Water Resources Systems

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Lecture No. # 24

Multi-reservoir systems

Good morning, and welcome to this the lecture number 24, of the course, Water Resource Systems - Modeling Techniques and Analysis. And now in the previous lecture, we went through the reservoir operation problem, we started with the standard operating policy; recall that the standard operating policy deals with the release policy whereby, we meet the demands to the best extent possible from the amount of water available during the current time period.

We do not look beyond; we just look at the current time period; and look at the amount of water available, which is given by S plus Q, where S is the storage and Q is the inflow during the period, compare it with the demand, and if we have adequate amount of water to meet the demand, you just meet the demand, and if you in periods, where you do not have adequate water, which means S plus Q is less than the demand, then you empty the reservoir, provide all the water to meet the demand, and then empty the reservoir; that is the idea of the standard operating policy of course, you can also include the evaporation losses there, S plus Q minus E is what we look at in the standard operation.

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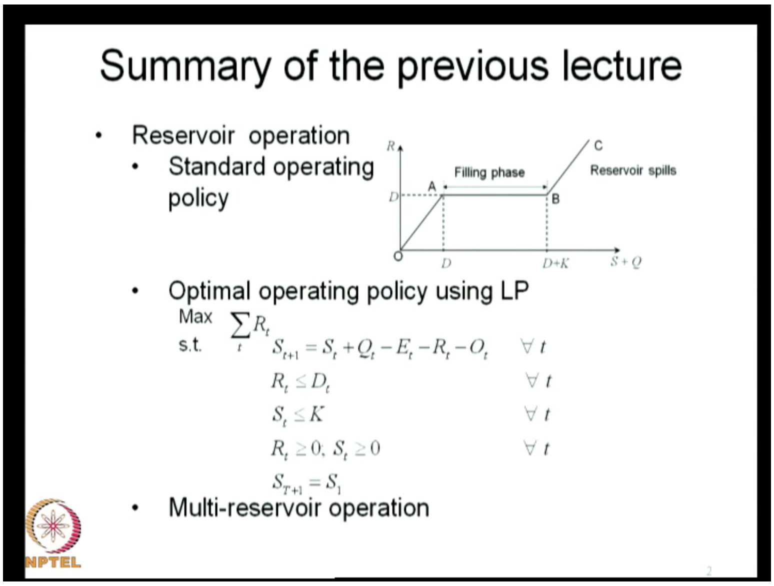

So, this is what we discussed? This is the phase in which the reservoir will be empty after you meet the demand, D is the demand, and this is the S plus Q ,which is the storage plus the reservoir, reservoir inflow. During this period it is a filling phase, because after you meet the demand, you still have some water left that will contribute to the filling of the storage reservoir, beyond this point, there will be reservoir spills; that means, you have met the demand and at the end of the period, you still have excess water; that means, even after meeting the demand, you have excess water that goes the spill, because you can only store up to the capacity.

Now, that is the standard operating policy, as I mentioned in the previous class, the standard operating policy is not an optimal policy, because you are not looking at other time periods, then we discussed, how to formulate an optimal reservoir operating policy for the same given data, the simplest form that we discussed was the LP formulation linear programming problem formulation, where we are looking at maximization of the realization R t over all the time periods, so it means essentially, we are looking at maximizing the some of the releases during all the time periods subject to all the standard constraints, this is the continuity equation.

And then we set R t less than or equal to D t, because we are maximizing R t, you put an upper limit to make sure that R t approach is that upper limit, and then what is the upper limit, upper limit is the demand itself, and these problems typically we are solving for one single objective; namely the water supply, where the demands are all known.

And therefore, you would like to meet the demands to the best extent possible in all the time periods, compare this with what we did in the standard operating policy, in the standard operation, you did not look at any other time periods, when we are making a decision during the particular time period; whereas, in the optimization, you arrive at a certain operating policy, in terms of the releases to be met as well as in terms of the storage to be maintained in the reservoir by looking at the entire year. So, typically we solve it for one year time horizon by looking at the entire year, you will arrive at the optimal operating policy; that is the major difference.

And therefore, the standard operating policy will not yield an optimal policy; whereas, the optimization techniques that we have dealt with in this course can be used to obtain the optimal operating policy, then we towards the end of the last lecture, I introduced a simple multi - reservoir system, and we are interested in obtaining the release policies, for such multi - reservoir systems by looking not only across time periods, but also looking across space; that means, what is likely to happen at a particular reservoir, because of the decisions that we take at some other reservoirs. So, we will look at multiple reservoir systems, multi - reservoir systems as an integrated system, and then arrive at decisions; typically, the optimal release policies at each of the reservoirs, now we will continue the discussion. So, for completeness, I restate the problem, and then we will formulate the mathematical problem.

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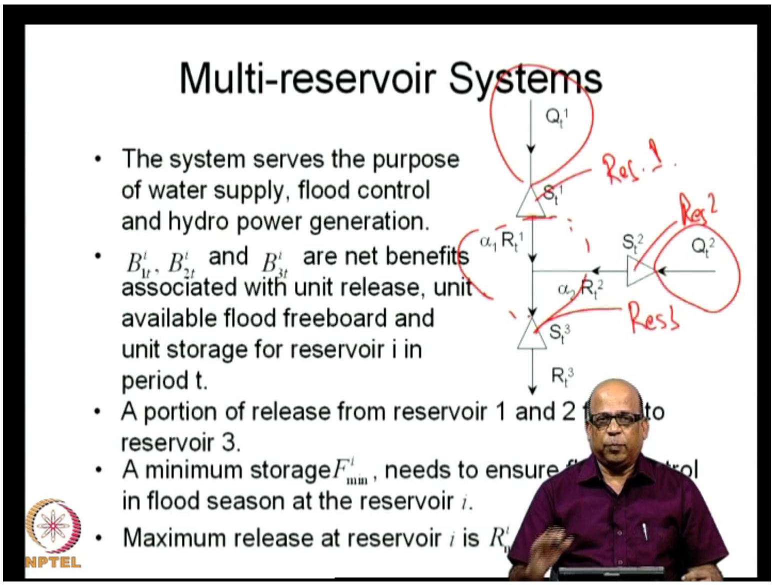

So, this is the system that we were talking considering, there are three reservoirs, I encourage you also look at the previous lecture, where I have discussed is in some detail, but I will just give the summary. There are three reservoirs here, each of these 2 reservoirs; that is the reservoir 1 and reservoir 2, this is reservoir 1, and this is reservoir 2, and this is reservoir 3.

The reservoir 1 receives it is own inflow through it is catchment, reservoir 2 receives it is own inflow through it is catchment, and the reservoir 3 receives not only the natural flows from this catchment, but also the controlled flows that have coming from reservoir 1 as well as from reservoir 2. Now, the fraction of the control flows that contribute to reservoir 3 from reservoir 1 is alpha 1; that means alpha 1 R 1 t comes to reservoir 3 in period t; similarly, alpha 2 R 2 t comes to reservoir 3 in period t.

Typically, you can imagine this to be return flows from the irrigation or you will let some amount of water into the hydro power after some losses, the hydropower tail race comes and joins the stream. So, there is certain amount that is lost or is already used form the releases that you made and only the remaining amount of water comes and joins the stream. And therefore, joins the downstream reservoir.

Now for this system now, we will look at, how the formulation of a optimization problem, by which we can derive release policies at all these 3 reservoirs, in an integrated model by which I mean, when you are deriving the reservoir operating policy at reservoir 1, you are also looking at the consequences of this reservoir policy on the reservoir 3; similarly, consequences of operating policy at reservoir 2 on the operation of reservoir 3 itself.

Now, we are considering that each of these reservoirs supplies, serves, the purpose is of water supply, flood control, and hydro power. So, at each of this reservoirs, you have three purposes, now we introduce these terminologies B 1 t, B 2 t i, B 3 t i; i refers to the reservoir; t refers to the time period; and B 1 refers to the unit net bene[fit] your net benefit corresponding to unit release; B 2 refers to the net benefit corresponding to unit available flood freeboard; and B 3 refers to the unit storage that is the benefits associated with unit storage; remember we have said that the storage directly determines the hydropower. So, which means that the benefits associated with the hydro power may be in terms of the amount of power that you can generate is directly related to the storage itself in reality.

However, recall what I said last time; that it also depends on the release through the penstocks, because the power generated is proportional to Q into H; Q is the discharge; and H is the head; head is determined by the storage; and Q is determined by the release policy; however, for this simple problem, we will just take it as storage. And therefore, we associate benefits associated with the storage, and these benefits can be typically hydro power generator, at particular reservoir storage in particular time period t.

Then this is the continuity now; that is, how the water is flowing from one reservoir to another reservoir etcetera, then for the flood control, we will specify a minimum buffer storage to be available in each of these reservoirs 1, 2, and 3; during the flood season, which means, we will say that; there must be a free storage space available at reservoir i; and that we specify as F min of i, this is the flood free board to be made available at reservoir i during the flood season, then the release that we are making from each of these reservoirs is limited by the canal capacity itself or the river capacity and so on.

So, you may put a higher bound on the release that is to be made from the reservoir. So, these are the conditions, one is that you want to meet the hydro power to the best extent possible, you want to maximize the power that is generated, you would like to at the same time maintain the flood free board to absorb the flood waters, and you would also like to make sure that your release are such that they do not exceed a maximum value.

This is the problem, and then you also have the net benefits associated, as I keep repeating, it need not be just the economic monitory returns, but it may be in terms of the physical outputs that you may get in terms of crop yield, in terms of the hydro power that is generated, in terms of the you know, the flood control volume that you could achieve or the flood control that you could achieve and so on.

This problem now, we will formulate as a simple linear programming problem and look at various terms here, and various constraints. This is just similar to the single reservoir problem except that we are while writing the continuity; we are looking at the continuity of the flow from upstream at any particular location.

So, when we are writing the continuity at reservoir 2; for example, the reservoir 2 does not have any control structure upstream of it; and therefore, the nature flow come from the catchment itself is the inflow to this; whereas, when you come to reservoir 3, it has the natural inflow Q 3 plus the control flow coming from reservoir 1, which is alpha 1 into R1 t in time period t, and the control flow that is coming from reservoir 2,which is alpha 2 R 2 t.

So, you just look at the continuity, formulate the continuity equation at each of these reservoirs those become constraints, and you formulate the objective function. Now, the objective function for this is that you are looking at maximization of the net returns, out of this reservoir policy, operating policy; and we have defined at reservoir i, where i is equal to 1, 2, 3; the net returns that you acquire out of releases, out of maintaining a flood control storage and out of maintaining a particular storage.

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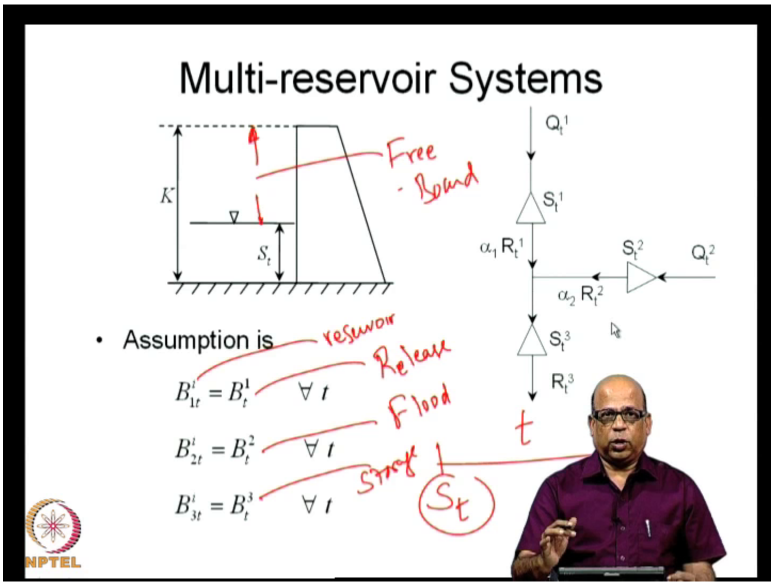

So, let us see how we formulate this? Now for this example now, you understand that the storage at any particular time and we refer to the storage S t as the storage at the beginning of the time period. So, this is the time period t; and this is the storage at the beginning of time period t; now this storage must be always less than the capacity K; further this is the free board that we are talking about; that means, if you have a storage of S t, and the capacity is K, the associated freeboard that you have for absorbing the floods is K minus S t, and then, what we do is for this example, we will say that the B 1 t i, this i refers to the reservoir.

We are assuming that at all the reservoirs though constants B t remains the same. So, this is for release, and this is for storage. Let me just take this B 2 t is for flood freeboard, and this one is for hydro power or the reservoir storage itself. So, we take out the super script i here, to indicate that these remain the same for all each of the reservoirs. Now, of course, in a in a general form you can also include i, but let us understand the formulation for a simple, simplicity problem first.

And now, we will look at, how we formulate the objective function? You are looking at one year time horizon, which means t is equal to 1, 2, 3 etcetera; there are several time periods. If you are looking at monthly time periods, you may have 12 time periods. If you are looking at seasonal time periods, you may have 2 or 3 for example, you may have a monsoon season or non monsoon season or you may have a summer, monsoon and winter season and so on.

So, your time t here, depends on the time horizon and the intra seasonal periods for which would like to make the decisions, if you are specifically interested only in irrigation and hydro power, you may still reduce the time period to 10 days; and if you want to operate it specifically mainly for flood control, your time periods can be even so smaller.

Let us say, one day time periods 6 hours time periods and so on. Now, these these details or these kind of sophistication, we can include, when we are going for real time operation. So, let us look at, how we determine the reservoir operating policy for this kind of system looking at all the reservoirs together.

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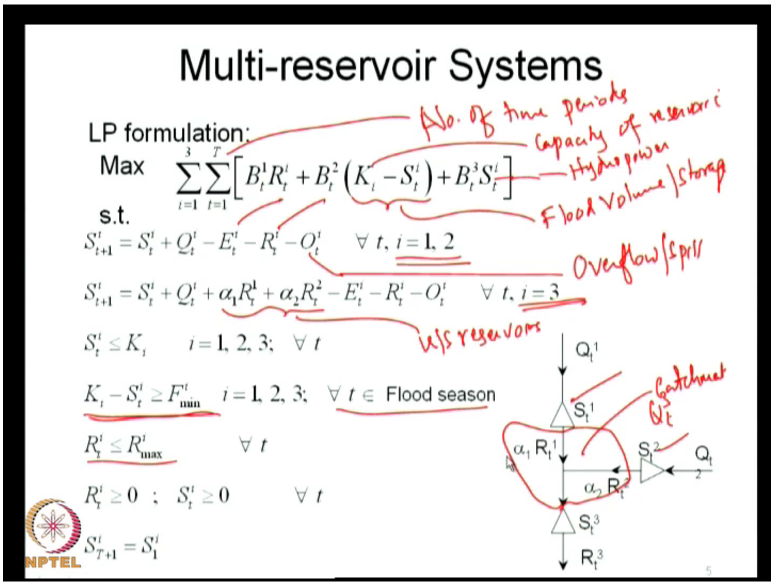

You will formulate this as LP formulation, just understand these terms correctly, we have t time periods. So, this is the last time period in the year or the number of time periods.

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Number of time periods in the year. So, if you are looking at monthly time periods t will be 12, and this is number of reservoirs i is equal to 1, 2, 3. So, all you are doing is across time periods, across space over all the reservoirs, you are summing up the net returns that you get from the reservoir release policies. So, at the reservoir i, you are making the release of R t i, i is reservoir reservoir number, and t is the time period, R is the release and associated with the release, you get a benefit of B t 1; you have removed the superscript i for the benefits, we are assuming that the benefits remain the same across all the reservoirs.

Then you come to the second one, you understand this term correctly, this is the flood freeboard, flood volume or flood freeboard or flood storage available in time period t at reservoir i, this is K i is the capacity of reservoir i. So, if you have a storage S t in reservoir i, you have the flood freeboard available of K i minus S t i; and associated with this flood freeboard, you have a return of B 2 t per unit storage that this made available and therefore, you multiply this B 2 t into i.

Then the storage at the reservoir i is S i t, and this determines the hydro power, and associated with the unit storage, you have a benefit of B t 3 in time period t. So, you are summing over time periods and you are summing it over all the reservoirs. So, this is how you formulate the objective function? I repeat the objective function as formulated here, is the sum of the returns that you get out of the release that are made from each of these reservoirs 1, 2 and 3.

And the flood freeboard that you have made available in each of these reservoirs in time period t, and also the hydro power that you are generating, which is directly related to the storage in each of these reservoirs. So, this gives this objective function as written here, gives the net benefits across time periods within a year; that means, over all the time periods within a year, over all the reservoirs together. So, that is a lumped objective function that we are talking about and in achieving this; that is in achieving the maximum returns, we need to satisfy all the constrains.

And specifically, the flow of water from one point to another point is what we have to account for and that is given by the storage continuity equations, the storage continuity equations we write for reservoir 1 and reservoir 2 together, because they do not have any control structure upstream of it; and therefore, you write for these two together. So, we will write first for reservoir number 1 and 2 for time period t, we write for reservoir number 1 and 2 together, because they do not have upstream control structures here. So, this would be a straight forward continuity equation, starting with a storage S i t plus Q i t minus evaporation losses minus the release that you have made at reservoir i; and this is the spill or the overflow. So, this is overflow or spill in time period t.

When you come to reservoir 3; i is equal to 3, you have to account for, what is coming from the upstream, alpha 1 R 1 and alpha 2 R 2; these are the terms that we include here, this is the flow that is coming from upstream reservoirs, and this is S i t plus Q i t, this is the storage in the reservoir number 3 and the inflow that is coming to reservoir 3, remember this Q i t is, because of the natural flow contributed by the catchment, free catchment. So, this is Q 3 t that is the intermediate catchment between the reservoir 2 and 3 as well as reservoir 1 and 3 and the additional catchment that you may have.

So, this is how you write the continuity for reservoir number 3? And of course, all other constrains that you have looked at namely; that the storage must be less than the capacity and look at this constraint specifically, we write this, because you want to maintain a minimum flood freeboard or flood storage. And K i minus S i t; S i t is the storage, look at this figure, S i t at particular reservoir i is the storage and K is the capacity.

So, this becomes the flood free board, we would like to maintain this above a certain limit, which means that we specify a minimum flood free board to be provided; and then say that in each of these time periods, this term here must be greater than the minimum flood freeboard for all the reservoirs i is equal to 1, 2, 3; we say for all t belong into the flood season.

Let us say, that you are talking about a monthly operation in a monsoon country like ours; and you may have periods such as August, September, October, as flood periods. So, you may specify this minimum flood storage only for those periods, and for all the remaining periods, you would like to have the storage as high as possible, because the storage has benefits associated with the hydro power generation as well as for water supply.

And therefore, you may want to have as high as storage as possible. So, you may put these minimum conditions, only for the flood season and for all the remaining seasons you may put this as 0. And this is the condition that we specify, which restricts the release that you make will be limited to which restricts the release to the maximum capacity of the canal itself or the river itself. So, this is the maximum release constraint and all other constraints are similar to what we have done earlier. So, this is the linear programming problem, because the B t s is known here, this is the capacity which is known.

And therefore, these are linear functions of the decision variables R and S. R is the release, S is the storage. When you solve this, you are leaving the boundary conditions open, in the sense that you are not specifying the storage either at the beginning of the time period at any of these reservoirs or at the end of the year at any of these reservoirs leaving everything free, you are looking for the optimal combinations of the storage is at each of these reservoirs as well as the releases, and that is what you would get, when you solve this example.

Now, in this general form, you can add several details as you progress. So, we will just solve a simple example using this model and then, see how you can add keep on adding details; for example, we said that the hydropower that is generated is a function of storage; and therefore, we associated the B 3 t with S i t itself, but you can also add to the hydropower, the discharge that is going through the penstocks and develop the hydro power returns, in terms of discharge as well as the storage, storage that defines the head, and to relate the storage with the head, you can also use the area capacity elevation relationships. As, we have discussed earlier, and then include those details here.

Then you can also add other details corresponding to this alpha 1 and alpha 2, which we have taken as constants, they may be varying with respect to time, because if the alpha, the whatever that is coming from this reservoir is, in fact irrigation return flows that can be time varying; similarly, if it is during the flood season to maintain certain amount of flood storage, you may want to make additional releases your alpha 1 can be different and so on. So, many physical details of the system can be incorporated into these models to the desired extent; obviously, the model start becoming starts, model will start becoming more complex, as you add more and more details; however, the principle remains the same.

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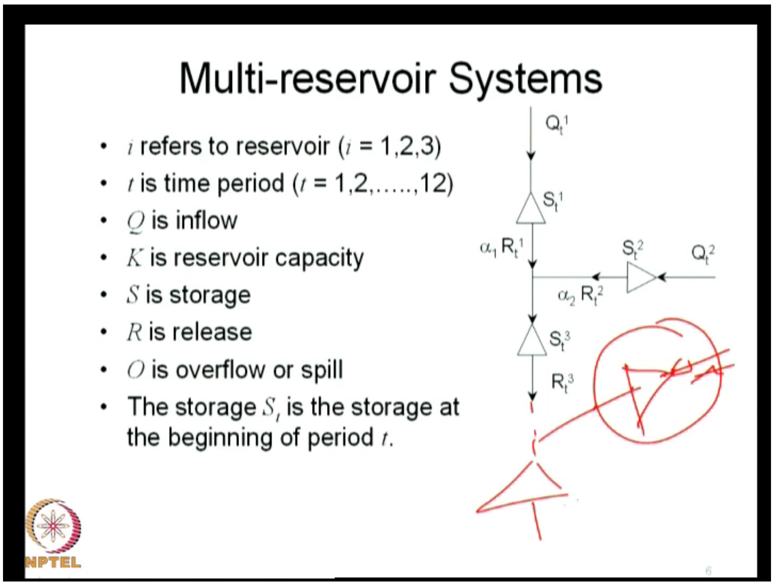

So, these are all explained now. So, this is for completeness shake, I have given all the details here, all the definitions, the spill here overflow that we have included in the continuity equation, may create some problem, when you are solving the problem, because solving the this particular optimization model, because of the continuity equations.

So, you may have several continuity equations, in this particular case, we may take three or four for in demonstration, but in general you may have 12 time periods at least associated with the months, 12 months, and you may have not 3, but 14 or 15 reservoirs; and therefore, the number of equality constrains including the overflows, as the decision variables related to the storage as well as the release will be large, the number of the equality constraints will be large.

And therefore, in general when may whatever algorithm that you use for linear programming may failed to give a feasible solution, in such a situation, you may have to use the integer variables, as I have expressed, explained in the last class. And then accounts for the spills.

Now, all these details are already explained. So, we will just look at, how we solve this problem for a 3 reservoir problem? If you have another reservoir, let us say that from third reservoir, you are also going to downstream and then putting another reservoir here, and then to this reservoir, there is another reservoir that is contributing. Let us say that you have this kind of a reservoir coming here.

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So instead of 3, if you have 5 or 6 or 14 or any number of n numbers of reservoirs, the model structure remains the same, as I just explained, except that you will keep looking at each of the reservoir, and then add constraints associated with that taking into account the continuity of the flow.

Let us say, you added one more reservoir here, you just write the continuity for this separately, if there is no upstream control structure here, and then this contributes to the the downstream reservoir and R 3 t is coming from here. So, you add these 2 and put a continuity equation for this and so on.

Also at each of these reservoirs, you may have different objectives; this may be only for flood control, whereas this may be for flood control and hydropower together and so on. So, all of these kinds of specific systems details can be incorporated in the general model structure that I have shown, by accounting for the coefficients that you have used the net benefit coefficients; that I have shown here and so on.

And by rewriting the continuity equations taking into account, the continuity of the flow from where it is coming - where it is getting added - how much of it is being extracted - how much of it is being lost - what is the kind of overflows or the spills that are occurring at each of this reservoirs and so on. So, you have just maintaining the mass balance at each of these node points, and then writing the constraints. So, the structure of the model remains the same, irrespective of reservoir, you have three reservoirs, four reservoirs, n number of reservoirs.

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Example – 3

Consider the data given in table below for a three period, three reservoir system.

Reservoir	Inflow			K	F_{min}			B_1^*	B_2^*	B_3^*
	t = 1	t = 2	t = 3		t = 1	t = 2	t = 3			
1	25	10	15	10	3	2	7	50	10	25
2	10	30	15	15	2	3	4	60	10	30
3	20	12	15	20	2	3	5	70	10	35

$\alpha_1 = 0.2$ and $\alpha_2 = 0.3$

We will just take a simple example with three reservoir systems. So, that you understand the problem correctly. Now, this is the same configuration of the three reservoirs. So, essentially, I am looking at these three reservoirs now. We will just take out this. So, this is the system that I am now considering.

And for this system, we will use some data, you have three reservoirs 1, 2 and 3; there are 3 time periods 1, 2 and 3; the inflow during each of these 3 time periods; for each of the reservoirs is given, in some units, volume units 25, 10 and 15. K is the capacity of the reservoir i. So, first reservoir reservoir number 1 has the capacity of 10 and so on 15, 20.

The F min is the minimum flood freeboard that you need to maintain, in terms of the volume units, in each of the 3 time periods at each of these reservoirs. So, in time period t is equal to 1 you would like to maintain a minimum of 3 units of flood freeboard, and that is how this data is given.

The B 1 star that is shown here, is the net benefit associated with the releases, at reservoir 1 it is 50; at reservoir 2 it is 60; at reservoir 3 it is 70; and so on. The B 2 star is associated with the F min; that means the flood freeboard that you are providing and the B 3 star is associated with the storage.

Now, these terms are the same as, what I have used here, except that I am taking out the t here for in the example problem, to indicate that the net benefits remain the same across all time periods. So, I am simply putting it as B 1 star, B 2 star and B 3 star. And further, we use alpha 1 as 0.2; and alpha 2 as 0.3, which means the fraction of the release that joins the downstream reservoir from the reservoir number 1 is 0.2, and this is 0.3.

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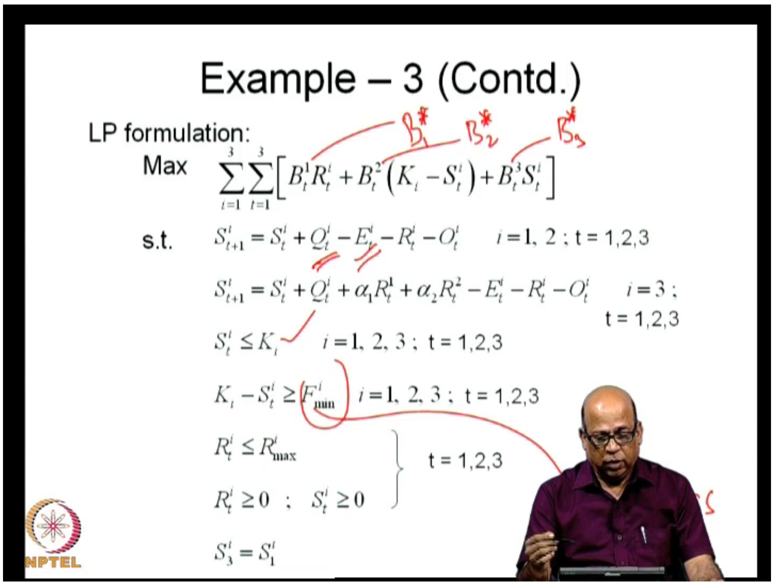

We will solve this problem, we will again rewrite the problem for t is equal to 1 to 3, there are 3 number of time periods, there are 3 number of reservoirs, and this B 1 t and B 2 t etcetera are in fact, B 1 star and B 2 star. So, I can write this as B 1 star, and this as B 2 star, and this as B 3 star. Essentially to indicate that across all time periods, the coefficients remain the same.

And we write the continuity equation, I just explain this for i is equal to 1 and 2, and also for i is equal to 1; i is equal to 3, where which receives contribution from the upstream reservoirs. We write this constraint here, for t is equal to 1, 2 and 3. We have specified, what is the minimum flood freeboard; that needs to be maintain and the flood freeboard at reservoir i in period t is simply K i, which is the capacity of the reservoir minus S i t, which is the storage at the beginning of the time period t at reservoir i; and this should be greater than or equal to F min of i; and F min of i in each of these time periods is given.

So for completeness shake, we can also write this as F min t of i, for t belong into the flood season; that means, for each of these time periods in the flood season, you may have different F min specified as data, you imagine for for example, a large system like a

Narmada reservoir system, where you may have 14 or 15 reservoirs; and at each of these reservoirs, you may specify a different flood freeboard during different time periods.

Let us say, in July you may specify something at let us say, Burgee reservoir, and then at Narmada Sahara reservoir, you may specify some other free board in July, but in August these two values can be different and so on. So, in general you can include this as F min t, which varies form time period to time period at the reservoir I; for t belonging to flood season, we may say and for t not belonging to flood season this can be just 0.

So, this is the model that we will solve for the example given earlier, what is the data that will be necessary here, the flows at each of these reservoirs, this is the inflow, and this should be given. The evaporation rates may be specified, if you are ignoring the evaporation, then this term does not exist, and then you have to specify the storage capacities, you have to specify the F min i.

So, all of this data is given. So, the inflow is given at all the reservoirs for all the time periods, the storage capacities are given at all the reservoirs, you have specified the minimum storage or minimum flood control storage at each of this reservoirs; for each of the time periods and of course the benefit functions.

And the alpha 1 and alpha 2, which are the fractions of the releases from the upstream reservoirs that join the downstream reservoirs, so that is also specified here. This is the LP problem, because the objective function is linear, and all the constraints are linear. So, you can use linear programming software, and then solve this problem.

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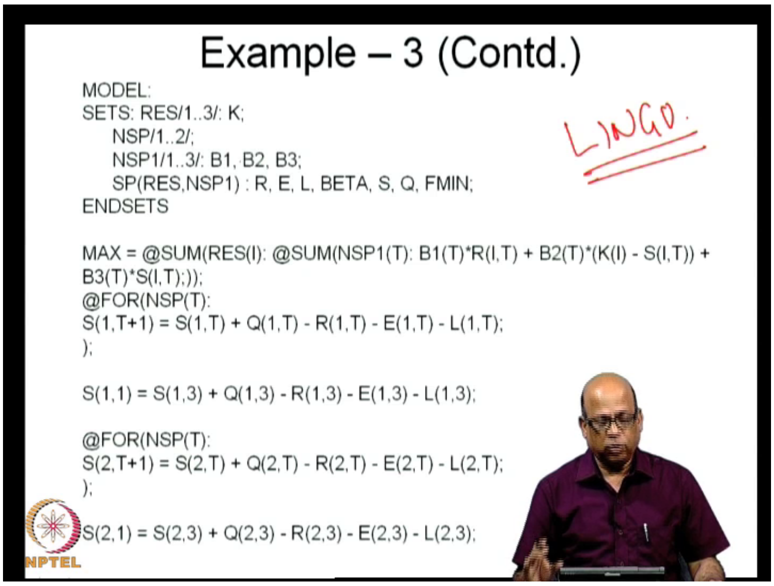

Now, I am giving you here, the code for the LINGO software, which as I mentioned in the previous class, you can use it freely for educational purposes with limited number of constrains, limited number of variables and so on. So, this is the code, I have explained earlier, how to write - how to use the LINGO for single reservoir, we only extend it to multiple reservoirs, I do not want to spend too much time on this. Let us go further.

So, we can use the continuity equations for time periods 1 to 2, and then your T plus1 at the end of the time period becomes let us say, you have 3 time periods, the T plus 1 at the end of third time period becomes 4 and that you can account for in defining your continuity form one time period to another time period.

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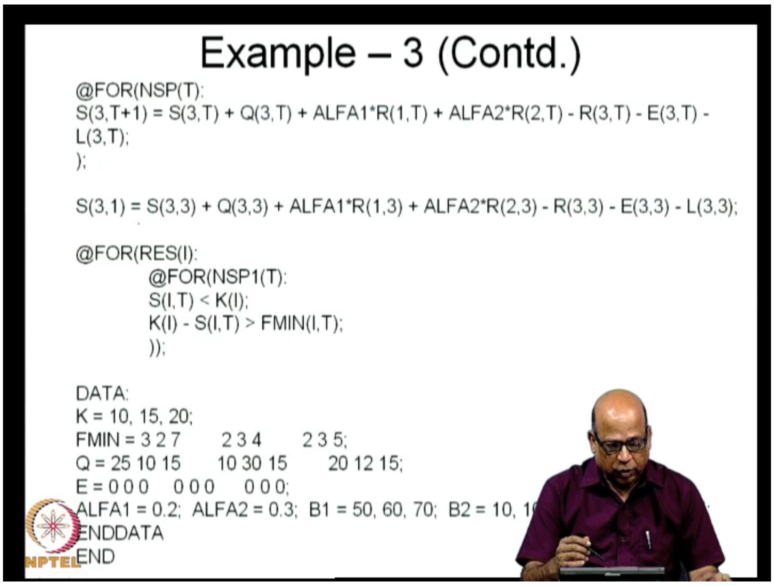

So, this is the simple code, I want go into the details of this, all we are doing is rewrite the constraints in the form of using the for loops and so on. And this is the data that we specify at each of these reservoirs, because F min would have been defined as an array here, you would have defined F min as an array consisting of reservoir as well as the time period, you will write F min for each of the reservoirs for each of the time periods. So, this is the format.

The syntax and other things of LINGO can be writing from a manual. So, this is just a simple may be about 15, 20 statements of the LINGO problem, remember here irrespective of your number of reservoirs, number of time periods etcetera, this program remains the same.

That means, as your number of reservoirs increases, perhaps you may structure it slightly better. So, that the continuity equation is written in a more general form, I have used it only, because it is only three reservoir problems, I have used it slightly in a elegant way of writing it, but you can make it more general, so that the problem can be solved for any number of reservoirs, for any number of time periods.

When you solve this, as I mentioned, you should get a reservoir operating policy, which means, you should get the optimal storage is to be maintained at the reservoirs, and the associated optimal releases from each of the reservoirs, such that together something good happens to the system, in terms of the best objective function that you get and so on. So, you will get the solution like this.

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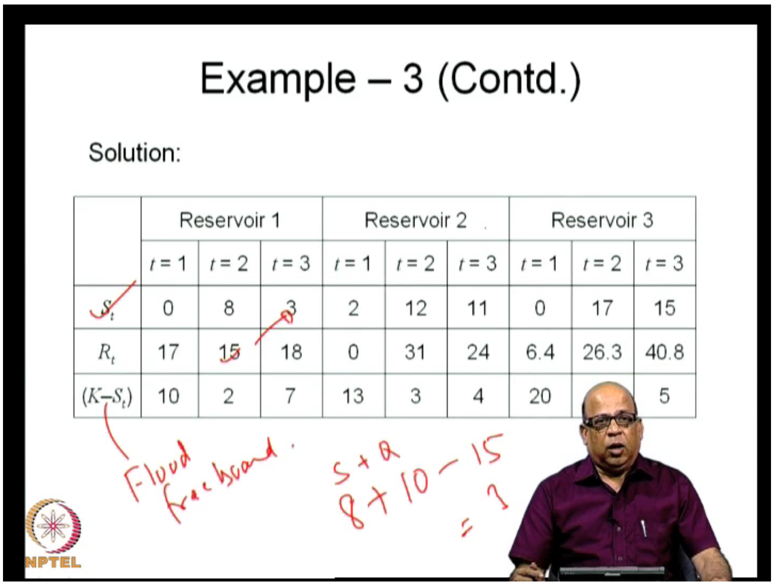

You will get the storages to be maintained, at each of the reservoir during each of this time periods, remember S t is the storage at the beginning of the time period and similarly, at reservoir 2 the storage to be maintained; at reservoir 3 storage to be maintained and so on. And associated releases, and the associated flood freeboard, this is the flood freeboard. So, this defines the optimal policy; that means, at at period number 2, let say at reservoir number 1, you maintain a storage of 8, and then during the period 2, you make a release of 15, and then you end up with certain storage at the end of the period.

Let us say, 8 is your reservoir storage at the beginning of time period, t is equal to 2, and then you have an inflow of 30. So, 8 plus 30 is 38, and you make a release of 15. So, 38 minus 15 that will be the storage at the beginning of next time period, we are looking at reservoir 1 I am sorry. So, at reservoir 1, you look at the flow during the time period t is equal to 2, which is 10. So, you have time period t is equal to 2; 8 plus 10, which is the storage plus inflow and minus you make a release of 15 and you will end up with a storage of 3; that is the this 3.

So, the storage continuity that is the mass balance has to be maintained even in the optimal solution, and in general when you have large number of constrains, large number of reservoirs and so on. It is better to put a check on final optimal solution; that you obtain to ensure that the mass balance is in fact, maintained in the optimal solution, why this is important is, because of the overflows; overflows in general as equality constraints cause some difficulty in large linear programming problems, you must be alert to the situation; that the optimal solutions that you obtain finally, may have overflows occurring even, when the storages are not at the capacity.

So, the overflows must be non zero only, when the reservoir has reach the capacity and that you must be, you must make sure, at the reservoir 3, when you are checking for the continuity, you must make sure that you use the alpha 1 and alpha 2, because the reservoir 3 is also receiving water from upstream reservoirs. So, that fraction has to be accounted for, when you are looking at, when you are checking for the mass balance at reservoir number 3.

So, to summarize, what we did just now, is to arrive at optimal operating policies for multiple reservoir systems, multi - reservoir systems, where the decision that you make at one particular reservoir is going to affect the decision at other reservoir, typically the downstream reservoirs or typically the reservoir to which this particular reservoir is connected and you are then looking at the integrated decision, integrated policy of how to operate the reservoir across time periods, such that it is part of a larger system in which several other reservoirs are are also located, and then you are looking at a comprehensive objective function or an aggregated objective function for the entire system together, while at the same time maintaining the particular objectives of individual reservoirs.

So, individual reservoirs may have their own priorities for the example, flood control may be higher priority at a particular reservoir, you assign the associated coefficients there the B star that I use there, can be different for the different reservoirs; can be different for the different time periods; can be different for different objective functions.

So, in general when you have very large systems, keep in mind system like Narmada Sahara system. Narmada system, where you may have 14 or 15 major reservoirs apart from a larger number of minor reservoirs, you may have different objectives to be satisfied, at each of these individual reservoirs.

At at that same time, you may want to have a large systems view point, and then want to maximize or want to optimize something for the entire system together in an integrated way, and that is where we formulate models like this optimization models like this, and arrive at optimal operating policies, at each of the reservoirs, such that the policies are linked together at across each of the time periods. This was using the linear programming.

Recall that in one of the earlier lectures, I also introduced the dynamic programming for reservoir operation, what we did there was? That for one year period and the specific example that I took numerical example consisted of four time periods in a year, may be refer to lecture number 16 and 17. What we did there was? That we define the reservoir operating policy as a set of sequential decisions to be taken based on the state of the system at the particular time.

So, the sequential decision, as the function of the state of the system, and the state of the system in that particular example that I have discussed was in fact, the storage; given the storage at the reservoir, how do we operate in a sequential manner, such that at the end of the time horizon, and typically the time horizon is one year, the objective function is maximized. So, whenever we were we are talking about sequential decisions, the dynamic programming is much more elegant, a much more handy compared to the linear programming problems.

So, we will now revisit the dynamic programming, and pose the problem slightly differently, then what we did in the earlier case? Where we did the dynamic programming, and talk about, what is called as the steady state policy or the stationary policy, we say a policy is steady state, when you keep on applying the policy over over and over again across time periods, over many years, you get the same returns, may be in terms of the hydro power, may be in terms of the flood control, may be in terms of the agriculture produce and so on.

You keep getting the same returns, as you apply this particular policy, which is the steady state policy over a long period of time and that is the intuitive understanding of the steady state policy. Typically, to derive the steady state policy, the dynamic programming is well more, well suited, because you can solve this problem over a large number of years.

Let us, say that you solve this problem of reservoir operation for 5 years 10 years 15 years 20 years and so on; and as you get the steady state policy, how to how to get the steady state? I will presently discuss as you apply the steady state policy over a long period of time, the net returns that you get every year, we will remain the same and the policy also converges to a certain steady state policy, which you can keep on applying based on the state of the system. And this is the problem that we discussed now.

So, this is either called as the stationary policy or the steady state policy, because you do not alter the policy, once you derive the policy by policy you recall, what I mean is the decision on the releases to be made at each of these reservoirs for a given state of the system, and the state of the system typically is defined by the storage and the inflow together, and as I mentioned earlier, you can keep on adding more and more sophistications to more and more sophistications to the problem, in terms of details regarding soil moistures, details regarding the rain fall and so on, to define the state of the system.

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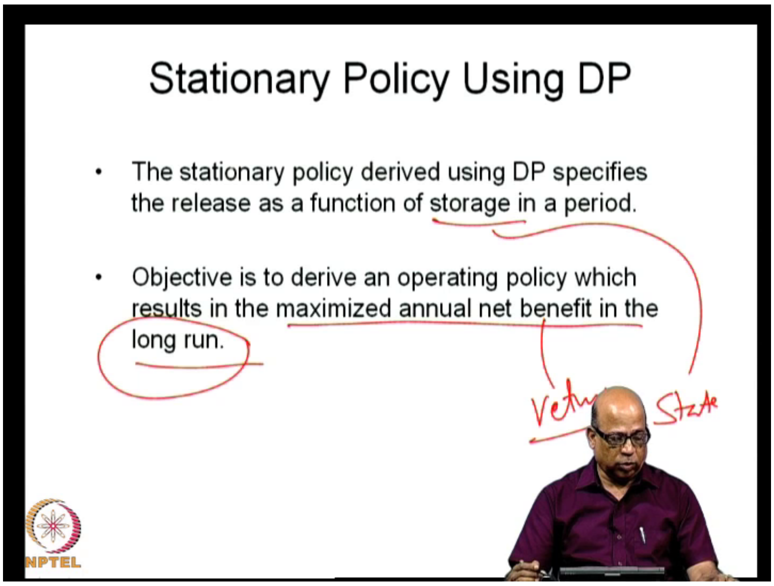

So, the stationary policy derived using dynamic programming, Specifies the release as the functions of storage in a period. In in this particular case, we used the storage as the state variable; that is the state of the system is defined by the storage. Now, the objective is to derive an operating policy, which results in the maximized annual net returns, we will say benefits is also return. Do not always confuse the benefits with monitory benefits; do not always relate to the benefit with monitory returns, it is in general the returns that you get in terms of the hydro power, in terms of the irrigation etcetera. So, keep the physical picture in mind when we are doing the optimization.

Now, this in the long run is what is important, as I said the steady state policy is typically applied over a long period of time over five years, ten years and so on. As against the policies that we discussed earlier, which were typically for one year; that means, you take a one year time horizon optimize that and then use that for that particular year. Now, that need not be optimal, in the sense of long term returns that you get; and therefore, we go for, water called as the steady state policies or the Stationary policies.

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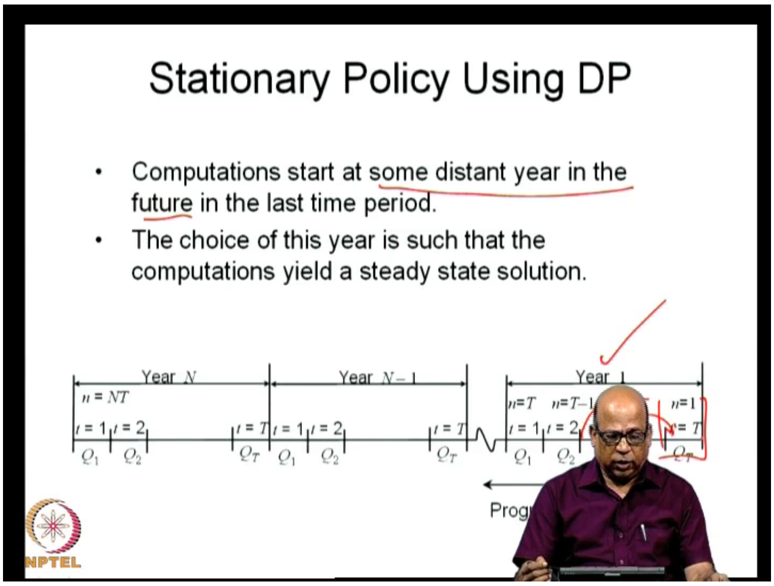

What we do in the stationary? In deriving the stationary policy is that we start from some time in future and proceed backwards; proceed in the backward direction, now you please revise what we did in the dynamic programming earlier. So, the last time period in a year is capital T, we start in the backward direction and these are the stages. So, n defines here, small n defines the stage in a, in the dynamic programming. The Q 1, Q 2 etcetera are the flows, inflows.

So, you start the computations in the backward direction, each time taking one time period, and then next time you look at connection between this time period and this time period; next time we looked at the connection between this time period and this time period and and so on. So, you progress in the backward direction until, you are sure that the steady state has reach.

So, this is some distant year in the future, do not worry whether it is year number 20, year number 30 and so on. So, this distant year in the future, you have starting as the starting point and then progressing backwards, keep doing the computations in the dynamic programming until you reach a steady state, a steady state, I mean the end the state, where the annual returns will remain constant.

The policy converges to a certain policy; that means, there would not be any further changes as you keep on doing this again and again, this happens, because your flows remains constant, this is the deterministic problem. You are not addressing any uncertainty associated with the flows therefore, the flows remains constant and you are solving it for year after year over the same intra seasonal time period; and therefore, after some time the policy, in terms of the releases that you need to maintain converges to certain steady state values steady state value. So, that is what we call as steady state policy.

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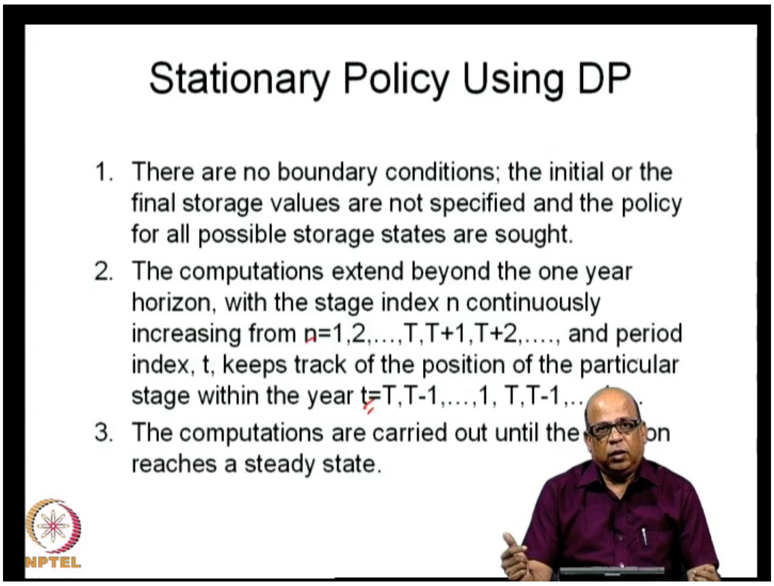

Now, there are certain features of this steady state policy that we must understand, you go to may be lecture number 16 or lecture number 17, where I discuss the dynamic programming algorithm for arriving at the reservoir operating policy for a year. Now, there we applied a boundary condition saying that my storage at the beginning of the first time period was a specified value.

Let us say, in that example, we took S 1 to be equal to 0; however, when we are deriving the steady state policy, you do not apply any such conditions at either at the beginning or at the end; leave everything free and let the algorithm choose, what are the best storage is to be maintained or in this particular case? We may say that if the storage is in fact, the state of the system, for a given state of the system, what is the optimal route that you need to take?

So, we do not specify any boundary conditions; that is the first condition, then we use two indexes here, one is n and another is t, much the same way as we did in the dynamic programming earlier, the n keeps on keeps the track of the progress of computations. So, n is the stage and keeps on keeps on increasing. So, n is equal to 1, 2, 3 etcetera 12, 13, 14, 15 and so on. It keeps on increasing, the T there keeps track of the intra seasonal period corresponding to the stage; for example, n is equal to 1 corresponds to T is equal to 1; 2 corresponds to T is equal to 2.

If you are doing it for monthly time period, n is equal to 12 corresponds to T is equal to 12; n is equal to 13 corresponds to T is equal to 1, because a year consist of 12 time periods. So, T goes from 1 to 12, 1 to 12, 1 to 12 etcetera; whereas, n keeps on increasing to keep track of the stage in the dynamic programming.

And then, we proceed in a backward direction typically, for steady state operation; and then we carry out these computations until a steady state is reach. We will see how we identify the steady state in the dynamic programming algorithm. So, essentially then in today's class, we started with the reservoir operation problem that we were discussed in the previous class.

We introduce the multi - reservoir systems, where the continuity equations of the mass balance equations, we write taking into account the control flows that comes from the upstream reservoirs, and also taking into account the intermediate catchment flow at each of these reservoirs. The releases that we talked about from an upstream reservoir only the part, only a part to or a fraction of the release may actually add to the downstream reservoir, and this is what we account for in writing writing the continuity equation at the downstream reservoir.

What I discussed through a three reservoir problem, can be generalized to any n reservoir problem, at every reservoir, you look at the conditions to be satisfied, in so for as the mass balance at particular that particular reservoir its come from; and also the specific objectives that that particular reservoir has to serve; and then integrate all of these in a single model.

And we have looked at the linear programming model for multi - reservoir systems subsequently, when I discuss the applications, we will also see, how we can simulate such reservoirs? Such large and complex reservoir systems using the auto correlation simulation techniques, and then generates several possible outputs, and then, how do we screen this outputs through a linear programming problem and so on.

Then towards the end of the lecture, then we solved the simple example of the three reservoir problem using the LP, recall that in the last lecture, I have introduce the software LINGO, which can be downloaded freely for education purposes, you can use the small code that I have given for multi - reservoir systems, and make it more general for any general reservoir system. So, towards the end, I just introduce the concept of the steady state policy, we will continue our discussion on the steady state policy using the dynamic programming in the next lecture. Thank you for your attention