**Water Resources Systems**

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**Module No. # 05**

**Lecture No. # 22**

**Reservoir Capacity using Linear Programming (2)**

Good morning and welcome to this lecture number twenty two, of the course water resource systems, modeling techniques and analysis. Now, over the past few lectures, you have been now talking about reservoir systems. So, initially we started with the determination of the reservoir capacity using the sequence peak algorithm and then we went on in the last lecture to determine the optimal capacity using the linear programming. So, recall that. We first started with the optimization where we looked for the minimum required capacity of the reservoir for a given sequence of inflows, to meet a given sequence of demands.

So, initially we started with not accounting for the evaporation losses. And then, subsequently in the last lecture we introduced a technique, by which you write the reservoir continuity equation by accounting for the evaporation losses as storage dependent losses. How did we do that?

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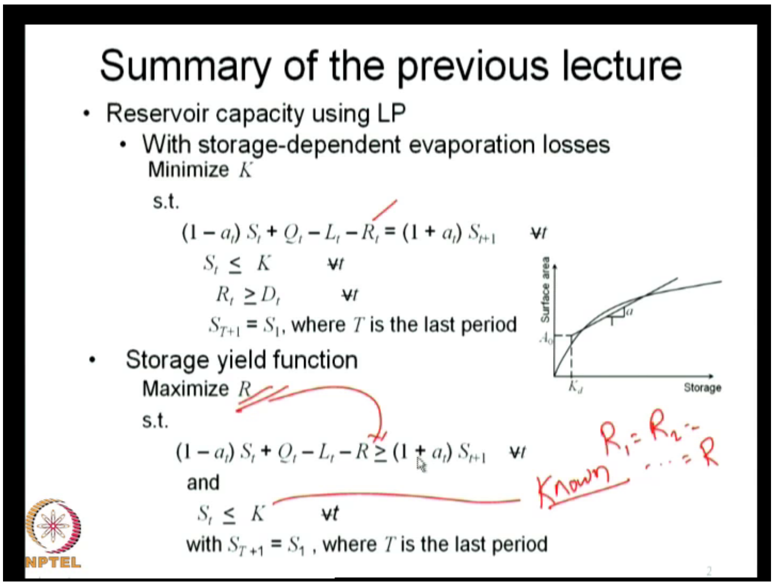

We looked at the area capacity relationship. And then, beyond the dead storage level, we approximated the area capacity by a straight line. Look the slope of that straight line, which is actually surface area per unit storage beyond the dead storage. And then, rewrote the continuity equation.

So, this is the continuity equation that we wrote and used in the linear programming problem, where we are essentially looking at obtaining a minimum storage to meet these demand patterns d t. And, that is why we wrote R t greater than or equal to d t; where R t is the release from the reservoir. And then, we solved this example to obtain the minimum capacity. So, this is the first level of exercise that we do in the reservoir systems, where for a given sequence of flows, we want to determine the minimum reservoir capacity. And, remember we are talking about the active reservoir capacity or the live capacity of the reservoir. Then, we went on to ask the question of what is the maximum yield that we can expect from a reservoir of a given capacity.

So, we fix the capacity of a reservoir, the inflows sequence is known. We ask the other question related to, what is the maximum constant release that we can make from the reservoir for a known capacity and for a known sequence of inflows. Remember, all these problems that you have been talking about are deterministic optimization problems. In the sense that there is no uncertainty associated with any of the variables that we are talking about. They are all known or they are determined in a deterministic sense. For example, the storage at the reservoir is deterministic, release is deterministic and the inflows are deterministic and so on.

So, there is no uncertainty associated with any of the variables that we are talking about. In the storage yield function problem, where we are looking at maximum constant yield or maximum constant release that can be ensured form a reservoir of known capacity. The formulation will look like this. So, initially we had minimized K when we are looking at the storage capacity determination. In the yield determination we fix K. So, this K is known or given. So, for a given storage, we determine a constant release. Remember here it was R t, which was varying from time period to time period. But, here we are looking at a constant release all through the year.

So, we are saying R 1 equal to R 2 equal to R 3 etcetera up to R t, and that will be equal to R. So, this is the constant release. And, we are looking at maximization of release. The physical picture you should keep in mind always that for a given reservoir of known capacity, what is the maximum release, constant release that you can meet from the reservoir all through the year? There may be twelve time periods like this, if you are looking at monthly operation. So, all through the year you want to maintain a constant release. Such problems should be important when you are looking at releases for irrigation. Or, let say that this system is part of, this reservoir system is part of a major multi reservoir systems in which you want to optimize the releases at this particular reservoir, maintaining a constant maximum release all through the year.

So, that is the problem here. And then, we started talking about an example related with the storage yield function. So, the yield is the maximum constant release that you can maintain from the reservoir all through the year; that is the yield. Obviously, the yield will be a function of the reservoir storage or the capacity of the reservoir, for a given sequence of inflows. So, typically what we do is, to generate the storage yield function, you start with the particular reservoir storage, keep increasing the reservoir storage. Associated with the every reservoir storage, you get one value of maximum yield; which means, you solve this optimization problem over and over again, every time changing the value of K.

And, the yield will keep on increasing up to a maximum point of R. Obviously; you cannot keep on increasing the yield. If you keep on increasing the reservoir storage, what will be the limiting factor there? The limiting factor will be the supply itself. So, the inflow becomes the limiting factor. And, these we call it as inflow limitation; that is, the yield will be limited by the flows. On the other hand, the yield can be also limited by the storage. So, we will see an example now, of how we generate the storage yield function. So, this is what we discussed in the last class. We will proceed further now. And, look at how we generate the storage yield function for a given hydrology. By given hydrology, I mean the sequence of flows are known, the rate of evaporation is known and the area capacity relationship is known; so that, you can determine the constants a t here, as well as the constant L t here. Refer to the previous class how we do that?

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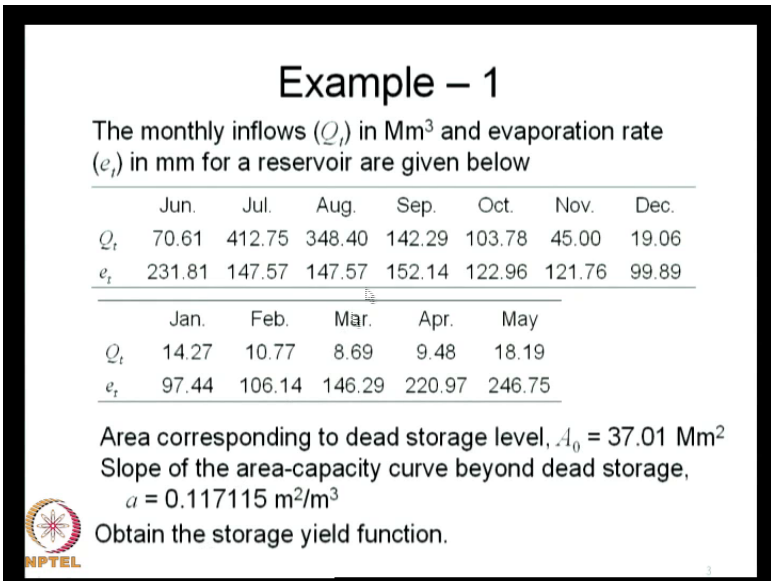


Alright. So, we will take the flows here. Remember, unlike in the previous section or unlike in the problems that we discussed in the previous class, where the demands D t were also specified. In the problem dealing with maximization of releases or problems dealing with yield from the reservoir, you do not specify the demands are priory. You are saying what is the maximum demand, maximum constant demand that I can meet from the reservoir? That is the problem that we are talking about. And, therefore the data that you will need is in terms of the flows and in terms of the rate of evaporation, e t is in millimeters and flows are in million cubic meters. This is the same data that I have used earlier.

So, this is the data and then we have the area capacity relationship. So, A naught is given, this is the constant and the slope a is given, which is 0.117115. Remember, a has units. It is not a unit less quantity because we are talking about the slope here, slope of area capacity or area storage relationship. Now for this data, we will obtain the storage yield function. We will use linear programming for this purpose.

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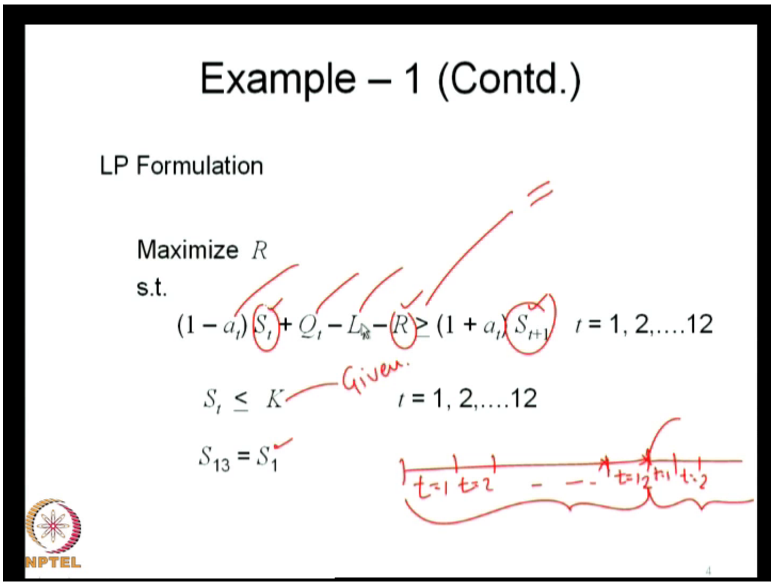


So, like I said in the previous class, you can write the continuity equation in this form. Typically the continuity equation, strictly speaking, should be an equality constraint here. But, this as you solve examples of linear programming you will understand, as you have large number of equality constraints. Let us say, there are twelve number of equality constraints. Normally, creates some difficulties in solving linear programming problems; often, it gives you infeasible solutions because here there are decision variables S t, R t or R in this case and S t plus 1. If you put an equality constraint and then those things you have to meet number of times, that is the equality constraints, number of equality constraints is , the left hand side has to exactly match the right hand side for all the twelve cases. And therefore, you may often get infeasible solutions. To avoid that, I have explained in the previous class, there are different ways of handling this problem. And, one of the ways is simply make it greater than or equal to here.

So that, whatever excess is there, is observed as into R here. So, you are actually allowing for some latitude in the constraints or some flexibility in the constraints. So that, if there is additional water, it can be accommodated in one of the terms because you are saying left hand side can be greater than or equal to, it need not be exactly equal to the right hand side. So, this is how we write the continuity equation. a t can be determined, Q t is known, L t can be determined and other variables are decision variables. So, this S t plus 1 is the decision variable, R is the decision variable, S t is the decision variable. And, K is given. So, for a given K, we solve this particular problem.

And, like I mentioned in the previous class, if there are twelve periods, the storage at the end of the thirteenth period or the storage at the end of the twelfth period, I am sorry, let say t is equal to 1, t is equal to 2, etcetera, and t is equal to12. At the end of the twelfth period, the next cycle starts. Therefore, t is equal to 1 here, t is equal to 2. This is how we solve the problems in deterministic sense, where the sequence will keep on repeating. And therefore, the storage at the end of the time period twelve will be storage at the beginning of the thirteenth time period. And, that we set it as storage at the beginning of first time period. So, S 13 is equal to S 1. This is what we do. So, this is the problem now. We will solve this using the data that is specified here. So, this data we will use and solve this particular example. For that, we need to determine a t and write down all the constraints. We also have to determine L t.

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## Example – 1 (Contd.)

$e_t$ converted to 'm'

| Month | $Q_t$ ($Mm^3$) | $e_t$ mm | $a_t=a*e_t/2$ | $L_t=A_o*e_t$ ($Mm^3$) | $(1 - a_t)$ | $(1 + a_t)$ |
|---|---|---|---|---|---|---|
| Jun | 70.61 | 231.81 | 0.01357 | 8.58 | 0.9864 | 1.0136 |
| Jul | 412.75 | 147.57 | 0.00864 | 5.46 | 0.9914 | 1.0086 |
| Aug | 348.4 | 147.57 | 0.00864 | 5.46 | 0.9914 | 1.0086 |
| Sep | 142.29 | 152.14 | 0.00891 | 5.63 | 0.9911 | 1.0089 |
| Oct | 103.78 | 122.96 | 0.00720 | 4.55 | 0.9928 | 1.0072 |
| Nov | 45 | 121.76 | 0.00713 | 4.51 | 0.9929 | 1.0071 |
| Dec | 19.06 | 99.89 | 0.00585 | 3.70 | 0.9942 | 1.0058 |
| Jan | 14.27 | 97.44 | 0.00571 | 3.61 | 0.9943 | 1.0057 |
| Feb | 10.77 | 106.14 | 0.00622 | 3.93 | 0.9938 | 1.0062 |
| Mar | 8.69 | 146.29 | 0.00857 | 5.41 | 0.9914 | 1.0086 |
| Apr | 9.48 | 220.97 | 0.01294 | 8.18 | 0.9871 | 1.0129 |
| May | 18.19 | 246.75 | 0.01445 | 9.13 | 0.9856 | 1.0144 |

So, exactly the same way as we did in the previous class for completeness sake, I will show the calculations again. They will remain the same. e t is in millimeters, we convert into meters. So, whenever we are using e t anywhere, we are converting into meters. And then, using it, which means that here actually you were dividing it by 1000.

And therefore, you get a t here. And, L t is A naught into e t. A naught is given. A naught is the data here 37.01. And, this is how; you get L t and then 1 minus a t, 1 plus a t, all of these I have explained in the previous class. So, from this, now we write the continuity equations.

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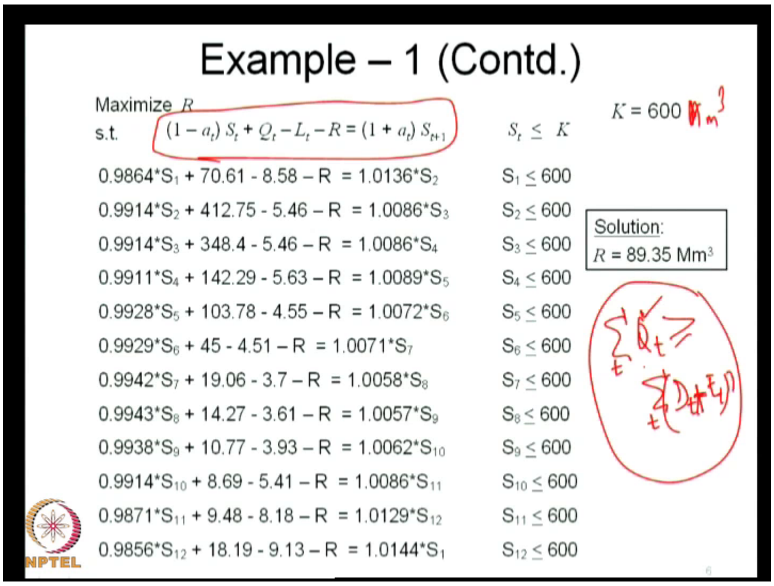


Now to begin with, we will start with equality constraints. Although we wrote greater than or equal to, strictly it should be equality constraints. So, let us begin with equality constraints here. Do not get confused between, whether we use equality, greater than or equal to, less than or equal to, etcetera. Remember that the continuity equation is strictly is in this form. This is the form of continuity. This is actually the mass balance. It has to be met in this particular form. But, remember in this form, we have not explicitly accounted for the spills or the overflows. What happens after the reservoir has reached its capacity? That has not been explicitly incorporated here. And, that may cause problems several in many situations. And, that is why we use either greater than or equal to or some integer variables and so on, which presently I will discuss. But, right now, we will use the equality constraints as they are and then, solve the example. So, this is twelve numbers of constraints. We have put S 13 is equal to S 1. Therefore, the twelfth constraint will be in terms of S 1 on the right hand side.

And, K we are starting with 600. And, for this we obtain the solution as R is equal to 89.35 million cubic meters. This means that, for a given storage of 600 cubic meters, this is in million cubic meters, it is a… for a given storage of 600 million cubic meters, for this given sequence of inflows the maximum constant release that you can maintain from this storage is 89.35 million cubic meters. So, this is the yield corresponding to the storage of 600. If you reduce the storage, what will happen? The yield will also reduce; if you increase the storage the yield may increase up to certain point. Why I say may is that, let say instead of 600 million cubic meters, you have 10000 million cubic meters, would it increase by that much amount? No, because it will be limited by the flows that are coming.

So, you can only meet the demands up to certain point because they will be limited by the flows. Specifically as I mentioned in the last class, the condition that you need to meet is that sigma Q t must be equal to, must be greater than or equal to sigma d t minus, let say I will put e t; where e t is the losses over all t in a broad sense. That is, the total flows that you have must be greater than or equal to the total demand plus, this is plus the evaporation losses. If this is not met, then you will get infeasible solutions in this. So, no matter what capacity you provide, you will be able to meet a constant demand, a maximum constant demand only up to certain point. Beyond that, whatever capacity that you are building will be a waste.

So, let us examine that. Let say K is equal to 600 million cubic meters, you got 89.35. We will solve this example retaining the same constraints sets, but with different capacities. So, I will change from 600, let say I will start with 200 million cubic meters, I will check what is the capacity; 300, what is the capacity; 400, what is the capacity; etcetera like this in discrete units. Let us, keep on rerunning the problem with different storage values, to do that we need good, elegant software. And, there are many types of softwares available for linear programming and very elegant softwares available.

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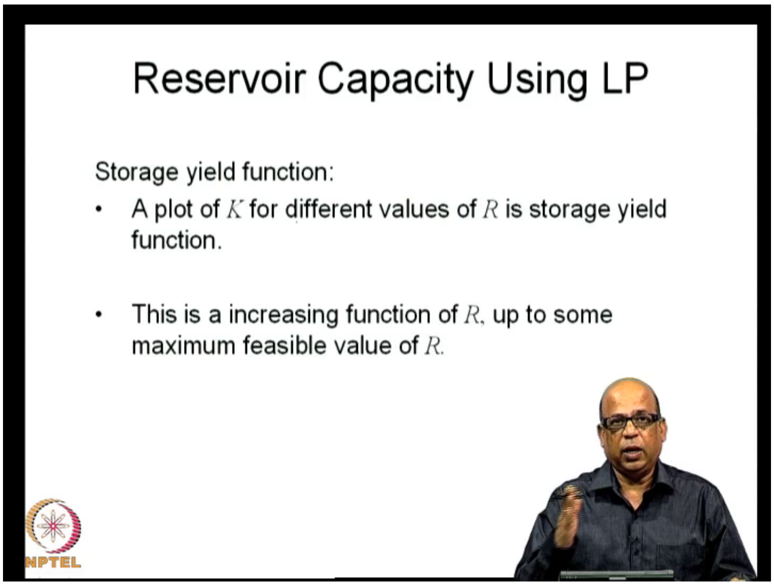


I used in this course; I used the software called LINGO for educational purposes. It is available freely with very limited number of variables and limited number of constraints. You can just download it and then use it. But, for actual applications, large scale applications, etcetera, you may have to purchase the software. But, I will just explain what we essentially do in the storage yield function using that particular software LINGO. So, we need to develop a function, which relates K versus R; that is, for a given capacity what is the yield? So, this is the storage yield function. That is what we do. And, as I said it can only go up to certain point, beyond which it may remain constant; which means, no matter how high you build the dam, how large is the capacity of the reservoir, there is only a maximum limit up to which you can meet the demands. And, that will be limited by the inflows. So, it is called as flow limitation.

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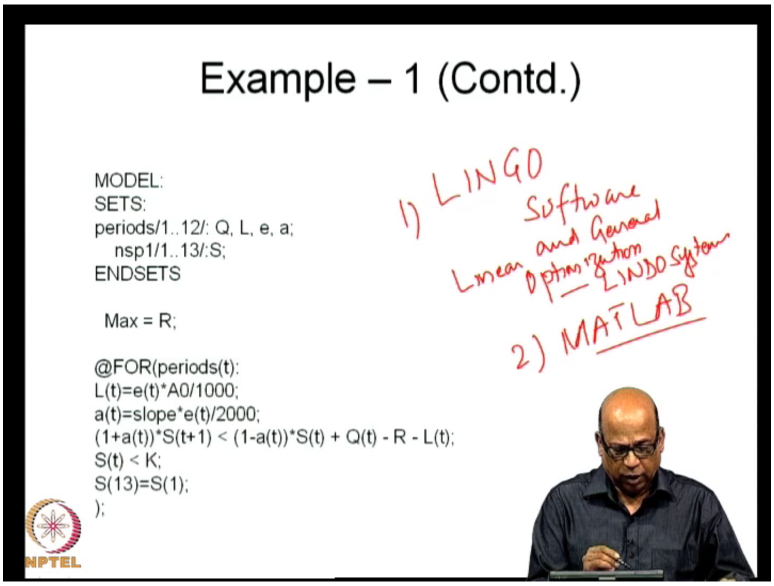


Now, this is the format of the software. As I said, this is the LINGO software. And, there are other softwares also available. From MATLAB, you may have some general optimization softwares. This is linear and general optimization. And, this may be for educational purposes. You can download it from LINGO systems. A very useful user’s… is also available. So, if you are using only the linear programming, and perhaps the integer programming, quadratic programming, etcetera, which are the variants of linear programming, this is a good software. Otherwise, any nonlinear programming, linear programming softwares, etcetera, is also available in MATLAB.

So, there are large numbers of very handy softwares available for linear programming. You can choose any of them. I would encourage the teachers of this course to introduce to the students some simple software like LINGO or through MATLAB etcetera. Where, assignments can be solved using this kind of softwares. For the LINGO, the entire model that I have wrote here, all of these can be written in a very compact form like this. Now, I will not go into the details. You define essentially Q, L, e, a etcetera as arrays of dimensional 12. S has dimension 13 because you need S 13 is equal to S 1. All of these terms here can be just written as; at for periods t. It is like a loop, which you are defining for t.

And, the periods t we have defined as going for 1 to 12. And then, you write all of these constraints and that is it. So, this completes the formulation here, and then we specify the data. For example, e t is a data, A naught is data, slope is the data, small e t is the data, and then you determine a t here. And, K is the data, as for as this problem is concerned and Q t is a data, so all of these need to be specified as data and that we do here.

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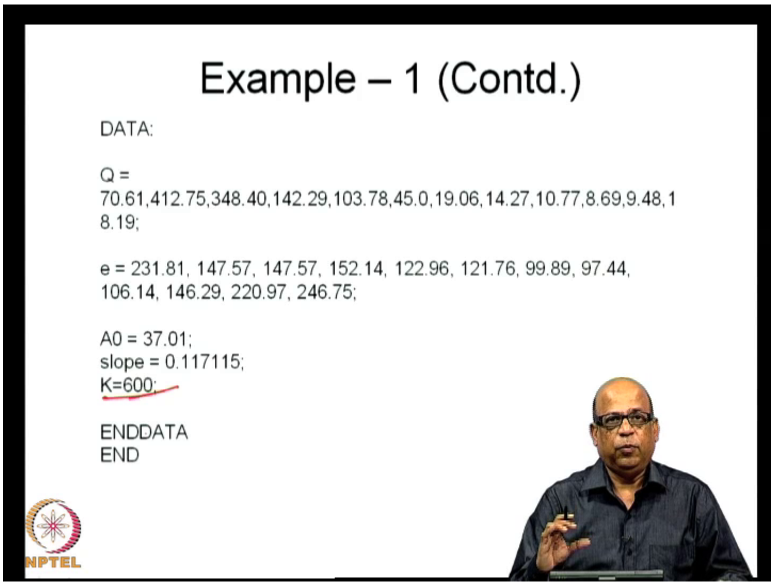


The format is like this. Data, you specify Q twelve values, e twelve values and A naught like this and slope is this and K is equal to 600. Remember here this 1000, that is coming is because we are converting e t into meters. Similarly, here it was e t by 2 and then have divided by 2000.

So, this is the simple problem, simple model that you can use the LINGO software, and then solve using this particular format. Now, this max equal to R is the objective function. So, I encourage you to solve this problem. Every time, you change the value of K, let us say K is equal to 600. Everything else remains the same. We change value of K and get the value of R. There are also other decision variables like S 1, S 2, etcetera that we will not worry.

Simply look at the R which is the yield. So, for a given storage you get the yield. Like this, you do it for several values of K.

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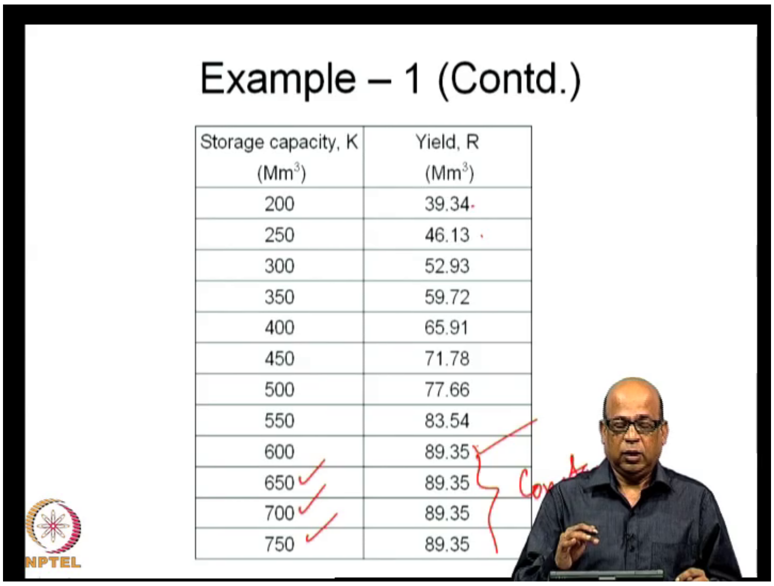
Example – 1 (Contd.)

| Storage capacity, K ($Mm^3$) | Yield, R ($Mm^3$) |
|---|---|
| 200 | 39.34 |
| 250 | 46.13 |
| 300 | 52.93 |
| 350 | 59.72 |
| 400 | 65.91 |
| 450 | 71.78 |
| 500 | 77.66 |
| 550 | 83.54 |
| 600 | 89.35 |
| 650 | 89.35 |
| 700 | 89.35 |
| 750 | 89.35 |

So, for 600 you got 89.35, for 200 you get 39.34, for 250 you get 46.13. Remember what I am doing is, I am simply changing the value of K, running the software, running the optimization problem and getting the value of R.

So, this is the yield R, for a given storage capacity K. For 200, I get 39.34; for 250, I get 46.13; like this for various values I get the yield. Once I reach this 89.35, no matter how much I increase, this remains constant; which means that, if you build a reservoir beyond 600 million cubic meters, it is all a waste as long as for as meeting the yield is concerned. That means the yield from that is the reservoir will not be any more than 89.35. No matter how big the reservoir is. Now, this 89.35 is the constant release that you can maintain all through the year from that particular inflows sequence.

So, no matter how big the reservoir is. You will not be able to maintain a yield more than 89.35. You can verify how you got this 89.35. 89.35 is constant release. 89.35 in to 12, is the amount that you are using, plus you add all the evaporation losses that you have got. This should be equal to the inflow, the total flow that is coming because the total volume of water that is available is simply sigma Q t. And, in that evaporation losses are going and then in you are meeting the constant release of 89.35. That is the idea there.

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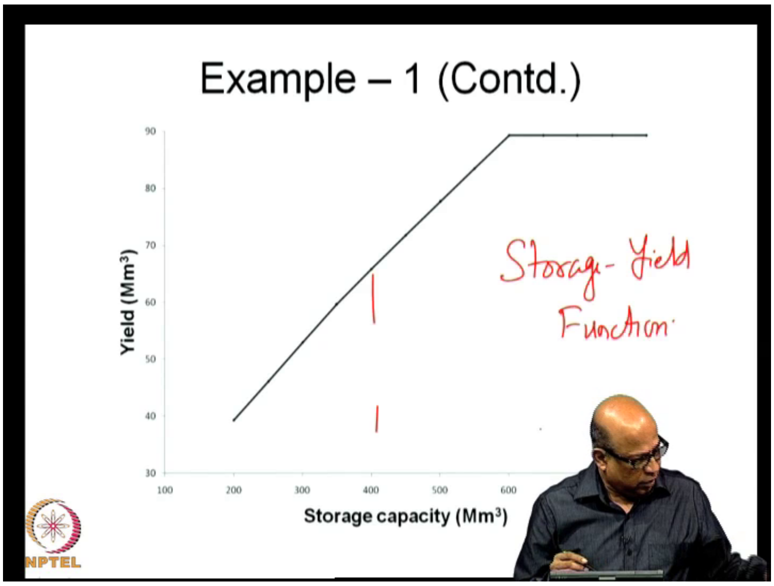


So, we generate a storage yield function. And, look at the pictorial representation of this. So, this is the storage capacity and this is the yield in million cubic meters. Now, this kind of analysis will be useful when you are planning for a reservoir. That means you are asking the question, should I build a reservoir for 400 units, 500 units, 600 units and so on. What is the constant maximum release that I can maintain for a given hydrology? So, this is the storage yield function. Now, there were some subtle issues that I was mentioning when we determine the storage capacity. Specifically, the continuity equation that we dealt with, there are something… you must know or keep in mind related with the continuity equation. What does continuity equation do?

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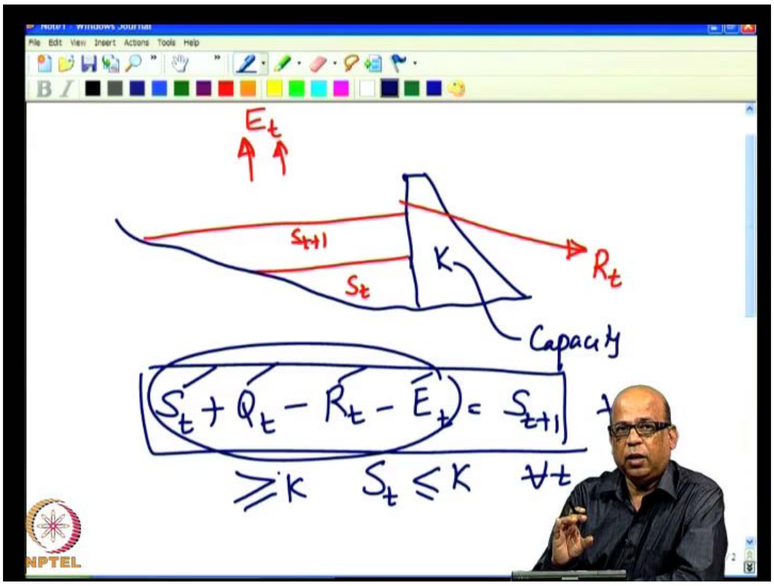

You have a reservoir and by reservoir I mean there is a storage structure. This is the dam and then upstream of that you have stored the water and this is the reservoir. Then, from time period to time period the storage is changing. Let say that, this is S t and this is S t plus 1; so, like this the storage is changing, and then you are taking out a release R t. This may be constant or changing from time period to time period.

There is a evaporation that is taking place, that is e t and the capacity of the reservoir is known, this is K. K is the capacity. Now, this is what we wrote as a continuity equation as S t plus Q t minus R t minus e t should be equal to S t plus 1. So, this is the storage continuity that you have been writing. And, this we write it for all t. And, because the storage at any point should not be greater than the capacity, we introduce the constraint S t is less than or equal to K for all t. This simply states that the storage cannot be greater than the capacity. Now, how does the program know, how does the linear programming problem know that it has to accommodate for the additional water?

Let us say that your Q t is so large and your storage initial storage plus Q t is such that, this term here is greater than or equal to K; which means that the end of the periods storage that you are getting by accounting for S t plus Q t minus R t minus e t, this term is greater than or equal to K. Then, you are saying that S t plus 1 must be less than or equal to K because you are writing this for all t. Then, what happens to the difference S t plus 1 minus K, that is S t plus 1? So, obtain minus the reservoir capacity that has to be accommodated as spill.

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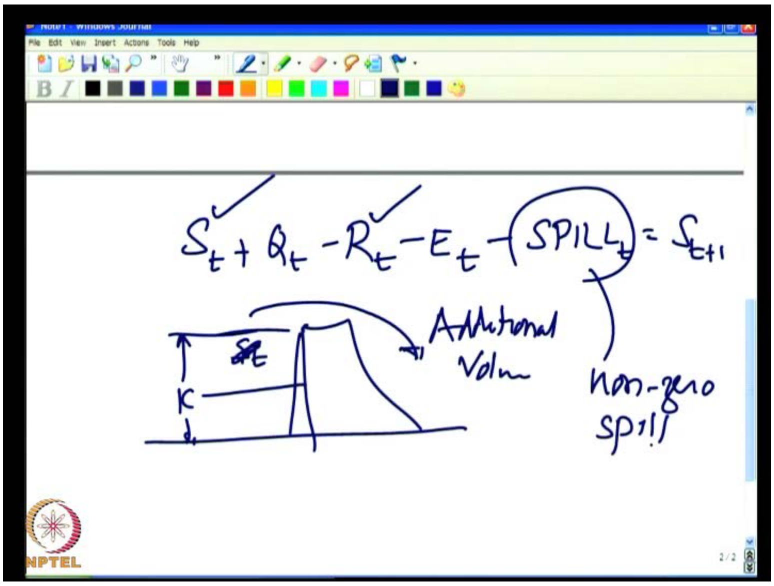


So, we may include then the term spill. So, I may write here, S t plus Q t minus R t minus e t. I may say minus spill. Let us say spill is the overflow, t is equal to S t plus one. So, what is the difference between what I did here and what we are writing here? In this case, if you had specified R t, let say that a specific demand has to be met from the reservoir and therefore you could have specified R t as R t equal to d t. Then, it would have been extremely restricted you. And therefore, you would end up with infeasibilities because you are specifying this and then saying that your S t should be always less than or equal to K. And, if your Q is very large in certain periods, there is no way the optimization can account for the additional water. And therefore, you put spill t as another term. So that, the optimization problem can account for the additional water through spill t.

Now, you understand what will happen the moment we put spill t. Our concept of including the spill t was that as soon as the storage exceeds the capacity, this is the capacity in terms of the volume; the additional volume should go as spill, only the additional volume. But, if you write the continuity like this you have the decision variables R t, you have the decision variables S t, or if you specify R t, your decision variable will be S t and K. But, in the yield function, we are talking about decision variable as R t for a given K. How does the program know, how does the optimization know that is, this spill t should occur only when we reach this capacity and not before that. If you do not specify additional constraints what will happen is even when your storage is here, you may get a non-zero spill.

So, you must remember the subtlety of modeling, where you have to specify to the program in a mathematical constraints sense that the spill t that we are introducing now, should occur only when the reservoir storage reaches the capacity. So, this is what you keep in mind. In the last lecture, I introduced one penalty function for the objective function in the objective function; so that, the spills whenever they are occurring will be penalized. So, that was one way of doing it. We were talking about minimization of storage in that case.

So, let us see what we do in the maximization of R. That means in the storage yield function when we are doing it, how do we account for the spill? So, this is the problem. Remember whenever you are putting it in mathematical forms; you must remember that the spills can occur because of the way we write the constraints, the spills can occur even when the storage has not reached the capacity. And, you should be alert to such situation. When you look at the output or when you look at the results of the model, you must specifically see whether the spills are occurring, only when the storage has reached the maximum storage or the capacity. So, this is the problem that we will now address. That is, how do we account for the spills in the storage continuity equation? There is a problem. And then, there is some very nice way of doing it, very elegant way of doing it. This is by using the integer variables.

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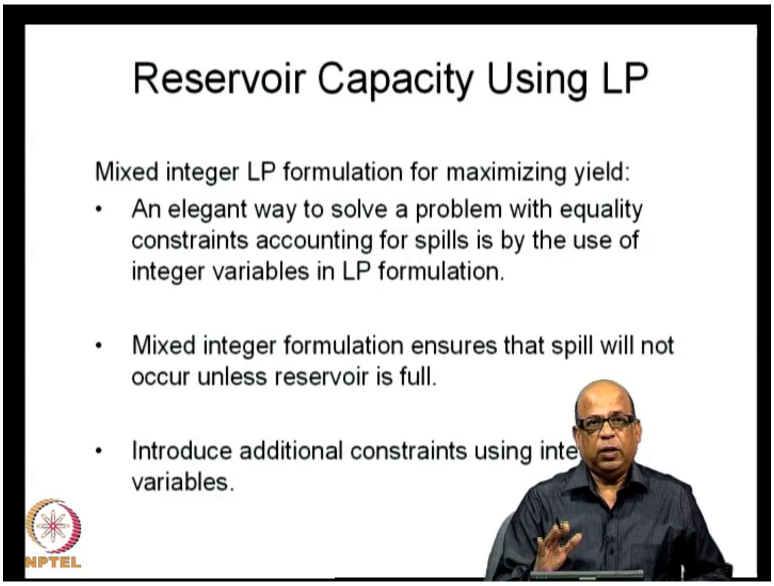


So far, whatever linear programming problems that we dealt with all the variables that we considered were real variables, floating points as floating variables, as we call it in programming languages. But, we now introduce an integer variable in to the linear programming problem. The integer variables can only take on integer values like 0, 1, 2, 3, etcetera. Further, we introduce what are called as the binary integers. That means they can take on values only between 0 and 1. So, we introduce these variables in to the problem now to account for the equality constraints, specifically the storage continuity equation and accounting for the spill and to ensure that the spill will occur, only when the reservoir has reached its capacity. So, this is the problem that will do. And, this is what we do in what is called as a mixed integer formulation. That means some variables we introduce as integers, and all other variables would be real variables. So, this is what we call as mixed integer problem. And, we formulate this to ensure that the spill will not occur unless the reservoir is full. So, typically what we do is, we introduce some additional constraints to ensure that the spill will occur only when the reservoir has come to its capacity. This is very nice and interesting way of doing it.

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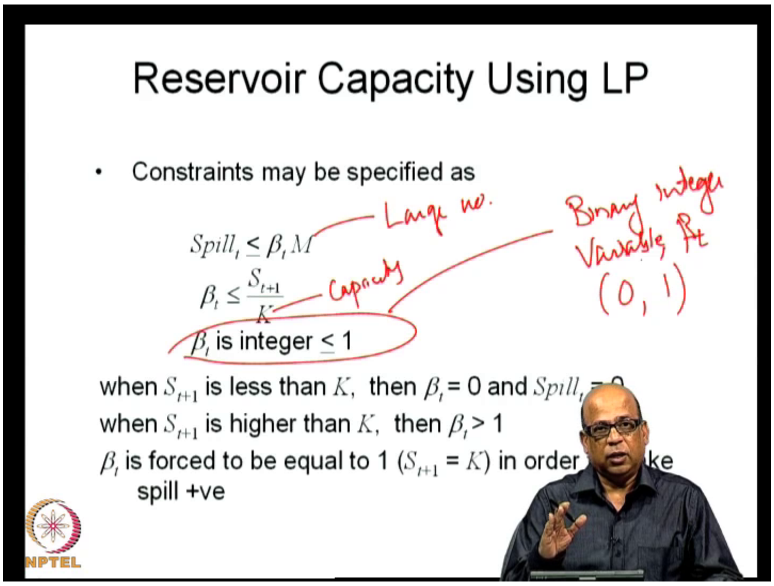


Just let us go through this. So, what will we do is that, we will introduce beta as an integer. This is a binary integer variable beta t. Now, there is a way of specifying it in any linear programming problems. Specific variables can take on only integer variables. And, further we can also specify that it takes on only two values 0 or 1. Now in this particular case, we will do it explicitly. So, we say that beta t is an integer variable and it is less than or equal to 1, how many integers are below 1 is because beta t also has to be non-negative because we are using it in a linear programming problem.

So, there are only two variables, two values possible for beta t namely 0 and 1. These are the only two variable values that are possible for beta t because we are saying, it has to be an integer variable. Now, which means that beta t can take on only values between values of 0 or 1. Now, you look at this constraint now. We will say beta t must be less than or equal to S t plus 1 divided by K. This is the capacity. And, S t plus 1 is the storage at the end of the period t or at the beginning of period t plus 1, which is obtained from the continuity equation.

If S t plus 1 is less than K, what happens? Then, you are saying that, if S t plus 1 is less than K, this value will be less than 1. And then, beta t can only be 0 because it can only take values of 0 or 1. So, beta t will be 0, whenever S t plus 1 is less than K. If S t plus 1 is greater than K, from the continuity you obtained S t plus 1 and that is greater than K capacity, then what happens? This value will be more than 1. And therefore, because of this constraint beta t can be only 1. So, beta t can take on a value of 1, whenever S t plus 1 becomes more than K. And, it will take on value of 0, whenever there is S t plus 1 is less than K. Now further, what we will do is, we want to also penalize spill t. That means whenever there is a beta t, beta t of 1, we will say that spill t must be less than or equal to beta t into a large value. Just to make sure that the spill t is just limited to meeting that continuity equation.

So, we penalize the spill t by saying that, it should be less than or equal to beta t into some larger amount of value, that is, large number. So, the beta t is forced equal to 1, in order to make the spill positive whenever there is a spill that is occurring. So, the spill will occur only when S t plus 1 is greater than or equal to K. So, this is how, we introduced the binary integer variable beta t to make sure that you are accounting for the spills. And, for that the spills will have a non-zero value, only when the reservoir reaches its capacity.

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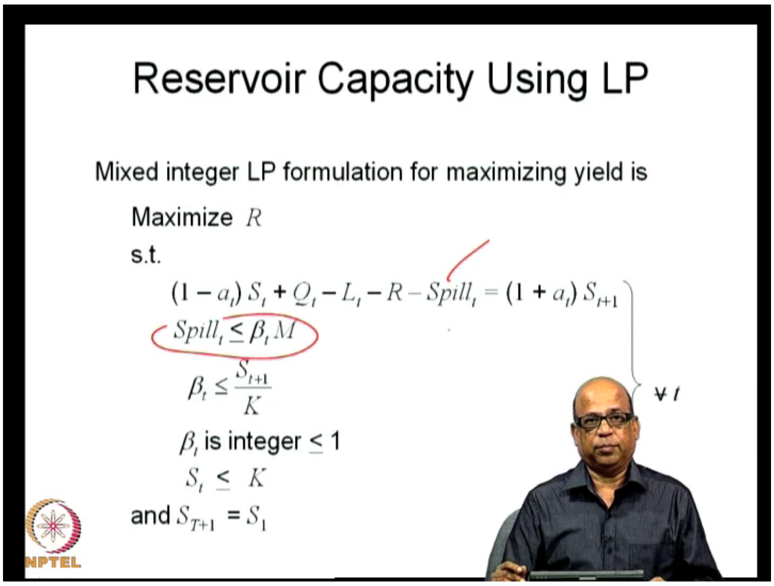


So, the formulation will look like this. You will have 1 minus a t. This was up to this point is a continuity equation that we introduce. Now, this is the additional term that we will introduce. Now, minus spill t and the right hand side remains the same. We write the spill t as less than or equal to beta t into M. To make sure that the spill t is, let us say, you have 1. Here beta t was 1, when beta t is zero spill t will be equal to 0. But, when beta t is 1, spill t will be limited to meeting this particular exercise, this particular constraint. You can examine deleting this constraint, what happens to the spill t. These are you know by trial and error, you can just make sure that by just deleting this constraint and resolving this problem, what happens to the spill t?

We are thought of fooling the problem by introducing this to make sure that, this constraint as an equality constraint is exactly met by putting, when you put beta t is equal to 1, spill t will be less than or equal to a large quantity. And therefore, you are actually building in a flexibility to make sure that you fill up the spill t exactly. And, this will ensure that the beta t will take on a value of 1, whenever the storage exceeds the capacity. And, we are saying beta t is integer and all other condition as there; where T is the last time period in the year for a monthly time. Monthly times step, T will be 12. And therefore, S 13 will be equal to S 1. So, this is how, you formulate the problem to make sure that you are accounting for the spill, over and above the storage reservoir storage.

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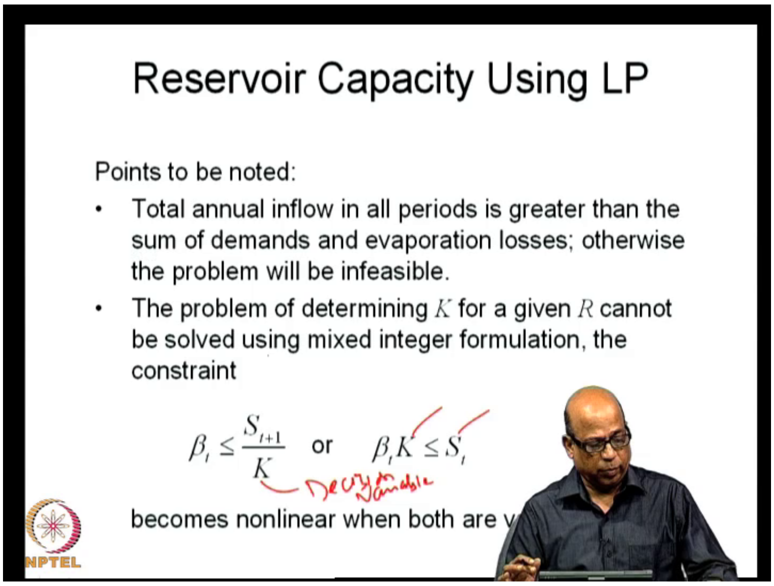


Let us do this example now. But, when you are doing this, just let us recapture what you have done in these formulations when you are talking about storage yield function. You remember this kind of formulations, they will yield a infeasible solution or the lead to infeasibility, if your total inflow over all the time periods is less than or equal to the total demand plus the evaporation. So, that is the first check that you are needed to make when you make a formulation and then run the linear programming. And, if you get infeasibility, it immediately indicates that there is some problem with your inflow and the demand plus evaporation.

If you have the inflow less than the total demand, then no matter what kind of reservoir you build, you will not able to satisfy the demand pattern. So, the infeasibility almost always indicates that your problem is, that is, the flow is inadequate to meet the particular demand. Then, the problem of determining K, let say that you wanted to use this logic that I just explained about the integer variables, let say that you wanted to use it not for the storage yield function, where you are looking for maximization of R. But, you wanted to use it for minimization of K, in which K would have been a decision variable.

If this was the decision variable, then what happens? You would have written beta t is less than or equal to S t plus 1 by K, but this leads to a non-linear constraints because beta t in to K is less than or equal to S t, is what you are arrived. And, this is the decision variable, this is and this cannot be accommodated in a linear programming problem. So, the problem of determining K for a given R, it cannot be solved using the mixed integer formulation. So, these are some certain points that you remember.

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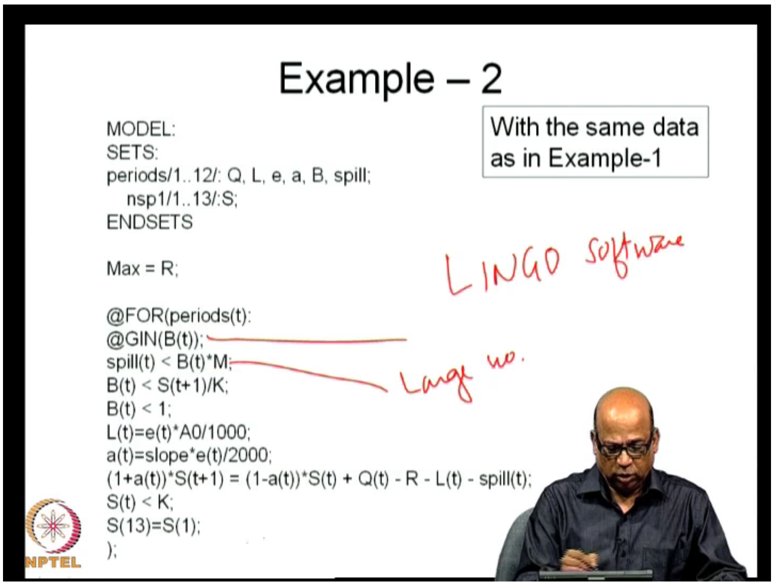


Now, we will go to this problem and solve this problem with the same data that you have used earlier. How do we do this? We use the same data first and I am using the LINGO software for this. So, this is the code for the LINGO software.

And, this is the format. First define the sets. And, that is, all these are arrays of dimensions 1 to 12. And, that set we call it as periods and then S has the dimension of 13. We are looking at maximization of R and then we define all the constraints. And, that is how it is done. Now, this is the format for integer. So, at GIN b of t, we are saying that b of t is an integer. And then, we are writing these constraints. Spill is less than or equal to beta t slash M. So, spill t is less than or equal to beta t star M. And, we define M to be a large number. And, beta t is less than S t plus 1 divided by K and beta t is less than 1. So, essentially we put all these constraints in this and the all the remaining constraints remain the same as earlier.

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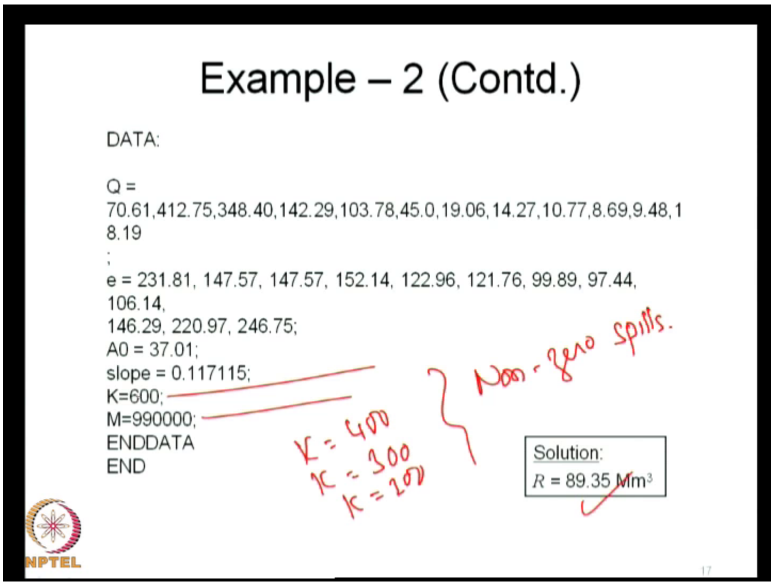


So, for given K we solve that now. And, all this is data; remember M is a data, K we are specifying as data and all other data remain the same as the previous problem. We get the solution 89.35 for storage of 600, which is the same as what we got earlier. What did we do? Here, we have used spill t and we have used the actual equality constraint. Now, in the detailed solution which I did not provide here, in the detailed solution you will see that for this particular combination of the storage being at 600 and the inflow being of this type, there were no periods in which spills occur for this. However as you reduce this capacity, let us say that K is equal to 400, K is equal to 300, 200, etcetera, you will get non-zero spills. For this you did not get any spill, which means beta t in the solution had a value of zero always.

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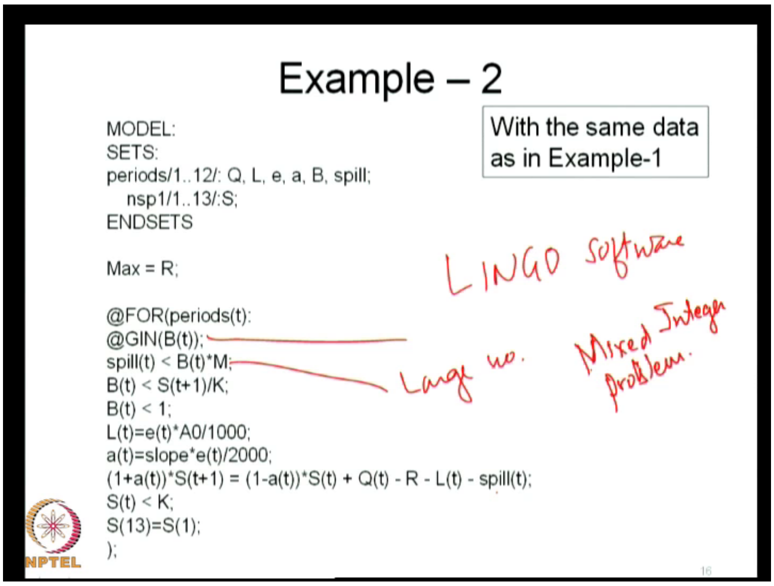


However, this is the way in which you introduce integer variables. And, this is called as a mixed integer problem. Why mixed? It is because there are some variables, which are integers; some variables which are not integers. So, that completes the topic on deciding on the reservoir capacity, where we are looking for the minimum required capacity for meeting a certain demand pattern with the inflow sequence is pre specified for a given rate of evaporation losses that are taking place.

So, that was the problem that we dealt with first reservoir capacity determination. Then, subsequently we talked about storage yield functions, where we are interested in obtaining the maximum constant release that can be made from a reservoir of known capacity. So, initially we determined the capacity. Capacity was not known. Then, we talked about storage yield function, where the capacity was known for a given sequence of inflows. You are talking about maintaining a constant maximum release from the reservoir. And, that is the yield. Then, we generated storage yield functions.

As you can appreciate now, both these are planning exercises. Where we are thinking about building a reservoir and then we are asking what kind of capacity that you need to build. Even the storage yield function, you are saying that if I build reservoir of 200 units, then what is the type of demands that I can meet? Constant demands that I can meet? If I make it 400, what is the constant demand that I can meet? So, the storage yield function is also typically used for planning purposes.

Now, we go one step further and then start looking at the operational problem, reservoir operation. And, in water resource systems, this is the most important problem that we address. So, we will now start talking about the reservoir operation problem. I will first introduce what is the issue that is of interest now. What is the problem that will be talking about?

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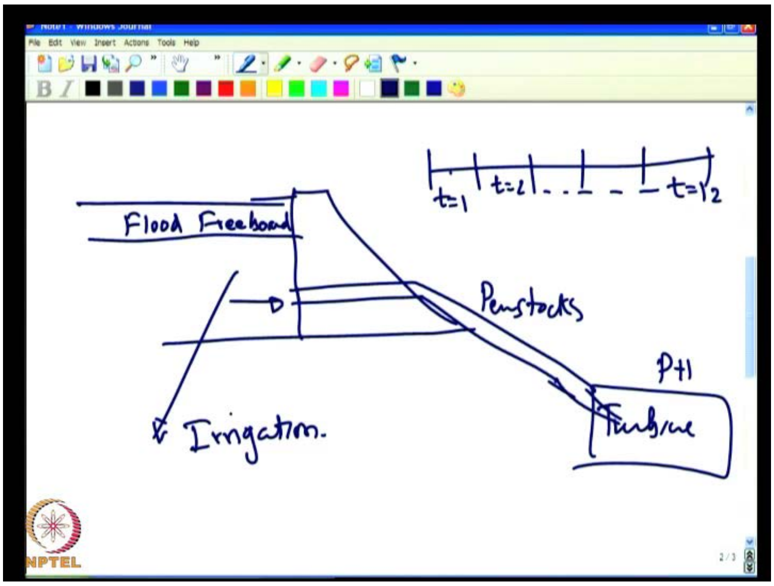


Let say that, you have a reservoir of known capacity and then you are operating this reservoir. The reservoir is typically operated through its gate. Let us say, you are using the water for irrigation purposes, you operate the reservoir through its gate. Or, if you are operating the reservoir for hydropower, then you will have penstocks and then you will have a power house. And, you want to make the releases through the penstocks. And typically, you will have a turbine here and the power house. Then, you may also have flood control storage, flood free board and then you may also have water for irrigation. And, let say that your operational time periods R, let say monthly time periods; t is equal to 1, t is equal to 2, etcetera, t is equal to 12.

So, monthly time periods, you want to operate such a reservoir. What is the operational problem? You have to specify a priory. For a given storage during a certain time period t, how much release should be made from the reservoir for irrigation, for hydro power or for any other purpose… let say municipal and irrigation releases or how much storage to be maintained during the flood season in the reservoir to accommodate the flood volumes? So, this is the operational problem. That means, you are saying at the beginning of time period t, which may be a month at the beginning of let say August month, what should be my storage level or what should be the release given the storage level? So, this is the problem that we are talking about. So, the sequence of releases to be maintained in a year or the storages that needs to be maintained across time periods in a year, as a policy of operation. So, this is what we define as reservoir operating policy.

If your storage is so much during a particular time period, how much you have to release during that particular time period? Is what, you specify through a reservoir operating policy. So, one is we specify the reservoir operating policy; another is the actual real time operations. So, distinguish these two. One is defining a reservoir operating policy, which will maintain to the best extent possible as you go from time period to time period, another is the actual reservoir operating policy which will depend on, which will also use the forecast for the inflows and so on.

So, that is the different ball game, altogether will come to that subsequently. But, right now will focus on defining a sequence of releases to be maintained from the reservoir depending on the state of the system at those time periods. Now, the state of the system can be the storage at that particular point. Let say, you may specify the release as a function of the storage at that particular time. Or, you may specify the release as a function of both the storage as well as the inflow during that particular time period. So, essentially we are talking about operating policy for the reservoir and these are called as the reservoir operating policies.

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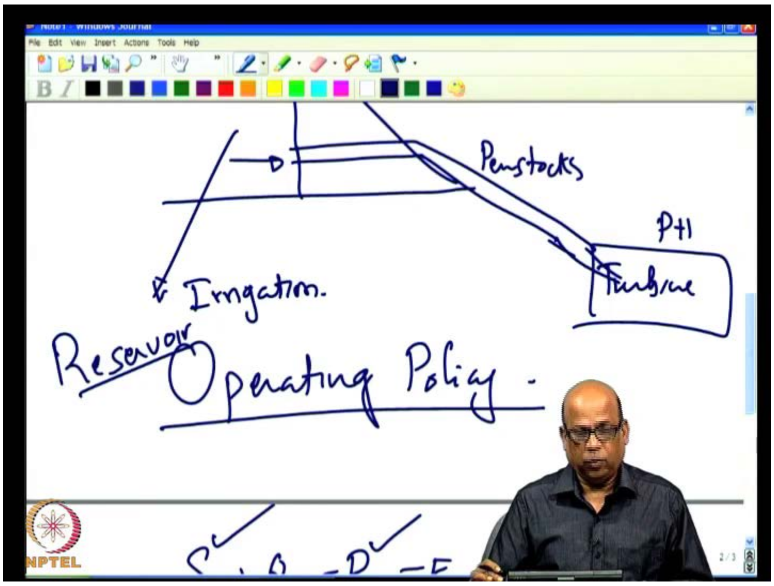

So, water resource systems deals with the significant problem in water resource system, is one half determining reservoir operating policies. We will start with the most commonly used reservoir operating policy, which is the standard operating policy; so called, standard operating policy.

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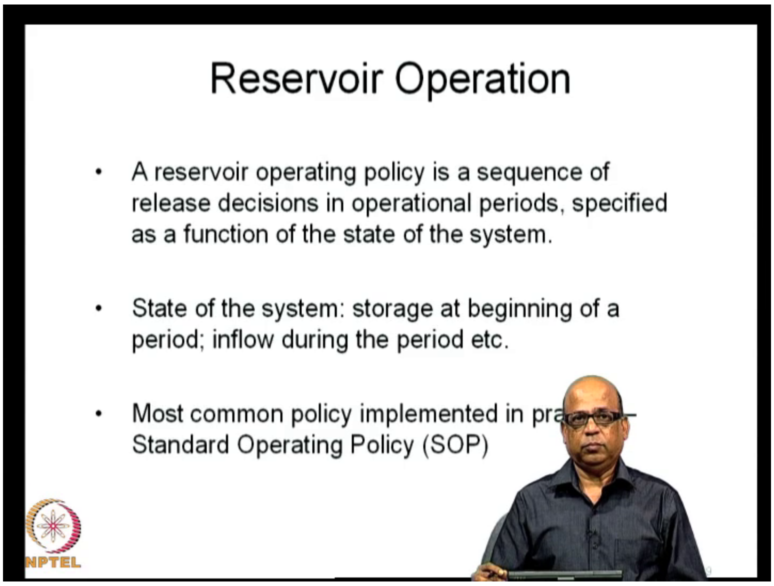


So, we will formally define now, what we mean by reservoir operating policy. Reservoir operating policy is the sequence of release decisions in operational periods. Now, these operational periods can be, depending on that purpose for which you operate the system. It can be ten day periods for example, in irrigation systems. You may operate typically for ten day time periods in flood time periods. You may operate it on daily basis, perhaps on hourly basis. So, you may want to develop your real time operating policies on hourly basis or in large planning purposes, you may operate it for monthly sequences. Typically, we derive the reservoir operating policies on monthly basis or sometimes you may want to just examine the seasonal policies. When you are looking, when you are interested in the overall performance of the system, you may talk about seasonal operating policies; that means in the season in the kharif season, rabi season or in the dry season, wet season, what kind of storage is needed to be maintained and so on.

So, the operational periods will depend on the specific purpose for which the water resource system is being operated. And, as I said, you may want to specify the release sequences as a function of the state of the system. And, the state of the system typically can be defined completely by the storage available in the system, available in the reservoir at the beginning of the time period or both the storage as well as the inflow. Or, if you want to include more details; storage, inflow and the soil moisture in the command area; storage, inflow and the rainfall in the command area and so on.

So, you can build in as much sophistication as you desire in to this model. And then, specify the release sequences. How do we operate the reservoir for a given state of the system? And, in the absence of any of these policies derived from systems techniques, typically what most of the reservoir systems adopt intuitively is the so called standard operating policy. In one of the examples in the previous lectures, I discussed about the standard operating policy. We will cover it in more detail now because we are now starting with the reservoir operation problems. It is like meeting the demands to the best extent possible in each and every time period. That is a standard operating policy. That means we do not look beyond, we simply look at this current time period, look at the storage available accommodate or account for the inflow that is likely to come during this time periods. So, that will give you S plus Q. There is storage plus the inflow, which defines the total amount of water available during that time period. If the total amount of water that is available is less than the demand during that time period, then you empty the reservoir completely, release everything that is available to meet the demands to the best extent possible. Not to the full extent because it is not possible to meet the full extent possible.

So, you release the entire amount that is available. Bring down the reservoir level to zero. You are talking only about the active storage. Bring down the reservoir level to zero. If the total amount of water that is available is more than the demand during that time period, then you release up to demand, store the remaining amount of water. Like this, you keep on building the storage whenever you have excess water and empty the reservoir whenever you have deficit amount of water. This is what, is essentially the standard operating policy.

And, when you have excess amount of water over and above the reservoir capacity you allow for the spill. That is all there is to the standard operating policy. We will discuss a standard operating policy in some detail because to understand the reservoir operating policies, the optimal reservoir operating policies, which will be interested in. First, we must understand the standard operating policy, which is intuitively practiced by master of the reservoirs especially in our country.

So, in today’s lecture we discussed essentially the storage yield functions, where we are looking at the maximum constant release that can be maintained from a given reservoir with known capacity. And, we developed the storage yield functions by solving the linear programming problem for various specified storages.

As you increase the storage, typically the yield will keep on increasing up to certain point. And, beyond that, the yield will remain constant because of the inflow limitation. And then, we looked at some subtlety or nuances of the storage continuity equation, where you want to include the spill explicitly and you want to maintain the continuity equation in an equality form, then how we use the integer variables and, formulated the problem as a mixed integer problem. And then, we solved for the maximization of yield. Remember that problem formulation, we cannot use it for minimize storage; when the storage itself, the storage capacity itself is the decision variable because that leads to non-negativity. Sorry. And, it leads to non-linearity. And therefore, we cannot use that particular constraint in that form.

So, towards the end, I introduced the problem of reservoir operation and specifically the standard operating policy. So, reservoir operation policy, reservoir operating policy is the policy of maintaining a sequence of releases as a function of the state of the system. And, typically the state of the system is defined by the storage that is available. And, in many cases the storage that is available plus the inflow that is coming. So, S plus Q, depending on S plus Q we define the release sequences to be maintained in a year. So, this is a reservoir operating problem. We will continue this discussion in the next lecture. Thank you for your attention.