Water Resource Systems

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Module No # 05

Lecture No # 25

Stationary policy using Dynamic Programming

Good morning and welcome to this, the lecture number 25, of the course, Water Resource Systems, Modeling Techniques and Analysis. Over the last few lectures, we have been now discussing about the reservoir operation problems and the specific type of problems that we are talking about are the deterministic type of problems, where we do not consider the uncertainty associated with the random inflow. We are taking the inflows as a deterministic given sequence and for this given sequence of inflows, we are obtaining the reservoir operating policies.

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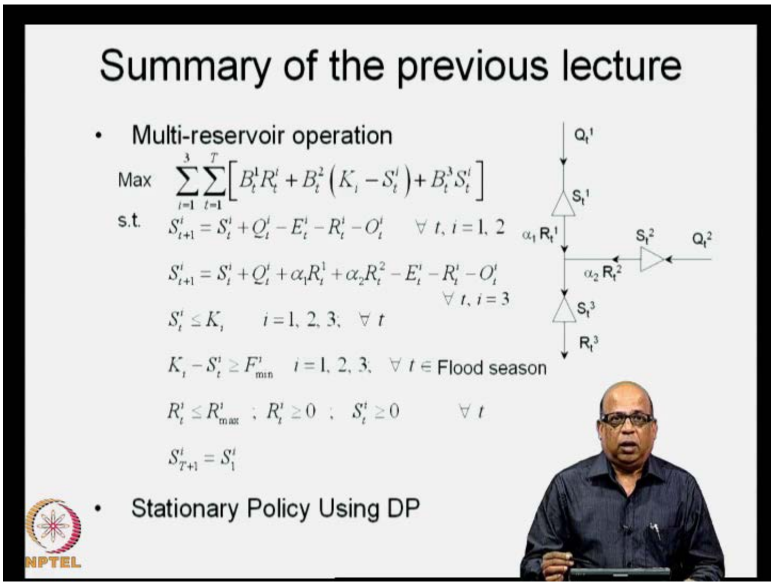

In the previous lecture specifically we dealt with the Multi-reservoir operation problem where we formulated a linear programming problem for a given configuration of the system as shown here. And then, we obtain the optimal operating policy for such a system and showed also through a numerical example. As I mentioned, this can be generalized to any n number of reservoir systems. All you have to do is write the continuity equations correctly, looking at where the water is coming from and where it is going. So, just account for the mass balance at every node, by node I mean it can be a reservoir or it can be a user point let us say that you may have a lift irrigation scheme somewhere along the river and you want to include that lift irrigation also as part of the mass balance. At that location you write the mass balance again.

So, like this you write the constraints depending on the type of configuration of the reservoir system that you have and then also add additional constraints as the system demands. For example, in this particular formulation we also added the minimum flood free board required during the flood season. And such system dependent constraints can be added and also you can decide on the type of objective function specific to the problem. In this particular case we added returns or benefits associated with the release that is made for water supply, the flood free board that you keep and also the hydro power that is generated which we directly related to the storage that is available.

Like this for a given system, you can identify the objective function and write the constraints, the major constraints there, which are common to all systems, are the mass balance equations or the reservoir continuity equations. So, the reservoir continuity equations you write, looking at where the water is coming from, where it is going, for what purpose you are using and so on., how much of it is going and contributing to the downstream reservoir and such system dependent features you build into the reservoir continuity equation. And in addition you may also have constraints depending on the specific purpose is for which the individual reservoirs are operated.

And then we went on to discuss a sort of introduce in the last class, towards the end of the last class, the stationary operating policy using dynamic programming, where we will continue the discussion today on the stationary operating policy. What we mean by stationary operating policy is that given the state of the system, let us say the state of the system is defined by storage at individual reservoirs, if you are talking about a multiple reservoirs, if you are talking about a single reservoir, you reckon the storage as you reckon the state of the system as the storage to begin with.

Given the storage of a reservoir at a particular time period you will have a operating policy for that particular period in conjunction with the other periods. What I mean by that is, but, when you implement the policy for that particular time period, your state gets transform to another state and with that particular state you have another, you have the next time periods policy. Like this sequentially you are operating the reservoir and the stationary policy is a steady state policy in the sense that if you operate the system using this particular policy over a long period of time, the annual net returns that you get will remain constant. So that is when we say the steady state has been reached.

Because the reservoir operating reservoir operation is a sequential decision problem, it is ideally amenable to be modeled by dynamic programming. And one example of use of dynamic programming for reservoir operation, we have seen earlier, perhaps in lecture number 16 or 17, you please refer to those lectures. You will see how the dynamic programming algorithm can be used to solve a reservoir operating problem problem.

In the earlier discussion on dynamic programming for reservoir operation, we stated the boundary conditions; that means, initial reservoir storage at the beginning of the time period 1 of a particular year was specified. And then for a given specific reservoir storage we solve that example. In the case of steady state operating policy or the stationary operating policy, we do not specify any boundary conditions. So we leave everything free and then allow the algorithm to choose for a given state of the system what is the optimal path that has to be followed. We will do that exercise now and relate this with what we did earlier for a 1 year operation using the dynamic programming.

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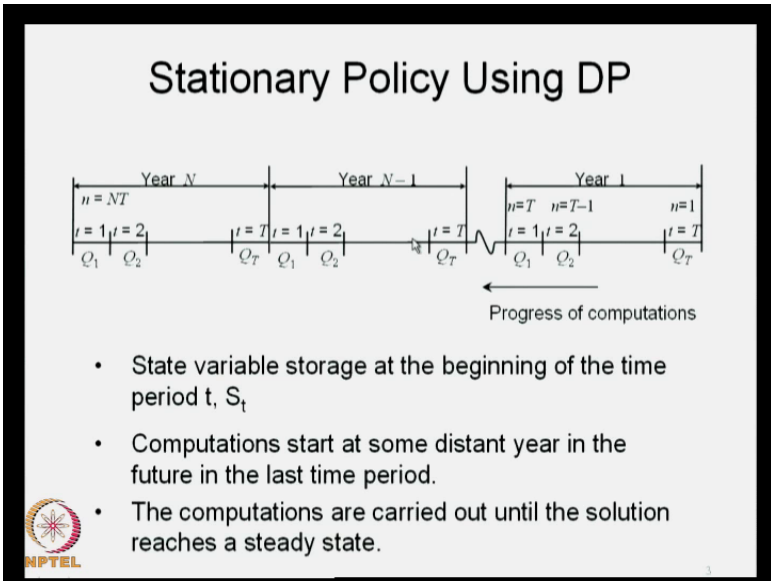

In the first 1 year operation what we did was, we consider just 1 year, any 1 year and then the inflows Q 1, Q 2 etcetera Q T, where all known are given and then we solve the example for solve the problem for only 1 year. When we are looking at stationary policy; however, what we do is we do not solve the problem for 1 year, but, we solve it from a distant future, keep coming back in the backward direction, keep coming back and keep solving this year after year; that means, for year after year you keep solving the same algorithm, until a steady state is reached.

We will see presently, what we mean by the steady state and how we identify the reaching of the steady state. The steady state is assured for these problems because the inflow pattern is remaining the same and your net benefits etcetera the functions remain the same. And therefore, when starting with the distant future you keep solving the algorithm in the backward direction year after year after year etcetera. A stage will come when the steady state is reached and that is where you stop.

Often the students get confused how distant is should be this year or what is that year number from which we start, this has no relevance. Simply start with n is equal to 1 some some time in future, because your inflow pattern remains the same simply solve it, over and over again across several years until steady state is reached. Do not worry about that is year number 50 year number 60 etcetera does not matter because your inflow pattern is fixed, irrespective of which year you consider the same inflow patterns you are using and therefore, do not worry where you are starting.

So, look at this diagram now. You start in a distant year in future and call that as year number 1 and keep going back like this. Now any year N, this is the general year N. So, you keep on coming back like this solve for first year then go to the next year, using the first year optimized solution , solve for the next year next year and so on., Recall that in the dynamic programming , we define the stage to be the time period. So, n is equal to 1 in this particular case we are moving in the backward direction, n is equal to 1 corresponds to the last time period of year number 1, in the computation. n is equal to 2 to t is equal to capital T minus 1, capital T is the last time period in a year. If you are looking at monthly operation T would be 12.

Like this we go and then at n is equal to T, capital T, you have t is equal to 1. Then you go to the next year. In the next year again capital small t is equal to capital T, T minus 1

T minus 2 and so on., it keeps on going and then you go to the next year and so on., So, you keep continuing this computations without halting at year number 1, you go to year number 2, go to year number 3, the same stages you keep on continuing. So, your n if you look , n keeps on increasing n is equal to T, then n is equal to T plus 1, n is equal to T plus 2 etcetera. n keeps on increasing whereas, the small t will always vary between t is equal to 1 to t is equal to capital T.

What I mean by that is if you are looking at monthly operation let us say, you have 12 number of time periods which means your capital T is 12. So, the small t which keeps track of the time period within the year will always vary between t is equal to 1 to 12 because months cannot be anything other than t is equal to 1 to 12. So, the small t keeps track of which month of the year you are solving that problem. Whereas, the small n which is an indicator that is used to keep track of the computations will be continuously increasing. So, n is equal to 1, 2, 3 etcetera up to 12 then 13, 14, 15, 16 etcetera. It keeps on going.

So, small n keeps track of the computations, the small t keeps track of the time period within the year. Now with this now with this understanding we will now solve for all or formulate first for achieving the stationary policy. To begin with we will take the storage at the beginning of the time period as the state variable. Like much like what what we did earlier in the case of dynamic programming for reservoir operation, As I said the computations start at some distant year in future and we carry out the computations until the steady state is reached.

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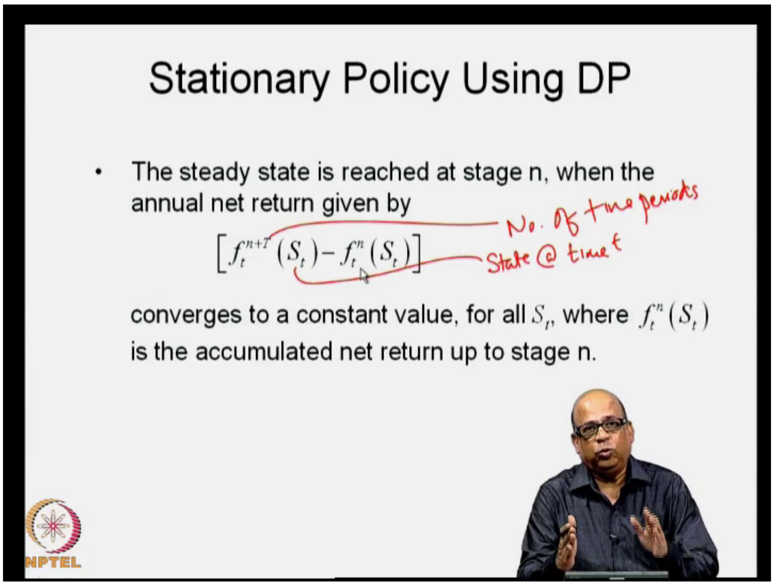

How do we recognize the steady state, you have the system performance measure I request you to refer to lecture number 16 and 17, where we have discussed this algorithm and these notations are all introduced there. So, this is the accumulated system performance at particular stage, n plus T is the stage here. And S t is your storage at the beginning of the time period p and this is the state variable.

So, for a given state variable, if you achieve this condition, that is namely f t n plus T, T is the number of time periods in a year. So, this is number of time periods in a year for a given S t which is the state variable. This is the state at time t.

Now look at this, this term here what does it give this shows that f t n which is for time period t which corresponded to a stage n for a given state S t, you go to the next year in the computation, when you go to the next year the stage will be n plus capital t. Let's say you had 4 time periods in a year and you are referring to the same time period in the next year. How much will be the stage difference it will be n plus 4. So, that is what it indicates. So, this is the same time period t, but, the stage of computations has move to the next year and therefore, we are referring to n plus t here.

So, f t n plus t for a given S t minus f t n S t, this quantity now, what is shown in the brackets, this quantity gives you the annual system performance for a given state variable S t, why annual system performance because you are looking at 1 year apart for the same time period. Let us say you are considering monthly operation and you are looking at the

June performance in the computations. June performance that is f t associated with June for a given S t of a particular year and the June performance for a given S t in the next year in the computation. So, this difference gives you the annual system performance for that particular ah S t.

Now if this remains constant across all a time periods and across all S t's then we say that the system has reached or the policy has reached the steady state and when the policy has reached the steady state, it means that for a given storage which is the state variable you just adopt that policy over and over again.

Let us say that the policy says that for a given storage you make this particular release in in a deterministic sense. Then you keep on following that policy over a long period of time you will get the best annual benefits and these remain constant that is a implication of the steady state policy or the stationary policy. So, this is how we identify the steady state when we are computing the system performance using the dynamic programming.

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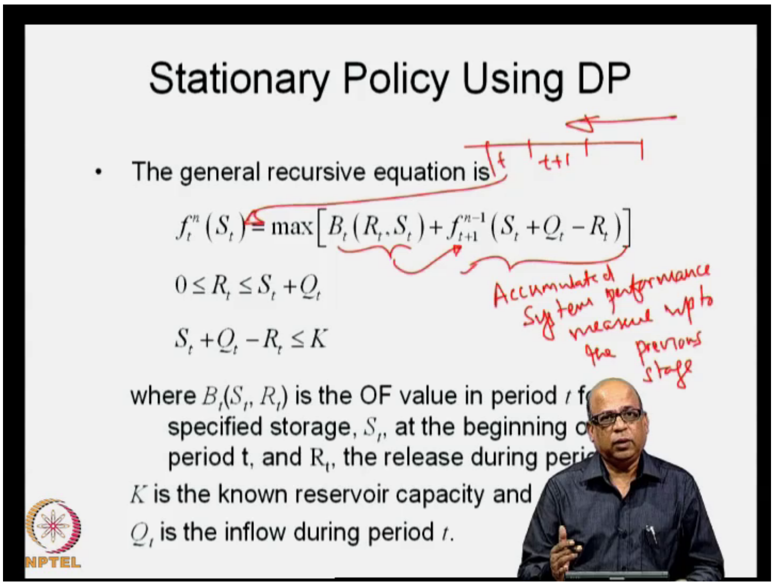

Now, if you recall your recursive relationship again I request you to go to lecture number 16 and 17, where we have introduced this. But, you just understand that you are solving for time period t which corresponds to stage number n and S t is your state variable which is the storage at the beginning of the time period t, you get a immediate return which is in general a function of the release that you make as well as the storage. For example, you may have returns associated with water supply which is for release, you may have returns associated with hydro power which will depend both on the storage as well as on the release and so on.

So, in general the benefit of the returns that you get out of the operating policy will be the functions of both the release as well as the storage. In in a simple (( )) example like this it can be a functions of several other aspects right now we will not go in to the other details, but, just understands that the benefit that is accrued or the returns that is accrued in the time period t, plus this term, second term there, is the accumulated benefit up to the previous time and previous time in the backward direction. So, it will be t plus.

This is accumulated I will not call it as benefit; I will say system performance measure (No audio from 17:12 to 17:20) up to the previous stage. So, this is how you are relating the present time period with the next time period in the backward direction. So, you are moving like this. So, t plus 1 comes first and then t comes. So, you are moving in this direction and you are writing the recursive relationship for the time period t. So, this is for time period t.

And of course, you will have these conditions that is the release must be limited to the water that is available in that time period S t plus Q t. You can also detect the evaporation from this S t plus Q t minus E t, but for the simple formulation will take will ignore the evaporation and this is S t plus Q t minus R. After making the release whatever amount of water that is left in the storage must be less than or equal to the capacity K is the capacity.

Now this is the general recursive relationship that we formulated for (( )) formulated for dynamic programming algorithm for reservoir operation. We will use the same recursive relationship, except that we will not stop at the end of the first year. We will keep progressing on the computations we will after first year you go to the second year, after second year go to the third year and so on, in the backward direction. Your n keeps on increasing t will always vary between t is equal to 1 to, t is equal to capital T. That is all the result, that is all is the difference between what we are doing for stationary policy and what we did earlier for a single reservoir operation.

And remember here we are not specifying any boundary conditions in the sense that we are not pre specifying the storage at the beginning of the time period or storage at the beginning of the first time period in a year or storage at the end of the last time period in a year. We are not specifying anything we are leaving everything free and then seeking solutions for what should be my policy for a given time period, if the storage at that time period is specified. And storage at that time period is in fact the state variable.

And we know now how to identify the stationary stationary policy or how to identify that the algorithm in fact, reaches the steady state. Now we will take the example, in fact, this is the same example the data is the same as we introduce for the dynamic programming in perhaps lecture number 16 or 17, I encourage you to go to those lecture and see this example, this particular example. We are just picking up the examples and then solving if for several years until we reach the steady state.

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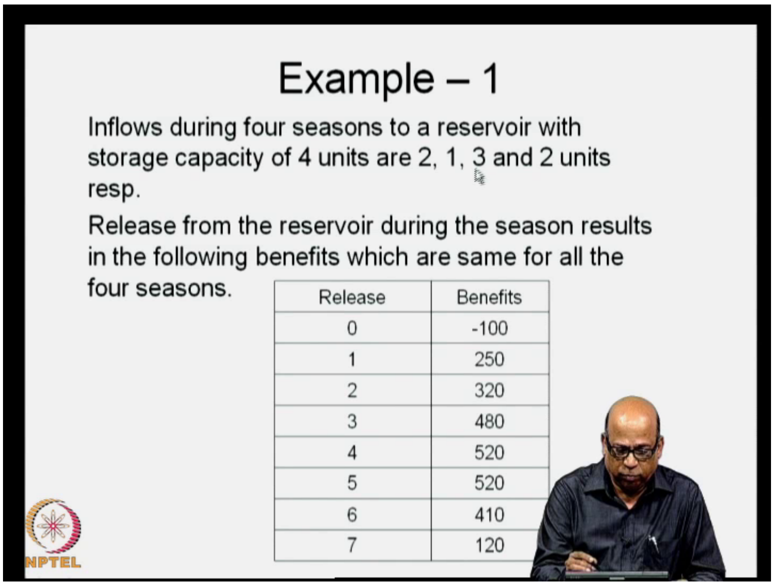

So, the benefits are the same there are time 4 time periods that we called it as 4 seasons and the reservoir capacity is 4 units. You have the inflows to 2, 1, 3 and 2 units and we have the benefits associated with only release. For this numerical example we will not worry about benefits associated with the storage, but just associated with the release. And these are the benefits.

Remember this I keep on repeating the benefits need not always be monitory benefits. In fact, in systems formulations we use different types of objective functions for example, you may specify a target demand to be met and then we may consider the objective function or the system performance to be just the deviation of R t minus D t or the absolute value of R t minus D t and then you may want to minimize this. Or if you are talking about hydro power you may talk about the actual amount of power that is generated and you may want to maximize the power.

So, the benefit function that I am mentioning here can be made as as system representative as you desire it is not just a monitor benefit this you keep formally in your mind. So, this is same data that we use you can go up to 7 because the maximum flow is 3 and the storage capacity is 4. So, 4 plus 3 you go up to 7, 7 units and then associated with 7 units you have the returns or the system performance measure given here and this table remains the same across time periods which means that the benefits associated with the particular release are the same in all the 4 seasons that we are considering for simplicity.

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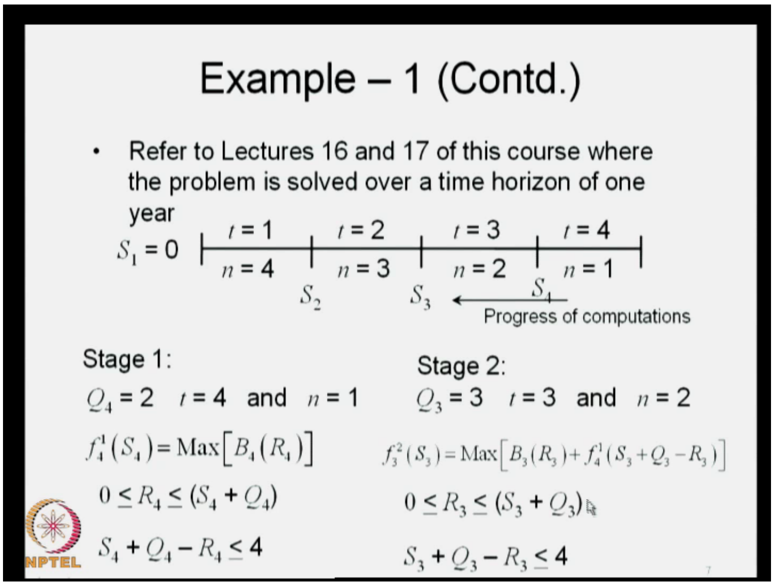

What we did in that example just to recollect, is that we specified the beginning of the time period storage S 1 is equal to 0 and then we progressed in the backward direction starting with t is equal to 4, this is the last season in the year and then we progressed in the backward direction for example, stage 1 which corresponded to t is equal to 4 and that had a flow of 2 we wrote the recursive relationship here, because there is nothing to look beyond. Remember in that case we solved it only for 1 year. I am showing here the problem that we have already solved in lecture number 16 or 17 where we solved it for 1 time period, 1 year I am sorry 1 year.

And then we went on to stage 2. In stage 2 we related with the system performance of the previous time period which is t is equal to 4 in that case previous stage F 1 and then S 3 plus Q 3 minus R 3 defines the state at that particular time period. This is the storage at the beginning of the time period 3 plus the inflow during the time period 3, minus the release that you have made during a time period 3, which is the decision variable. So, this is how you relate from 1 time period to another time period and in of course, these conditions.

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For example, Stage-2 calculations $Q_3 = 3$ $t = 3$ and $n = 2$

S_3	R_3	$B_3(R_3)$	$S_3 + Q_3 - R_3$	$f_4^1(S_3+Q_3-R_3)$	$B_3(R_3) + f_4^1(S_3+Q_3-R_3)$	$f_3^2(S_3)$	R_3^*
	1	250	4	520	770		
	2	320	3	520	840		
2	3	480	2	520	1000	1000	3, 4
	4	520	1	480	1000		
	5	520	0	320	840		
	2	320	4	520	840		
	3	480	3	520	1000		
3	4	520	2	520	1040	1040	4
	5	520	1	480	1000		
	6	410	0	320	730		
	3	480	4	520	1000		
	4	520	3	520	1040		
4	5	520	2	520	1040	1040	4, 5
	6	410	1	480	890		
	7	120	0	320	440		

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Refer to Lectures 16 and 17 of this course

So, when we solve this remember as a sample example, the sample calculations, I will show you for S 3 is equal to 2, 3 and 4. You got the system performance measure all of these computations are simple computations you will look at the immediate benefits and then look at what is the water available for the next time period and that defines your system performance for the next time period and so on., you add this 2 up and then pick up the maximum if you are looking at the maximum value and then get the R 3 star value.

So, this is all that we have done in the earlier lectures 16 and 17 like this you keep on doing until you will reach the last stage which is n is equal to 4 that corresponded to n is equal to that is t is equal to 1 in that particular case. And at t is equal to 1 in the example that we solved earlier, your storage was fixed. We said that S 1 is equal to 0 for that particular problem. So, when you go to t is equal to 4, t is equal to 1 I am sorry n is equal to 4 in the backward direction, you will start with S 1 is equal to 0 and solve it only for AS 1 is equal to 0.

When we come to stationary policy; however, we do not stop the computations and we carry it further and then see when the steady state reaches. So, the computations nature of computations till remains the same, but you keep moving forward forward in the in the backward direction keep moving ahead in the backward , until the steady state is reach. So, this you must understand correctly we do the same computations, but we do not stop at t is equal to 1, that is n is equal to 4, you go to n is equal to 5, which corresponds to t is equal to , n is equal to 6, which corresponds to t is equal to 3, n is equal to 7 which corresponds to t is equal to 2 and so on., like this you keep on repeating the same type of computations until the steady state is reach let us do that now.

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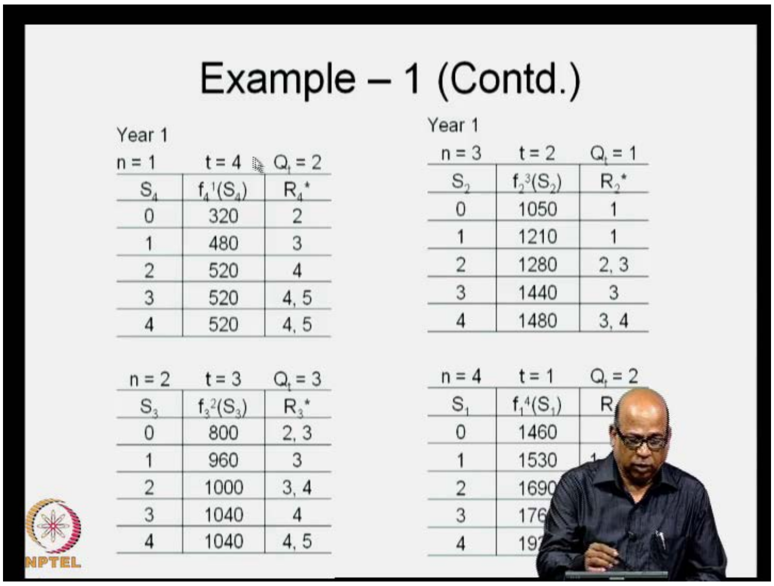

So, for completeness sake now I will show for year number 1, I am just showing these values now the F values associated with the particular stage and particular time period and F 3 star values or the R star values for that particular time period. All the intermediate calculations are used only (( )) to get this. So, we will only look at these values now. So, for year 1, n is equal to 1 and Q t is equal to 2, my f values will be like this 3, 24, 80 etcetera, we are solving for all the storage states 0, 1, 2, 3, 4 etcetera and the associated release values are like this.

Then for year number 1, n is equal to 2 we go and then resolve we get these values then you go to n is equal to 3, n is equal to 4 and so on., So, you keep on repeating the calculations always relating with the previous stage. For example, this relates to this and this related to this this stage and this stage relates to this stage and so on.

Now at the end of year number 1 you have reach t is equal to 1 and the stage is 4. Do not stop the computations at this point also see that we are solving it for all possible storage states S 1 is equal to 0, 1, 2, 3, 4 unlike what we did earlier where we solved only for S 1 is equal to 0.

Now we progress to year number 2. So, what we are looking at is this diagram here. So, year number 1, we are progressing in backward direction go to year number 2, solve for year number 2, go to year number 3, solve for year number 3, always relating with what has happen in the previous stage this is what we are doing.

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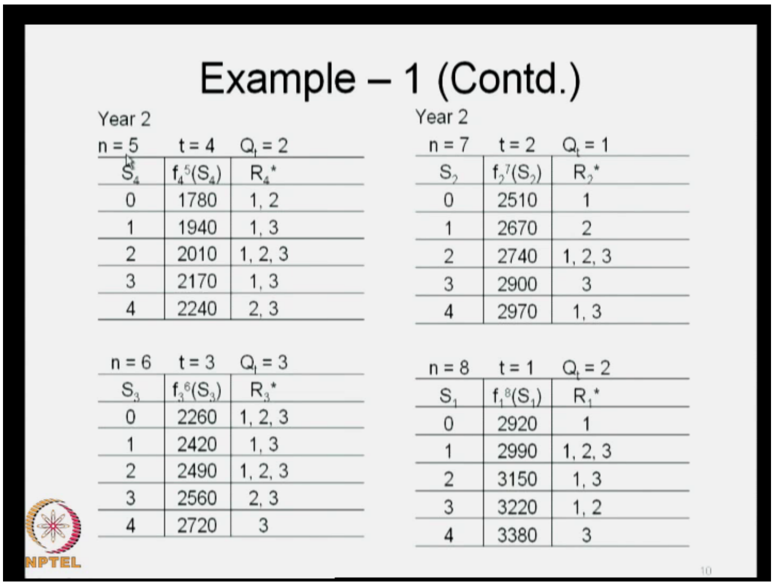

Example – 1 (Contd.)

Year 2

n = 5 t = 4 $Q_t = 2$

S_4	$f_4^5(S_4)$	R_4^*
0	1780	1, 2
1	1940	1, 3
2	2010	1, 2, 3
3	2170	1, 3
4	2240	2, 3

n = 6 t = 3 $Q_t = 3$

S_3	$f_3^6(S_3)$	R_3^*
0	2260	1, 2, 3
1	2420	1, 3
2	2490	1, 2, 3
3	2560	2, 3
4	2720	3

Year 2

n = 7 t = 2 $Q_t = 1$

S_2	$f_2^7(S_2)$	R_2^*
0	2510	1
1	2670	2
2	2740	1, 2, 3
3	2900	3
4	2970	1, 3

n = 8 t = 1 $Q_t = 2$

S_1	$f_1^8(S_1)$	R_1^*
0	2920	1
1	2990	1, 2, 3
2	3150	1, 3
3	3220	1, 2
4	3380	3

So, after year number 1 calculation here, we go to year number 2, which means n is equal to 5, see n is equal to 4 and then we go to next stage n is equal to 5, but that corresponds to t is equal to 4 and we solve this this relates to this f 4, 5 here relates to f of the previous stage which corresponds to time period 4. So, we relate this with this value now, f 1 4. And then we solve this we get the f values we also get the corresponding R 4 values.

See observe that for several states here there are multiple solutions for example, when S 4 is equal to 2, you may have R 4 as 1, 2 or 3 like this there are several states for which multiple solutions exist. Now you can carry on for year number 2. So, you went up to n is equal to 8 because there are 4 time periods. So, every year will have 4 time periods, t is equal to 4, 3, 2, 1 in the backward direction, but, n keeps on increasing 5, 6, 7, 8. Like this you go to year number 3, n will be 9 9, 10, 11, 12, but, t will be still 4, 3, 2,1 like this you go year 3, solve for year 3, go to year 4, solve for year 4.

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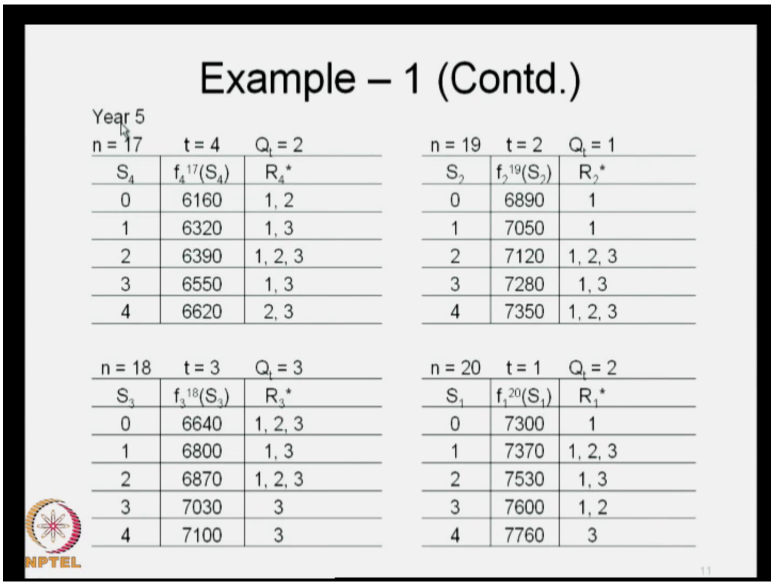

Example – 1 (Contd.)

Year 5

n = 17 t = 4 $Q_t = 2$

S_4	$f_4^{17}(S_4)$	R_4^*
0	6160	1, 2
1	6320	1, 3
2	6390	1, 2, 3
3	6550	1, 3
4	6620	2, 3

n = 19 t = 2 $Q_t = 1$

S_2	$f_2^{19}(S_2)$	R_2^*
0	6890	1
1	7050	1
2	7120	1, 2, 3
3	7280	1, 3
4	7350	1, 2, 3

n = 18 t = 3 $Q_t = 3$

S_3	$f_3^{18}(S_3)$	R_3^*
0	6640	1, 2, 3
1	6800	1, 3
2	6870	1, 2, 3
3	7030	3
4	7100	3

n = 20 t = 1 $Q_t = 2$

S_1	$f_1^{20}(S_1)$	R_1^*
0	7300	1
1	7370	1, 2, 3
2	7530	1, 3
3	7600	1, 2
4	7760	3

Like this we keep on going I show here year number 4 calculations 17, 18, 19 and 20. So, you have solved it for year number 5 now. And these are the accumulated system performance measure that we are getting. Now at every stage when you are solving, when you reach let us say the third year, you should start looking for the steady state whether steady state has been reached or not. You have to start examining whether steady state has been reached or not; obviously, this we cannot do for 1 year, you have to do at least 2 years and then go to the third year and then look at between third, year a given state and the second year, a given state, that value remains the same as second year and first year.

So, at least you must do these calculations for 3 years and then start looking at whether steady state has been reached. But, in general it is advisable that you carry out at least 5 or 6 years and then start looking at the steady state and the steady state will be reached fairly early within 4 years 5 years and so on, as I will show. So, we have solved it up to 5 years now we will examine whether the steady state has been reached.

How do we examine this? You look at this expression here. In fact, this part you have to understand correctly. You are looking at a particular time period f t and we are looking at the system performance associated with the given state variable S t, given state S t between stage n and stage n plus t, which means for the same time period, you are looking for 1 year behind, in terms of our computations.

So, n plus t minus n is actually t which is the 4 which is the t number of time periods within that year and in the example there are 4 number of time periods. So, you pick up a particular time period t let us say t is equal to 3 and for a given t let us say S t is equal to 2. You look for that particular stage and n plus t behind or n plus t ahead in the in the sense of stages, increasing stages.

And see this value that you get which is a annual system performance measure for that particular storage state. If this annual system performance measure remains constant as you change n your capital t is the same, but, as you change n if this system performance measure that I have shown here remains constant then the steady state is reached. Let us examine what happens after we solve for 5 years here. So, this is a year number 5, I have not shown computations for year 3 and year 4. So, you continue and year 5 you will get these values here.

Now let us say that you want to examine for t is equal to 2, now t is equal to 2 corresponds to n is equal to 19 here and what is the other one that you have to consider 19 minus 4 which is 15.

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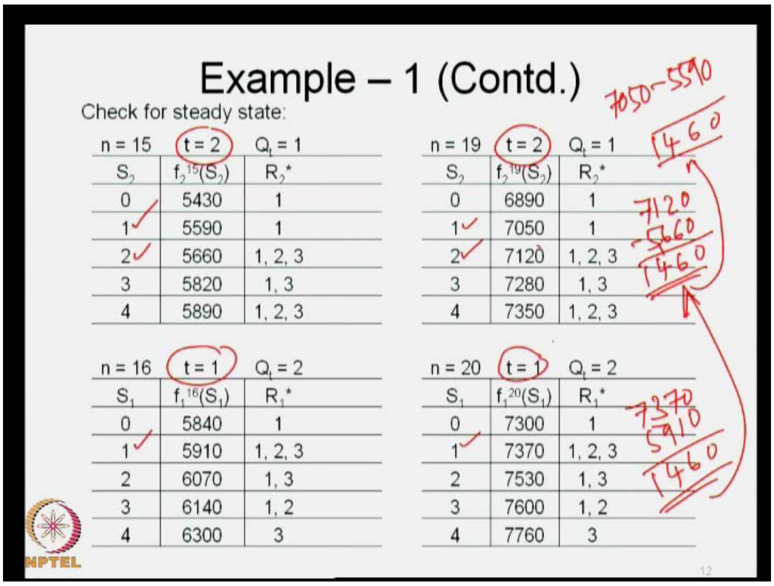

So, let us see n is equal to 19 which corresponds to t is equal to 2 the last time we visited t is equal to 2 in our computation was when is n is equal to 15, which is the previous year. So, we consider these 2 tables now. n is equal to 19 and n is equal to 15 both of which corresponds to the same time period t is equal to 2 and therefore, the difference between the f values that you get for a given state is the annual system performance measure.

So, let us say that you look at this difference now. So, S t is equal to 1 let us say, S t is equal to 1 here, n is equal to 15, t is equal to 2 here, t is equal to 2 here. So, for the same time period you are looking at 1 year system performance measure for a given storage state. So, S 2 was equal to 1 here, S 2 is equal to 1 here. This is 7050 minus 5590. So, this would be 0 6 and 4 14 60.

Similarly, for 2 we will see, this is the 2 here, this is the 2 here for same t is equal to 2, 7120 is what you are getting here minus 5660 14 60. So, this 14 60 is the same as this 14 60. So, we have examine for S 2 is equal to 1 as well as S 2 is equal to 2, like this you examine for all given values of S 2 for t is equal to 2. You get the same value of 14 60.

Similarly, you come to t is equal to 1. So, t is equal to 1 in year 5 corresponded to n is equal to 20 and t is equal to 1 year in the previous year 4 corresponds to year n is equal to 16. So, look at these 2 values now, S 1 is equal to 1 let us say 7370 that is I am considering S 1 is equal to 1 for this it is 5910. So, this is 0 6 4 and 14 60. So, this is the same as this 14 60 this is same as 14 60.

So, like this now when we solved for 5 years time period when you look at the same time period across different years in the computations, the accumulated system performance measure you pick up and then calculate the annual system performance. This difference here in fact, corresponds to the annual system performance measure corresponding to that particular state. This remains constant, as you can see for any state you pick up for any time period the annual system performance remain remains the same namely in this particular case 14 60. And that is where we say the steady state policy has been reached or the steady state has been reached. And the policy associated with this is in fact, the stationary policy.

So, what is the stationary policy now for t is equal to 1 if you specify the storage you follow this particular policy as this shown here. So, let us say that the stationary policy in fact, will now be for t is equal to 1 if S 1 is equal to 0, R 1 is 1, if S 1 is equal to 1, R 1 can be either 1, 2 or 3 if S 1 is equal to 2, R 1 can be 1 or 3 if S 1 is equal to 3, 1 and 2 and so on., So, this is for t is equal to 1. Then for t is equal to 2 for a given S 1, S 2 you have the associated R. Like this you can do it for t is equal to 3 t is equal to 4 and so on.

Remember irrespective of the time period, irrespective of the storage state if you follow this policy specified for that particular time period for that particular storage if you specify follow this policy the annual returns will remain the same 14 60, 14 60 etcetera. So, irrespective of your storage state and the time periods the annual returns will remain the same, if you follow this policy over and over again in a long period of time. Now that is what is a steady state policy. So, this is what I have shown here how to examine whether the steady state policy has been reached.

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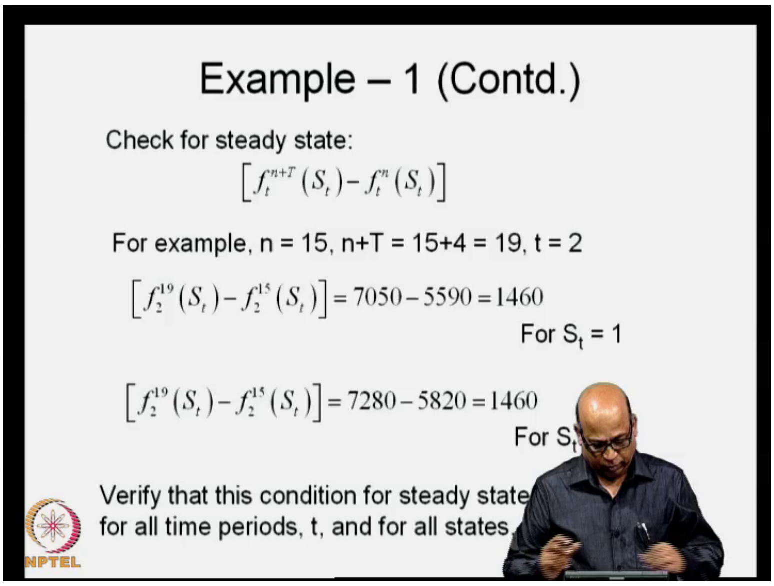

So, n is equal to 15 n plus t is 19 and for t is equal to 2 we get a 14 60 and for S t is equal to 1 this is. Similarly, for S t is equal to 3, you get 14 60, you can verify that for any S t's here you will get the same return 14 60 and that is where the steady state has been reached.

Remember we are able to achieve the steady state in this particular case, because the inflow patterns remains the same, you are not changing the inflows from 1 year to another year. So, the same inflow pattern is repeating and therefore, you will get the steady state also your nature of objective function still remains the same. So, you are simply accumulating the objective function associated with different state variables and then looking at the difference in terms of the annual returns that you get corresponding to that particular state variable. And that is why you are able to achieve the steady state.

If your inflows were changing from 1 year to another year let us say that you were solving if for 10 years and every 10, every year the inflow pattern was changing then at the 10 years you will not get a steady steady state policy. You are able to achieve the steady state remember I repeat, only because the inflow patterns remains the same.

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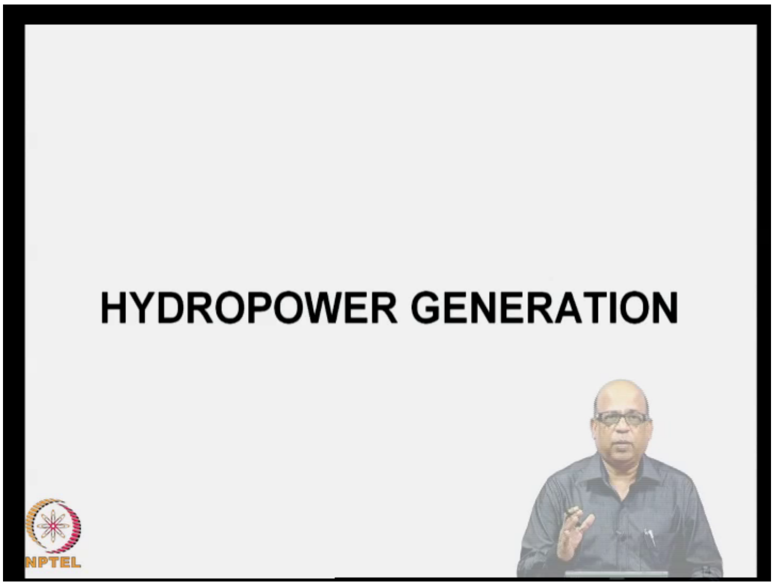

Alright, now we will move to a different topic, but, just let me summarize what we did in the optimal reservoir operation. Initially we considered a single reservoir and then looked at the standard operating policy and then formulated a linear programming problem, after going through the sequent peak algorithm etcetera.

So, the linear programming problem specified an optimal operating policy for a given objective function; that means, you you specified the objective of operation and then for that particular objective, you achieved a operating policy optimal operating policy in terms of the end of the year, end of the period storage that needs to be maintained.

Then we went on to look at how we formulate for multiple reservoir systems then so for in today's class I just discussed the steady state policy using the dynamic programming. The dynamic programming is ideally suited for reservoir operation problems because it is a sequential decision making problem and therefore, we use the dynamic programming problem with the same sequence of inflows repeating, because it is a deterministic problem we take the same sequence of inflows from year to year, may be this may be average inflows for various time period within the year, but the same sequence repeats and then we solve the dynamic programming over a number of years until the steady state is reached and I've just demonstrated how the steady state is reached.

So, form this now we move to a slightly different problem of hydro power generations still sticking to the reservoir operation. In in the reservoir operation problem that so for I discuss in the problems that I discuss so for, we did not look at the details of a particular purpose for which the reservoir is operated. For example, we did not look at the irrigation, how the cropping pattern changes, how the water demand changes from time period to time period, how the soil measure changes from time period to time period and so on. We did not look at those details.

Now we will… what we will do is, that we will start introducing some details associated with hydro power generation, which means that you are looking at the operation of the reservoir only for hydro power generation. So, what we will do now is that we will look at details of hydro power generation and see how we obtain the reservoir operating policy or how we simulate to begin with let us simulate this for hydro power generation.

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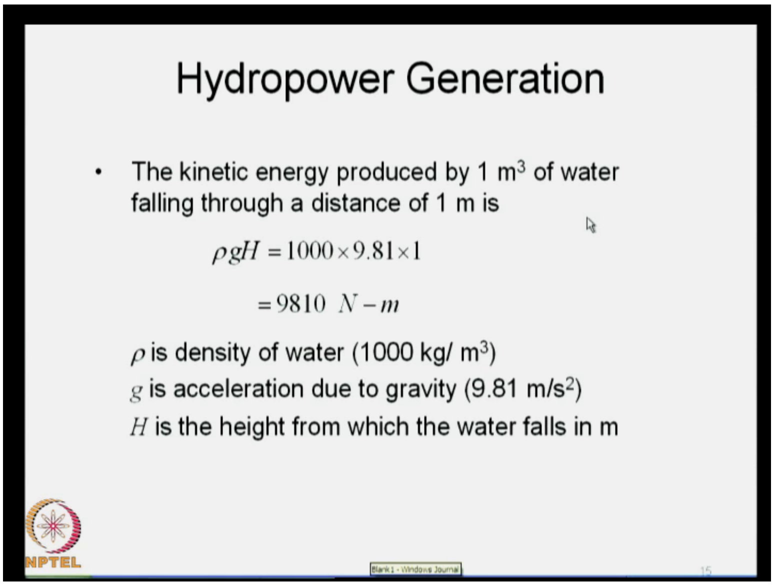

From your early physics you know that you have, let's say.

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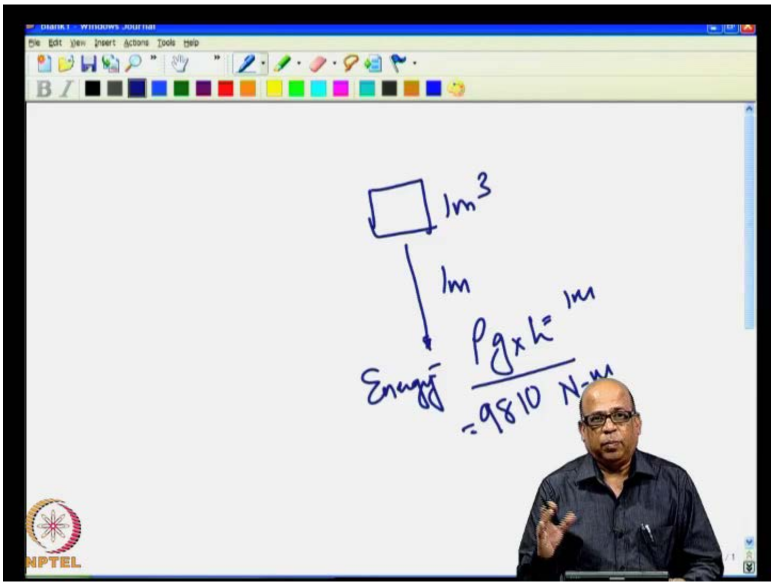

One million 1 meter cube of water falls through 1 meter. Then it generates it has a kinetic energy,this is row into g into H, H is 1 meter. So, H is the height through which it falls. So, it has an energy of row g h. This is 1 meter cube of water falling through 1 meter height and this has a energy of row g H and H is in this particular case 1 meters. So, this is will take 9810 Newton meters, g is 9.8 and this is 1000 kg per meter. So, 1000 k g per meter cube and therefore, you get an energy of 9810 Newton meters.

The power is energy per unit time, energy produced per unit time is the power. So, this is what we know from your basic physics. So, 1 meter cube of water falling through 1 meter height produces a power of 9810 watts if it is per second; that means, it has an energy of 9810 Newton meters and then per second it will be 9810 watts.

Starting with this basic principle now, we will in fact, this is what is this is the energy that is used in producing the power, hydro power. Starting with this basic principle now, we will look at how we handle the power generated from a river and also from a reservoir. Let's see how we do that.

So, as I said row g H is, row is 1000 k g per meter k g per meter cube 9.8 1is your g, H is 1 meter. So, you get a energy of 9810 Newton meters. Now if this water is falling per second; that means, continuously same amount of water is falling at at a uniform rate then the power that is produced is 9810 watts.

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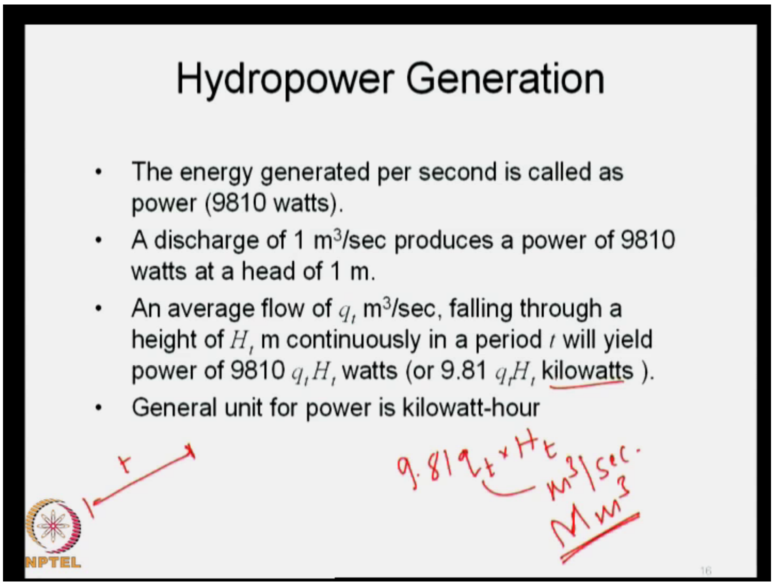

So, this is energy generated per second is power that will be 9810. Now this is what we did for 1 meter cube of water falling every second over a 1 meter distance. So, the discharge there is 1 meter cube per second. So, this is the discharge of 1 meter cube per second that is falling through 1 meter height, this produces 9810 watts.

Now let's say that you are passing a discharge of Q t meter cube per second from 1 meter cube per second we have gone to Q t meter cube per second in a time period falling through H t meters; that means, not from for 1 meter, but, H t meters then; obviously, the power will be 9810 into Q t into H t, because 9810 for was for 1 meter cube per second for 1 meter. So, it will be 9810 into Q t into H t watts or divided by 1000. You write it as 9.81 Q t H t kilo watts generally we use kilowatts mega watts and so on. So, we divide this you get 9.81 Q t H t kilowatts.

You must be aware that a common unit that we use for power is the kilowatt hour that is, so, many kilowatts produced across hour. So, if you multiplied by number of hours in a time period then you will get kilowatt hours. Fr example, in 1 day continuously you are producing this power that will be these many kilowatts into 24 hours. So, if you multiply the kilowatts by hours you will get the unit which is kilowatt hour.

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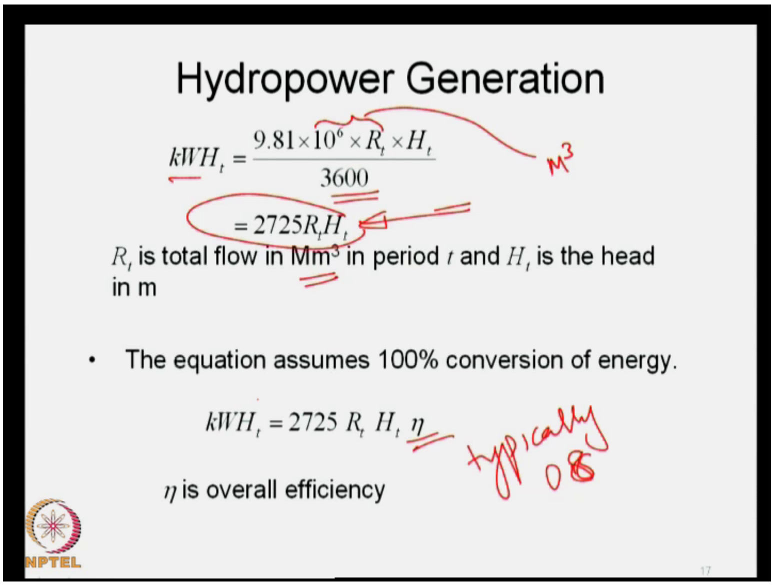

So, starting with this now, let's say that your your kilowatts was 9.81what we had here was Q t meter cube per second. Ok let us derive this. So, that there will be no confusion, what we have here is that you have 9810 Q t into H t watts or 9.81 Q t into H t kilowatts 9.81 Q t into H t kilowatts is what we have.

Let's say your time period is some 1 day or number of days and so on. We have to convert this into the number of hours that this time period t has and also in most of the cases we are talking about the discharge in million cubic meters. This was in meter cube per second. But, we will be talking about million cubic meters. Let's say that you are making the release through the pen strokes in million cubic meters over this particular time period, let's say this time period is 1 day or 2 days and so on. So, you are making a total discharge of so many million cubic meters over this time period p.

Using this now expression 9.8 1 Q t into H t kilowatts, we will convert this into megawatts. Let us say this was 9.81 into 10 to the power 6 into R t. So, which means this is in meter cube and divide by number of seconds. So, you will get if your R t is in million cubic meters convert that into meter cube first by taking 10 to the power 6, divide it by 3600 to make it meter cube per second, then you will get 9.81, this is Q t now into H t. That will be 2725 R t into H t. So, this is the expression that we use for kilowatt hours that is produced.

Now R t is the total flow in million cubic meters in period t. So, this million cubic meters is what we have converted into meter cube here. And meter cube per second is what we want therefore, we have use 3600 and therefore, you get k kilowatt hours is 2725. Now this assumes that 100 percent of the energy is converted into power, but, that will not be the case, you will have a efficiency for example, you may have conveyance losses, you may have losses of loss of head in the turbines itself, there will be turbine efficiency of converting kinetic energy into converting the kinetic energy into electric energy and so on.

So, you will have an overall efficiency associated with it and that we multiply. Typically it may be of the order of 80 percent or 0.8, 0.83 and so on. So, we we will have to convert that use that and look at the kilowatt hours. So, eta is the overall efficiency here, typically of the order of 0.8.

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There are ways of determining this, but, just remember it may be of the order of about 80 percent. So, this is the the way that you get it in kilowatt hours.

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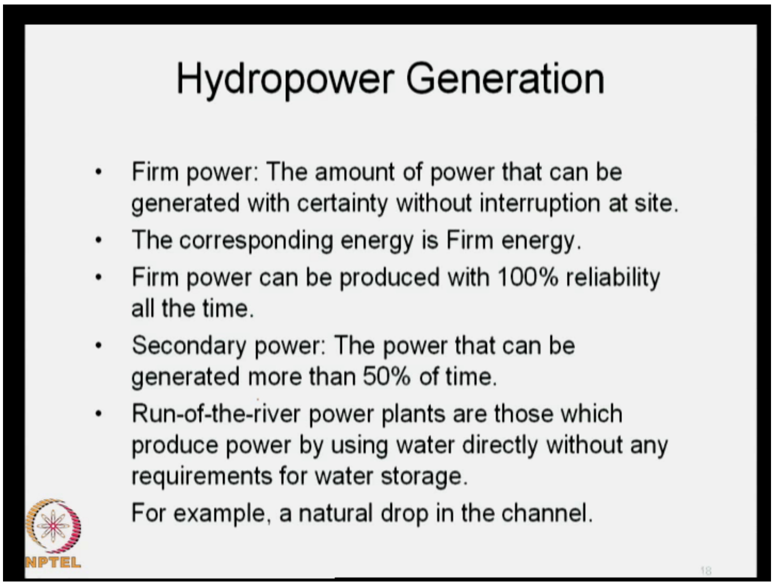

Alright now the firm power, there are some concept that we have to be clear on this. Let me just go through this. There is a concept of what is called as a firm power or the minimum power that you can produce at a particular site. This is the power that is available with 100 percent reliability. Remember our power that is produced is a function of both the discharge as well as the head.

Look at this expression this is the discharge and this is the head. At given site, depending on the water that is available and depending on the head that is available in a particular time period, you will have a minimum power that can be produced. Now this is the assured power. So, this is power that is available with 100 percent reliability and then that is a minimum power that you can produce and this is called as the firm power.

We also have a concept called as the secondary power secondary power is that power which is available with 50 percent reliability. Obviously, the secondary power you can… let's say that you are promising an industry, from a particular power house you are promising an industry that such an such a power is available with 50 percent reliability.

Now this power is generally priced much lower than the firm power. Firm power is making you are making a firm commitment that 100 percent of the time I will supply this particular amount of power to a particular user, let's say industry or municipal municipal locality and so on. So, that is made at 100 percent assurance. So, that is a firm power.

Secondary power is over and above the firm power you are able to produce additional power which is not available 100 percent of the time, but, it is available some (( )) time. So, 50 percent of the time it is available and that is called as a secondary power. Then we have a concept of the run of the river system. So, let us look at the way the run of the river systems is produced, run of the river systems operate.

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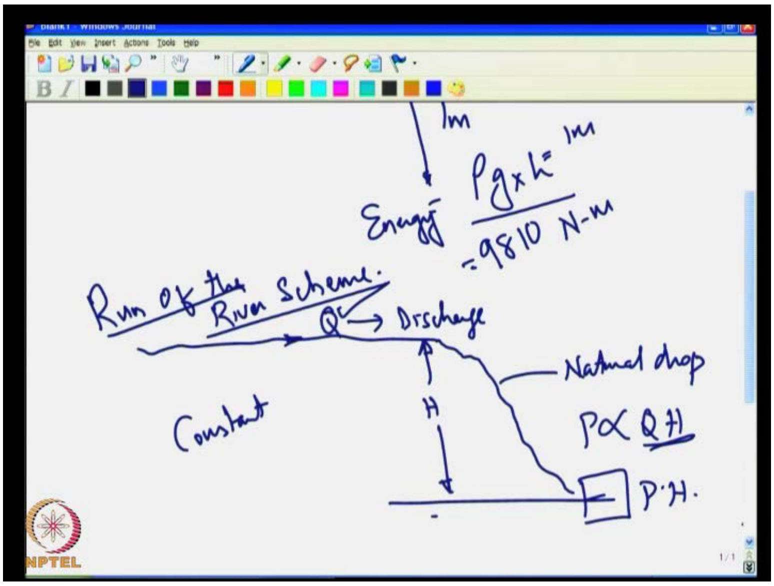

Let us say there is a river and then it enters a terrain where there is a natural drop. So, this is a natural drop that is available and you put a power house here. So, you do not build any reservoir or any structures major structures here to produce a power. So, this is the head that is governing. Because there is the natural drop you know the depth of the flow here is negligible compared to the drop that is available and therefore, what happens is, this head is always available to you. So, there is a constant head near constant head that is always available to you for hydro power generation.

And your power is proportional to Q into H, where Q is the discharge that is coming here, it is a discharge. And H is the head which is in meters. So, discharge is in million cubic meters, H is in meters. So, the power is proportional to Q into H. In fact, it is a equal to gamma Q H as we saw, because H remains constant here, in the case of run of the river schemes.

Remember in the run of the river scheme, we do not construct any reservoir or a dam to build up the head and storage. Simply we use the natural drop that is available, use the water that is coming through the river and put a power house, put a turbine here and produce the power that is all and therefore, because we are talking about the natural drop the head remains constant.

Typically these are small head power systems, where the head will be of the order of 30 meters, 40 meters, 50 meters and so on. It will not be as large as 200 meters, 30 300 meters and so on. And also the discharge that is coming here is the natural river flow and therefore, typically the natural run of the river schemes will produce not very large amount of power, they are limited in their power generation capacity.

So, when you have the head constant the power that is generated is mainly decided by the Q, which is the discharge that is coming at that particular point. So, this is the point at which you are generating the power. So, we will look at how you produce the run of the river power.

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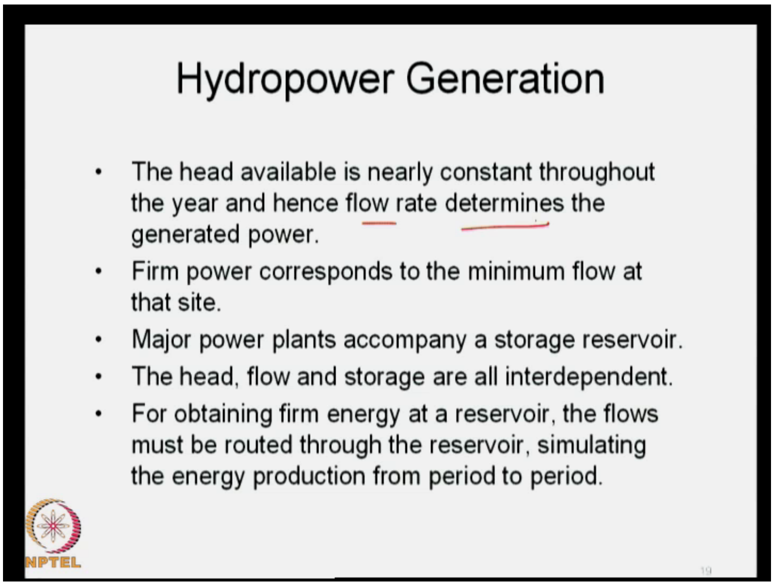

Now the… because the head available is nearly constant the flow rate determines the generated power. Now in such situations remember what I said about firm power, firm power is the minimum power that is available from a particular generating station. Or the firm power is available with 100 percent reliability. Now here H is constant and the power generated is dependent on Q and H because the H is constant and you are looking at the minimum power or the firm power what will be the firm power. The firm power will correspond to the minimum flow that comes in the river.

So, the minimum flow determines the firm power in the run of the river system, because your head remains constant. So, this is what we do in hydro power generation, systems techniques using for hydro power generation. So, first we are looking at the river run of the river schemes where the head is constant then subsequently will also go at, go to the reservoirs where we build up the head, build up the storage and then create a head and pass the discharge through the pen strokes, where there is a controlled flow through the pen strokes.

We will continue the discussion in the next class. So, essentially in today's class I discussed about the steady state policy or the stationary policy for a reservoir using the dynamic programming. We discuss how to achieve the steady state, how to examine for the reaching of the steady state and what is the implication of the steady state, in terms of its operation, recall that the annual returns remain constant when the steady state is reached.

And then we went on to introduce the concepts basic concepts for hydro power generation, essentially we are using the fact that the water falling through a particular height produces energy and this kinetic kinetic energy is converted into electric energy through turbines and and so on.

And we are looking at the principles of how the how to account for this energy generated through the discharge as well as the head. In the run of run of the river systems, we do not create any reservoirs there, we use the naturally available drop head and then produce the energy. So we will continue this discussion in the next class. Thank you for your attention.