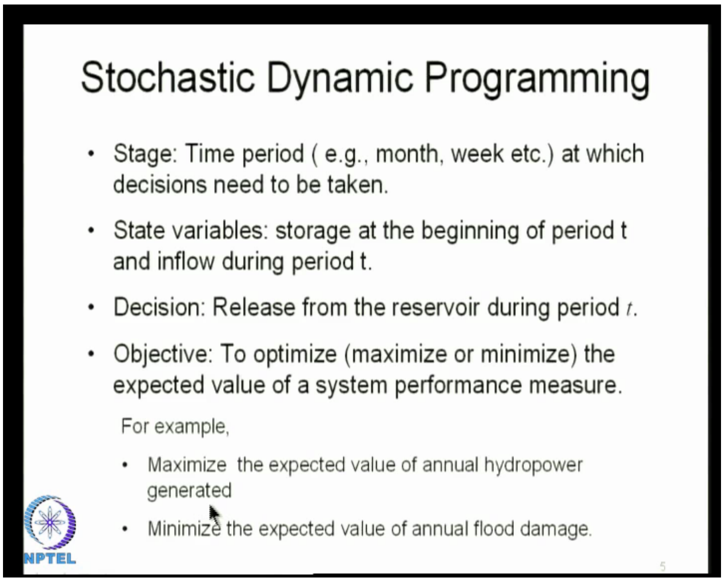

So, these are the three important concepts. One is that the inflow as well inflow is divided into a number of class intervals. Each class interval has its own representative value. Storage is divided into a number of class intervals. Each storage class interval has its own representative value and the inflow is treated as a stochastic process. With this now, we will start formulating the dynamic programming problem, in fact the stochastic dynamic programming problem. Much similar to what we did in the deterministic dynamic programming, we consider the stage of the dynamic programming as a time period in which the decisions are necessary.

For example, you may be operating the reservoir for monthly time periods. Then, a month is the stage, June month, July month, August month etcetera. That becomes a stage. Again I refer to the deterministic dynamic programming development for reservoir operation. So, go through the stationary policy that we derived through the deterministic dynamic programming. We are just building on that. So, there also we define the stage to be a month or a time period during which the decision is made. Now, these typically may be of the order of then daily time periods, monthly time periods, seasonal time periods, and so on. Then, the state of the system now in your earlier deterministic dynamic programming exercise that we did, we said the state of the system is completely defined by the storage at that particular time period because we were treating the inflow to be deterministic. That means, the inflow value is given and it was not changing in the deterministic case.

In the stochastic dynamic programming case, because the inflow varies randomly, you cannot hold the inflow to be constant. Therefore, the state of the system at a particular stage is defined by both the storage as well as the inflow. So, we start with state variable definition as the state variables will be. I repeat that. The state variables at a given time period are given by the storage at the beginning of the time period t, that particular time period and the inflow during the time period t.

Now, the inflow during the time period t was not considered to be a state variable in our earlier deterministic case, remember, but now because the inflows vary randomly, they constitute a stochastic process. We need to consider the inflows also as a state variable. So, we have the concept of a stage, we have the concept of the state variables, which means the state of the system at a particular time period t is completely defined. If you know the storage at the beginning of the time period t, and the inflow during the time period t, then we make the decision. So, we define the decision as the release made from the reservoir during the time period t.

So, we have the stage, we have the state variables, we have the decision. Then, we operate this system with a particular long term operation. I again emphasise the probabilistic methods that we are using for optimization or essentially to obtain the steady state policies. These are essentially long term policy because you have incorporated the probability distributions of the inflows into the optimization problem. Therefore, when you operate the system with the optimal operating policy derived from these techniques over a long period of time, the expected value or the objective function value gets optimized.

Therefore, when you are defining the objective function for the stochastic dynamic programming, we refer to the expected value of a system performance major. Now, the system performance major can be for example, it can be annual hydropower generated. If you are operating the reservoir for hydropower, it can be the expected value of the annual hydropower or it can be expected value of let us say, the flood damages control, control of flood damages and so on. So, when you are referring to the objective function as the or the performance major as annual flood damage, we may want to minimize the expected value of that or we are referring to the performance major as annual hydropower, we may want to maximize the expected value and so on.

So, typically, the objective function is stated as the expected value, either minimize or maximize depending on the system performance major, the expected value of the system performance major. So, the objective function for the stochastic dynamic programming will be typically minimize or maximize the expected value of the system performance major which is the function of the state variables. As you can see if you are looking at the hydropower, the hydropower will depend on the storage as well as on the release and the storage will decide the head that is available and therefore, storage, head, hydropower generated and the release are all related and the storage is being regulated because of the random inflow and therefore, all of these become random variables.

So, typically the system performance major set we are talking about in reservoir operation problems will depend on both the storage as well as the inflow. In fact, if you want to include more details, which perhaps in the application I will talk about. If you want to include more details, let us say at the irrigation feeds, if you want to look at the soil moisture also as one of the state variables, then the system performance major will also depend on the soil moisture if you are talking about the crop field optimization and so on.

So, the system performance major that we are now talking about will be system dependent. It will be dependent on the objective for which you would like to operate the system and typically, it depends on the storage as well as the inflow and the other state variables that may you want to use. The other state variables can be soil moisture in the command area, the rainfall in the command area and so on. So, depending on the type of systems that you have, you may want to add several state variables. In fact, while on the topic of the state variables, I can also mention that if you have a multi-reservoir systems, let us say a 3 reservoirs, 4 reservoir systems and you want to use the stochastic dynamic programming, then the storage at each of these storage reservoirs will become a state variables, the inflow at each of these reservoirs will become a state variable and so on.

So, look at the system and then, define the state variables, define your system performance major based on the objective with which you are operating the system, and the objective or the system performance major depends on the state variables. Also, the decision that we are making in the multi-reservoir system cases, you may want to make the reservoir release at each of the reservoirs as the decision variables. So, this is some of the things that you must remember before we go to the formulation.

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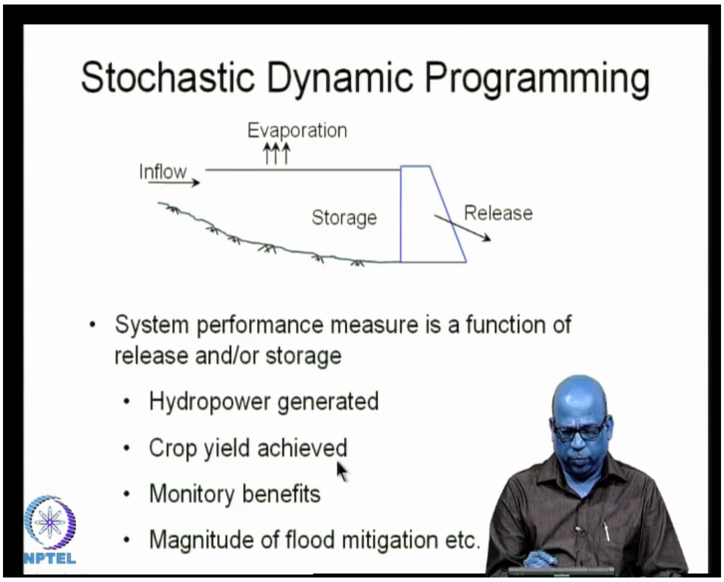

So, if you look at this, now again the same diagram, we may have the system performance major as function of hydropower generated or the crop yield achieved or monitory benefits or magnitude of flood mitigation and so on. So, depending on the type of function for which the reservoir is operated, you may form the system performance major.

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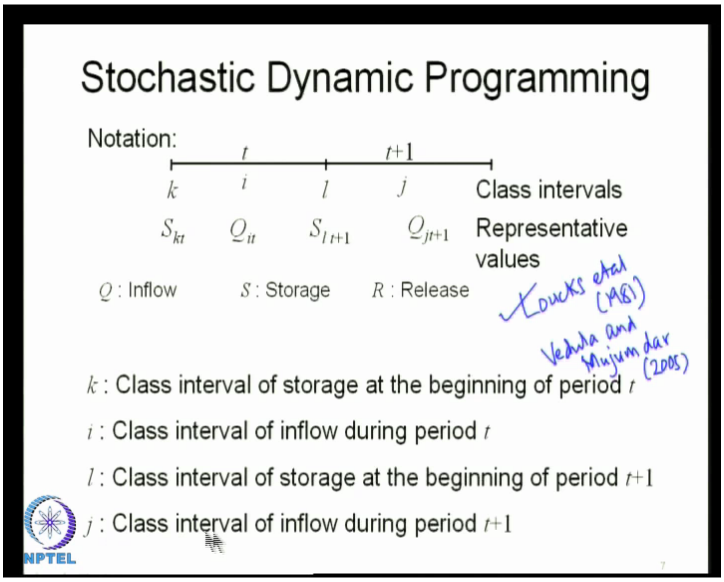

Now, we use certain notations and we will understand the notation carefully, so that the further development of the SDP becomes easy. I would encourage you to go through the classical books by Louck’s etal 1981. Now, this explains SDP at length and it is a classic book. I would encourage all the students to go through this. If this is not available, then at least you go through this book, Vedula and Mujumdar. This is 2005. This is the Indian book published by Tata McGraw Hill. So, both these books have explained this notation correctly and clearly. So, it is important for us to understand the notations clearly.

Now, keep in mind always that we are talking about storage in discrete class intervals, inflow in discrete class intervals because of which the evaporation that we determine will be in discrete class intervals, because of which a release that we determine will be in our discrete class intervals. So, we will take two time steps, adjacent time steps t and t plus 1. So, these are the two adjacent time periods t and t plus 1. With respect to this we will define now.

The storage at the beginning of the time period t is a state variable, the class interval to which the storage at the beginning of the time period t belongs is denoted as k. The representative value of that particular class interval is given, is denoted as S kt. So, this is the class interval and this is the representative value. We are talking about the time period t now. The inflow during the time period, where the storage is a point process at a particular point you indicate the storage. So, we are talking about the storage at the beginning of the time period t, whereas the inflow is the continuous process which happens all through the period t.

So, we denote the class interval to which the inflow during the time period t belongs as i. So, i is the inflow class interval for time period t. The representative value of that particular class interval i is denoted as Q it. Q is because we are referring to the inflow. Here we used S because we were referring to the storage, k is associated with S, i is associated with Q. Then, we come to the end of the period. The end of the period storage belongs to a class interval l. So, we denote by l the class interval to which the storage at the end of the time period t belongs, and the associated representative value of that particular class interval is denoted as S lt plus 1.

Why we use t plus 1 here? Because the end of the time periods class interval is also the beginning of the time period class interval for the next time period which is t plus 1. So, starting with S kt, we have come to S lt plus 1 which denotes the storage at the end of the class interval k, which is also equal to the storage at the beginning of the time t plus 1. I repeat that because I used k there. Starting with class interval k, the storage transforms into a class interval l because of the inflow that has come here and the release that we make and the evaporation etcetera. So, S kt which is the representative value of the storage at the beginning of the time period t transforms into S lt plus 1, which is the storage at the end of the time period t, which is also at the beginning of the time period t plus 1.

So, this is what you must understand correctly. So, k goes to l or S kt goes to S lt plus 1. Now, look at what is happening to the inflow. We have an inflow i here in the time period t denoted by Q it. This is the representative value, this is the class interval. In the next time period t plus 1, we denote this as class interval j. So, when we are in time period t, the inflow during the next time period, we denote it as class interval j. The class interval of the inflow, we denote it as j and the representative value of the inflow during time period t plus 1 is denoted as Q jt plus 1.

So, these are the important notations, k and S kt denote the storage at the beginning of the time period, k being the class interval, S kt being the representative value of the class interval, i and Q it denote the inflow during the time period t, i being the class interval, Q it being the representative value of the class interval, l and S lt plus 1 denote the storage at the end of the time period t or the beginning of the time period t plus 1. 1 being the class interval, S lt plus 1 being the representative value, j and Q jt indicate the inflow during the time period t plus 1, j being the class interval and again Q jt plus 1 being the representative value. Now, the representative values are typically chosen as the mid-points of the particular class intervals.

So, this is how we define and this definition you must keep in mind, understand correctly because all over subsequent discussion will be based on this particular notation. So, k is a class interval of storage at the beginning of period t, i is the class interval of inflow during period t, l is a class interval storage at beginning of period t plus 1, j is a class interval of inflow during period t plus 1 and these are the associated representative values.

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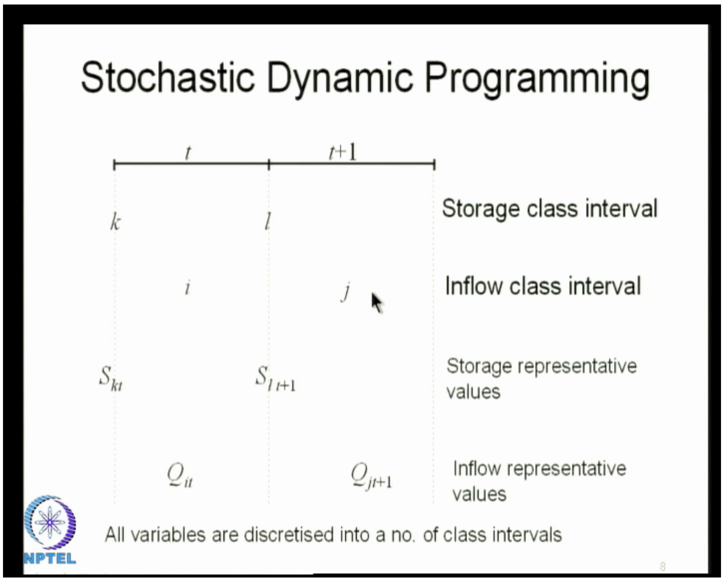

So, you understand this diagram. It is essentially summarizing what I just discussed. In time period t, you have the storage class intervals k and l and from k you are going to l. The inflow class interval in time period t is i, in time period t plus 1, it is j. The storage representative values will be S kt here, S l t plus 1 here and the inflow representative values Q it and Q jt plus 1. Now, these will have units of storage and inflow, whereas these are simply class intervals. For examples, these may be k is equal to 1, k is equal to 2 etcetera, i is equal to 1, i is equal 2 and so on, j is equal to 1, j is equal to 2 and etcetera, whereas S kt will be 100 units, 120 units and so on. Q it will be 500 units, 600 units and so on.

So, these are the inflow and the storage. These will have the inflow and the storage units, whereas these are simply class intervals. Now, all variables, especially the state variables now we are talking about will be discretized into a number of class intervals. How we discretize these is a slightly involved process, but we will right now assume that simply look at the historical data for the inflows and then, during each of the time period, you use your judgment and then, divide it into a number of class intervals. This will also have implications on the computational requirements and so on. We will see all those subtleties of the SDP as we progress. Now, with these notations, now we will start looking at the inflow which is a random variable. As I mentioned Q t is a time series, in fact Q t is the stochastic process and we make the assumption that the inflow follows what is called as the Markov chain.

Now, the Markov chain is an important assumption in the stochastic dynamic programming development. We will just understand what Markov chain is. You again refer to the basics of probability theory that we have covered and so on. So, we are now dealing with X t as the random variable. In fact, in our SDP, Q t becomes a random variable, but in general if X t is a random variable, then we define the Markov chain as a particular stochastic process. You must refer to the NPTEL course on stochastic hydrology in which Markov chains are discussed at length.

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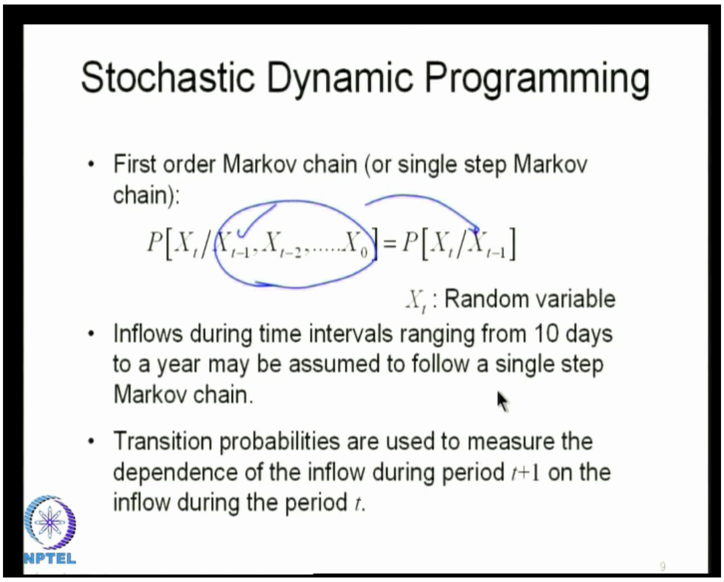

So, some background on Markov chain, but will use just the definitions now. You look at this definition. This is the conditional probability. If we can write probability of X t, given X t minus 1, X t minus 2 etcetera, X t minus 3, X t minus 4 and so on, X 1, X 0. If we can write this as equal to probability of X t, given X t minus 1. Now, this is the conditional probability. If we can write the conditional probability of X t, given the entire history of the process, that means, the complete history of the process is given by this. If we can write that as conditional probability of X t, given just the previous value X t minus 1, then it is called as a first order Markov chain or single step Markov chain. Single step because we are saying that X t in some sense depends only on X t minus 1 and not on the entire history, which led to X t minus 1.

So, in some sense, the memory of the process is limited to only what has happened during the previous time period and not in the periods before that. So, that is the idea here and in general, inflows, for reservoir inflows typically ranging from 10 days to 1 year. For example, 10 days, 15 days, we may have 1 monthly operation or seasonal operation etcetera; these can be assumed to follow Markov chains. That means, the inflows during such intervals can be assumed to follow Markov chain, but for a more regress validation, what you may do is, you can plot the correlogram.

Again refer to the stochastic hydrology course if you are any student is more interested in this topic. You need to plot the correlogram and then, in fact for single step Markov chain, the theoretical correlogram must be exponentially (( )). That means rho k which is the auto-correlation at k will be given by rho 1 to the power k, the first auto-correlation to the power k. Right now do not worry too much about it. What we will do is we will assume that the inflow during the time period, inflow Q t, which is the stochastic process, constitutes a single step Markov chain. That is an assumption that we used.

Now, once we make that assumption, we introduce the important concept of the transition probabilities and the transition probabilities are the single most important requirements for the Markov chains. Let us look at the transition probability now, and understand the transition probabilities correctly.

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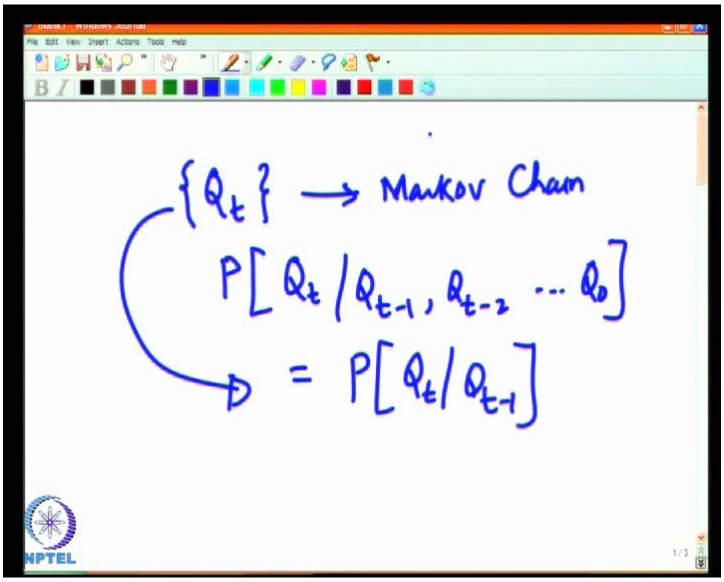

So, what we are saying now is that Q t, which is an inflow during the time period t is a stochastic process and it follows a Markov chain. Therefore, we are saying now probability of Q t, given Q t minus 1, Q t minus 2 etcetera. Q 0 is given by probability of Q t, given Q t minus 1 and this is a requirement of single step Markov chain.

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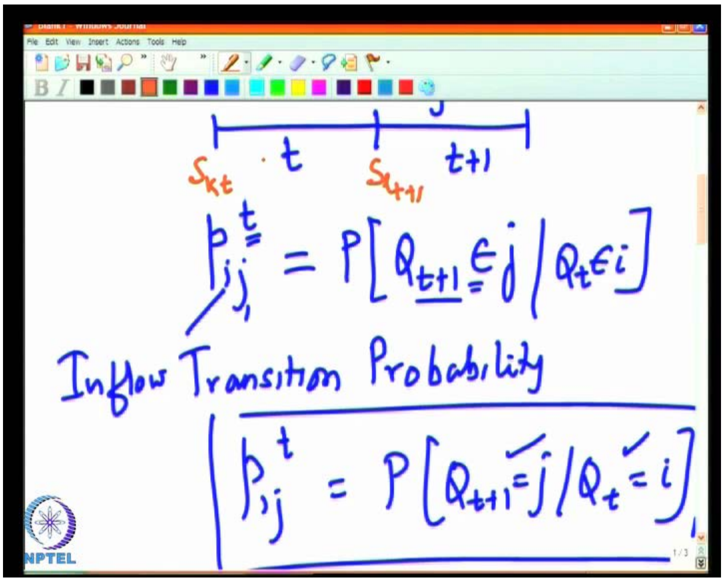

Now, if you consider two time periods, two adjacent time periods, let us say, we write t and t plus 1 here. From our notation what we said? We said i is the inflow class interval in time period t and j is an inflow class interval in time period t plus 1 and because inflow is a random process, there is a probability associated with the transition of the inflow from class interval i in period t to class interval j in period t plus 1. This is what we define as the transition probabilities P ij t. So, P ij t is the probability that Q t plus 1 in time period t plus 1. The inflow in time period t plus 1 first will say belongs to class interval j, given that Q t in time period t belongs to class interval i. So, the superscript t that I am using here indicates a transition between t to t plus 1.

So, we are writing for the transition from the current time period t to the next time period t plus 1. You understand this again. Our notation is P ij t. That means, starting with class interval i in time period t, it goes to class interval j in period t plus 1. That is a notation here and this is given by probability of Q t plus 1 belonging to a class interval j, given that Q t belongs to class interval i. That is a definition of transition probability. So, this is called as transition probability and because we are referring to the inflows, it is also called as; it is also generally denoted as inflow transition probability. Further, this belonging to sine we delete and then, simply say that, for simplicity we say probability of Q t plus 1 is equal to j, given Q t is equal to i. So, this notation equal to, it indicates that it belongs to the Q t plus 1 is in class interval j, given that Q t is in class interval i.

So, this is the notation that will be using. Now, this is an important concept because this is what relates the inflow during time period t to inflow during time period t plus 1. Now, you look at what is happening to storage. So, this is how the inflow transitions are governed from i to j in period t to t plus 1. The inflow goes from i to j with a probability of P ij t. Now, what happens to the storage? The storage at the beginning of this time period was known. Let us say, this was S kt. This is the storage and it has gone to S lt plus 1. How this transition is governed? Now, this transition is governed by simply the storage continuity or the mass balance that we use to start with a particular storage, you add some inflow and then, you take out the reservoir release and then, you end up with the end of the period storage.

So, both the inflow as well as the storage which are in fact the state variables, we know how the transition takes place from one time period t to the next time period t plus 1. We will use this now and then, formulate the SDP recursive relationship. So, we use the transition probabilities to major the dependents of the inflow during period t plus 1 on the inflow during period t. That means, the probability that the inflow transits into a class interval j in the next time period t plus 1 starting with the inflow being in class interval i in period t. That is a transition probability.

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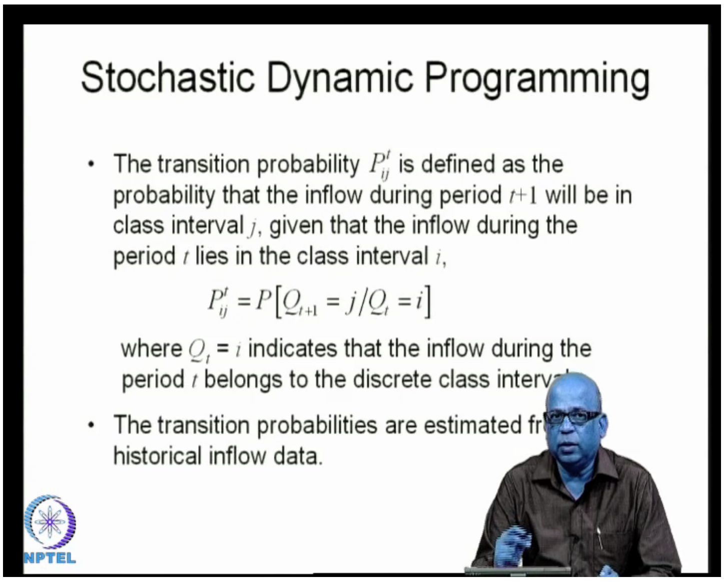

We denote as I just now mentioned P ij t, which as the transition probability as probability of Q t plus 1 equal to j, given Q t equal to i. Remember equal to i by this notation, we indicate that the inflow during that particular time t belongs to the class interval i or belongs to class interval j as the case may be.

Now, we are talking about the inflow transition probabilities typically at a reservoir side we will have the major historical flows. We use the major historical flows to estimate the transition probabilities by the relative frequency approach. We will see how these are estimated in fact from the historical data, may be through simple examples or (( )) later on, but remember that the historical data for each of the time periods, adjacent time periods use, put them side by side and then, look at from which class interval it went into which class interval in the next time period. Then, based on a relative frequency approach, you can determine the P ij t.

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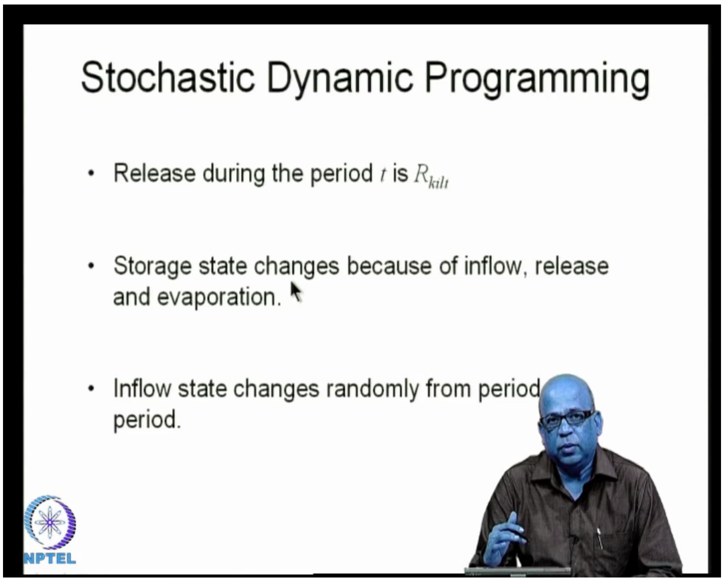

Right now we will not worry too much about it, but just know that the historical data that you have in fact can be used to determine the inflow transition probabilities. The storage state now changes because of the continuity. That means, you have the release made and you also have the evaporation that is taken place, you have on inflow. So, starting with a particular storage, you may have now this storage continuity.

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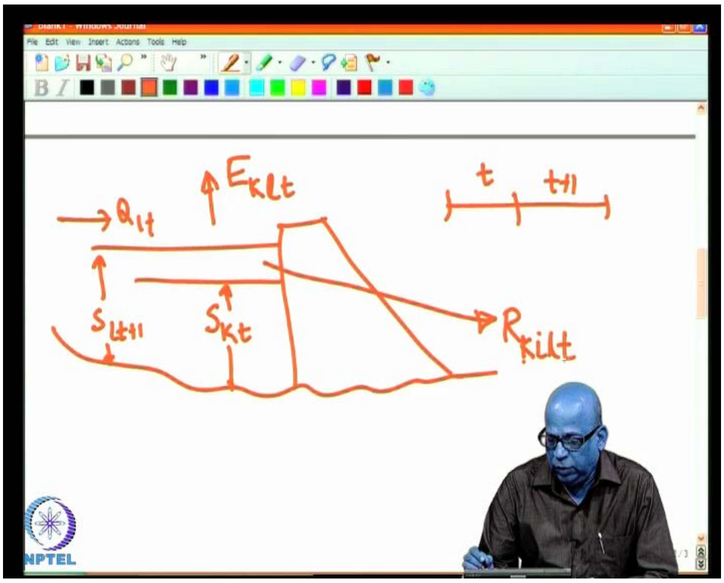

Again you need to understand correctly the way we write in SDP. It is slightly different because we are talking about the class intervals and the representative values. We have the storage S kt which is the storage at beginning of the time period t. It goes to storage S lt plus 1 in time period t plus 1. We are talking about two adjacent time periods t and t plus 1.

Now, this transition or this transformation of the storage from S kt to S lt plus 1 depends on how much release you have made and the release that you have made will be a function of k, which is the initial storage, i which is the inflow during the time period t l which is the end of the period storage and the time in the time period t. So, we denote the release as R kilt indicating that, the release here is the function of the storage k or the storage class interval k, the inflow i, the end of the period storage l and the time period t.

So, the continuity here will and then, you also have the evaporation losses. We will indicate the evaporation losses by e. It depends on the initial storage k, the final storage l and the time period t. So, the evaporation loss will have three subscripts, k, l and t. Why does it depend on k and l? Because we are wakening the evaporation loss based on the average storage during the time period t, which will depend on the storage at the beginning of the time period t, and the storage at the end of the time period t. Therefore, it depends on k and l and we are indicating that we are referring to time period t.

So, these are the notations that we use for the variables. R being a function of kil and t, e being a function of ekl and e being a function of k l and t. Now, that is what we are using you know. So, the release during the time period t is denoted as R kilt. Now, the storage changes because of inflow release and evaporations starting with S kt. It goes into S lt plus 1. We write the continuity equation and inflow state changes randomly because it is a random variable and then, it is governed by the transition probabilities. So, the inflow transition is governed by the inflow transition probabilities. The storage transformation is governed by the continuity.

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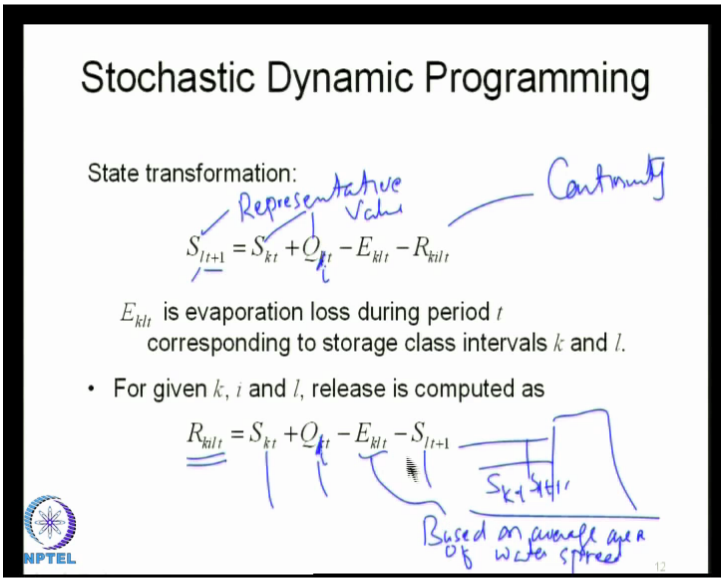

So, the state transformation now for the storage, we will write it as this is simply the reservoir continuity. Continuity but written in the discrete form using the class intervals. What is the S lt plus 1? It is not the actual storage. Remember S lt plus 1 is the representative value of the storage in the time at the beginning of the time period t plus 1. So, storage at the beginning of the time period t plus 1 belongs to a particular class interval and S lt plus 1 is the representative value of that particular class interval.

Let us say that the storage class interval to which the storage belongs is l is equal to 3 and l is equal to 3 may have a, will have in fact a particular representative value of let us say, 500 million cubic meters. So, this is the representative value of the storage class interval l in time period t plus 1. So, this is the representative value. Similarly, all these are representative values.

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So, S kt also is a representative value. This is Q it here. Q it is also a representative value. So, I make the correction. This is Q it, this also a representative value and E klt is determined based on k and l and R klt can be defined based on kil in time period t. So, this is just the storage continuity. What we are saying is storage at the beginning of the time period t, which is storage at the beginning of time period t plus 1 is equal to storage at the beginning of time period t plus 1, the inflow during the time period t minus evaporation loss that has taken place during the time period t minus the release during the time period t. All of which are functions of the time period t and k and i, both of these, that is E as well as R or functions of k and l and R in addition is also function of the inflow.

So, this is the storage continuity equation and this represents the state transformation as far as storage is concerned. Now, just so that we do not lose sight of what we are doing in the dynamic programming problem, that is the deterministic dynamic programming, remember what we did. We examined for various values of storages in a particular time period what is the optimal release that is necessary. This is what we examined. Exactly the same thing we do here. For a given storage k and an inflow i because our state variable, state vector will consists of two variables, now k and i. For a given initial storage k and a given inflow i, what should be my release. R kilt is the question that we are asking and therefore, we must be able to write the continuity in terms of R kilt.

So, we will write the same continuity equation to determine R kilt is equal to S kt plus Q it again. This is Q it minus E klt minus S lt plus 1. So, this is how we determine R kilt. For a given k, for a given i, for a given l, we are determining R kilt that is the idea here. That means, we are making search and then, in making the search, we pre-specify k, we pre-specify i, we pre-specify l, and for that we are determining the release.

The idea here is that for a given k and i, what should be my optimal l which determines R kilt and what should be my optimal release is the question that we are asking. As we solve the examples, as we write the recursive relationship, this becomes much more clear and therefore, for a given k, i and l, we should be able to determine R kilt using this expression. Now, there is E kilt sitting there in the continuity equation. E kilt is the evaporation loss associated with the beginning of the periods storage S kt, which has the particular area of water spread and the end of the periods storage S lt plus 1, which has an area of water spread associated with this and we reckon the average area during the time period t. Then, based on the average area, we get the loss, average area of water spread.

This is exactly what we did earlier. Except that, now we are regaining with respect to the discrete class intervals k and l and therefore, the evaporation loss also will take on only discrete values associated with k and l in time period t. So, for a specified k and for a specified l, knowing i, we should be able to determine R kilt and we also determine evaporation. This will be based on the average water spread and then, we relate it with rate of evaporation and so on and then, obtain this as (()). All of these are in (()).

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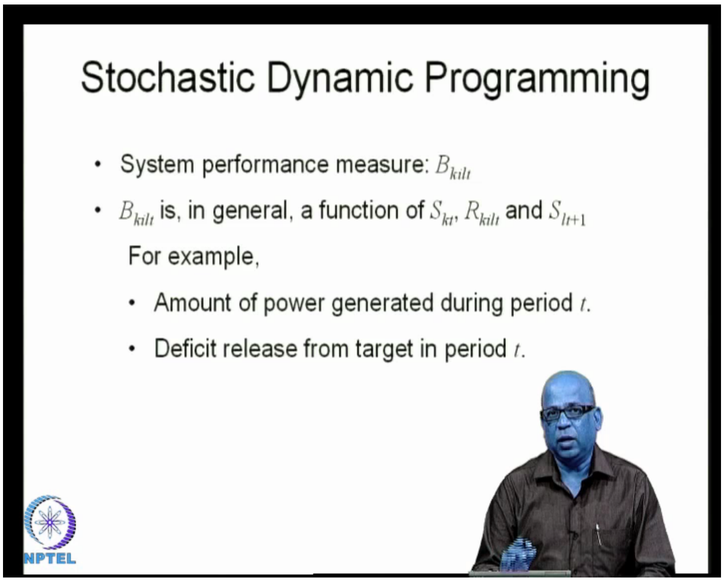

Now, we come to the system performance major. Like I said earlier, the system performance major will depend on the state of the system and you know the particular objective. For example, if you are looking at the maximization of hydropower. Now, if you are looking at maximization of hydropower in time period t, the system performance major will depend on the initial storage k, the final storage l because both of these will determine the R kilt, which is the release as well as on the inflow i because that is what determines your storage transformation from k to l.

So, in general, the system performance major is also denoted with all the 4 indicators, indices, that is k storage at the beginning of the time period, i which is the inflow during the time period, l which is the end of the time period storage and the time period t itself. So, the system performance major, we denote it as B kilt and B kilt is in general a function of S kt, R kilt and S lt plus 1. For example, if you are looking at amount of power generation during the time period t, it will obviously depend on the head as well as the release. Therefore, it depends on both, S kt, S lt plus 1 and also it depends on the inflow during the time period t. Therefore, we indicate the system performance major as B kilt and t.

Similarly, if you are looking at deficit release from a target in period t, so you are looking at R kilt. How high or low is R kilt from with respect to a particular specified target release. Therefore, it will also depend on all the 4 indicators there. Therefore, we indicate it as B kilt. For a given combination of k i and l in period t, we must be able to determine B kilt because we would have specified the objective function or the system performance major. For example, we may be saying that we would like to have the maximum expected value of the hydropower generated. So, in time period t, if I know the storage, if I know the inflow and if I know the end of the period storage, I should be able to get the hydropower that is generated in time period t. That is the idea there. So, B kilt can be determined once k i and l are fixed in time period t.

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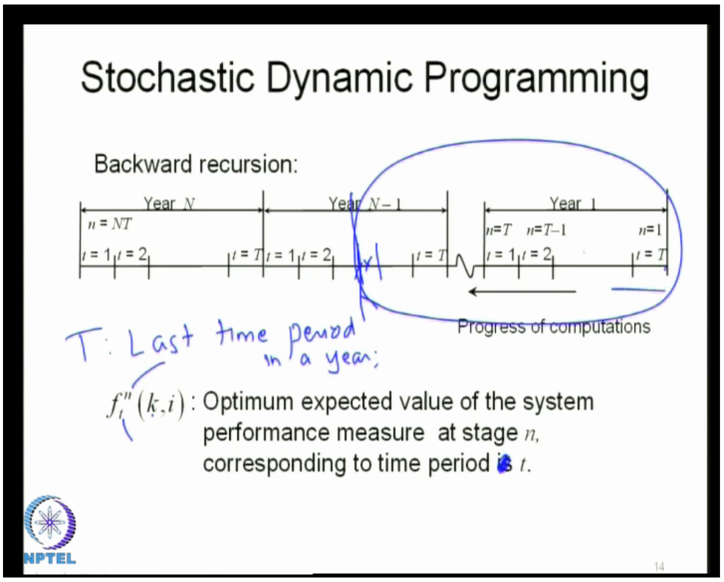

With all these notations now, we will start writing down the recursive relationship much the same way as we did for the stationary policy using the dynamic programming. We start far into the future and then, proceed backwards. We start from a period far into the future and then, proceed backwards until the steady state is reached. How we identify the steady state? It will come to presently it is much similar to what we have done for the deterministic case. So, we start at some distant time and we denote the last time period in a year. T is last time period in a year. For example, if you are looking at monthly operation, capital T will be 12 and the small t, excuse me, small t varies from 1 to 12 in that case. We use the notation N to indicate the stage in the dynamic programming.

So, n is equal to 1, 2 etcetera goes up to t. We are proceeding in the backward direction. So, t is equal to capital T, t is equal to capital T minus 1, capital T minus 2 etcetera. It goes up to 1. In the year 2, n keeps on increasing, n is equal to t plus 1, t plus 2 etcetera, whereas t will again be capital T, capital T minus 1 etcetera. So, the small t that I have used here will keep track of the time period within the year. So, it cannot be anything other than t is equal to 1, 2 etcetera up to t, whereas the small n here keeps track of the stage in the dynamic programming. It will be progressively increasing. As you proceed further, it will be progressively increasing.

So, if you do it for n years now like this, the n will be equal to n into capital T because there are so many time periods there. So, with that now, we define the system performance major. This is a notation that we have used earlier in dynamic programming. There we had only one state variable. Now, we have two state variables. So, understand this notation f t n for a given k, i. We define this as the optimum expected value of the system performance major at stage n corresponding to time period t. So, we are standing in any particular stage, let us say here and then, we have come backward in the backward direction up to this point. We have come up to this point.

So, the expected value of the system performance major, when we have come from n is equal to 1 up to this point is called as, is denoted as f t n k i. That means, n is the stage in the dynamic programming, t is the corresponding time period, k is an initial storage and i is the inflow. For a given initial storage and the inflow in time period t, the expected value of the performance that has been obtained up to that particular stage is called as, is denoted as f t n k i.

So, with these notations now, we will start writing the recursive relationship. We will start with exactly what we did in the last, in the dynamic programming case. We will start with a last time period, write the equation for that, then go to the next time period, next stage, relate with what is happening, what has happened during the previous stage and so on, and then progress in the backward direction until we reach the steady state. Now, this discussion we will continue in the next class.

So, essentially today's class we have introduced the important concept of the stochastic dynamic programming and we are talking about the stochastic dynamic programming for reservoir operation. We introduce the notations that we use, remember k i l are the important notations that we have introduced. K is the storage class interval at the beginning of the time period t or k is the class interval of the storage at the beginning of time period t. So, k refers to time period t always and l is the storage at the end of the time period t, which is also storage at the beginning of the next time period which is t plus 1. So, l refers to t plus 1, i is a inflow class interval and j is the inflow class interval in time period t plus 1, i is in time period t, j is in time period t plus 1 and we have the associated representative values.

So, each of these state variables, namely inflow as well as storage are discretized into a number of class intervals. Each of the class intervals has the representative value and the representative value will have the units of the variable. Whereas, the class intervals will be simply class intervals 1, 2, 3 and so on. With that, we also introduced the notation for release R kilt as well as the system performance major B kilt. With these notations in place, we used the dynamic programming algorithm and proceed in the backward direction to obtain the steady state operating policy.

There is an important concept of Markov chain that we have used in the process, we will be using in the process and I just defined what the Markov chain is. If we can write for a random variable X t, if we can write probability of X t, given X t minus 1, X t minus 2 etcetera X naught as equal to probability of X t, given X t minus 1, which means the entire memory of the process is limited to what has happened during the immediate previous time step. We call it as a single step Markov chain. So, we will use all these concepts in formulating the SDP problem for reservoir operation. We will continue that discussion in the next lecture. Thank you very much for your attention.