Water Resources Systems

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Model No. # 05

Lecture No. # 23

Reservoir Operation

Good morning and welcome to this lecture number twenty three, of the course water resources systems, modeling techniques and analysis. In the last few lectures, we have been talking about reservoir systems. So, initially we started with capacity determination that is, the minimum capacity required for meeting a certain demand patterns for a given sequence of inflows. And then, we went on to discuss the storage yield functions, where we are looking at the yield in terms of the maximum constant release that can be maintained from the reservoir for a given sequence of inflows. And then, we looked at the optimization problem by which we can still achieve the storage yield function by maximizing R.

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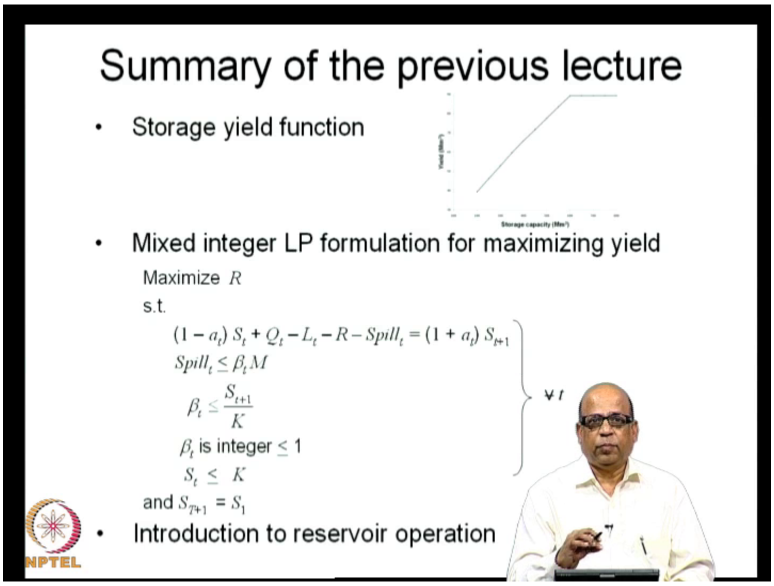

So, essentially in the last lecture, what we did is, that we solved an example for storage yield function with respect to the storage, that is, the capacity storage, capacity of the reservoir, what is the yield that we get for a given storage capacity, is what we did. And, through this example we showed that in general, the yield keeps on increasing as it increases the storage capacity. But, after the time it remains constant. So, indicating that no matter how high the storage provide beyond this particular point, the yield will still remain the same. And, as I mentioned in the last class, this is because of the inflow limitation. And then, we went on to solve the same example using a mixed integer L P formulation. This is where we stopped last time. So, let me just go through it.

So, what we are looking for is the constant release, which is the maximum constant release is what we are looking for. So, maximize R and then we put this R here, remember it is not R t but R which means it is the constant across time. And then, we included explicitly the spill term here, to make sure that the spill as you obtained form the solution is non-zero, only when the storage reaches the capacity. We included ensuring that condition, we included the integer variables. So, beta t here is an integer variable. And, typically it is the binary variable, which means it can be on either 0 or 1.

Just look at this formulation. I explained this in the last class, but for completeness sake let me repeat it. What we are saying is that beta t is an integer which is less than or equal to one, which means that beta t can take on a value either 0 or 1 because being integer less than or equal to, it cannot be assumed either 0 or 1. And, look at this condition S t plus 1 divided by k. And, S t plus 1 is storage at the end of the time period or at the time period t or storage at the beginning of the time period t plus one. So, if that is greater than k, what happens? This will be more than 1 and because of these conditions beta t will take a value of 1. So, whenever the storage exceeds the capacity beta t takes a value of 1; if this is less than K, then this ratio will be less than 1. And therefore, beta t has to take a value of 0. Now, when beta t is zero, which means that the storage is less than the capacity, you do… want any spill there. Beta t is zero, you put a by virtual these conditions beta t is less than or equal to beta t into m. When beta t is zero, spill t has to equal to zero and beta t is zero, only when storage is less than the capacity. And therefore, you will make sure that spill is zero. When beta t is M, you want a non-zero value for spill which will satisfy this. And, to make sure that you get a non-zero value of spill, you multiply beta t by some large number M.

So that, you have the flexibility to choose spill t to make sure that this is satisfied correctly. So, beta t into m; where M is the large number. In fact, spill max or the maximum value of spill that is possible at reservoir etcetera, you can use. So, this will be spill t is less than or equal to some maximum value. That is the idea there. So that, you will get at beta t is equal to

1, which happens only when S t plus 1 is greater than or equal to K. As determined from this, you will get a non-zero value of spill. So, this is how we include and all other condition explained earlier.

We know these are same condition that we use everywhere a stage less than or equal to K and so on. Then, towards the end, I just introduced the problem of reservoir operation. Recall that the reservoir operation, which is the single most important problem in water resources systems, is the one dealing with deciding on the release sequences across time periods. Remember, you are talking about large volumes of water here. And therefore, how we operate the reservoir is an extremely important problem on the field. And therefore, at the planning level that is, you plan the reservoir operation in a certain manner, keeping in mind or keeping in view the likely inflow sequence is that are coming in the year. So, typically we use the historical data and then derive the reservoir operating policy.

By reservoir operating policy we mean, sequence of reservoir release is across time periods as the function of the state of the system. Now, state of the system can be storage alone or storage and inflow together or storage, inflow and rainfall in the command area; storage, inflow, rainfall and soli measure in the command area, etcetera. So, as much sophistication, as decided can be put into these models. So, that is what we defined as reservoir operating policy. And, the simplest of such reservoir operating policy, which commonly is adopted intuitively in most of the reservoirs especially in India, is so called standard operating policy. Or, the standard operating policy is simply to meet the demand in the particular period to the best extent possible without looking at other periods.

So, simply look at the water available in this time period. At the beginning of this time period, which is typically S t plus Q t that is the storage, and the inflow, of course the inflow will occur during the time period. But, while deriving the operating policy will stay S t plus Q t is known; that means the storage at the beginning of the time period, plus inflow during the time period. Look at the total water available and then compare it to the demand. If the total water available is more than the demand, simply meet the complete demand; if the total water available is less than the demand, you empty the reservoir put all the water to meet the demand. So, which means meeting the demand to the best extent possible from the available water is the standard operating policy. And, whenever the water is excess, you store the water. So, water the level in reservoir keeps on increasing until it hits the capacity. We are still talking only about the life storage. Remember, always whatever term that I am using here like S t, K, etcetera, they all refer to the live storage.

Once the reservoir level hits the capacity, anything beyond that after meeting the demand will go as spill. So, there is a spilling phase and there is a spill phase. And, there is a period in which, not period really, there is a phase where S plus Q is less than the demand that is the total amount of water available is less than the demand. That is the point where the reservoir will be always empty. So, this is how the standard operating policy is derived. Let us go through the standard operating policy, understand correctly what it means in terms of the actual operation. And, standard operating policy that we will presently see is not an optimal operating policy. It is just as a rule, as an operating rule. For example, look at the storage, look at the inflow that is likely to come and then make the release compared to the demands. That is all. There is no optimization involved here.

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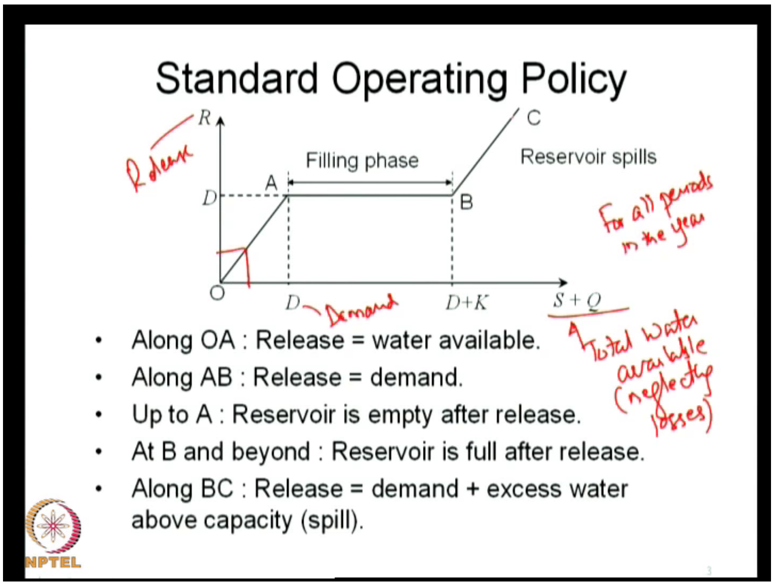

So, let us start with the standard operating policy. Typically it is shown as a graph like this. On the x axis, you see S plus Q. This is storage plus the inflow. This is the total water available, neglecting losses there is. Now, this is for any period, for all periods in the year. Now, S plus Q and D is the demand. Let us say you are looking at some irrigation, water allocation, some there is a rare or municipal and industrial water allocation and so on. So, the demands are known. So, as long as you are in this region that means, your water available is less than the demand you make the complete release; which means whatever is the available you release, which means is a 45 degree line. So, this will be equal to S plus Q itself.

Whatever is available, you make the demands. So, demands were here, you are here; which means demands is much higher than the water that is available. So, you make the complete release. And therefore, after making the release here, this phase in OA reservoir will be empty. There was an initial storage, you got some inflow and then you made release of the amount equal to S plus Q, whatever was in the storage plus, whatever was as inflow. And therefore, the reservoir release the reservoir level will be at the bottom. Therefore, reservoir will be empty. So, the reservoir will be empty after release in this region. Then, once S plus Q is greater than or equal to D, what will happen? You will still make the release D, only just to meet the demand. But, because S plus Q is higher than D, you are left with some water. And, this water we start filling the reservoir.

So, as you start going to the right of D there is total water available to the right of D, the reservoir level starts filling. So, this is the filling phase you keep on making the release D, only equal to D. On, the y axis is the release shown. So, this is the release. So, you are saying that release will be equal to D, whenever your total water available is greater than D. So, the reservoir start filling up. Until, what time it fills? Until your S plus Q which is the total water available, after you meet the demand D is greater than the storage itself, the storage capacity. So, until D plus K, which means this is the storage equal to K. So, you have filled the storage up to the capacity. Beyond the capacity, it will go as spill after meeting the demand. So, you meet the demand, but still there is an excess and that excess go as spill. So, this is the standard operating policy. Meet the demand to the best extent possible, fill the reservoir whenever there is excess water and then spill over the reservoir whenever the reservoir is full. You make the demand and then still there is excess water. That is all.

You are not looking at what is likely to happen to the next time period and so on. So, simply look at this present time period. Look at the current state of the system in terms of the S plus Q, which is storage plus inflows. Look at the demand and meet the demand to the best extent possible. So, as I said along OA release is equal to water available, along AB release is equal to demand and up to A reservoir is empty after release, at B and beyond B reservoir is full after release and then along BC there is a release equal to demand plus excess water above capacity, that is, the spill. So, this is the standard operating policy.

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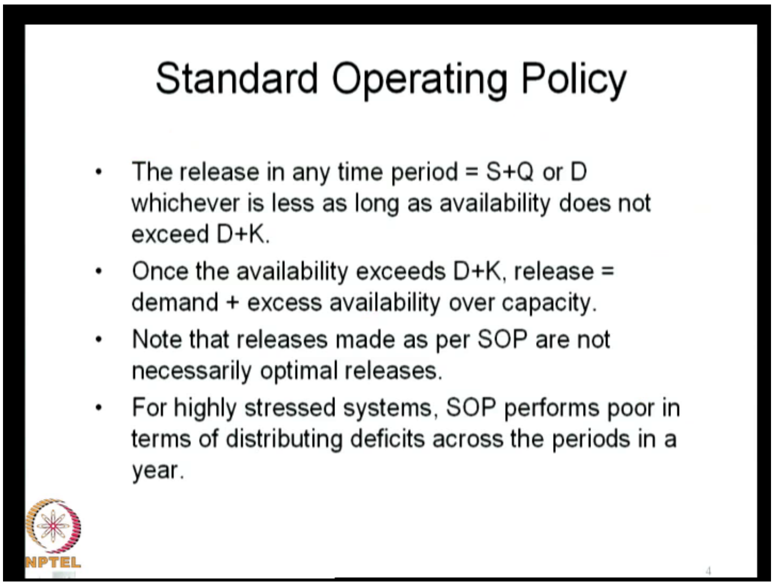

Now, the same thing is explained here. I will not go through all these details, but it is available for going through. Now, exactly the same thing as I explained is written here.

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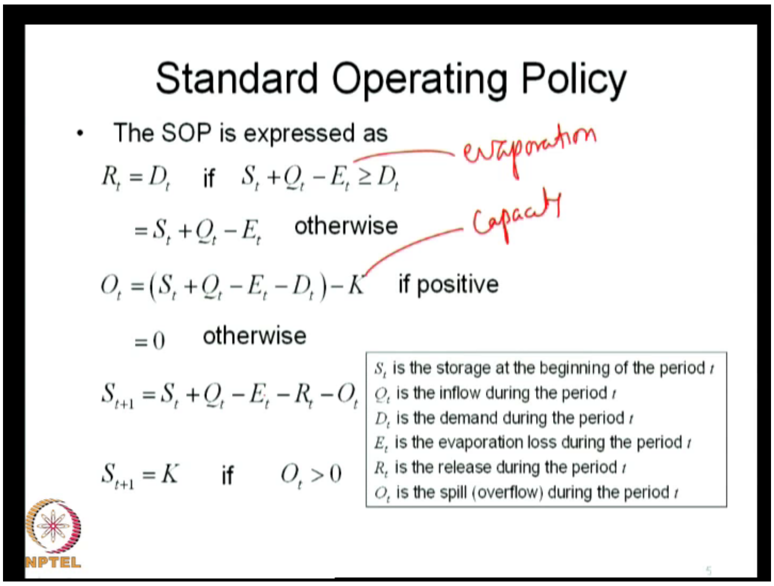

Now in mathematical notations, we write it as follows. So, if S t plus Q t minus E t, this is the evaporation which was not included there in this term here. Here, we neglected evaporation, but if you include the evaporation you can write it like this. So, the storage at the beginning of the time period plus the inflow during the time period minus whatever are the losses, if it is greater than the demand, then you make R t is equal to D t. If this water available after taking out the evaporation is less than the demand, then the release is equal to water available itself;

which is, S t plus Q t minus E t. the overflow will be S t plus Q t minus E t, which was the water available minus D t, which is the demand when you meet the full demand from this. If this is in excess of the capacity, this is the reservoir capacity. If this is value in excess of the capacity, then deduct the capacity and that will be the excess and that will be the overflow.

So, what we are saying is simply that after the reservoir comes to the full level, after you meet the demand, whatever the excess that goes as spill. So, that is the overflow. And then, we obtain the storage, then if this is negative, then obviously this will be 0; that means there is no spill, when the reservoir level is lower than the capacity level or reservoir storage is lower than the reservoir capacity. Once we get O t, we get the storage at the end of the next time period. S t plus 1 is equal to whatever we started, which is the storage at the beginning of the time period plus, inflow during the time period minus, evaporation that is going out of the reservoir minus, the release that you made minus overflow. So, this is the algorithm which defines the standard operating policy.

We should be able to simulate the operating policy, the operation of the reservoir using the standard operating policy. And, of course that because of this condition now, what happens? S t plus 1 will be equal to K, if whenever O t is greater than 0 because of the way we calculate the O t. So, this is the algorithm by which we simulate the standard operating policy. I have seen of course, shown in one of the earlier lectures, how we simulate using the standard operating policy, when I introduced the concept of simulation. But, again for completeness sake, let us quickly go through another example because you are now talking with the reservoir operating policy. So, we will start with the standard operating policy and then move on to optimal operating policy.

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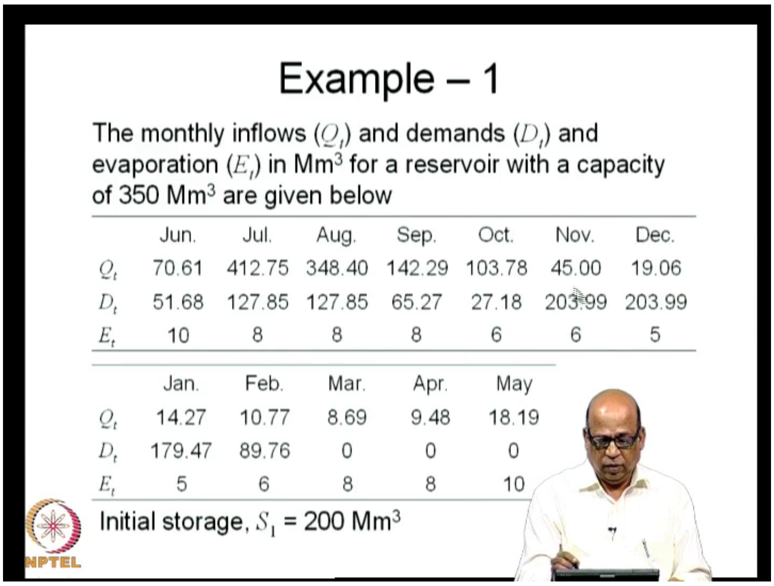

So, let us take this example, where we have the monthly flows Q t and you have the demands given here and then the evaporation average, evaporation rates are given here. Now, when we specify E t like this, for example E t is also in volume that is, the evaporation losses are also in volume, in general what we are doing is that an average evaporation from the reservoir is what we were giving you are not specifying the evaporation as storage dependent losses, which can also be done as I had shown in the previous classes.

One of the previous classes, when we take this E t volume in units for a particular month let us say, June month has evaporation of 10 million cubic meters. It means that over a period of time, you have taken the average of evaporation from reservoir over a long period of time and then, that is what you are putting here. Alright. Now with this, now we will simulate the reservoir operation with the standard operating policy. Now, for a simulation of operating policies, typically we specify the storage at the beginning of the first time period. So, we will say we will start with S 1 is equal to 200 million cubic meters, the reservoir capacity is 350 million cubic meters. So, we are starting with 200 million cubic meters and then the inflow will start. And then, we will release according to the standard operating policy and then see how the reservoir behaves.

Look at this mismatch between the inflow and the demand, here the flow is more compared to demand, flow is more here, flow is more here, flow is more here. This is the month on period June, July, August, September, October etcetera. But, the actual water required is in the other season, when the flows are smaller. So, you have 45 million cubic meters inflow.

But, the requirement is 203, 19 here requirement is 203 like this. So, there will be a period typically in monsoon regions like ours, after the monsoon is over your flows will start reducing, in fact they approach zero very fast in most of the rivers. But, the demand will start increasing because that is the rabbi season, where the irrigation demand will be higher. So, this is what the typical picture is where the flows are smaller. You have large demands and this mismatch is actually managed by the reservoir.

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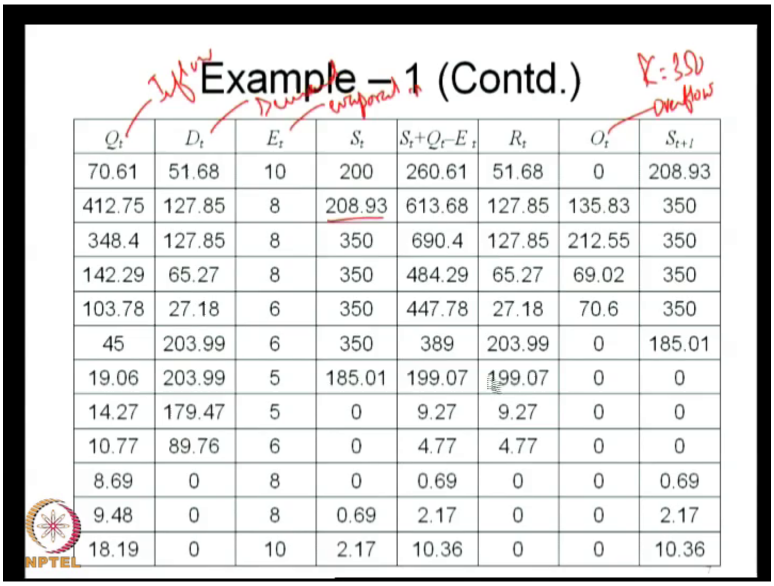

Example – 1 (Contd.)

Q_t	D_t	E_t	S_t	$S_t+Q_t-E_t$	R_t	O_t	S_{t+1}
70.61	51.68	10	200	260.61	51.68	0	208.93
412.75	127.85	8	208.93	613.68	127.85	135.83	350
348.4	127.85	8	350	690.4	127.85	212.55	350
142.29	65.27	8	350	484.29	65.27	69.02	350
103.78	27.18	6	350	447.78	27.18	70.6	350
45	203.99	6	350	389	203.99	0	185.01
19.06	203.99	5	185.01	199.07	199.07	0	0
14.27	179.47	5	0	9.27	9.27	0	0
10.77	89.76	6	0	4.77	4.77	0	0
8.69	0	8	0	0.69	0	0	0.69
9.48	0	8	0.69	2.17	0	0	2.17
18.19	0	10	2.17	10.36	0	0	10.36

Alright. We will do the standard operation now. This is the data, Q t is the data, this is the flow, this is inflow and this is demand and this is evaporation and so on. And, S t is the storage at the beginning of the time period t, which is given. So, at the first time period, we start with reservoir storage of 200 million cubic meters. And, this we add Q t 200 plus 70.61 and take out the evaporation 10 units that comes to 260.61. Now, this is the water, amount of the water available in time period t after the accounting for evaporation losses. Then, compare this with the demand 51.68 because this is more, you meet the demand to the full. So, R t will be equal to the demand 51.68. And, 260.61 minus 51.68 is less than the capacity which is 350. So, K here is 350. And therefore, the overflow will be 0 because your storage at the end of the time period will be 260.61 minus 51.68, which will be 208.93, which is less than the capacity 350. Therefore, the overflow will be 0. This is the overflow or the spill.

Now, this storage at the end of the time period becomes the storage at the beginning of the next time period. Again, look at S t plus Q t minus E t that comes to 613.68, and then look at the demand 127.85 because you have enough water here. You make R t is equal to D t, 127.85. Then, 613 minus 127.85 are greater than 350. And therefore, overflows will occur. You compute the overflows, which will be 613.68 minus 127.85 minus the capacity 350. That will be 135.85. And therefore, you will end up with the storage of 350. Whenever there is the overflow here, your storage at the end of the time period must be equal to the capacity 350. So, make sure that you will get a capacity of 350, whenever there is the overflow. And, the overflow is computed from; this is the total water available after accounting for the evaporation, minus the release that you have made. And, whenever there is the water available is more than the demand, you make release equal to the demand itself. After making the release, you look at the water available and comparing it with the capacity 350. If the water available after making the release is more than the capacity, that excess you account for as overflow; so that, the end of the storage period will be equal to the capacity itself. So, this is how you simulate from time period to time period. And, the end of the time period storage becomes the beginning of the time period storage for the next time period and again you carry out like this.

So, 350 come here and so on. And then, suddenly you hit a low inflow period. You look at this point here, the inflows were decreasing and then suddenly you hit a very low inflow period. But, the demands are very large and you simulate it and you look at the demands. Let us say demand is 203, but you have only 199.07, so you release 199.07. Now, you have only 9.27, whereas demand was 179. You release only 9.27. And therefore, there is a huge mismatch here that occurs. In standard operating policy, there is no way you can project. For example, if you had stored some amount of water, you could have met more demand here in this periods; that means what I mean is, by sacrificing on meeting demands, certain locations, certain time periods when you had huge amount of water, we could have stored more amount of water and increase the storage. So that, that storage will be available for the next time periods. But, in the standard operating policy, this cannot be done. And therefore, in the standard operating policy, you simply keep looking only at that particular time period and then meet the demand to the best extent possible.

And then, you can do several analyses using the simulation that you did. I have shown it only for a few periods here. In fact, I have taken only one year period and shown the operation. But, typically what we do in simulation of using standard operation? Is that, you take last, let say, twenty years to thirty years data, your demands may be constant for different time periods within the year. So, if you have thirty years of inflow data, the demands may be varying form D 1 to D 12. Only the same thing, same demand will keep on repeating from year to year. But, the flows are different and you apply the standard operation in the way as shown here. Simulate the data and then you will be able to look at how the system performs under standard operation. What do I mean by performing? The performance of the system, you may look at the release and look at how many times you are able to meet the demand fully. Let us say you are able to meet to fully here, fully etcetera, like this you keep on looking at this. And then, once you come to this point, the demands are not met fully. Like this, you can evolve several performance majors. We touch upon the performance majors subsequently in the course. But, this is how you simulate the reservoir operation using standard operation, an operating policy. However, standard operating policy is not an optimal operating policy. So, typically we will be interested in an optimal operating policy.

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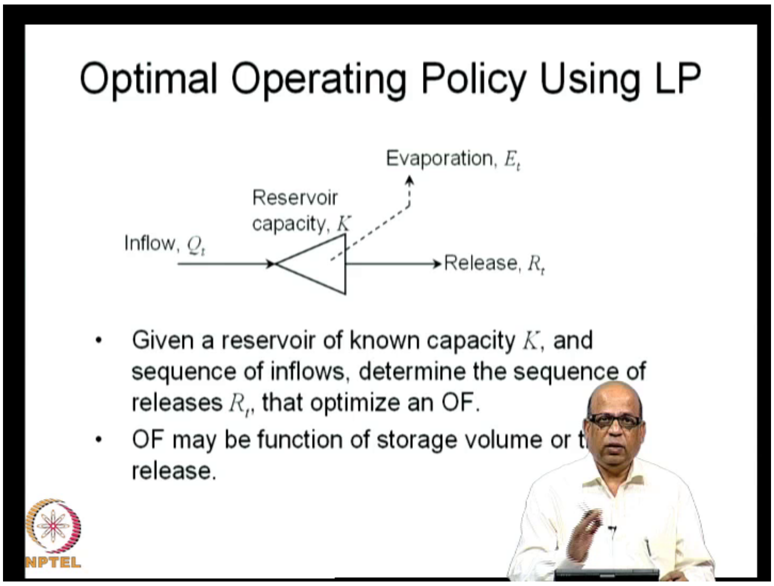

And therefore, you will look now at optimization of the operating policy, where you will have a purpose of operation, purpose for which you want to operate the reservoir. For example, you may have the release going only for irrigation with its returns associated with the releases in terms of, let us say crop yield or in terms of the monitory benefit that you can account for or in terms of the revenue that you get or in terms of the intangible benefit of providing the water to the farmers and so on. So, you may have certain objective with which you are operating the reservoir. This objective, you quantify and put it in the form of an optimization problem, objective function as we have studied earlier.

So, the problem now is given a reservoir of known capacity K and the sequence of inflows. So, Q t is known, K is known, determine the sequence of releases R t that optimize an objective function. Now in general, when you are talking about reservoir operating policies, the objective function in a time period, we know what we are looking at is the optimization of the reservoir operation over a time horizon. And typically, we begin with one year as the time horizon, the objective function which determines the value for the release as well as the value for the storage is in general a function of both the storage as well as the release. For example, you may make the release for irrigation, you get a return, but your storage level may be contributing to the head for the hydropower and therefore the storage also has certain values and so on. So, you may have in general, the objective function as a function of both the release as well as the storage.

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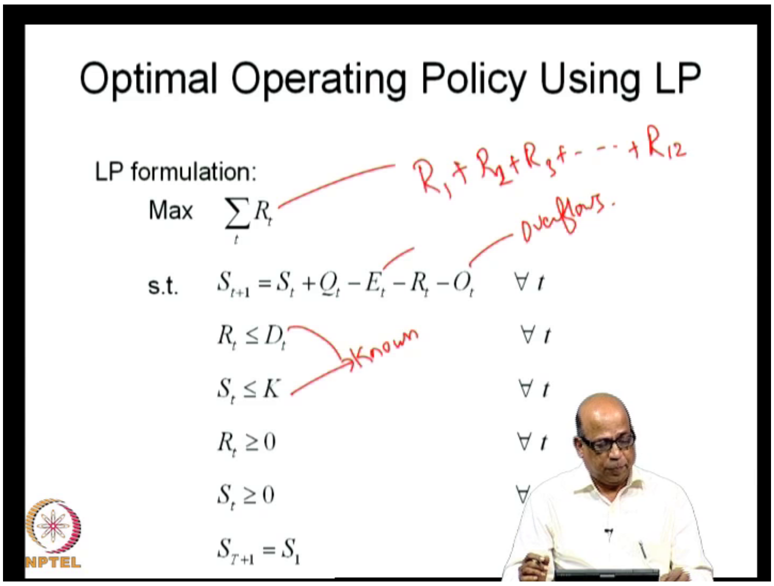

Ok. We will look at one of the simplest optimization problems, where we are looking at deciding on the release policies. There is a certain demand pattern or demand sequence that is known. So, you have D t which is the demand. This is known, and the capacity is known, that is, the reservoir capacity is known. We are looking at meeting the demands to the best extent possible. And therefore, I will look at maximization of R t; which means that, if you have, let us say twelve time periods R 1, R 2, R 3, etcetera. This is the function that I will look for.

And, we meet the storage continuity, this is the evaporation loss. This can also be made as storage dependent losses as I have discussed earlier. But, for the time being, we will not worry. We will take average evaporation losses during the time periods. So, this is the storage continuity. These are the phase overflows in the actual solution of the problem. You may want to put the integer variables as we discussed just in the last class, also at the beginning of today's class.

For overflows to meet this continuity equation as an equality constraint in the linear programming there are several ways of handling as I have discussed in the previous classes. You can use either an integer variables and then you use the overflow or the spills. Now, I put R t is less than or equal to D t. You know, some of these constraints the students must be careful. Many times, mechanically we write R t should be greater than or equal to D t because demands are there. And, we would like to make releases more than the demands. But, when we are looking at maximization of summation of R t, if you make R t greater than or equal to D t, most of the time what will happen is the reservoir will be empty because we are saying R t should be greater than or equal to. So, because we are making maximization of R t, R t will be increased to the best extent possible. But, we put an upper cap of demand; which means that in a no time period, R t should be more than the demand. There is a cap that we put; so that, we have water available for the other time periods.

And, we meet the continuity equation and all of these are standard conditions. And, S t plus 1 will be equal to S 1 that is, T is the last time period. We say that the last time period storage at the end of the last time period must be equal to storage at the beginning of the first time period. These are the constraints. So, we will run this. When we run this, remember for S 1 we need not put a starting value here. You can do solve this problem for various starting values of S 1 or leave S 1 free. And then, let the optimization problem decide what is the sequence of storage is and the associated sequence of flows. Now, Typically, this is what is done in defining the reservoir operating policy; that means you leave everything free and then let the optimization problem decide, what should be the storage at the beginning of the time period one, beginning of the time period two, etcetera and then, the associated release sequence. That is what we define as optimal reservoir operating policy.

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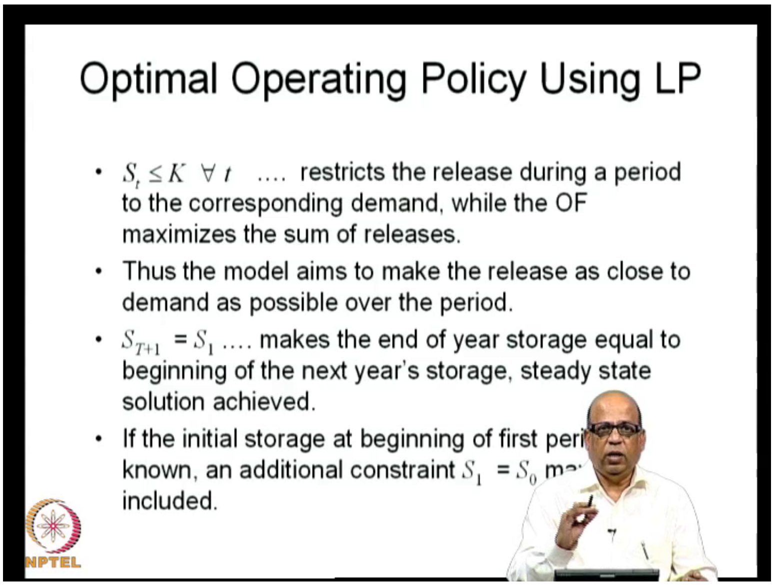

Now, all of this I have told earlier already. So, typically we may start with a given storage at the beginning of the time period. That is, you specify like we did in the standard operating policy. We can specify that the capacity is 350, but my initial storage is only 200 or my initial storage is only 100, then you start building the operating policy.

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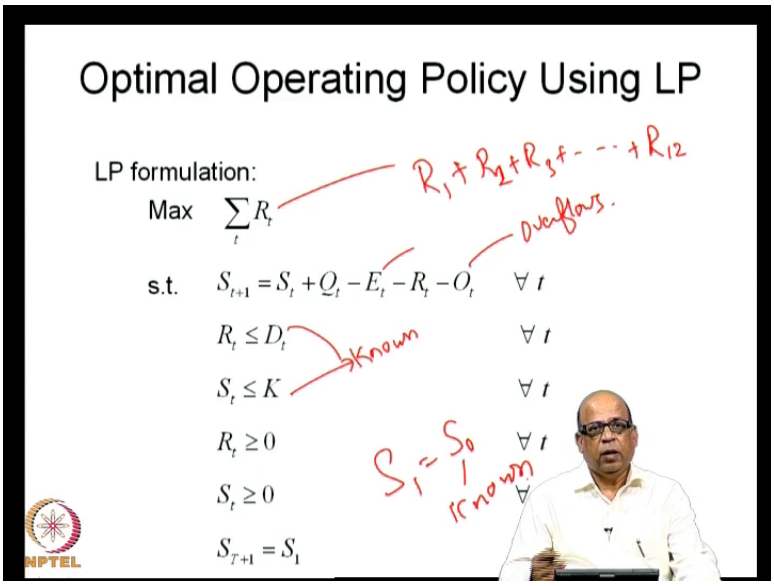

In this case, what we do? We add an additional constraint here, saying S 1 is equal to 200 or some instance. So, S 1 is equal to some S naught known value or some… We can put this condition. There is lots of variance for possible on this. We will just understand the simplest form first and then see what else can do this, and do over that.

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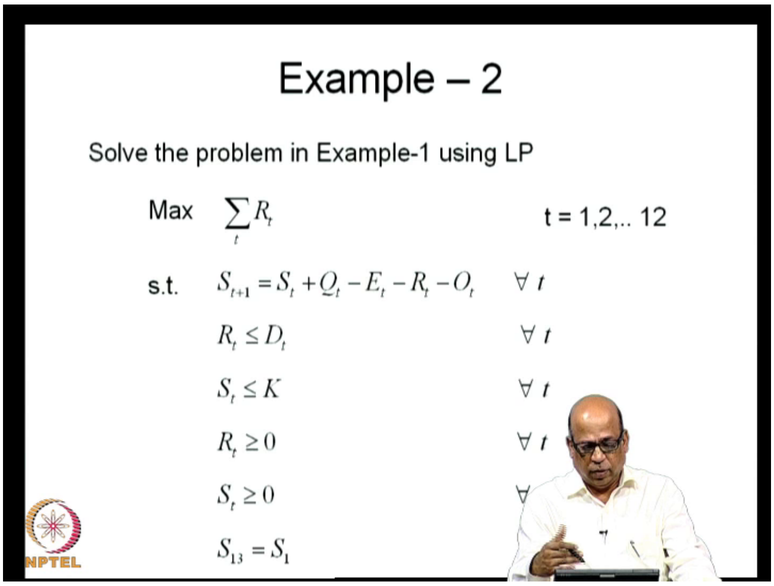

Now what we will do is, we will take the same data as we did for the standard operating policy and solve the LP. So, the data that I have used for standard operation, the same data, this data we will use and solve LP without taking S 1 as 200, as we did in the previous case. So, we are not putting that condition that the storage at the beginning of the time period is 200. We will just relax that and then solve this optimization problem.

There are two things that you must note. That, one is the evaporation is the average evaporation, we are not making it storage dependent losses, you can also make it. The overflows, we are just including it as the continuity equation. If for the specific problem, if you come across problems handling the difficulties, handling the overflows in the continuity equation, you can use the integer variables, which I am not doing for this simple problem. Alright. Then, we get the solutions. So, what we did is that we solved this problem sigma R t and then put the continuity equation and put R t is less than or equal to D t and all other conditions and solve this LP problem. And, t is equal to 1 to 12, there are twelve time periods. We use the LINGO software for this particular example and then solve this.

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Example – 2 (Contd.)

Solution:

Soln from optimization model.

t	Q_t	D_t	E_t	S_t	R_t	O_t	S_{t+1}
1	70.61	51.68	10	10.36	51.68	0	19.29
2	412.75	127.85	8	19.29	127.85	0	296.19
3	348.4	127.85	8	296.19	127.85	158.74	350
4	142.29	65.27	8	350	65.27	69.02	350
5	103.78	27.18	6	350	27.18	70.6	350
6	45	203.99	6	350	39.00	0	350
7	19.06	203.99	5	350	108.87	0	255.19
8	14.27	179.47	5	255.19	179.47	0	84.99
9	10.77	89.76	6	84.99	89.76	0	0
10	8.69	0	8	0	0	0	0.69
11	9.48	0	8	0.69	0	0	2.17
12	18.19	0	10	2.17	0	0	10.36

We get the solution. These are all data. The solution will consist of S t, R t, O t and of course S t plus 1. So, this part of the table is shown here. This part is the solution from the optimization model. So, we come with storage of 10.36 at the beginning of the time period. Remember in our model, we left the storage is free; that means let the optimization choose, what are optimal storage is that needed to be maintained and so on.

So, we come with this storage 10.36 and release is 51.68 because your flow is more and so on. So, you can just take for each of the time period. You can check the storage continuity, whether it is maintained or not. That is, 10.36 plus 70.61 minus 51.68 minus 0 must be equal to minus ten; of course which is the evaporation loss must be equal to 19.29. So, these are the optimal storage is that you get and these are the optimal release is that you get. Notice that this S t plus 1 will be equal to S t here, much the same way as we needed for the standard operation. Now, in this is a simple example. You may get a solution just similar to S O P. For example, your storage if you started, do with storage equal to 200, perhaps you will get identical result as you obtained in S O P because this is the very simple optimization problem, where we are simply looking at maximization of R. We are not saying that the… should be minimized or some…. simply we are looking at maximization of R. Now, this will define the policy. How do we define?

How do we communicate the policy to the operators is, by saying that on an average in a year, you have this kind of inflow pattern. Now, this may be average inflows for the different time periods June, July etcetera. For this kind of average inflow pattern, you maintain these reservoir storages 10.36, 19.29 etcetera and make this kind of releases as the flows come. So, the policy is indicated in general as the end of the periods, storage that needs to be maintained for each of the time periods. And, you provide it in terms of a rule curve. That means, you maintain the rule curve, shows that you maintain this kind of reservoir levels or the reservoir storage is at the end of given time periods. Now, that is what we do from this. Let us say that, you start with time period t is equal to 1, desired reservoir release or storage volume at the end of month. So, storage at the end of the month is what we are showing. So, at the end of the month you may have 19.29. And, this is how you show the storage levels. And, at the end of twelfth time period, you may get a 10.36. And, there may be certain periods in which you may have storage of 0.

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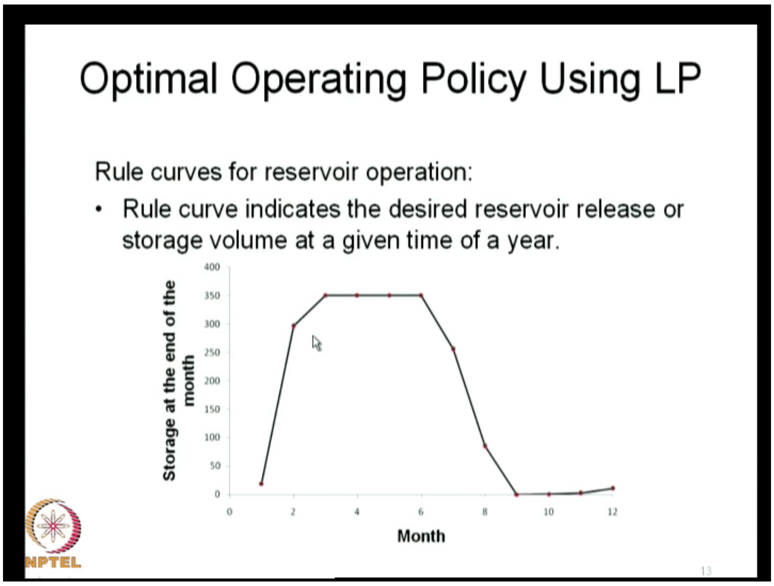

So, this shows the policy. Of course remember that, this is, by, for the simplest optimization problem that we solved. But, in general this is the procedure that you adopt. You solve the optimization problem, obtain the storages and then specify the policy of maintaining the storages. Associated with these storages remember that, there are also optimal releases that you are maintaining for meeting this kind of demand patterns. So, this is how you specify the operating policy. Now in general, in realistic situations you will not have such a simple demand patterns. You may also have flood season; you may also want to meet the demands, industrial demands during certain time periods, which may have higher weightages; you may want to meet municipal demands for drinking water purposes during certain periods, which may have much higher weightages and so on. So, you may have multiple objectives.

Keeping all those things in mind, you can put the optimization problem. You can formulate the optimization problem and then arrive at the reservoir operating policy. But, the general principle remains the same. That, you formulate the optimal objective policy, I am sorry. I will repeat. You formulate the optimization problem with certain objectives, objective function that reflex the objectives of your reservoir operation. Solve the optimization problem; you will get the storages as well as releases in general. And then, formulate the policy of maintaining the reservoir storages at the end of the time period, across different time periods within a year.

So, in the end, you must be able to specify what is the reservoir level that you need to maintain at the end of each of the months let us say, June month and July month. That is all. So, this is what we call as optimal operating policy as you progress and perhaps in case study that I have discussed, I will indicate how complicated or how complex the problems can be. But, the principle still remains the same. You formulate the optimization problem correctly and then solve it and provide the reservoir operating policy, in terms of the storages as well as the in terms of the releases.

Ok. Now, so far we have been talking only about single reservoir systems. We discussed the problem of determining capacity for single reservoir. We also discussed at some length now, the reservoir operating policies; both the standard operation as well as the optimal operation. and, in earlier, one of the earlier classes where I introduced the multi objective optimization, I have talked about a single reservoir having different objectives; flood control, hydropower, irrigation, etcetera which may be in general conflicting with each other. We know how to address multiple objectives, which are typically conflicting with each other in an optimization problem.

But, so far we have not discussed, except by way of mentioning in the simulation topic. We have not discussed specifically the multi reservoir operation problems or multi reservoir systems. Now, we start looking at multi reservoir systems. Remember, we are still talking about deterministic problems where there is no uncertainty associated with either the inflows or the storages and so on.

Now, let us look at multi reservoir systems. And, typically when we are talking about water resources development, water resources management and so on, our scales are special scales are large. And, we are typically talking about several reservoirs may be two reservoirs, four reservoirs, five reservoirs, fourteen reservoirs, eighteen reservoirs and so on. If you look at case like Narmada River or Tennessee valley authority, Damodar Valley Corporation or California valley corporation reservoirs, California valley authority reservoirs, I am sorry, it is a TVA, that is, a Tennessee valley authority or California valley corporation reservoirs and so on. When we are looking at such systems, you typically have large number of reservoirs. And, these are the reservoirs together have to be optimized as a single system.

Some action that you take at one of the reservoirs is likely to affect some other reservoirs; some action that you take simultaneously at several two, three, two or three of different points, may affect the reservoirs down stream of that adversely; and some actions that you take during certain time period may affect, may have its effect in some other time period within the year. And therefore, the multi reservoir systems have to be looked at as an integrated single unit, where the flow is not necessarily the river flow, but the flow of actions, flow of different components is all integrated together. So, let us look at one simple example.

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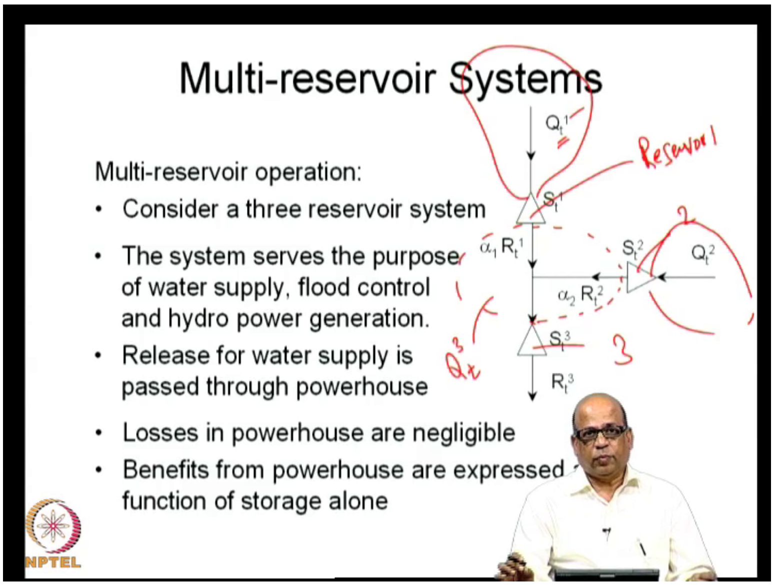

We will start with the simple example, just to understand what complications can arise in multi reservoir systems. So, we will look at three reservoirs here. There is a main stream of the river, on which there are two major reservoirs. We call it as reservoir one here and reservoir three here. And, on the tributary from the side, there is another river that is governing the tributary joining this main stream. And, on that you have another reservoir here. So, this is the reservoir two. So, we will we will start with a simple reservoirs systems consisting of three reservoirs. Two and three. Now, we will just understand the continuity, very simple, continuity there is the inflow that is coming and adding to the storage at the reservoir one. This inflow is because of the catchment of this reservoir.

So, catchment of this reservoir, so there is a catchment of this reservoir. And, that is what is contributing to this inflow. Similarly, there is a catchment here. That is, contributing to inflow. When we come to reservoir three, so what happens at reservoir one here, is that flow joins. This is the flow in time period t at reservoir one. So, we are indicating Q 1 t as the inflow in time period t at reservoir one. Similarly, Q 2 t is the inflow in time period t in reservoir two, then you also have… because of the catchment here, there is also an intermediate catchment. All of this is an intermediate catchment for reservoir three.

So, the reservoir three receives water not only from the reservoir two and reservoir one, but also from its own catchment. So, you may have Q 3 t here, as the flow in time period t at reservoir three due to its own catchment in between. In addition to, it also receives water from reservoir one. Let us say you made release of R 1 t from the reservoir one, part of it may come and join this reservoir in general because some of them, some of it may have been utilized. And, only some part may come and actually join this reservoir.

So, we will indicate as alpha 1 R 1 t; where alpha 1 is fraction of the release that joins the down steam reservoir. Similarly, from the release that you make from the reservoir two, some fraction comes and joins this river and subsequently the reservoir three. So, alpha 2 into R 2 t is the amount that is coming from the reservoir two and joining this particular reservoir. Now, we will look at this reservoir, as the, this system as an integrated whole and formulate an objective function, formulate an optimization problem for this. So, we are looking at the simultaneous operation or the integrated operation of these three reservoirs. The reservoir one may have its own objective, reservoir two has its own objectives, and reservoir three has its own objectives. But, because the reservoir one and three, I am sorry, one and two here in this case are contributing to reservoir number three, the objectives there are all linked. Because what we do at reservoir one while affecting reservoir three, what you do reservoir three will affect reservoir three. And therefore, you need to look at this operation as this system as an integrated unity unit.

So, we will see here. What we will do is for the sake of formulation, we will say that this system serves purpose of water supply, flood control and hydropower generation; that means, at each of the reservoir you have water supply. Let us say you have irrigation water supply, then you also have flood control, which means that during the flood season you may want to retain certain flood free board; so that, you can observe the flood waters and also you have hydropower generation at each of the reservoirs. And therefore, the individual reservoirs has the own purpose to be met.

In addition, they should meet the total systems objectives, and then release for water supply is passed through powerhouse. In general, it may happen that, part of a release that you make for a irrigation also goes through the powerhouse, then we can neglect the losses in the powerhouse, although you can also account doing it. It is not the great deal. You can just account for the losses i. Some percentage of the water that is released 10 to the power of and so on. For this simple problem what we will do is, benefits from powerhouse are expressed as a function of storage alone in general. And, in fact this problem I will explain, in perhaps in the next class.

In general, for hydropower generation what happens is, the benefits in terms of the power that you can generate is a functions of both storage as well as release through the… because you can recall gamma q h is the power that is generated; where q is the discharge and h is the head and gamma is the unit weight of water. And therefore, power is proportional to, power that is produced is proportional to both q as well as h whereas in this particular problem for demonstration, what we will say is that the benefits are the returns that you get from hydropower is simply related to the storage. We will not worry about the release as far as the reckoning of the benefit is concerned.

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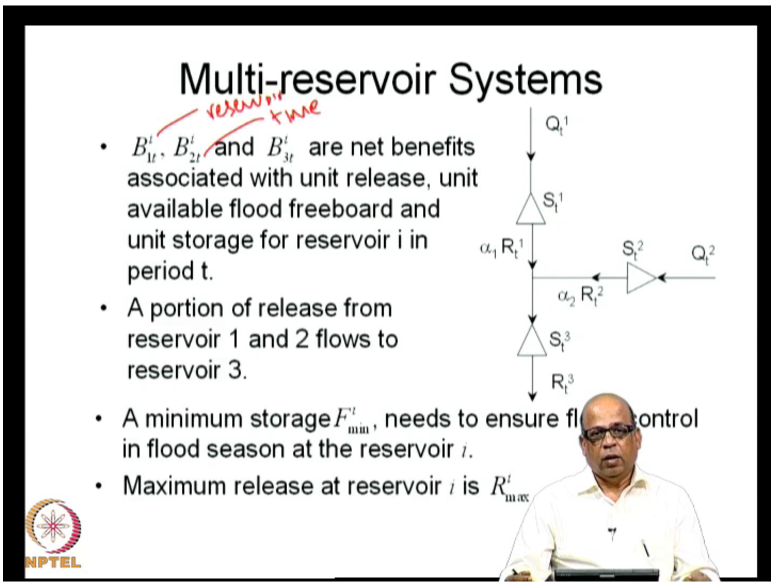

Now, we will say B 1 t, B 2 t, B 3 t are the net benefits associated with unit release, unit available flood freeboard and unit storage for reservoir i. So, I will put a super script i here to indicate that it is a reservoir and t is the time. And, this one and two are the reservoir, I am sorry, one two and three are associated with the purposes. For example, one is for the release and two is for the flood freeboard and three is for the storage. And, as I just mentioned the storage determines the benefits that acquire out of hydropower. Now, this is for unit volumes. For example, you need to release, this is the benefit and unit flood control storage or flood volume that you make available in the reservoir, this is the benefit and so on.

Now, a portion of release from reservoir one and reservoir two flows through to reservoir three. As I explained alpha 1 R 1 t and alpha 2 R 2 t, they come into reservoir three. Further we will say that, there is a flood period. For example, in country like ours monsoon country you may say August, September, October and perhaps in some places November also may be flood periods or may be July, August, September and October, four months are flood periods. Now, during these four months, we may want to retain a certain minimum flood zone in the reservoir, flood absorption zone.

So, we want to make sure that a minimum storage of f min i, i is the reservoir. At reservoir i, why we want to maintain a minimum storage, buffer storage; which means, that the storage level should not be more than the certain points, so that, you have adequate buffer to absorb the flood waters. And, that is what we are indicating. Here at each of the reservoirs, we specify that during this flood to a season, which may be different for different reservoir system during flood season. We want to retain a certain flood freeboard. Further, the release that you make from a reservoir has an upper limit. And, lower limit is of course zero. But, it also will have an upper limit because if you are making the release through a canal, then the canal capacity determines the upper release. So, each of this reservoirs has a maximum limit on the release. So, R i max is a maximum release that is possible from reservoir i. So, you have the flood control requirement. You have the maximum release requirement and you have the benefits associated with the release from the reservoir, the storage available in the reservoir as well as the flood freeboard that you keep in the reservoir.

Now, this is the typical problem which we will formulate as an optimization problem now. Associated with unit release, you have the benefits from each of the reservoirs; associated with the flood freeboard that you maintain during the flood season, you have a benefit; associated with unit flood storage that you maintain, you have a benefit; associated with the storage because of the hydropower, you have benefit at each of the reservoirs. Now, these benefits need not allow economical benefits. They can be intangible benefit; they can be physical quantity and so on.

But, right now, we just assume that they are returns that are provided or something good that happens because of these particular variables like release storage, flood control storage and so on. With these, this understanding of the system, we will formulate an optimization problem which will decide the release policy or the reservoir operating policy at each of the reservoirs: reservoir one, reservoir two, reservoir three. Not separately, but together. That means, when I am saying this is the optimal release at reservoirs number one, it has taken into account its implication or the operating policies implication on the down steam reservoirs and on all the other reservoirs in the system. So, that is how we formulate the problem as an optimization problem and solve it to obtain the reservoir releases at each of the reservoirs in the system. What I demonstrated through a simple three reservoir system can be generalized to any n number of reservoirs because the principle remains same. You look at the continuity; you look at the objective function and then formulate the optimization problem.

So, we will continue with this discussion. And, in the next class I will start with formulating the optimization problem for this. So, essentially in today's class what we did is that, we started with the LP formulation, that is, linear programming formulation for obtaining the storage yield function. That is, we maximize R and then look at the storage function, which was done in the last class. I just explained it today. But, we specifically discussed the reservoir operation problem at some length today.

In today's lecture, we started with the standard operating policy, where we recalled that in standards of operating policy for reservoir we do not look at other periods. We simply look at the current time period, look at the water available, storage plus inflow, compare it with the demand and meet the demand to the best extent possible. That is the standard operating policy, without bothering about what is likely to happen in the other time periods within the year.

Then we formulated a linear programming problem for the same issue of obtaining the reservoir operation or reservoir operating policy, where we look at maximization of R t, some of R t, the releases are the time periods within the year subjected to all the standard constraints. And, obtained the storage that is needed to be maintained at each of the time periods and the associated releases during each of the time periods; this will specify the operating policy for the reservoir. And, we provide typically what is called as the rule curve for the reservoir operators. The rule curves will indicate the reservoir storage to be maintained at the end of the each of the time period, so for example at the end of the June, at the end of the July, and so on. And then, towards the end of the lecture, I just introduced a multi reservoir system where we are looking at integrated operation of multi reservoir systems. You may have multiple purposes at each of the reservoirs and each of the reservoirs has its own objectives of an operation. And, you look at this particular system as an integrated whole, formulate on objective optimization problem and obtain the reservoir operating policy for each of the individual reservoirs.

In the next lecture, we will start with this problem and formulate an optimization problem and typically, a linear programming problem and solve it to get the reservoir release policy at each of the reservoir. Thank you for your attention.