

Water Resources Systems
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Lecture No. # 26
Hydropower Generation

Good morning, and welcome to this the lecture number 26, of the course, Water Resource Systems - Modelling Techniques and Analysis. Over the last few lectures, we have been discussing about reservoir operation problems and specifically the deterministic reservoir operation problems, where the sequence of inflows is known and we are not adding any uncertainty in the modelling.

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Summary of the previous lecture

- Stationary Policy Using DP

Progress of computations ←

The diagram shows a timeline from Year N to Year 1. Year N is divided into periods with inflows Q_1, Q_2, \dots, Q_T and indices $i=1, i=2, \dots, i=T$. Year N-1 and Year 1 follow a similar pattern. An arrow labeled 'Progress of computations' points from Year 1 back to Year N, indicating a backward recursive process.

Steady state:
 $[f_t^{n+T}(S_t) - f_t^n(S_t)]$
 remains constant $\forall S_t$
- Hydropower Generation

$$kWh_t = 2725 R_t H_t \eta$$
 - Firm power ; Secondary power ; Run-of-the-river power plants

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Specifically, in the last lecture, we discussed the stationary policy using the dynamic programming, recall that we formulated the problem as dynamic programming problem. The reservoir operation problem as the dynamic programming problem, and then started the computations for into the future; in some year for into the future proceeded in the backward direction, and kept solving the problem in a recursive manner, you seen the recursive relationship until a steady state **until the steady state** is reached.

And the steady state, we examine by this, this is the accumulated performance of the major performance major for a given state S_t , in the stage n ; and this is the same system performance at n plus t , where t is the number of periods in the year. So, this actually indicates the annual system performance corresponding to the state S_t , if this annual system performance remains constant for all S_t , in for all given time periods t , then we say that the steady state is reached.

And typically, in most realistic problems, the steady state will be reached fairly soon is within about four or five cycles; and that is where we specify the policy by policy, I mean for a given storage state, we specify the release to be made or the end of periods storage to be maintained, this is how we specify the reservoir operating policy, and when we apply the steady state operating policy or the stationary operating policy over a long period of time.

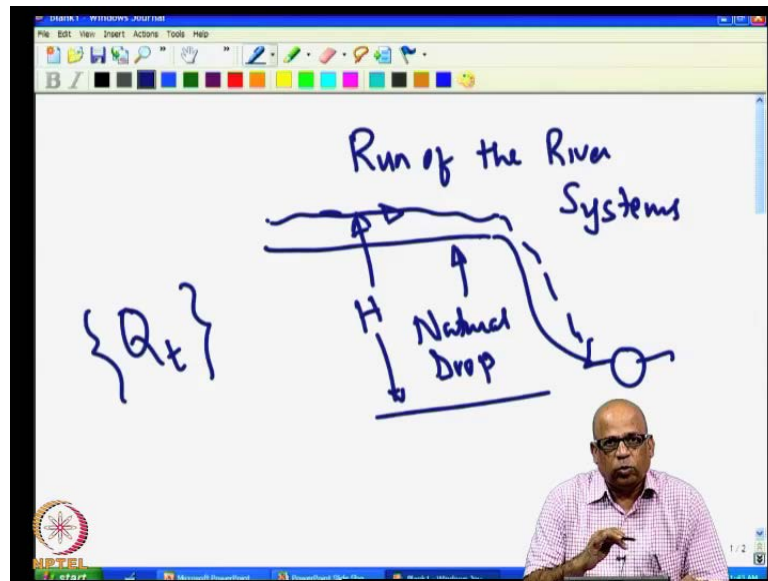
The annual system performance major remains constant; and also it is the optimized manual power system performance major. So, this is the concept of stationary policy remember, we are still talking about deterministic optimization; and when we go to stochastic optimization, the same stationary policy will also derived using the randomness inflows inflows also accounting for the randomness in inflows.

Then, we went on to introduced a concept of hydropower generation, recall that the kinetic energy possessed by water falling through a particular height, it is converted into hydropower using our turbines generators and so on. So, typically the power that is generated is a function of the discharge that comes in as well as the net head that is available.

So, we derived the expression for hydro power generation as $2725 R_t H_t \eta$, where R_t is the discharge through the penstocks that is reaching the turbines, this is in million cubic metres; and H_t is the net head in meters; and η is the overall efficiency. So, this takes into account the turbine efficiency and efficiency of generators etcetera, overall plant efficiency is accounted for in η .

Now, the H_t , which is the net head available for power generation is after accounting for the losses, head losses in the penstocks and other losses; and specifically, when we come to hydro power generation at the reservoir sides, we also account for the tail race water level in recurring the H_t .

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Then, we went on to introduce the run of the rivers systems, where if you recall, we are talking about a natural drop that is available; that is the river is flowing at a certain point; and then there is a sudden drop that is available; that is the natural drop; and therefore, the water falls through this natural drop, and then you may have a turbine at this point. So, these systems are called as run of the river systems.

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Where we use the natural drop, and in general we do not construct any major structures here, to build up the head. So, we just use on natural drop that is available, and then construct, and then generate the hydro power. Now, because this natural drop remains constant more or less, and then this depth is negligible compare to the total head, the net head remains constants. So, this in fact, determines the net head that is available, and this head remains constant.

And therefore, when we are talking about furan power, which is the minimum power that can be generated at a particular point. The furan power would then be decided by the minimum flow, because the head remains constant, the minimum flow that is occurring at that particular side, itself determine the furan power.

And therefore, if you are analysing for furan power, for a run of the river system, what would you do? You will collect the historical data. Let us say, the historical data of flow

is Q_t ; and then pick up the minimum flow over the historical period; let us say, you have monthly flow data for last 50 years or something; and then you pick the minimum flow, the power that is generated associated with the minimum flow, itself becomes the furan power in a deterministic sense.

In the sense that you are **you are** assuming; that the flow sequence repeats itself into the future. And therefore, the minimum flow that occurred in the historical past, the same minimum flow appeared in the future; and therefore, the power generated the minimum power that is generated will correspond to the minimum flow, at that particular location.. So, this is what we did in the run of the river system.

We also introduced a concept of secondary power, recall that furan power, I mentioned as the power that is available with 100 percent reliability, which means the minimum power that is available for a particular power project, be the run of the river project or a reservoir power project.

The secondary power, we mentioned as the power that is available with 50 percent reliability. So, obviously the furan power is a minimum power that is available all through and the furan power, the secondary power will be higher than the furan power, but it is available at lower reliability. Let say, 50 percent of the time the secondary power is available. In fact, we define the secondary power as that power, which is available 50 percent of the time; and typically, when you are supply in the power to the industries and so on.

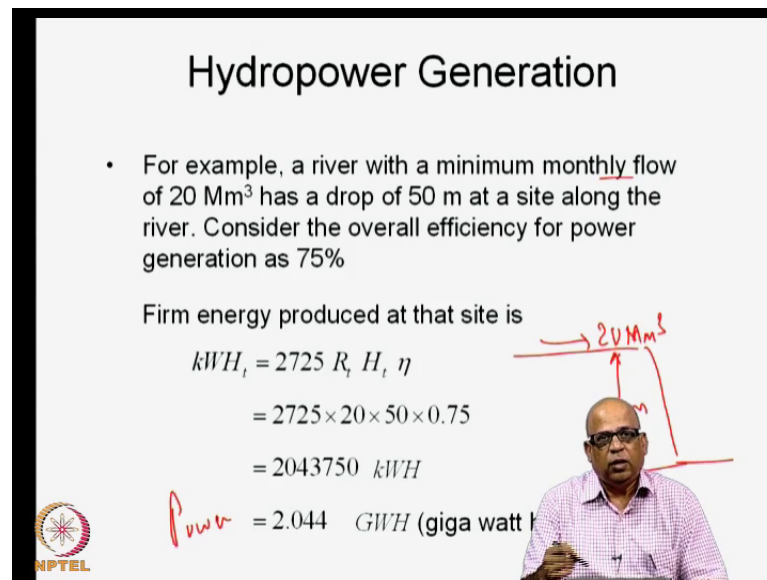
The secondary power is charged differently corresponding to the furan power, which is assumed power 100 percent of the time. Now, for the run of the river problems, it is the computations are fairly straight forward; you look at the flow and look at the head available **head available** remains constant. So, corresponding to the flow, you should be able to generate the power.

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Hydropower Generation

- For example, a river with a minimum monthly flow of 20 Mm³ has a drop of 50 m at a site along the river. Consider the overall efficiency for power generation as 75%

Firm energy produced at that site is

$$\begin{aligned} kWh_t &= 2725 R_t H_t \eta \\ &= 2725 \times 20 \times 50 \times 0.75 \\ &= 2043750 \text{ kWh} \\ \text{Power} &= 2.044 \text{ GWH (giga watt h)} \end{aligned}$$


Let us look at a simple example. Let say, a minimum monthly flow of 20 million cubic metres is occurring; and then, you have a drop of 50 metres. So, you have a natural drop of 50 metres here, and then the flow that is minimum flow, minimum monthly flow of 20 million cubic metres. So, this is the flow that is coming. So, we want to get the firm power, remember the firm power for the run of the river systems corresponds to the minimum flow, and we have the overall efficiency 75 percent in kilo watt hours.

The power generated is given by simply $2725 R_t H_t \eta$, for details of derivation of this expression. Please, refer to the previous lecture. So, R_t is 20; H_t is 50; in consistent units, this is in million cubic meters, this is in metres, and this is the overall efficiency. So, we get these many kilo watt hours convert that into giga watt hours, kilo watts divided by thousand will give you mega watts; that divided by 1000 will give you giga watts. So, we get this as giga watt hours, this is not kWh you know. So, the power is 2.044 giga watts hours. So, this is how we determine for the run of the river system.

But more specifically, we will have reservoir projects downstream of which you have a power house. So, typically we build the head, and then generate the power from the reservoir projects. So, if you look at reservoir projects, the power generated will be a function of storage as well as the function of the discharge; that takes place through the penstocks, much similar to the run of the river system, except that the head which

remained constant in the run of the river system, will now be continuously vary, because it is dictated by the chain in the storage at the reservoir.

So, if you look at a reservoir project that is a inflow that is coming and contributing to the storage, you are letting out the discharge through the penstocks; and therefore, there is a chain in the storage that is taking place **place** from time period t to time period $t + 1$.

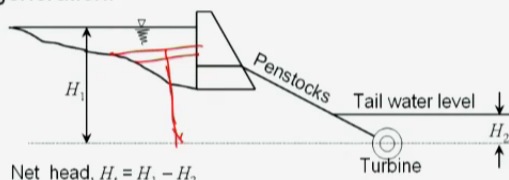
Let us say, you are operating it for on monthly time period basis, then for one month to the next month, there is a chain in the storage which happens continuously in fact, all through the month, because the inflow is continuous as well as the release is continuous, and also there is a evaporation loss that is taken place; that is continuous over time; and because of this fluctuation in the storage, the head available for the power generation is also continuously changing over the period of time.

Let us see, how we address this problem. So, the specific problem that I am dealing with this, to obtain for a specified power that has to be generated at a particular reservoir side, the necessary release required, which is associated with the head corresponding to the changing storage in a period t , and period t can be a month period. So, let us look at some details of this.

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Hydropower Generation


Simulation of reservoir operation for hydropower generation:



Net head, $H_1 = H_1 - H_2$

- The data required is
 - The inflow series at the reservoir
 - Storage-elevation-area relationships
 - Power plant efficiency.





So before that; we will understand, what we mean by the net head and other aspects. So typically, this is the set up for a power generation through a reservoir, you have a dam, then you have penstocks; and there is a natural drop or an artificially created drop at the particular side, then you have the turbine. So the, and then you will have tail water level; that means, from the turbine, you maintain a particular tail water level at this location.

So, let us say, H_2 is the tail water level, corresponding to the centre line of the turbine and H_1 is the head associated with the storage in the reservoir. Now, this is the storage and this is the centre of the turbine; therefore, this head is the H_1 , which is the total head. The net head is simply H_1 minus H_2 . Now, this net head is what is responsible for generating the power.

So, associated with any particular storage level here, which corresponds to an elevation, we should be able to get the net head. If the tail water level remains constant. So, H_2 will be specified in general for a given H_2 , we determine the net head corresponding to the H_1 associated with that particular storage level.

Now, when we want to analyse such a system for hydro power; typically, we start with simulating the system; that means, from one time period to another time period, we simulate, how the reservoir water levels change and associated with such a water level change - what is a change in the net head; and therefore, for a given power production for a power of given magnitude, what should be the release? And because of the release how the storage changes and so on.

So, from one time period to another time period, you keep on accounting for the change in the storage here, which you govern by the inflow that comes here as well as the release that you are making; and the release that you are making is to produce a certain given amount of power associated with the head, which corresponds to this storage, during that particular time period.

Now, as you can appreciate all these are interrelated; for example, the storage change here is governed by the inflow, which is given; and the evaporation losses that are taking place and the release that is made into this. Now, the release that is made into these penstocks, to produce a given power is decided base from the head, which is a function of the storage. The evaporation loss that takes place is decided based on the area of water spread, which is a function of storage.

And therefore, we need to devise a mechanism by which we will be able to simultaneously determine, the change in the storage, the net head required with the net head associated with that change in the storage, and the release associated with that storage and also the evaporation losses. So, all of these had to be determined simultaneously, and this for a given specified power, we will pose different variant of this problem subsequently, but this is a problem that were analysing to begin with.

You specify a particular power that has to be generated in a particular time period, and the time period can be of monthly intervals or ten day duration, even daily duration and so on. So, you can simulate these systems for different time periods; for that particular period, you specify the power that has to be generated, and then relate all of these variables namely, the storage, the evaporation loss that is taking place, the net head and the release during that particular time period, to generate that given power.

Remember, doing this obviously, the data that will necessities; that you should have a flow sequence, because that is what is the resource that is available to generate the power. So, the flow sequence data must be available then at the reservoir side, you must have relationships between the storage and the water spread area, which is used to determine the evaporation losses.

Then, you also need the storage and head relationship, as you can see these are the ground levels, which are leading to the reservoir here, and therefore, as the storage changes, your capacity changes; as the capacity changes, elevation changes. Lets us say, you are here, the head will be correspond to this, and then as the storage changes, your area changes as well as the net head changes; and therefore, you should have the relationship between the storage and elevation; and storage and area.

Typically, you will have at given side to storage **capacity** that is capacity, area capacity relationship as well as elevation capacity relationship; then you also need all details about the power plant itself. Typically, we need the power plant efficiency, the overall efficiency as well as you may need the capacity of the power plant itself, which puts a constraint on the maximum power that can generated at that particular location.

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Hydropower Generation

Simulation for monthly operation:

$$kWH_t = 2725 R_t H_t \eta$$

Rewrite the expression for a period of one month

$$P = \frac{2725 R_t H_t \eta}{1000 \times 30 \times 24}$$

No DB hours

$$= 0.003785 R_t H_t \eta \text{ MW}$$



Release in Mar³ into the penstocks

At R_t, E_t

S_t S_{t+1}

$$R_t = \frac{P}{0.003785 H_t \eta}$$

R_t is release to penstock in Mm^3 in period t
 P is power in MW, H_t is the net head in m in
 η is plant efficiency.

Now, let us say you want to simulate this system for a monthly time period, you started with this expression power in kilo watt hours is $2725 R_t H_t \eta$, now R_t is the, we will say now for reservoir problems, we will say it is release million cubic metres into the penstocks, remember we are talking about the reservoir operation only for power generation. So, do not mix this R_t with any other release.

For example, in our earlier problems that we consider for reservoir operation we were talking about R_t for irrigation release or **or** R_t for municipal and industrial supply and so on. You may have several purposes, for the time being we are focusing only on hydro power generation, the H_t here is the net head; and η is the overall efficiency.

So, when we are talking about a monthly time period, what we are specifying is, this is t which is an interval and you have a S_t here at the beginning of the time period; S_{t+1} at the end of the time period; this is storage at about the beginning of time period t ; and all other process are continuously happening during the time period. Let us say, your Q_t is continuously coming in this particular time period; R_t which is the release is a continuous process; and evapo transpiration is a continuous process during the time period.

So first, let us see, the total amount of power generated in mega watts. So, this was in kilo watt hours for a monthly time period, I will first divide it by number of hours in the time period. So, this is number of hours, in time period t , when time period t is of a

length one month. Now, from one month to another month, you can adjust this 30, 31 etcetera, but for simplicity will use 30 all through. In fact, when we are actually simulating for monthly operation, you can look at that particular month, and then adjust this 30, 29, 28, 31 and so on depending on the month that you are using.

So this, when I divide it by number of hours, I get it in kilo watts, I will convert that into mega watts by dividing it by thousand. So, the expression that we get for power in mega watts is $0.003785 R_t H_t \eta$; and therefore, I can write this as for a given power in mega watts for the monthly period, I can write this as R_t is equal to P , which is the power in mega watts divided by $0.003785 H_t \eta$.

So, this is the expression that will use. What does this give? This gives the required release through the penstocks for a specified power, when H_t is known; and η is given of course, but looks at H_t now, if you are looking at the time period t , which is a finite time interval typically, consisting of ten days, fifteen days, one month, even one day and so on. So, you have this storage continuously changing within the time period; and therefore, the head H_t is also continuously changing in the time period. So, we must have a mechanism of determining this H_t for the purpose of making the release, which is also constantly made all through the time period.

Let us look at, how this is done now. As I mentioned, the storage S_t changes to storage S_{t+1} at the end of the time period. So, for accounting for the change in the storage; and therefore, the associated change in elevation as well as associated change in area, we consider the average during this time period, which means essentially about we are saying is all the processes that are happening within this time period or happening uniformly. The release is made at the uniform rate, the inflow is coming at a uniform rate; the storage is changing at a uniform rate within the time period and so on. The evaporation is taking place at a uniform rate and so on. So, everything is happening within the time period in a uniform rate.

And therefore, to consider the head H_t , to determine the head H_t , we will consider the average situation within the time period, and then determine the head H_t , while determining the H_t the head H_t , you will also determine the evaporation loss, because the area of water spread that will be corresponding to the average storage. So, we must

be able to determine the head as well as the evaporation loss as well as the release corresponding to S_t and S_{t+1} .

However, there is another catch here, when we are simulating the reservoir system that is from time period to time period, we are simulating the reservoir system operation, the storage at the beginning of the time period t is in general known, because you are simulating, let say from the first time period of the first year, where you will specify the reservoir storage, and then determine the end of the period storage.

So, this will be known; whereas, this is not known; S_{t+1} is not known, S_{t+1} will be determined, only if you determine R_t , E_t and R_t and E_t , but R_t is also a function of the elevation, which is the function of S_t and S_{t+1} ; E_t is the function of the area, which is the function of S_t and S_{t+1} etcetera; and therefore, all of these are interrelated, you must solve for S_{t+1} , R_t and E_t in an iterative manner for a given power P .

So, we are specifying the power to be produced in that particular time interval, and determining H_t , which is necessary to determine R_t both of which are functions of S_{t+1} , which is also a function of E_t evaporation, this is done in an iterative manner, very simple iterative procedure, which converges very fast. So, I will just explain the iterative manner in which we simulate this.

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Hydropower Generation

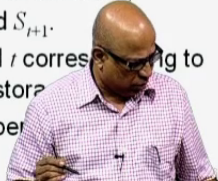

Reservoir storage continuity:

$$S_{t+1} = S_t + Q_t - E_t - R_t - Spill_t$$

where

- S_t is the storage at the beginning of the period t
- Q_t is the reservoir inflow during the period t
- R_t is the release required in the period t to generate the specified power corresponding to the net head, resulting from the average of S_t and S_{t+1} .
- E_t is the evaporation loss in the period t corresponding to water spread area at the average storage.
- $Spill_t$ is the spill (overflow) during the period t .

Area of Water Spread

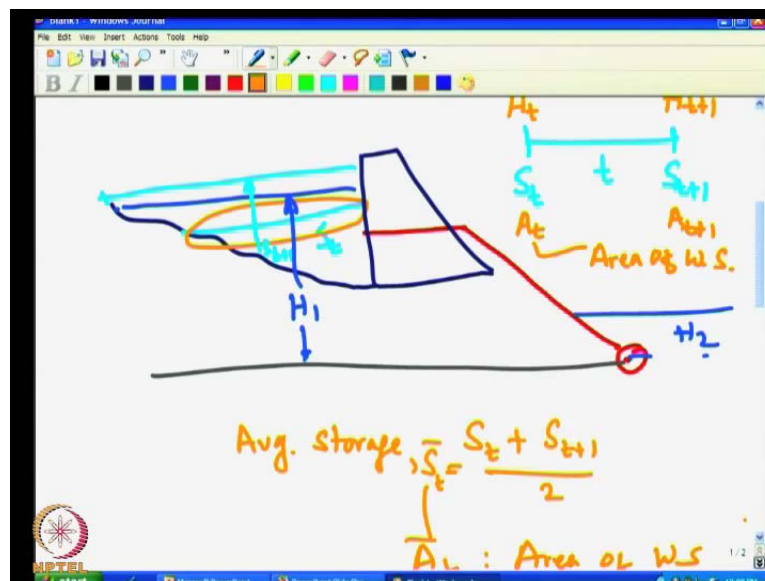


Now, this is the reservoir continuity $S_t + 1$ is equal to $S_t + Q_t - E_t - R_t - \text{Spill}_t$; spill is the overflow that happens in time period t . Now, the evaporation loss is dependent on area of water aspect.

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And R_t is the release required, here you just take R_t is the release required in the period t , to generate the specified power corresponding to net head result from the average S_t and $S_t + 1$. So, this is dependent on both $S_t + 1$ as well as S_t and $S_t + 1$ is not known; area of water spread itself depends on the storage is the average storage S_t and $S_t + 1$, average storage corresponding to $S_t + 1$ and $S_t + 1$, and spill t of course, we determine this from the end of the period storage.

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This concept, you understand correctly, because in my experience, I have seen that many of the students make a mistake in this algebraic procedure, iterative procedure for determining these variables simultaneously. Just understand this concept physically, what is happening? Now these are your contour **contour** levels; and let us say, the power house is located here, these are the **these are** turbine level and you have the net head, this is the water level, this is the gross head, and this is the tail water level, so H_2 . Now, this is the general set up here.

Now, let us look at, what happens between time period t and time period $t + 1$. Let us say that you were here in time period t , this is S_t , which is storage and time period $t + 1$, you went to this point $S_{t + 1}$, this $S_{t + 1}$; and this is the time interval. Let us say, you are talking about one month time period, because your S_t changes from S_t to $S_{t + 1}$ in time period t .

Let us say, I put this as S_t and this is $S_{t + 1}$, the area of water spread corresponding to the storage S_t at the **at the** beginning of time period t changed from A_t to $A_{t + 1}$, this is the area of water spread; similarly, the head change from H_t ; let us say, this was H_t to $H_{t + 1}$. So, head also change from H_t to $H_{t + 1}$.

So, what we do in this case is that we recon the average storage, which is $S_t + S_{t + 1}$ divided by 2, we write this as \bar{S}_t , which is the average storage in time period t , associated this average storage, we get let us say, the average area of water spread. We also get, let us say \bar{H}_t , which is the average head, average net head, we will be able to get this, if you know \bar{S}_t , because corresponding to \bar{S}_t you know, what is the area for the spread?

Let us say, this was the \bar{S}_t ; and then you know, the water area of water spread, you also know the net head; and therefore, you will be able to get everything associated with \bar{S}_t ; however, the difficulty is that we do not know \bar{S}_t , because this is not known; and therefore, \bar{S}_t is do not known, this problem we address by an iterative procedure by which we will be able to determine the particular \bar{S}_t , which would be also a function of $S_{t + 1}$.

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Hydropower Generation

Procedure for obtaining H_t, R_t, E_t and S_{t+1} iteratively:

1. Assume average storage $\bar{S}_t = S_t$ (Known)
2. Corresponding to \bar{S}_t obtain net head, H_t , and water spread area, A_t , from storage-elevation-area relationships.
3. Determine the release, R_t , required for generating the specified power, P , from
$$R_t = \frac{P}{0.003785 H_t \eta}$$
4. Obtain the evaporation loss from $E_t = e_t A_t$ where e_t is the evaporation rate in period t and A_t corresponds to the storage \bar{S}_t .

So, the way we will go about is that we start with an assumed value of S_t bar, remember S_t bar is an average **is the average** storage and we have done these computations for a known initial reservoir storage. So, this storage is known, what is this? This is the storage at the beginning of the time period t . So, we are doing this now for a given time period, starting with S_t , we want to determine S_{t+1} ; and also simultaneously, determine R_t, E_t ; and in the process we will also determine the H_t .

So, the way we go about this is that we assume S_t bar to be equal to S_t ; that is, we say that the average storage is equal to the storage at the beginning of the time period t ; corresponding to this S_t bar known, which is assumed we can obtain the net head H_t , the water spread area A_t , because we have the storage-elevation-area relationships.

And therefore, the moment you specify the storage, we know from these relationships, what is the elevation, and therefore, what is the net head, we also know, what is the area corresponding to that particular storage. So, from these we will be able to get H_t as well as A_t , once you get H_t , which is the net head, we can determine the release R_t associated with a pre-defined power P . So, from this expression **this expression**, recall we are getting it here.

So, we use this expression, because we know the H_t now, we can determine R_t , which is the release required to produce this particular power, because we know A_t , we can also determine the evaporation loss, and which is dependent on the rate of evaporation,

small e_t is the rate of evaporation and A_t is determined based on the average storage. So, we got E_t , which is e_t into A_t , now corresponding to the storage S_t , this is the storage S_t . So, we got for this particular specified S_t , we got now, what is R_t as well as what is E_t .

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Hydropower Generation

5. Get the end of period storage,

$$S_{t+1} = S_t + Q_t - E_t - R_t \quad \text{if } S_{t+1} < \text{reservoir capacity, } K.$$

$$= K, \quad \text{otherwise}$$
6. Get the average storage, $\bar{S}_t^* = \frac{S_t + S_{t+1}}{2}$
7. If \bar{S}_t^* is nearly equal to \bar{S}_t , the computed values of H_t, R_t, E_t and S_{t+1} are acceptable.

Else, set $\bar{S}_t = \bar{S}_t^*$ and go to step 2; repeat steps 2 to 7 until the computed values of H_t, R_t, E_t and S_{t+1} are acceptable.

Now, we apply this storage continuity equation S_{t+1} , which is not known is equal to $S_t + Q_t$, this is known E_t we have determine, and R_t you have determine, and then we limits this to K , which is the storage capacity, as we have been doing in the all the earlier classes dealing with the continuity equation.

So, we determined S_{t+1} , starting with a known value of S_t , which was assumed to be equal to S_t to begin with. So, starting with that we determined S_{t+1} , now we can determine the average storage; the average storage is $S_t + S_{t+1}$ divided by 2; again do not get confused, this S_{t+1} is as determine from this expression. Now, for a value of R_t and E_t determined as you for the assumed average storage S_t , now we got a new S_{t+1} ; and we can determine the average storage now, real average associated with these values.

So, we get average storage as $S_t + S_{t+1}$ divided by 2, now this average storage that you so compute, will be in general different from the average storage that you assumed to begin with, this was the average storage that you assume. So, if it is different from the assumed average storage, which is S_t , then we set S_t equal to this new

storage now, new average storage set \bar{S}_t equal to \bar{S}_t^* ; and then recalculate we go back to step number 2. Once you know, \bar{S}_t we know, how to calculate the head - how to calculate the area and from that you can calculate the release; and then obtain the evaporation losses, and again calculate S_{t+1} ; and therefore, again you calculate \bar{S}_t^* .

So like this, you keep on doing until the calculated \bar{S}_t^* becomes nearly equal to the \bar{S}_t ; that \bar{S}_t that you have assumed in the step 2; that is the \bar{S}_t that you started with in step 2 must be nearly equal to the \bar{S}_t^* that you compute after all these calculations, then we say the assumed the computed values of H_t , R_t , E_t and S_{t+1} are acceptable.

Remember, what we are achieving through this iteration, iterative procedure is that we are simultaneously obtaining corresponding to a specified S_t , which is the storage at the beginning of time period t , which is known associated with this, we are simultaneously determining the net head H_t , the release R_t required for producing a known power P and the evaporation loss E_t ; and therefore, the end of the period storage S_{t+1} .

So, all of these variables, we are determining simultaneously through this iterative procedure, and this iterative procedure commonly very fast. If you **if you** write a simple computer program or even if you do this calculations in a word sheet like a Microsoft excel or some other work sheet, it converges this fairly rapidly within three or four iterations it converges, which means then for a given storage at the beginning of the time period t , we are now able to determine, what should be the release; and the associated what **what** will be the net head associated with that particular storage and the evaporation loss and end of the period storage simultaneously, all of these we will determine simultaneously.

Now, this procedure we use in a simulation model; that is let us say that you have last thirty years of data or you may have a generated data for next fifty years or you may have several sequences of fifty year flows, how to do it will see subsequently, but you have a given set, given sequence of data, inflow data, then you want to simulate the reservoir operation for that particular sequence of data.

Then, what we do is? That you start with a known initial storage at the beginning of the first period of the first year; that means, S_1 at the beginning of first year is specified; and

then you know the Q_t ; the inflow is known during that time period, then we use this particular procedure to determine R_t, E_t and also the net head, which comes as a which determines in fact, the power that is generated for a given R_t or the other way around that which determines the release to be made through the penstocks for a given net head for a given power generation **sorry**.

So, all of these are determined simultaneously, and then you go to the end of time period storage **the end of the time period storage** becomes the beginning of the time period storage for the next time period, again you repeat the calculations and **and** so on. So, like this from period to period, you simulate the operation of the reservoir.

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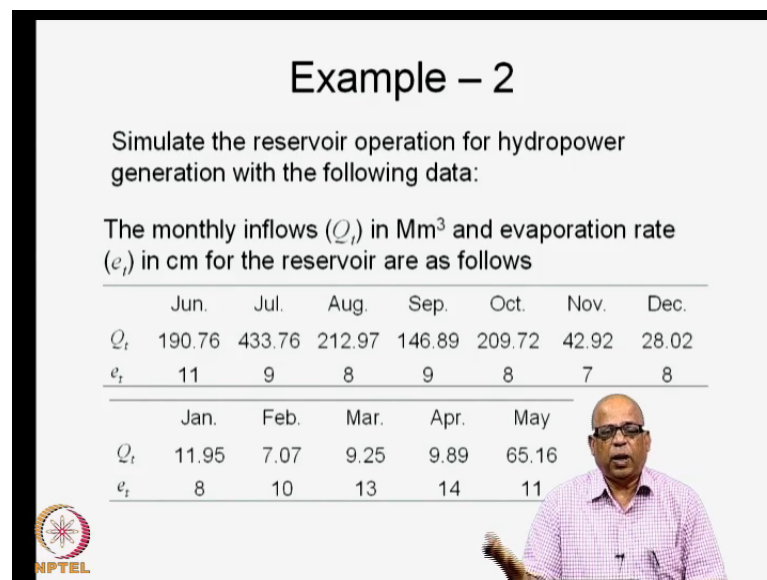
Example – 2

Simulate the reservoir operation for hydropower generation with the following data:

The monthly inflows (Q_t) in Mm^3 and evaporation rate (e_t) in cm for the reservoir are as follows

	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Q_t	190.76	433.76	212.97	146.89	209.72	42.92	28.02
e_t	11	9	8	9	8	7	8

	Jan.	Feb.	Mar.	Apr.	May
Q_t	11.95	7.07	9.25	9.89	65.16
e_t	8	10	13	14	11



Let us look at an example quickly, as I mentioned you need the inflow data. So, let us say, we take one year inflow data, I will show the example the calculation only for one year, but this can be repeated for several years, the e_t which is the rate of evaporation these are in centimetres. Now, as I mentioned earlier, the rate of evaporation is typically obtained from your **(())** operation data or any other way, may be water balance data etcetera. So, the historical average rate of evaporation is available.

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Example – 2 (Contd.)

Reservoir capacity,
 $K = 1226 \text{ Mm}^3$

Minimum power desired
in a month,
 $P = 73.5 \text{ MW}$

Plant efficiency = 81.54%
Initial storage = ~~824.63~~ Mm^3 ✓
Tailrace water level = 47 m

Capacity (Mm^3)	Elevation (m)	Area (Mm^2)
204.5	280	8.4
248.82	285	10
302.82	290	11.6
351.62	294	12.8
398.52	297.5	14
434.77	300	15
500.94	304.25	15
582.02	309	16.14
686.36	314.5	19.94
868.85	323	23
1013.03	329	25.06
1189.68	335.75	27.28
1226	338	28

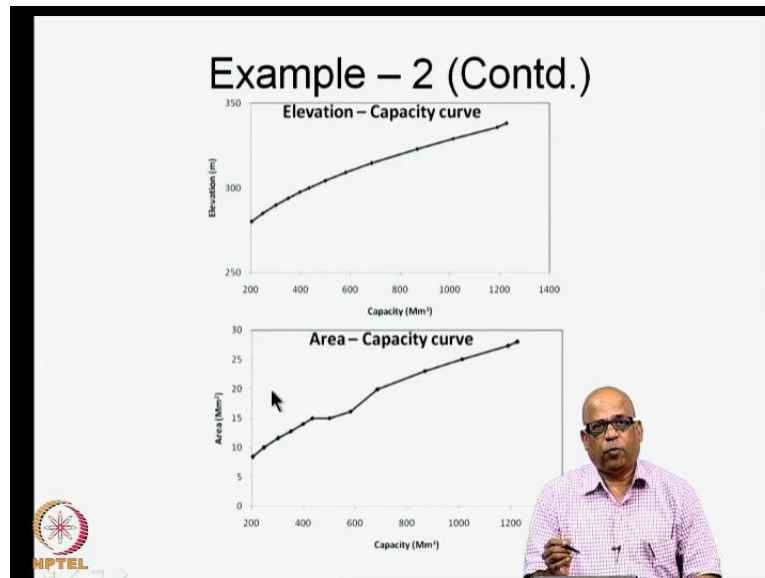


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Then, you also need the storage capacity, this is the storage capacity; and then we are specifying that we want to generate a minimum power of 73.5 mega watts in a month. We are given the plant efficiency, we will start with 824.63 million cubic metres storage; and the tail water level is given as 47 metres. So, when you are computing the net head, remember from the total head, you deduct the tail water level to get the net head.

You also need the area capacity curve as well as the elevation capacity relationship; and this is what is given here, elevation is in metres, capacity is in million cubic metres, and the area is in millions square metres. So, there all in consistent units; remember your e t here is in centimetres, so we need to convert that in metres.

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Now, just give a pictorial view. This is what we generally have at reservoir sides, you have the area capacity curve, this is capacity in million cubic metres, and this is the area in square metres, and this is the capacity in million cubic metres the same scale and elevation in metres.

Now, associated with any storage, this is capacity is storage. So, associated with any storage, we should be able to determine the elevation corresponding to this, which gives you the total; head and from the total head, you detect the tail water raise, tail water level; and therefore, you get the net head.

Similarly, associated with any storage, we should be able to determine the area, area of water spread from that we should be able to get the evaporation loss, because you know, the rate of evaporation. The evaporation loss in volume is simply equal to rate of evaporation in depth units multiply by the area of water spread in area units.

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Example – 2 (Contd.)

Illustration of calculations for June month (t = 1):


- Initial storage, $S_1 = 824.63 \text{ Mm}^3$; Assume $\bar{S}_t = 824.63 \text{ Mm}^3$
- Inflow for June month = 190.76 Mm^3
- From the Storage – elevation – area data corresponding to the initial storage, $S_1 = 824.63 \text{ total Mm}^3$, head = 320.84 m and $A_1 = 22.26 \text{ Mm}^2$
- Net head, $H_t = 320.84 - 47 = 273.94 \text{ m}$
- For $P = 73.5 \text{ MW}$ and $\eta = 81.54\%$,

$$R_t = \frac{P}{0.003785 H_t \eta}$$

$$= \frac{73.5}{0.003785 \times 273.94 \times 0.8154} = 86.935 \text{ Mm}^3$$

Total
Mm³

Tailwater level



So, the iterative procedure, I just demonstrated for time period t equal to 1. We started with a storage of 824.63 million cubic metres, which means we are saying that for June month, which is t is equal to 1. The initial storage is 824.63, which is given million cubic metres is given. Now, you should be alert that; you should not change this in the iterative procedure, because this is a given storage, which has to correspond to all the other variables that you have going to determine now.

So, 824.63 is fixed the storage at the beginning of time period one is fixed. Now, what we do is? We will assume that S_t bar is equal to 824.63 and entering that iterative procedure now; through which we will determine all other variables for the time period t is equal to 1, I am showing the calculations for time period t is equal to 1. So, we assume S_t bar, which is the average storage during the time period t as 824.63 itself, which is given, corresponding to this, anyway the inflows are given, all these other data are given here, inflows are given. So, June month inflow is 190.73. So, I will start with 190.76 I am sorry 190.76 million cubic metres

Now, we use the storage elevation area relationship; that is this is storage area and this is storage elevation data, corresponding to this average storage now 824.63, we determine the total head, as this is total head and this is in million cubic metres. So, what we do is with 824.63, you go to this point, we come somewhere here, 824.63 somewhere here and

go to the elevation point, read the elevation at that point, you get an elevation of 320.84 **320.84** here; and area of 22.26 million cubic metres.

So, you go here corresponding to 824.63, you read the area 824 that comes to 22 or something or you can use these relationships; and corresponding to 824.63 you are somewhere here, you interpolate; and then get the values. You can use actually to be accurate, you should use nonlinear interpolation, otherwise simply go to this curves; that you have drawn here, and use the curves to read the associated elevation as well as associated area. So, you got the ideas as well as the elevation.

Now, the net head here, this is the tail water level, net head will be 273.94 corresponding to this total head. We are given that we should produce a power of 73.5 mega watts; and therefore, to produce this power, I need a release associated with this net head 273.94 metres; I need a release of 86.94 million cubic metres by applying this expression here. So, I make this release now, we had a storage of 824.63, I have determine the release, I have also determine the area of water spread here, I have determined the net head corresponding to the average storage. And therefore, I should be able to determine also the evaporation loss

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Example – 2 (Contd.)

- Evaporation rate, e_t for June month = 11 cm = 0.11m
- Evaporation loss corresponding to $e_t = 0.11$ m and $A_t = 22.26$ Mm² is

$$E_t = e_t A_t = 0.11 \times 22.26 = 2.003 \text{ Mm}^3$$
- End of period storage, S_{t+1} is calculated as


$$S_{t+1} = S_t + Q_t - E_t - R_t$$

$$= 824.63 + 190.76 - 2.003 - 86.935$$

$$= \underline{926.45 \text{ Mm}^3} \quad S_{t+1} < K \text{ (1226 Mm}^3\text{)}$$

Average storage, $\bar{S}_t^* = \frac{S_t + S_{t+1}}{2} = \frac{824.63 + 926.45}{2} = 875.5$

$\bar{S}_t^* \neq \bar{S}_t$



So, evaporation loss is 11 centimetre, the rate of evaporation is 11 centimetre, which is 0.11 **milli** metres and area is 22.26 million cubic **million** metre square, then the

evaporation in volume will be simply 0.11 metres into this area of water spread, which is 2.003 million cubic metres.

So, I have determined all the necessary variables here, this is known, this is known, E_t I have determined, R_t I have determined, R_t is this. So, I will apply this and get an end of the period storage of 926.45 million cubic metres. Now, our K is 1226 million cubic metres; and therefore, this is below the capacity; and therefore, S_{t+1} is acceptable.

Now, I calculate the average storage, S_t which was 824.63; S_{t+1} , as I determined here; and the average storage comes to 875.5 million cubic metres, what was the average storage that we assumed to begin with we assume an average storage 824.63; and getting a average storage of 875.5, which is far away from what was assumed; therefore, S_t bar star as we compute from here is not equal to S_t bar; and therefore, I now set S_t bar is equal to this value now.

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Example – 2 (Contd.)

Second iteration for June month: $\bar{S}_t = 875.5$

- Corresponding to $\bar{S}_t = 875.5 \text{ Mm}^3$,
total head = 323.3 m and $A_t = 23.1 \text{ Mm}^2$
- Net head, $H_t = 323.3 - 47 = 276.3$
- For $P = 73.5 \text{ MW}$ and $\eta = 81.54\%$,

$$R_t = \frac{73.5}{0.003785 \times 276.3 \times 0.8154} = 86.2 \text{ Mm}^3$$

- Evaporation loss is
 $E_t = e_t A_t = 0.11 \times 23.1 = 2.08 \text{ Mm}^3$
- End of period storage, S_{t+1} is
 $S_{t+1} = 824.63 + 190.76 - 2.08 - 86.2 = 927.11 \text{ Mm}^3$
 $S_{t+1} < K (1226 \text{ Mm}^3)$



So, I set S_t bar is equal to 875.5; and then I go back to step 2 now, again corresponding to this newer S_t bar, I calculate the area, I calculate the total head; and then the net head corresponding to that net head, I calculate the release required, then I calculate the evaporation associated with this area, then I calculate the end of the period storage. Again check, whether this is less than K , if it is not less than K , you calculate the speed and set it to the capacity K itself, all of these routine exercise, we have done earlier, so you must be able to use those things.

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Example – 2 (Contd.)

Average storage, $\bar{S}_t^* = \frac{875.5 + 927.11}{2} = 901.3 \text{ Mm}^3$

$\bar{S}_t^* \neq \bar{S}_t$ *Set $\bar{S}_t = 901.3$*



Other iterations are performed in the same line for June month until

$\bar{S}_t^* = \bar{S}_t$ *t = 1*

Final solution for June month is

$R_t = 85.53 \text{ Mm}^3$ ✓
 $H_t = 278.45 \text{ m}$
 $E_t = 2.15 \text{ Mm}^3$ and
 $S_{t+1} = 927.72 \text{ Mm}^3$

Convergence



Now, associated with this S_{t+1} now, again we calculate \bar{S}_t^* , I come to 901.3; again this is not nearly equal to what we assumed, which was the 875.5; and therefore, I go to next iteration, I set again \bar{S}_t is equal to this now. So, \bar{S}_t , I set it as 901.3, this is million cubic metres; and then redo the calculation, associated this \bar{S}_t now; again I calculate the area, again I calculate the net head, the release required and so on. So, again I keep on doing this until we finally, achieve a convergence. Now, these are the values that are after the convergence is achieved.

So, we get for June month, this is still we are talking about t is equal to 1, for t is equal to 1. We get R_t of 85.53; H_t of 278.45; finally, S_{t+1} comes to 927.72 **meter** million cubic metres, then we set these as the storage at the beginning of the next time period, and compute, redo the calculations for the next time periods..

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Example – 2 (Contd.)

End-of-storage for June month is initial storage for July month.
 $S_2 = 927.72 \text{ Mm}^3$

Same procedure is followed for obtaining the H_t , R_t , E_t and S_{t+1} values for July month.

Final solution for July month ($t = 2$) is

$R_t = 81.84 \text{ Mm}^3$
 $H_t = 291 \text{ m}$
 $E_t = 2.52 \text{ Mm}^3$ and
 $S_{t+1} = 1226 \text{ Mm}^3$

So, we do it for July, again the same iterative procedure we get it for July; and because the storage at the end of the time period may come to the more than the capacity, we set it equal to capacity; and compute the spill. Like this, in an iterative way, we keep on calculating from one time period to another time period.


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Example – 2 (Contd.)

Iterative Procedure

Simulation results

Month	Q_t Mm ³	e_t cm	S_t Mm ³	H_t m	A_t Mm ²	R_t Mm ³	E_t Mm ³	$Spill_t$ Mm ³	S_{t+1} Mm ³
Jun	190.76	11	824.63	278.45	23.84	85.53	2.15	0	927.72
Jul	433.76	9	927.72	291.00	28.00	81.84	2.52	47.32	1226.00
Aug	212.97	8	1226.00	291.00	28.00	81.84	2.24	128.89	1226.00
Sep	146.89	9	1226.00	291.00	28.00	81.84	2.52	62.53	1226.00
Oct	209.72	8	1226.00	291.00	28.00	81.84	2.24	125.64	1226.00
Nov	42.92	7	1226.00	288.55	27.21	82.53	1.91	0	1184.48
Dec	28.02	8	1184.48	286.36	26.50	83.16	2.12	0	1127.22
Jan	11.95	8	1127.22	283.53	25.56	83.99	2.05	0	1053.13
Feb	7.07	10	1053.13	280.33	24.49	84.95	2.45	0	972.80
Mar	9.25	13	972.80	277.01	23.35	85.97	3.03	0	893.04
Apr	9.89	14	893.04	273.39	22.06	87.11	3.09	0	812.73
May	65.16	11	812.73	272.24	21.64	87.48	2.38	0	788.03



And tabulated like this, I shown it for one year with the inflow data given, e t data given, we are starting with known storage at the beginning of the time period one, which is June H_t , A_t , R_t and E_t ; these are all determined from the iterative procedure.

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And then, we get S_{t+1} associated S_{t+1} ; we use S_{t+1} also in determining these variables. The S_{t+1} at the end of this time period becomes the S_t for the next time period; and then again we redo this, calculate this, remember we calculate the spill only when the capacity is exceeded, which means the storage plus the inflow minus the release minus evaporation.

If these values, exceeds the capacity, then the remaining amount we are put it as spill; and then the end of the period storage must be equal to capacity. In the process we are determining the R_t ; remember all of these values of R_t and the associated values of H_t here determine will together produce the required power P , which is 73.5 million 73.5 mega watts.

Now, there will be certain. So, this is how we simulate, and like this you can go on for several years. As your inflow data is available, you can keep on repeating this for different years, different number of years. Let us say, you do it for fifty years, sixty years and so on. So, that you can get the reliability of the power generated is high, what do I mean by that you wanted to produce 73.5 watts; in certain periods, you may not be able to produce 73.5 watts, because of lack of available water and lack of available head. In which case you reconnect as the failure, we will discuss this problem further later on.

But right now, what is important is that let say that there was a point in which the release required to produce a certain power; that amount of water is not available in which case you apply this standard operation; and then release all the amount of water. That is what we do here.

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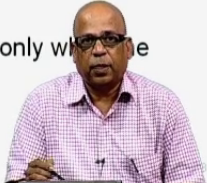
Hydropower Generation

Primary and additional power:

- When the power draft is adequate to generate the specified power P , the primary power is equal to P itself.
- When the power draft is less than that required to generate the power P , the primary power P is

$$P = 0.0030864 R_t H_t$$

- The additional power is generated only when the reservoir spills.

Let us say, when the power draft less than that required to generate the power P , the primary power; that means, you are not able to release the required amount of water, because of lack of storage, then you release whatever is available, in which case the power produced at that particular head will be less than the power that is required; and therefore, we determine that particular power, associated with the particular released.

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Hydropower Generation


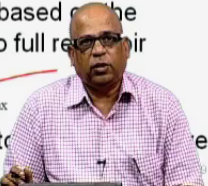
- The spill during a month is computed based on end-of-the-period storage as,

$$Spill_t = \begin{cases} 0 & \text{if } S_t + Q_t - E_t - R_t \leq K \\ S_t + Q_t - E_t - R_t - K & \text{otherwise} \end{cases}$$

- When there is a spill during a period, the end-of-the-period storage, S_{t+1} , is set to K , after computing the spill.
- The additional power is computed based on the spill with net head corresponding to full reservoir level as

$$P' = 0.0030864 Spill_t H_{max}$$

H_{max} is the net head corresponding to full reservoir level

Then, the other point is about the spill, as I said when you calculate S_t plus Q_t minus E_t minus R_t , which is starting with the given storage, we have added the inflow, we have

taken out the release, we have taken out the evaporation, now this net amount, if it is more than the capacity, then the balance is put as spill. And then, you calculate the... then you set the end of the period storage to capacity.

When the spill occurs sometimes, we calculate the power generated with respect to the spill also, because the spill also you can push it through the penstock, if the capacity is available. In general when we want to do this, we will do this at the maximum head; that means, the spill is push through the penstocks, when capacity is available and recon the power generated with respect to the maximum head that is available, which is corresponding to the storage capacity itself.

So, this is how we calculate the additional power. So, the additional power is corresponding to the spill that is occurred; and obviously, this should be for the amount spill and this is the H max, H max is the maximum head that is available, which corresponds to the capacity of the reservoir itself.


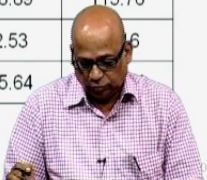
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Example – 2 (Contd.)

In the previous example, the spill occurs in the months of Jul., Aug., Sep. and Oct. $H_{max} = 291 \text{ m}$

Month	$S_i + Q_i - E_i - R_i$ Mm ³	K Mm ³	$Spill_t$ Mm ³	Additional power MW
Jul	1276.64	1226	47.32	42.50
Aug	1354.89	1226	128.89	115.76
Sep	1288.53	1226	62.53	56
Oct	1351.64	1226	125.64	

$P' = 0.0030864 Spill_t H_{max}$

So, in this example there were several periods in which the spill occurred; for example, 47.32, 128.89 etcetera. So, these are the periods in which the spill occurred we have produced the primary power which has 73.5 mega watts already. Now, corresponding to this spill, we will see the additional that can be generated.

So, this is the amount of spill and H_{max} is given, this corresponds to, the H_{max} corresponds to the maximum head. So, we use the expression 0.0030864 into spill into H_{max} to get the additional power. So, this is in addition to the power, the minimum power that you have specified; and this additional power is generated only during these periods, in which the spill was occurring. So, this is the spill and end **end** of the period storage must be equal to the capacity itself. So, this is how you simulate, the reservoir operation for hydro power generation.

So, in today's class all through, we discussed about simulation of reservoir operation for hydro power generation, remember in hydro power generation with reservoir systems in place; that means, a power house is generated, located downstream of a reservoir; and you are discharging the water through the penstocks.

And therefore, there is a change in the storage that is happening in any given time period. The change in the storage, which is govern by the inflow to the reservoir, which is govern by the discharge that you are making through the penstock, which is govern by the evaporation loss will govern this change in storage, will govern of power that is generated.

So, all of these need to be determined simultaneously, because starting with the known storage S_t , you do not know, what will be the end of the period storage S_{t+1} ; and S_t plus S_{t+1} together determine the average storage, which corresponds to the average area of water spread; and also corresponds to the average net head during that time period; and therefore, we have to do this iteratively.

And I have just introduced a iterative procedure by which all of these can be determined; and we have seen an example, through which we have shown, how to simulate the reservoir operation from one time period to another time period. Look at the minimum power that is generated; and then we also look at this those periods in which spill occurs and associated with the spill, we also generate additional power. So that completes the portions set out for deterministic inflows; that is reservoir systems with deterministic inflows. In the next class, we will start discussing on stochastic inflows or reservoir **reservoir** systems with random inflows. Thank you for your attention.