**Water Resources Systems**

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**Module No. # 06**

**Modeling Techniques and Analysis**

**Lecture No. # 30**

# Chance constrained Linear Programming for reservoir operation and design (2)

Good morning and welcome to this the 30 th lecture of the course, Water Resource Systems: Modeling Techniques and Analysis. In the last lecture, I introduced the concept of chance constrained linear programming. Recall that, when we treat the randomness in the inflows, we can no longer write the constraints in the deterministic form, because the inflows are random, we have the probability distribution associated with inflows and these probability distributions; we have to explicitly include in the optimization problem.

We are talking about the explicit stochastic optimization and the chance constrained linear programming problem is a optimization problem that belongs to the class of E S O or the Explicit Stochastic Optimization. And therefore, the constraints such as R t greater than or equal to D t, for example, the release being greater than or equal to demand, can no longer be written in that form, because the release is also a function of the inflows and be inflows being random, the releases will also be random. And therefore, any constraint that contains release will also have to be written in the form of the probabilistic constraint or the chance constraints.

And then, we went on to discuss the deterministic equivalent, because once you have the chance constraints; you cannot use the algorithms, the simplex algorithm etcetera straight away, because the chance constraints have to be converted into their deterministic equivalent; and then, you use your simplex algorithm or whatever software that, you have for solving the linear programming problem on this deterministic equivalent.

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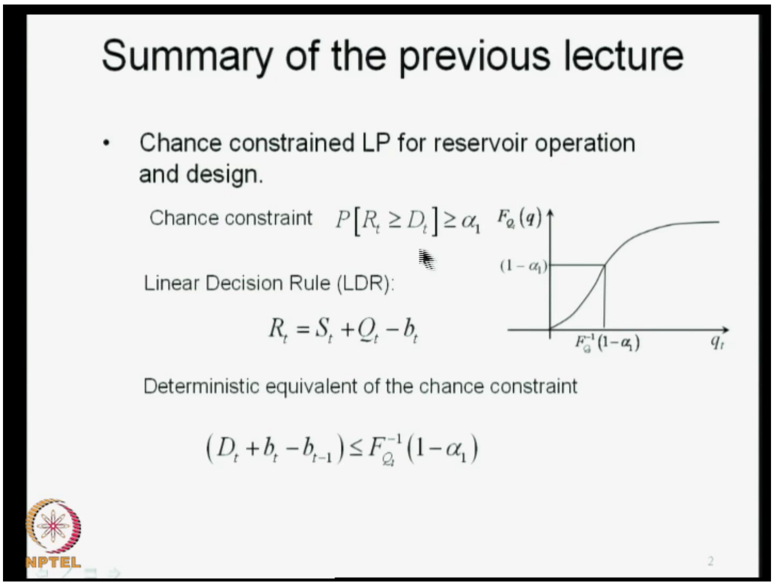


So, in the last class we checked, how we convert the chance constraint of the form; probability that R t greater than or equal to D t greater than or equal to alpha 1, this is the chance constraint. Your original constraint was simply R t greater than or equal to D t then, we are saying that to write it in the form of chance constraint, we are saying probability that R t greater than or equal to D t must be at least equal to alpha 1. So, you may specify alpha 1 to be 90 percent, 95 percent, and 80 percent and so on.

So, we are saying the probability that R t greater than or equal to D t is at least equal to say 90 percent say 95 percent and so on. And then, to convert this chance constraint into a deterministic equivalent, we used what is called as a linear decision rule, remember linear decision rule; we use mainly because we want to apply this chance constraints in the linear programming problems. And therefore, we want to have a linear relationship between the various variables that is the first reason.

The second reason is that, there are two random variables as you can see, R t is related to S t as well as Q t, the distribution of Q t is known, whereas the distribution of S t is not known by writing the linear decision rule, what we are achieving in a sense is that, the complete variability in Q t is transfer to R t straight away and without treating S t as a random variable, so that is what we achieve.

Refer to the previous class on details of this L D R; using such a L D R and using the distribution of Q t, we write the deterministic equivalent of this particular constraint as D t plus b t minus b t minus 1, the b t's are what are called as the decision parameters and these are deterministic by using such a form of the L D R we have also converted S t into a deterministic parameter recall in the previous class we said, S t is equal to b t minus 1 is what we get, as a result of this particular L D R.

Now, the deterministic equivalent of this constraint now, this chance constraint was written in this form D t plus b t minus b t minus 1; D t is the demand during the time period t, which is known, b t and b t minus 1 are decision parameters and these are deterministic. And the right hand side, F Q t inverse 1 minus alpha 1 is the particular value of the flow associated with a c d f of 1 minus alpha 1; so that is the interpretation of F Q t inverse 1 minus alpha 1. So, fix 1 minus alpha 1 and then, pick up the associated value of Q t on this particular c d f and that is what gives you the right hand side value.

So, the right hand side value will be a known quantity, if you have the c d f, D t is a known quantity, which has been pre-specified, b t and b t minus 1 become the decision variables, they are the decision parameters. So, this is how we converted the chance constraint into a deterministic equivalent, like that we converted all other chance constraints also.

For example, the the one associated with the maximum release; the one associated with the minimum storage, the one associated with the maximum storage etcetera, all these constraints we converted into deterministic equivalent. So, we will now write down the complete deterministic equivalent of the original problem.

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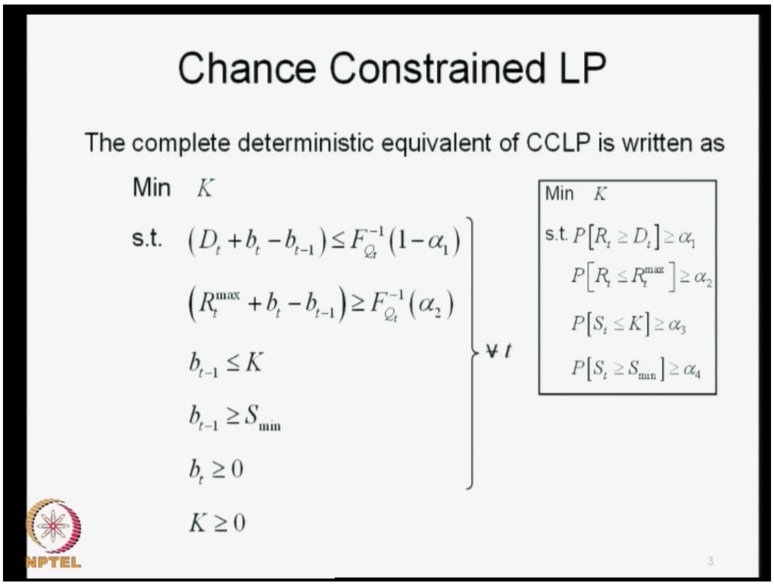


Now, the original chance constraint problem was this, we were looking for minimize K subject to probability of R t being greater than or equal to D t greater than or equal to alpha 1, probability of R t being less than or equal to R t max greater than or equal to alpha 2, probability of S t being less than or equal to K greater than or equal to alpha 3 and probability of S t being greater than or equal to S min greater than or equal to alpha K alpha 4.

Now of course, when we are using it in a linear programming form, then non-negativity constraints will also come in. So, this we converted into an equivalent deterministic equivalent; we say minimize K subject to this condition now has been converted into this constraint D t plus b t minus b t minus 1 less than or equal to F Q t inverse 1 minus alpha 1, this constraint is converted into this, now the constraints containing S t will become straight away deterministic, because S t is equal to b t minus 1.

We said that, the random variable S t is in fact, treated as a deterministic variable b t minus 1 and that is why these constraints, which are related with S t; they become deterministic straight away. So, we do not write alpha in terms of alpha 3 and alpha 4. And these are the non-negativity constraints, because b t is the decision variable, K is the decision variable, they are non-negative.

Now, there are certain small things that you must remember here, not really small, quiet important, but certain issues; one is we said these are alpha 1, alpha 2 etcetera, these are the probabilities. For example, we are saying that, probability of R t being greater than or equal to D t must be greater than or equal to a certain level of probability we may say it is (( )) and so on. We can change these probabilities with respect to t. So, I may write this as alpha 1 t, for different time periods, I may specify different reliabilities.

For example, if you are looking at a monthly time of monthly operation you may say that, the probability of meeting the demands during the deposit periods must be high or during the periods in which the demands have to be met are these are the critical periods you may want to place the reliability to be high of the order of (( )) and so on. But, there may be certain other periods, where you can afford to relax a bit and therefore, you can specify the reliabilities to be smaller. So, like this the reliability levels here, the probability levels can be different for different time periods that is the one thing, the second issue is, that we said b t is greater than or equal to 0.

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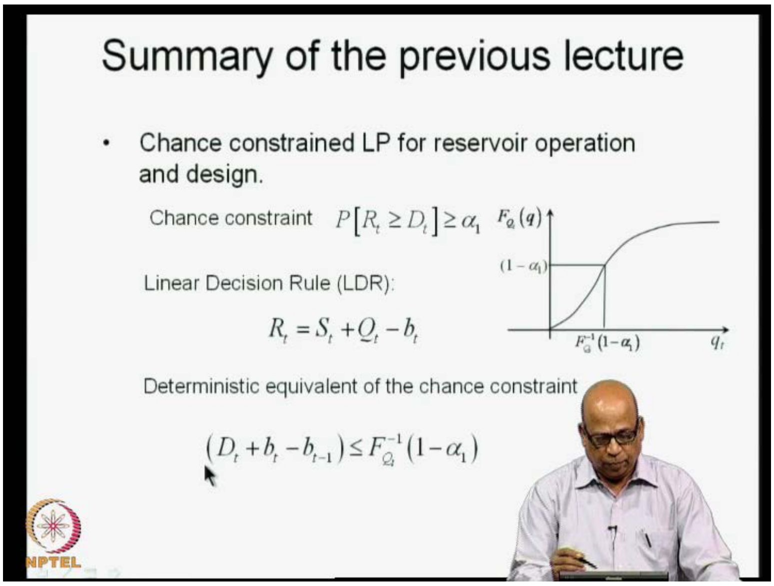


Look at the L D R; we have use this particular L D R, R t is equal to S t plus Q t minus b t, what we are saying here is that, the release is a function of the total water available S t plus Q t, we are ignoring the evaporation in all this discussions, it can be readily incorporated, but for clarity and simplicity, we will for the time being we will not worry about the evaporation. So, this is the total water available S t plus Q t. And you are saying that, out of S t plus Q t I will retain b t and then, release the remaining amount that is the idea there.

And therefore, b t has to be non-negative here, because there is a total water available, S t plus Q t out of which we are retaining certain amount and then, releasing the remaining amount, but there are certain other forms of the linear decision rule in which you can make b t to be non-negative also, you can allow b t to take values not to non-negative to to take negative values also, which means you can allow the b t to be unrestricted in sign. We will see one of the one of such a linear decision rule in the today's class.

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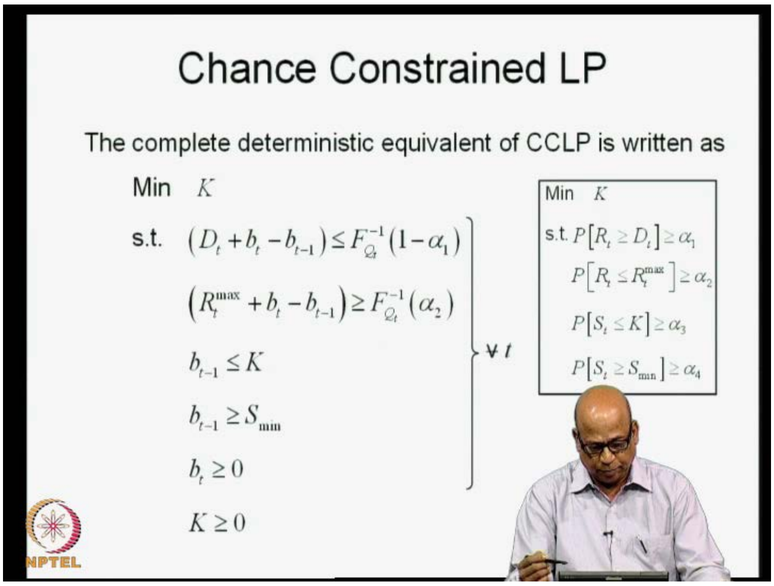


So, these are the two things. So, similarly, you may have S min as a function of t and then, alpha 2 will be a function of t and so on. So, you can have all of these changing across time periods of course, K cannot change with time period, we are talking about the capacity of the reservoir. So, that is one value, not to changing with respect to time. Now, this becomes the deterministic equivalent, you can solve this problem using any of the l p solutions.

So, you have to specify alpha 1, specify alpha 2, specify D t; which is the data demand, specify R t max, specify S min and then, you will be able to solve this example. These are constraints written for all time period, if you are talking about a monthly operation, you will have twelve constraint associated with each of these twelve constraints associated with each of the sets of constraints that I have written here.

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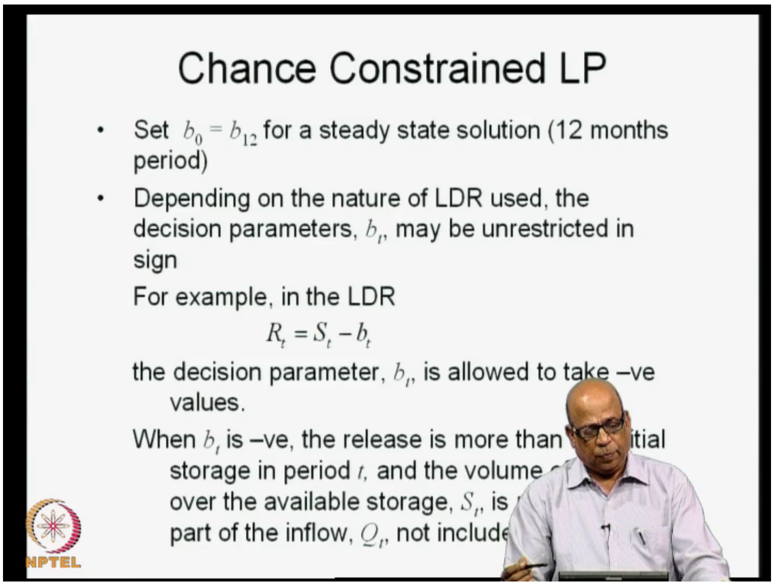


Now, when we are doing this problem, when we are solving this problem for a 12 month period; the last time period for example, when we are writing for t is equal to 1, you have a b t minus 1 here. So, when t is equal to 1, you will write this as b 1, when I will I will write that again what we are saying there is, so when t is equal to 1 , it becomes b 0 and b 0 will set it as b 12. So, remember this that, you have t is equal to 1, t is equal to 12 and then, this is t is equal to 1. So, the time period previous to t is equal to 1, is t is equal to 12, so we will write it as, b 12 that is one thing. So, for steady state solution we write it as, b 0 is equal to b 12 then, as I mention you can also set b t to be unrestricted in sign.

For example, you may take R t is equal to S t minus b t, this is the another form of the L D R; in which we are not accounting for Q t, all we are saying is from the available storage you retain b t and release only S t minus b t. Now in this case, b t can be made unrestricted in sign; which means that, b t can be allow to take negative values.

What happens, if a particular b t turns out to be negative in your optimal solution? It would indicate that, R t is equal to S t plus b t with that value of b t which means that, you are releasing more than what is available in the storage. How do we how is it possible? It is possible, because you are not accounted for the inflow. So, part of the inflow should take care of this negative value of b t. So, in the form of L D R that we are using, R t is equal to S t minus b t we can allow b t to be negative.

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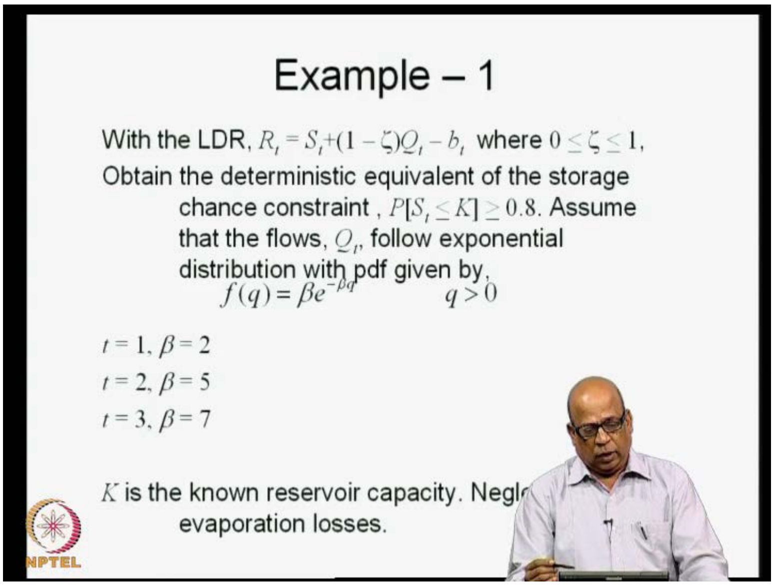


Now, what we will do is, we will solve a numerical example, these are all concepts that we have covered, we will solve a numerical example and then look at, how we obtain the reservoir capacity, when we incorporate all the constraints that involve either Q t directly or R t and S t, which are functions of Q t as chance constraints we form them as chance constraints and formulate the deterministic equivalent of that and then, solve the linear programming problem, that is what we will do now.

So, the first example that I will consider now is, that we will take the L D R as; R t is equal to S t plus 1 minus zeta Q t minus b t with zeta varying between 0 and 1. So, recall in the last lecture, I explained the general nature of the L D R; where we are accounting for a proportion of Q t in the linear decision rule. So, Q t is the inflow we are saying that, in making the decision R t, we will look at some proportion of the Q t, because zeta varies between 0 and 1 we may say that, zeta is equal to 0.5 lets say which means that, we are saying 50 percent of the Q t is taken into account while making the decision R t. So, that is the idea.

So, we use this L D R now and will obtain the deterministic equivalent of this constraint, probability of S t being less than or equal to K greater than or equal to 0.8. As I have been mentioning, Q t being the random variable that is the inflow during time period t being the random variable, we need the distributions of Q t, that is the C D F or the P D F from P D F you can always obtain the C D F in several cases. So, we need the distribution. So, in this particular case, we will assume Q t to follow an exponential distribution with the P D F, I will write it here more clearly f of q is equal to beta e to the power minus beta q, so this is the exponential distribution.

So, we will assume that in all the time periods Q t follows an exponential distribution, but the parameters are different, this has the single parameter beta. So, in time period t is equal to 1; beta is equal to 2, t is equal to 2; beta is equal to 5 and so on, so 2, 5 and 7. So, the beta parameters are given for these three time periods and therefore, the f of q is completely define for all the three time periods.

Now, this is the chance constraint we are writing; now K is a known reservoir capacity in this particular case. So, K will keep it as constant or deterministic and this right hand side, which is the alpha level or the reliability level is fixed, which is given as 0.8 and we will neglect the evaporation loss. So, for this problem, for each of these time periods, we must be able to write the deterministic equivalent.

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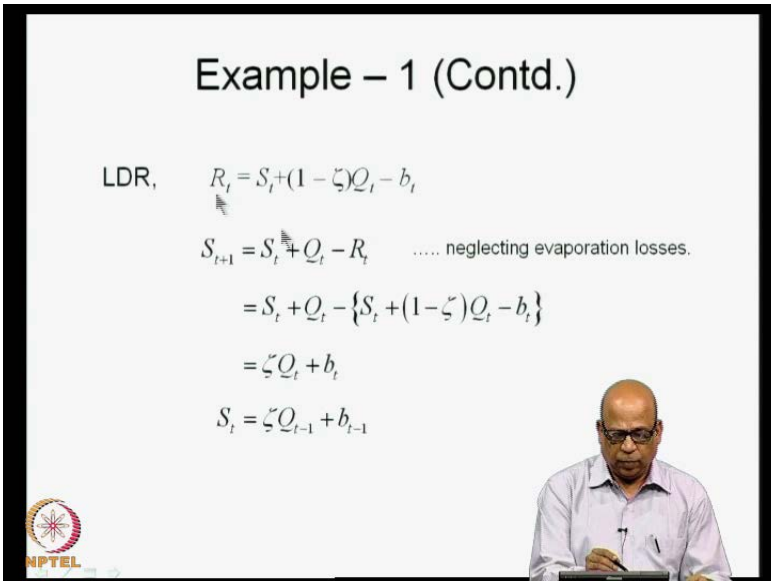


The L D R here is specified as S t plus 1 minus zeta Q t minus b t, we will use the continuity equation, this is the continuity equation by when we neglect the evaporation loss, so S t plus 1 is equal to S t plus Q t minus R t this is the continuity equation (Refer Slide Time: 17:45). So, I will use the L D R now, I will rewrite this as, S t plus Q t minus for R t, I will substitute the L D R and then, I get S t as t plus 1 as zeta Q t plus b t. So, S t plus 1 is this. And therefore, I will write S t as, zeta Q t minus 1 plus b t minus 1, because I am writing it for t. Now, zeta is a value between 0 and 1, this will be specified, when we are doing any application. So, right now written it as zeta, but remember that, zeta varies between 0 and 1. So, the L D R reduces to this form now.

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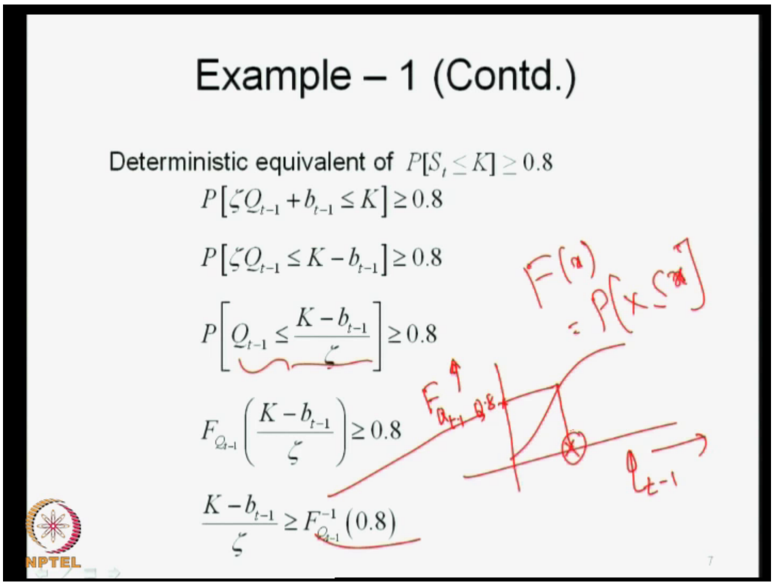


Now, we look at the deterministic we will formulate the deterministic equivalent of this, we are saying probability of S t being less than or equal to K is greater than or equal to 0.8. And S t is zeta Q t minus 1 plus b t minus 1. So, I will write this as, zeta Q t minus 1 plus b t minus 1, that is the probability of zeta Q t minus 1 plus b t minus 1 less than or equal to K must be greater than or equal to 0.8, this is what I have write.

Remember always that, you bring the random variable to the left hand side of the inequality, because you know the probability distribution of the random variable. Take out all the deterministic deterministic variables and the parameters to the right hand side of the inequality that is what we will do now. So, zeta Q t minus 1; Q t minus 1 is the random variable less than or equal to K minus b t minus 1 greater than or equal to 0.8.

Remember we are writing this for time period t, because we are saying S t must be less than or equal to K, but the movement we use this L D R, we have brought in b t minus 1 here, that is time period t minus 1. So, when we are writing the constraint for time period t, we are looking at the distribution of the inflow in the previous time period, t minus 1. So, these issues you must keep in mind.

So, I will write this as probability of zeta Q t minus 1 less than or equal to K minus b t minus 1 greater than or equal to 0.8 and I will write this as, probability of Q t minus 1 less than or equal to K minus b t minus 1 zeta greater than or equal to 0.8. Now, what is this term now? This term is, it reflects remember that F of x is equal to probability of x being less than or equal to x, so this indicates the C D F associated with this value.

Now, this value here is K minus b t minus 1 over zeta; K is deterministic, b t minus 1 is deterministic, zeta is the specified value and therefore, this is a deterministic value (Refer Slide Time: 20:44). So, we will write this as, F Q t minus 1 indicating that, we are talking about the distribution of Q t minus 1 of K minus b t minus 1 zeta greater than or equal to 0.8. Now, this we write again using our earlier notation, K minus b t minus 1 over zeta is greater than or equal to F Q t minus 1 inverse 0.8.

I repeat again because, students tend to make this mistake often what we mean by this is that associated with the F value of 0.8, you pick up the Q t minus 1 value, so these are the Q t minus 1 value and this is F Q t minus 1 on the y axis. So, this indicates that, it is that particular value of Q t minus 1, which leads to a F of F value of 0.8, that is the idea there.

So, this can be got, once the distribution is given. And this is is a known value or a decision variable, deterministic, this is a deterministic decision variable and this is specified therefore, this becomes a a deterministic equivalent (Refer Slide Time: 22:01).

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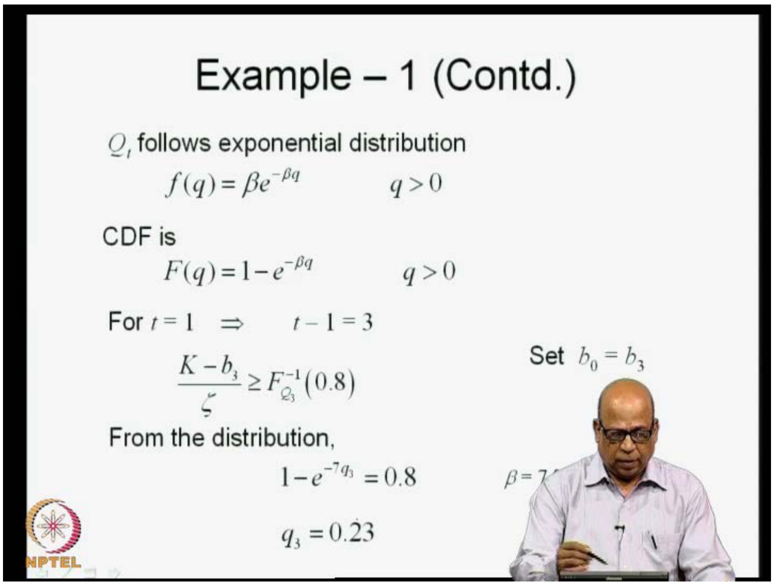

Now, it is given that, f of q is a exponential distribution. So, we have beta e to the power minus beta q, for q greater than 0 for all the time periods, but the beta is different for different time periods. Now, recall that, the exponential distribution has a C D F starting with this we can also integrate the C D F is simply minus infinity to x f of x d x this is F of x. So, in the previous lecture or lecture previous to that, we have discussed the exponential distribution. So, the F of q can be written as 1 minus e to the power minus beta q, q greater than 0.

Now, what is it that we are doing, we are we want to write this set of equations now, set of inequalities or the set of constraints, for all the time periods t. So, we need this value now F Q t inverse Q t minus 1 inverse 0.8, this is the value that we need to get this value we go to the distribution of Q t. Now, we have said that, Q t follows an exponential distribution in all the three time periods, but with different parameters, so beta 1 is different from beta 2 is different from beta 3.

So, we will write for t is equal to 1, now when t is equal to 1, the previous time period will be t is equal to 3, which is the last time period in the year. So, t is equal to 1 implies that; t minus 1 is equal to 3, this is the previous time period, when we are talking about the first time period. So, look at this expression now, K minus b t minus 1 zeta over zeta greater than or equal to F Q t minus 1 inverse 0.8 (Refer Slide Time: 24:07).

So, I will when I am writing for t is equal to 1, I will pick up these values corresponding to t minus 1 is equal to 3. So, I will write this as, K minus b 3 by zeta greater than or equal to F Q 3 inverse 0.8 by setting b naught is equal to b 3 like I explained just now, now which means for the distribution of Q 3 you go to 0.8, so we look at the distribution of Q 3 now.

So, you will look at F of Q 3 and then, go to 0.8 and then, pick up that particular value associated with 0.8 that is what you have to do. So, this is 0.8 we are talking about this particular value, which means from the F of q expression here you have the expression for F of q, you need F Q inverse 0.8 which means that, you are setting F of Q is equal to 0.8.

So, I will put this as 1 minus e to the power minus beta q 3 as 0.8 and beta for time period 3 is 7 that is the data that is given. So, beta is 7 for time period 3 and why I am using time period 3, because we are writing it for t is equal to 1, the previous time period t minus 1 is 3 and that is why I have to pick up for time period 3. So, 1 minus e to the power minus 7 q 3 is equal to 0.8 and therefore, q 3 turns out to be 0.23, which means this value here turns out to be 0.23 therefore, I write this as K minus b 3 by zeta is greater than or equal to 0.23.

Similarly, for t is equal to 2, when I am writing, it will be K minus b 1 divided by zeta greater than or equal to F Q 1 that is the distribution associated with the random variable Q 1 inverse 0.8 then, I apply the same thing and beta 1 is beta 2 is given as 5 and beta 1 is given as 2. So, I will write this as for time period 1, I will write it as 1 minus e to the power minus 3 q 1 is greater is equal to 0.8 and that you can verify that q 1 turns out to be 0.536. Similarly, for t is equal to 3, I do the same thing, I am referring to time period 2 then and then I use a distribution for time period 2 that is Q 2 and then, get q 2 is equal to 0.322.

And therefore, the deterministic equivalents for that particular constraint namely that, probability of S t being less than or equal to K greater than or equal to 0.8 for all t, that is t is equal to 1, t is equal to 2, t is equal to 3, this is the constraint that we are considering. The deterministic equivalents come to be K minus b 3 over zeta is greater than or equal to 0.23 and so on, this is what we obtained in the exercise. So, for one set of constraints like this, we write the deterministic equivalent during all the time periods, in this particular example we had three time periods. So, associated with each of these time periods, we write the deterministic equivalents.

Remember this is the known parameter here, this is known that is we would have specified zeta to be 0.5, 0.6 and so on, in the L D R and K is known or to be determine, but it the deterministic parameter b 1, b 2, b 3 have to be determine the deterministic parameters. And therefore, you are this constraint set is a is a completely deterministic you can use them in the linear programming problem. So, like this each set of constraints you convert them into deterministic equivalent and then, form the complete l p package, which will be a deterministic problem, you can use any of the software's to solve the problem.

We will now revisit our original problem which is we are looking at the minimum reservoir capacity associated with the chance constraints, subject to the chance constraints namely probability of R t being less than greater than or equal to D t greater than or equal to alpha 1 and so on. So, we will revisit that and solve a numerical example, so that you know from beginning to end, how to solve the problem. As I have been mentioning to solve the numerical example or to solve a chance constraint l p, you need the distributions of the Q t; Q t is the only random variable we are taking, but because Q t is the random variable, R t becomes the random variable; S t becomes the random variable.

And depending on the type of L D R that you use linear decision rule that you use you will get different forms of the deterministic equivalent. So, the first step is to see the to obtain to obtain the probability distributions of Q t, if you are talking about a 12 time periods, monthly time periods you should have the distribution of q 1, q 2, q 3 etcetera upto q 12, all these distributions may be different from one another or they may follow the same distributions, but with different parameters which essentially means that, the distributions are different. Then, there is a one first thing.

Second thing is you are saying that, probability of R t being greater than or equal to D t, so the D t set must be given, demand set must be given. Then, you are saying probability of S t being less than or equal to K; K is the decision variable which is the capacity this can be converted into deterministic equivalent as we did just now, like this whatever the constants that go into the problem must be determine (( )). So, we will solve an example now and then, after converting the problem into a deterministic equivalent, you should be able to use any of the l p algorithms, l p packages to solve the problem.

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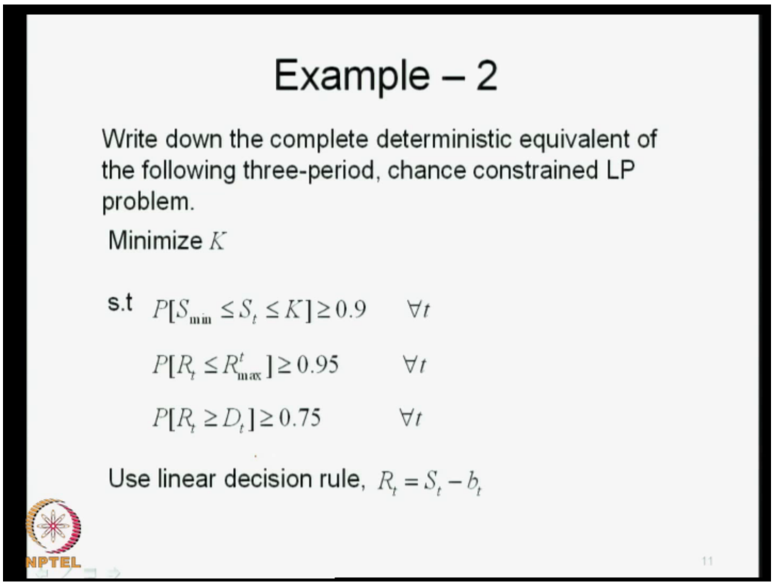


So, we will do a complete exercise now, starting with the original statement of the deterministic problem and then, converting this into the the chance I am sorry I will repeat that starting with a original statement of the Chance Constrained L P problems, CCLP and then, convert that into a deterministic equivalent, then use numerical values and then, solve the problem completely and interpret what it means.

So, this is our original problem that is, you are looking at the minimum capacity K subject to the S t; which is the storage at the time period at the beginning of the time period t must vary between S min and K. So, the probability of storage being in this range is greater than or equal to 0.9, this is the chance constraint, why we write it as chance constraint, because S t is the function of a Q t; Q t is the random variable, Q t is the inflow to our reservoir during the time period t, that is the random variable and therefore, S t becomes a random variable therefore, we write this in the form in this particular form in the chance constraint form.

Then, we have probability of R t being less than or equal to R t max; this is the maximum release during time period t must be greater than or equal to 0.95. Then, probability of R t being greater than or equal to D t, this is the demand during the time period t must be greater than or equal to 0.75, which means we are saying that, at least 75 percent of the time; my release is must be greater than or equal to the demands.

Now, depending on the type of a linear decision rules that we use as indeed we have shown in the earlier type of linear decision rule S t may turn out to be a deterministic parameter, but we will now see that it is in fact not so in general. So, we will use a particular linear decision rule of the form R t is equal to S t minus b t. The L D R that we used earlier was, R t is equal to S t plus Q t minus b t, but now what we will do is, we will simply say R t is equal to S t minus b t. So, we using this L D R, we will convert this chance constrained l p; now the way we are formulated, this is called as the Chance Constrained L P, CCLP this will convert it into a deterministic equivalent.

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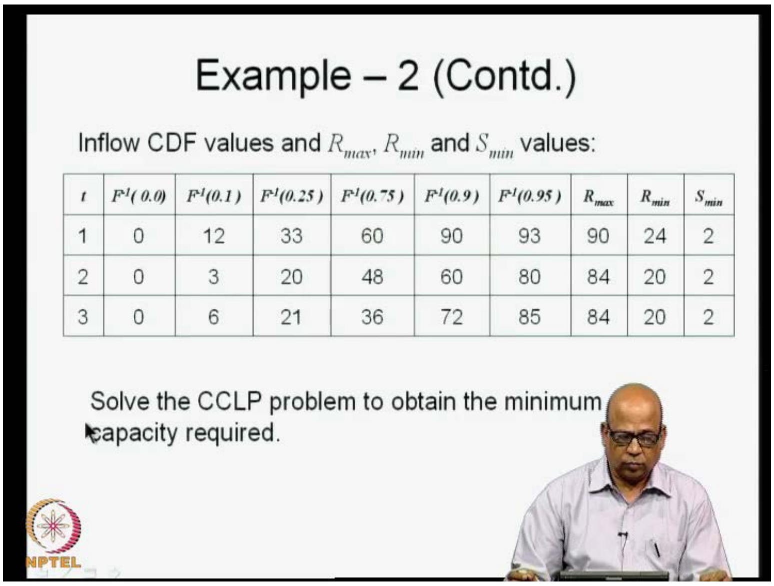


As I said for as to be able to convert the chance constrained l p into a deterministic equivalent, you need first the distributions of the flows, probability distributions of the flows. So, we have this here upto this point, the probability distribution values; that means, what does is indicate? That it indicates let say, for a particular time period you have a C D F and the values that we are giving here are associated with lets say this one 0.25, so I take 0.25 here and I come down this value will be 33 for time period t is equal to 1, this is 0.25, so this is what it means, F inverse 0.25 is 33.

F inverse 0.75 will be 60 let us say, 0.75 is here somewhere and then, this goes on like this. So, that is that is 60 and so on. So, this indicates for any time period t. So, I am writing it for time period t is equal to 1, these values are given now, which means the C

D F values associated with probability of 0.75, 0.25, 0.1,0.0 etcetera are given 0.9, 0.95 and so on are given.

Then, these are the constant values; R max, R min and S min, they appear here, S min appears here, R max appears here and D t is also D t is we have taken to be known, but I have not specified the D t values here, but S min that appears here is given and you also have R min; R min is the D t in fact, so R min is the minimum release which is not this one, this is the minimum release, this is I will write it as R min t minimum release or demand alright.

So, we need these values now namely D t; we need which is the R min given, R t max we need for each of the time periods, S min we need. So, these are the values that are given here. So, R max is given, R min is given; which is D t and S min is given. So, we have the probability distributions of the flows, we have all the constants that are necessary.

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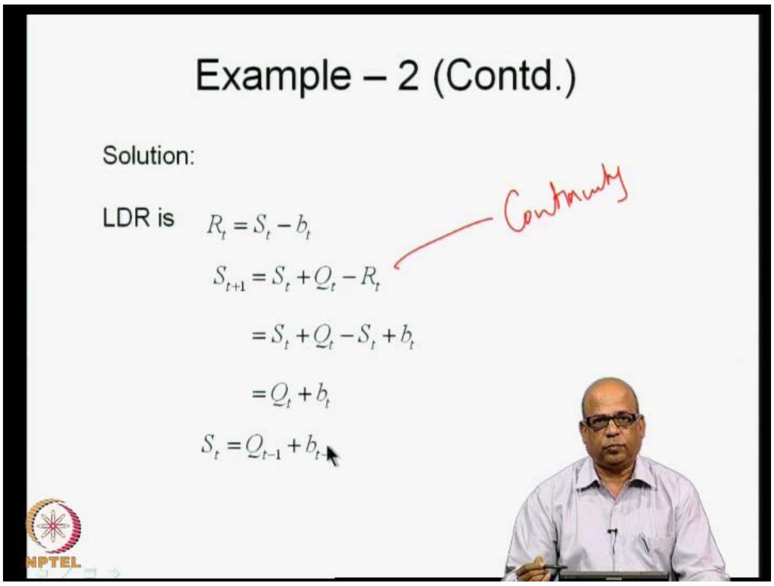


Now, we will go to convert into deterministic equivalent. Always we start with the linear decision rule, use the reservoir continuity equation and then, see how S t can be expressed in terms of Q t; Q t minus 1, b t, b t minus 1 and so on. So, we use the linear decision rule, which is in terms of R t with relates R t with respect to S t and Q t and Q t minus 1 with b t and so on.

So, we will start this R t is equal to S t minus b t and this this is the reservoir continuity. So, I will write this as, S t plus Q t minus R t; R t is S t minus b t. So, I write it as minus S t plus b t, so this turns out to be S t plus 1 is equal to Q t plus b t and therefore, I write S t is equal to Q t minus 1 plus b t minus 1.

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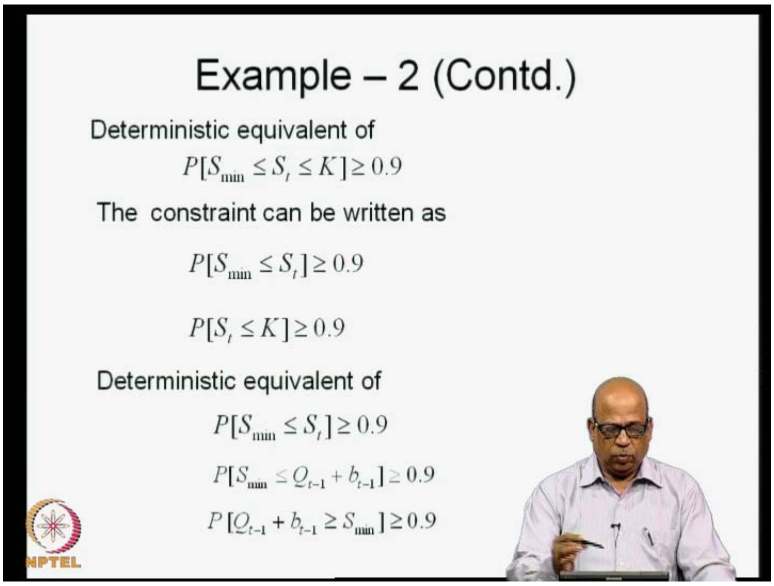


And then, we go onto write deterministic equivalents associated with each of the constraints; the first constraint here is written as probability of S t being in the range S min and K is greater than or equal to 0.9. Now, this will split into two constraints; one is we take the left hand side, so probability of S t being greater than or equal to S min is greater than or equal to 0.9.

The second one is probability of S t being less than or equal to K is greater than or equal to 0.9, which means we what we are saying is that, probability that S t is greater than or equal to S min as well as, probability of S t being less than or equal to K must be greater than or equal to 0.9. So, this is write as two different constraints.

Now, we will start with the first constraint now, we write probability of S min less than or equal to S t greater than or equal to 0.9, remember S min is known, this is known and S t is a decision variable in terms of Q t (( )). So, from here we have got S t as, Q t minus 1 plus b t minus 1. So, I write in terms in the place of S t, I write it in terms of Q t minus 1. Remember all the constraints, which contain the variables S t, R t etcetera, which are functions of the random variable Q t must be written in terms of the random variable Q t, because we know the distributions of the inflow Q t.

So, all these constraints have to be written expressed in terms of the inflows beat the Q t, Q t minus 1 and so on. So, you may depending on the type of L D R that you have used, you may get different terms associated with the inflows. So, here I am writing S t as Q t minus 1 plus b t minus 1. So, we bring the random variable to the left hand side of the inequality, so I write this as, Q t minus 1 plus b t minus 1 greater than or equal to S min greater than or equal to 0.9. Then, I will write this as probability of Q t minus 1 greater than or equal to S min minus b t minus 1, I have taken the b t minus 1 to the right hand side greater than or equal to 0.9.

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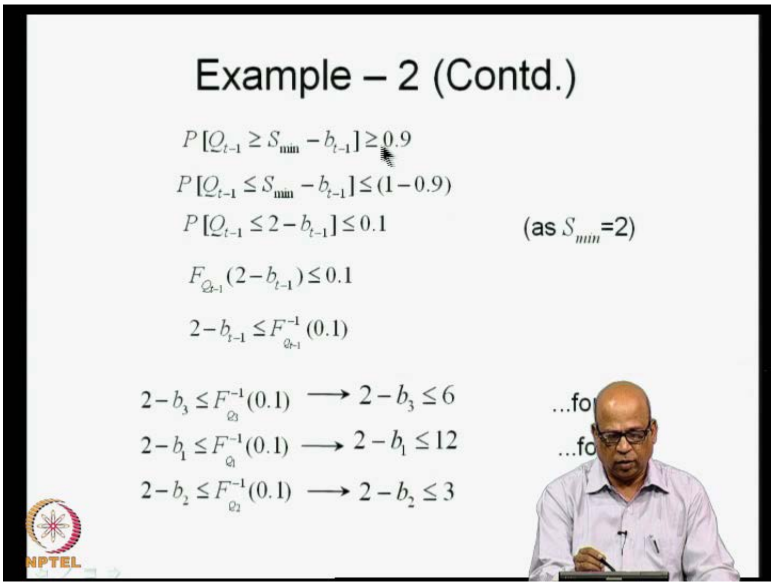


Then, I write this as probability of Q t being less than or equal to S min minus b t minus 1 less than or equal to 1 minus 0.9 I am changing the inequality sign inside therefore, this becomes 1 minus 0.9 refer to the last class, where I discussed this. So, because you change the inequality sign inside and you are talking about the probabilities, this will be this will change the sign as well as, this will be 1 minus of this value.

So, I will write this as, probability of Q t minus 1 less than or equal to S min is 2 minus b t minus 1 less than or equal to 0.1, now what is this now, this is probability of x being less than or equal to x, which means I will write this as, probability F of Q t minus 1 of 2 minus b t minus 1, this subscript Q t minus 1 indicates that we are talking about the distribution of Q t minus 1, 2 minus b t minus 1 less than or equal to 0.1 or we write this in our usual notation as, 2 minus b t minus 1 less than or equal to F Q t minus 1 0.1, now this is where we use the data on the distribution.

Now, this particular equivalent deterministic equivalent have to write for t is equal to 1, 2 and 3. So, when I write for t is equal to 1, your Q t minus 1 will be 3, because there are three time periods. So, Q t minus 1 will refer to q 3, R t minus 1 refers to 3 and we go to time period 3 and pick up that particular value associated with F Q t inverse 0.1. So, when I am writing for t is equal to 1, I am writing 2 minus b 3 less than or equal to F Q 3 inverse 0.1, now F Q 3 inverse 0.1 is given here in the table you just go there you are referring to time period 3(Refer Slide Time: 42:18) and 0.1, so this is 6.

So, that is what I pick up and write this as, 2 minus b 3 is less than or equal to 6. So, this 6 is obtain by distribution of Q 3 obtain from distribution of Q 3. So, this becomes the deterministic equivalent 2 minus b 3 less than or equal to 6. Now, I go to time period t is equal to 2, I am writing this expression for t is equal to 2 therefore, 2 minus b 2 minus 1, which is b 1 less than or equal to Q t minus 1 which will be Q 1 inverse 0.1 which means I go to the distribution of Q 1 and then, pick up F Q inverse of 0.1, this is given F Q inverse of 0.1, which is 12. So, I write this as, 2 minus b 1 less than or equal to 12.

Similarly, when I write for t is equal to 3, I pick up the value from distribution of t is equal to 2, which is 3 (Refer Slide Time: 43:22). So, I will write this as, t is equal to 3. So, like this the first constraint which is probability of S min less than or equal to S t greater than or equal to 0.9 is now converted into this set of constraints for t is equal to 1, t is equal to 2, t is equal to 3.

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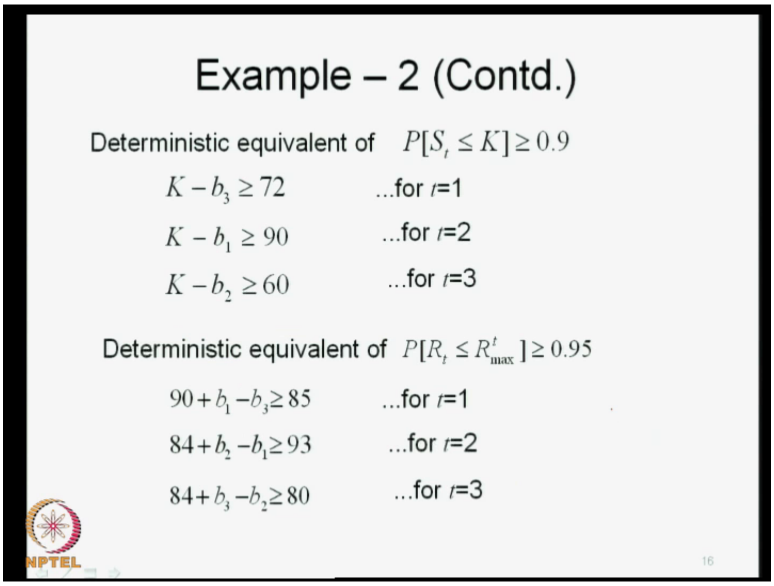


Similarly, we write this deterministic equivalent of S t less than or equal to K greater than or equal to 0.9, I leave it as an exercise, but this is what you get here. So, use the same logic and then, use the data, you will get K minus b 3 greater than or equal to 72, K minus b 1 greater than or equal to 90, K minus b 2 greater than or equal to 60 for t is equal to 1, 2 and 3.

Similarly, the deterministic equivalent of probability of R t being less than or equal to R t max greater than or equal to 0.95 this leads to 90 plus b 1 minus b 3 greater than or equal to 85, 84 plus b 2 minus b 1 greater than or equal to 93, 84 plus b 3 minus b 2 greater than or equal to 80, for t is equal to 3 just verify this.

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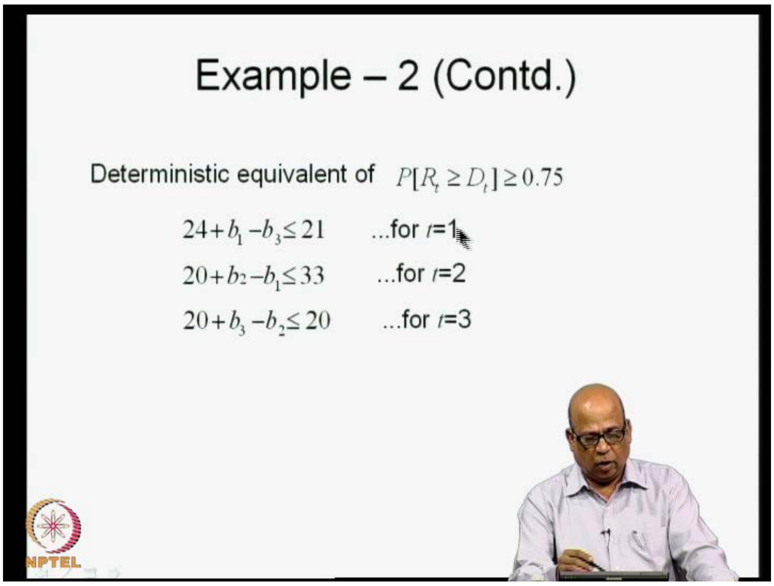


So, we have a written also for probability of R t being greater than or equal to D t greater than or equal to 0.75; this leads to 24 plus b 1 minus b 3 less than or equal to 21, 20 plus b 2 minus b 1 less than or equal to 33 and so on. We use the same procedure for each of the constraints, the probability constraints are expressed in there original form in terms of S t or R t. Using the L D R you express these probability constraints in terms of the random variable Q t, Q t minus 1, Q t minus 2 etcetera.

And then, using the associated distributions, probability distributions you can express them as deterministic equivalents. So, this is the general procedure. And that is how we obtained all these equivalent deterministic equivalents and then, you also put the data that is given to you and then, write them in terms of the numerical values.

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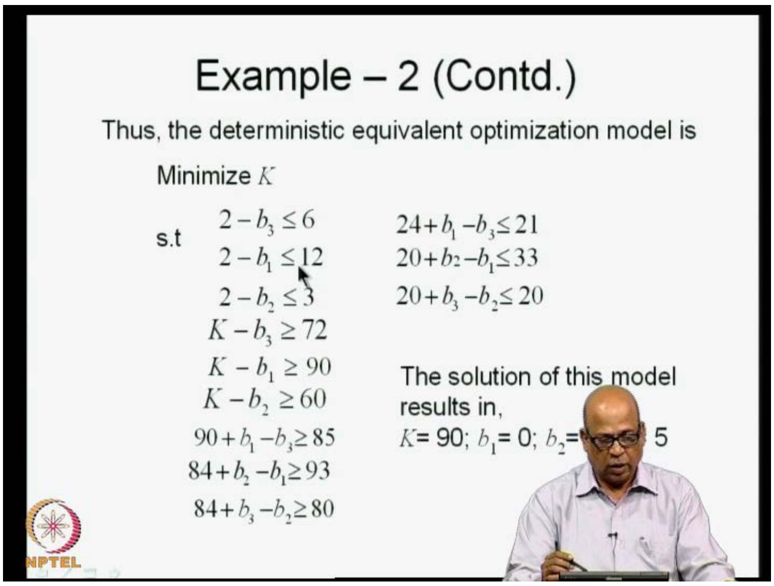

So, then if I write the complete deterministic equivalent; it looks like this, this is the set corresponding to the first set of constraints, this is second set of constraints, third set of constraints, fourth set of constraints (Refer Slide Time: 45:47). And then, you look at this we are looking at the minimize minimize value of K subject to all these constraints. So, what are the decision variables in this problem; the decision variables are b 1, b 2 and b 3 and K. So, you will obtain when you solve this problem, you will obtain the solution as K is equal to 90, b 1 is equal to 0, b 2 is equal to 9, b 3 is equal to 5. Now this is the solution that you obtain, when you solve this problem.

Remember the particular L D R that we used for this example is, R t is equal to S t minus b t, which means now the b t that you have used here can be left unrestricted in sign; that means, we you can allow the b t to also take negative values. So, I suggest you, do the problem both ways; first you take b t greater than or equal to 0; that means, do not allow the b t to take negative values and then, obtain the solution. You also allow the b t to take negative values and then, obtain the solution and compare the two solutions.

Remember when I when we write R t, the physical significant of this is, that when you write R t is equal to S t minus b t, you are you are being conservative, you are not allowing for the information content in Q t that is what is the likely value of Q t that is coming during the time period t is not accounted for when you are writing this. And therefore, when you allow b t to take negative values, you are saying that, R t can be greater than S t because, then it will become R t is equal to S t plus b t. So, R t can be greater than b t, in certain time periods; in which case you are allowing for part of the inflow to be also accounted for. So, that is the interpretation of this.

So, I encourage you to solve the problem on both with both conditions one; b t being non-negative and another b t being unrestricted in sign. And how to solve this problem I have discussed earlier, you can just solve the problem using both. Now, the optimal solution that you get here is K is equal to 90, b 1 is equal to 0 etcetera. Look at the optimal solution it says that; the minimum storage required is 90, 90 units and the associated values of b 1, b 2, b 3 b 3 are also given which means that, you need to operate this particular reservoir also in that particular form that is R t is equal to S t minus b t, where b t is specified.

Now, look at the storage S t minus b t is the release during that time period, so so you have the storage as well as, the release policy both built together in this optimal solution. Now, what does this mean this also means that, it is a a steady state solution in the sense that from time from year to year, you will you will adopt this particular policy of operation in that case, it becomes optimal in the sense that, you are able to meet the demands at least 75 percent of the time, you are able to meet the storage restrictions at least 90 percent of the time, 95 percent of the time etcetera, consistent with the probabilistic constraints that we stated in the problem.

So, that is the interpretation of the chance constraint optimization, remember just to summarize quickly the chance constraint optimization problem; in the chance constraint optimization problem, we specified the original constraints of the form R t greater than or equal to D t as, probabilistic constraints probability of R t being greater than or equal to D t greater than or equal to alpha 1 and so on. And then, we we obtain the solutions using the deterministic equivalents.

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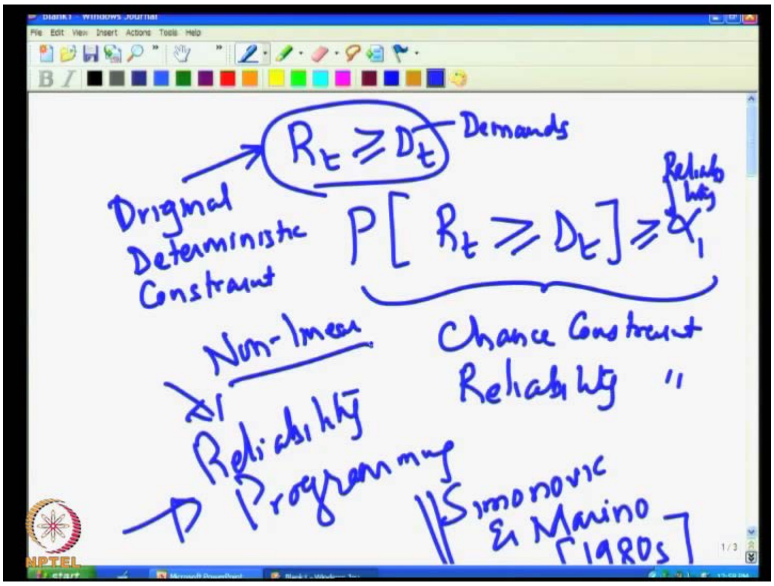


So, essentially what we did there is that a constraint of the form probability of R t greater than or equal to D t greater than or equal to alpha 1 let us say, now this is the chance constraint your original deterministic constraint was R t greater than or equal to D t; that means, you wanted to meet the demands during time period t with 100 percent reliability, this was the interpretation, this was the deterministic constraint; original deterministic constraint and this is the chance constraint or reliability constraint or probability constraint.

Now, what we did is, because R t is a random variable by virtue of it being a function of the inflows, which are random variables, we cannot write this in deterministic form and therefore, we wrote this as a chance constraint is of this form R t greater than or equal to D t greater than or equal to alpha 1 and then, we specified alpha 1. And then, we evolved methodologies of converting this chance constraint into deterministic equivalent.

Now, there are some interesting issues here; one is specifically for this particular type of constraint, where we are saying that, the release should be greater than or equal to the demand and therefore, the level alpha 1 that we specify here indicates the reliability of meeting the demands. So, this is the reliability what I mean by that is that, by writing this specifying this particular constraint we are saying that, the reliability of meeting the demand D t in time period t through the release R t is at least equal to a given level alpha 1.

We may say in as we did in this example we may say this is 0.75; that means, the reliabilities at least 75 percent, it can be more, but the minimum reliability we are specifying there, reliability of meeting the demand is at least 75 percent. Now, we were obtaining a given reservoir storage by specifying the reliabilities. You look at the problem in slightly different way, you have a given reservoir storage; that means, K is specified, you are solving this for an existing reservoir operation reservoir problem.

And then, we are saying that, probability of R t being greater than or equal to D t is greater than or equal to alpha 1, now there is the problem is slightly different, you may want to optimize the operation, optimize the reservoir operation such that, 75 percent, 85 percent etcetera are the minimum reliabilities with which we are able to meet the demands.

Now, I will posse the problem in a slightly different way that, you have the given reservoir and you want to meet the demands with highest reliability; that means, we are not pre-specifying the reliability, we will say that we want to maximize the reliability with which the demands can be met. So, that is the problem that is addressed by what is called as reliability programming, those who are interested can refer to some earlier papers by Simonovic and Marino, who introduce this concept I think in late early 1980's or (( )) somewhere on 1980's you refer to the papers; there is a series of three (( )) papers by Simonovic and Marino that introduces the concept of reliability programming.

You must understand, what is the concept here, when I write a constraint of the form probability of R t being greater than or equal to D t greater than or equal to alpha 1 or alpha let us say, now this alpha instead of pre-specifying the value of alpha, I will make alpha also as a decision variable and I will look for maximization of alpha itself in some sense. So, I will say that, I want to look at the maximum reliability that we can get from this system, maximum reliability of meeting the demands. Now this turns out to be a non-linear optimization problem, because R t is a function of Q t and when we are looking at alpha being a decision variable here, because of the because of the deterministic equivalents of this, you will get these constraints in terms of F inverse of alpha and those F inverse of alpha's will be all non-linear values, because you are talking about a C D F there.

And therefore, the reliability programming in general is a non-linear problem. So, this you just remember. So, we will close the discussion on the chance constrained linear programming now. What I would like to specify before I close, just as a matter of a closure of the topic, is that the chance constrained l p is in fact, a good handy tool as an explicit stochastic optimization technique; however, if you want to use linear programming, then you have to necessarily use linear decision rules and also because there are several random variables associated several random variables in involved in the constraints, writing down the deterministic equivalents without the L D R will be extremely difficult and involve, because you are talking about distributions of two random variables as the function of one random variable, Q t is the only random variable for which you can estimate the distributions, but these distributions information has to be transferred to two random variable S t and R t.

So, if you do not use the linear decision rule, then the problem becomes extremely complex, but the linear decision rule itself is a sort of not a very straight forward way of handling the random variables, because we are saying that, one of the random variables can be simply treated as deterministic equivalent deterministic variable and so on.

The chance constrained linear programming can be extended to linear reliability programming by also making the reliabilities themselves as decision variables. So for a given reservoir system, we may start talking about, what is the maximum reliability of meeting the demands. And these types of problems are especially relevant, especially important in the case of hydropower generation, where you have the infrastructure in place; the hydrology is random and therefore, you would like to maximize the reliability of hydropower production.

So, you will let the alpha itself as a decision variable and then, start looking for, what is the maximum reliability that you can obtain from such a particular reservoir system by operating it an optimal manner. So, the operating policy becomes the decisions to be made and the reliability becomes the decision variable. So, you will be essentially looking at the maximum reliability and this type of problem is typically address to the reliability optimization or reliability programming problems and these always turn out to be non-linear problems, you cannot use the linear programming for solving such problems.

And some literature is available on reliability problem programming problems the students, who are interested pursuing this pursuing along these lines must refer to the earlier papers by Simonovic and Marino somewhere around 1980's, they are the classical problems classical papers on reliability programming. So, in the next class, we will introduce the important topic of stochastic dynamic programming for reservoir operation, thank you for your attention.