Water Resources Systems

Prof. P. P. Mujumdar

Department Of Civil Engineering

Indian Institute of Science, Bangalore

Lecture No # 21

Reservoir Capacity using Linear Programming (1)

Good morning, and welcome to this the lecture number 21, of the course, Water Resources Systems - Modeling Techniques and Analysis. Now, we are talking about problems dealing with reservoir sizing. So, in the last lecture, if you recall, what we did is, we introduced the concept of the mass diagram and the ripple diagram that conventionally is used for determining the reservoir size. The problem there is that for a given sequence of flows and for a specified demand, what is the minimum reservoir capacity that is necessary.

So, this is the problem that we are dealing with, what is the minimum capacity of the reservoir that is necessary to meet the given sequence of demands for the given sequence of inflows. Now, classically this used to be done by the mass diagram or the ripple diagram, but the ripple diagram or the mass diagram it will be extremely difficult to get, accommodate the time varying demands and also to accommodate the storage dependent losses, and then we introduce the sequent peak algorithm.

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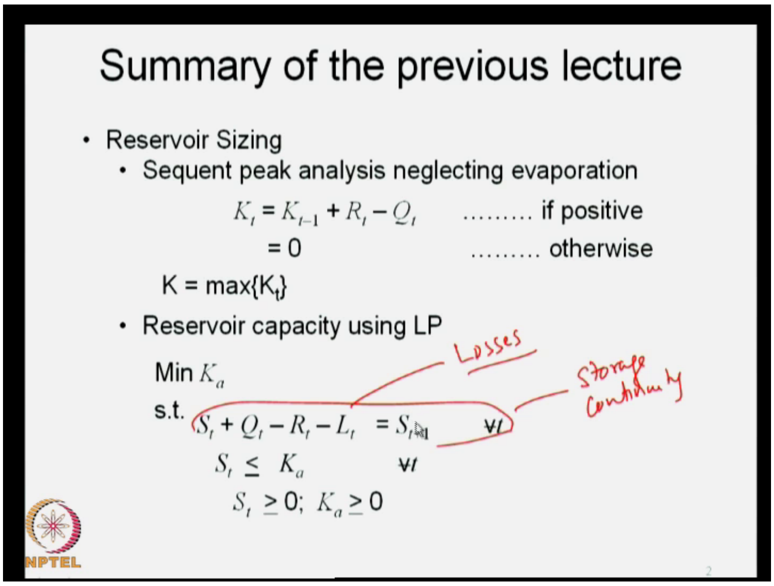

So, if you recall the sequent peak algorithm, we write it as K t is equal to K t minus 1 plus R t minus Q t; R t minus Q t in any given time period t. In fact, indicates the deficit that occurs during that time period, Q t is the inflow, R t is the demand or the release the from the reservoir, which we set it to equal to demand.

Therefore, R t minus Q t is the deficit, we are computing the deficits in a cumulative way, whatever was the deficit or the storage that was necessary during the previous time period, we add into this particular deficit in this t period, and then calculate in some sense, the cumulative deficit during a critical time, and K t minus K t becomes 0; if it is negative, which means that your K t minus 1 plus R t minus Q t; if it is negative, then it we set it as 0. Like, this we compute several all the K t s during all the time periods, and we pick up the maximum of such K t s and that is, what is the reservoir capacity? Recall that we do these computations typically for two cycles, just make sure that we do not miss the critical period; that is critical length of periods, time periods.

Especially, if the critical period is the occurring towards the end of the sequence, end of the inflow sequence, we carried out to carry the computations to the next cycle. So, typically, we carry it for a maximum of two cycles; in the two cycles will get the critical period. And therefore, we pick up the maximum of this K t s and this we call it as though minimum require capacity.

A minimum requires storage capacity, to meet the particular demands R t for a given sequence Q t. Again, here if you want to include the storage dependent losses, then it becomes slightly unyielding or slightly compare some and complicated; however, as I mention, there is an algorithm available developed by (( )) 1987; or something around that time, which have also incorporate the storage dependent losses in the sequent peak analysis. However, once we know the optimization algorithms, these types of problems can be very easily, readily, formulated as optimization problems.

So, let us look at the reservoir operation capacity determination using LP, this we covered in the previous class. What we essentially do is, that we write the storage continuity equation, S t is the storage at the beginning of the time period; Q t is the inflow during the time period; R t is the release that is taken out of the reservoir; minus L t is the losses, total losses during that time period; and this we write it us equal to the storage at the end of the time periods. So, this is just just the storage continuity, this constraint is storage continuity.

And then, we apply the constraint S t is less than or equal to K; to indicate that the storage must be less than or equal to the capacity; and of course, the non negativity conditions. And then, we minimize K, which means we are looking at the minimum reservoir storage, set of which will satisfy these demands for this given inflow patterns, with losses specify. Now these losses can be average losses during the time period and so on. So, this is how we determine the reservoir capacity, and we also solve that we get the same capacity as the sequent peak algorithm for a given demands and inflows.

Now, we will look at the losses, we said in this formulation; that the losses must be specified per period; that means, let say that we are talking about the volume units here, storage within volume, inflow volume, realization volume, and therefore, the losses was also in volume, typically in million cubic meters is what we talk about.

So, if you specify average losses, let say that you are talking about 12 time periods, monthly time periods, and then you may specify average loss during June month is so much; average loss during July month is so much, and so on. Based on your historical data, you would have estimated this.

However, notice that L t, which is the loss and specifically the evaporation loss, which is much more significant compute to any other losses such as (( )) loss and so on. So, we need to account for the losses as storage dependent losses, because as the storage changes, I mention this in the last class, but let us understand this correctly.

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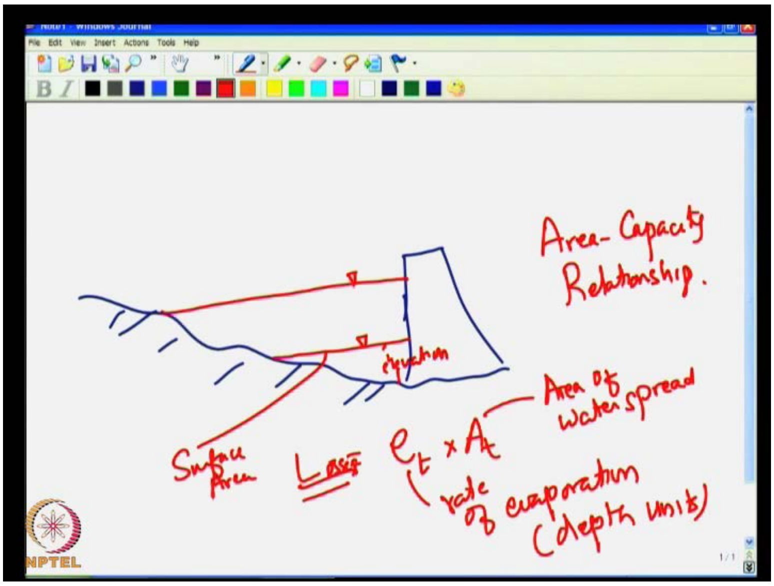

As the storage changes, the surface area changes, typically will have the reservoir like this, and then the upper upstream counter levels would be something like this, and then as the storage changes, let say that your S t was somewhere here, and then this will be the surface area.

So, the surface area; that is the water surface area here will depend on the contours upstream of the dam, and also on the storage level that is the elevation. So, typically you will have water called as the storage; that is area capacity relationships for any given reservoir, you have Area - Capacity Relationship, so from this storage, if you go to any other storage.

Let say, will go to this point, then this becomes the area in the third dimension here, which means the area will depend on the storage and the storage and elevation are dependent. So, as your elevation changes, your area changes, and as the area changes your losses will change.

So, typically we will have e t into A, the losses will be equal to e t into A, where e t is the rate of evaporation, this will be typically in depth units, and area is in area units, and this is area of water spread, and this area of water spread is related to storage, and typically we will be changing from time period to time period, because your area is changing, your storage is changing. And therefore, the area will also change.

And therefore, we need to account for the losses as storage dependent losses, because the storage is continuously changing by the area changes and therefore, the evaporation losses change. And therefore, we need to account for the losses as storage dependent loss. Let us see, how we do that.

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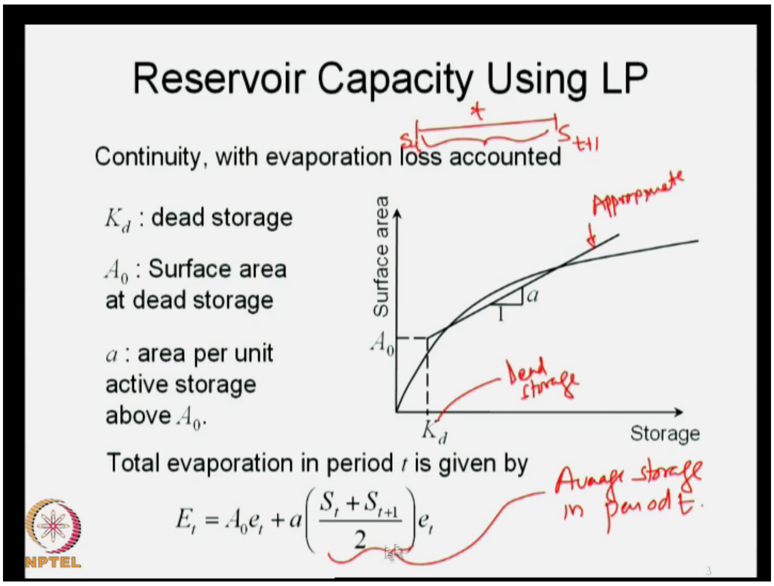

So, we want to now account for the evaporation, and rewrite the continuity equation.

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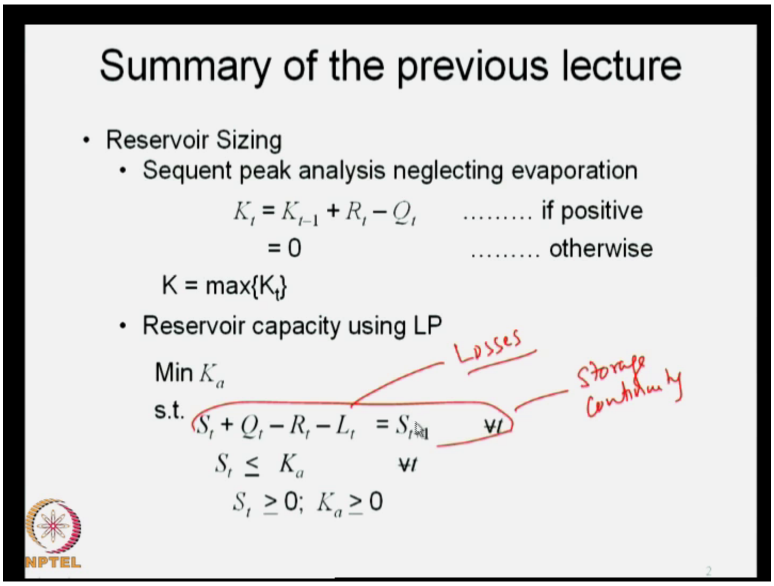

This was the continuity equation, S t plus Q t minus R t minus losses, where the losses were lumped, and then consider we considered average losses, but because the evaporation losses can be quite significant from time period to time period, and they are dependent on the storage is S t and S t plus 1. Specifically, in our country, like ours tropical weather, the evaporation losses have to be in accounted for accurately, because they can make a significant difference, as I will presently show through an example, in the capacity determination.

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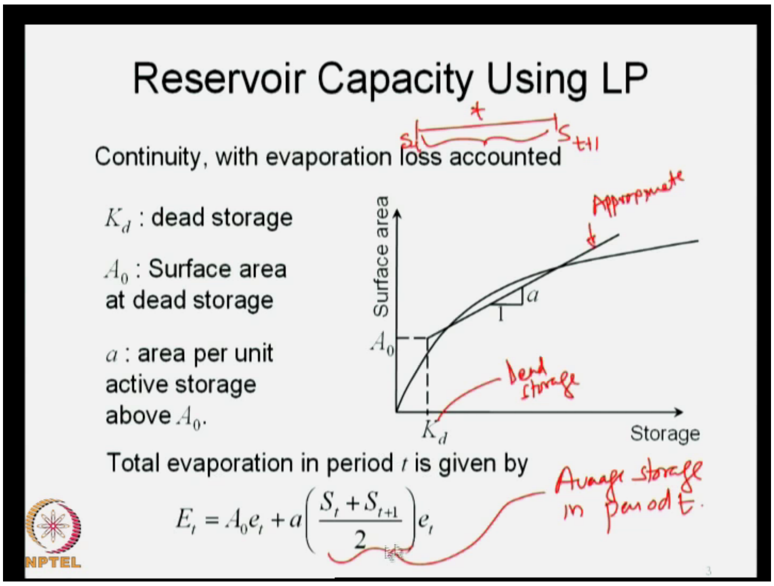

So, let us look at, how we account for evaporation as storage dependent losses? Now this is the area capacity relationship, this is storage on the x axis, surface area on the y axis, this is water surface area or water spread area, typically we will have this curve like this.

Now this is the area capacity curve for using linear programming, what we do is beyond the dead storage level? This is the dead storage, and we approximate the storage capacity relationship with a straight line. So, this is approximate straight line and then look at the slope of this, this slope is a is to 1. So, this is the slope of the area capacity relationship beyond the dead storage. So, this is the dead storage and beyond the dead storage, we fit a straight line and then call this as a is to 1. So, the slope is a is to 1.

Now look at, what is happening, corresponding to A naught, which is the dead storage there is a constant rate of evaporation. So, that will write it as A naught into e t, which means, now the total storage, we are reckoning as consisting of two parts, one is corresponding to the dead storage, and another is beyond the dead storage.

So, corresponding to do the dead storage, we have an area of water spread of A naught and that will give me a constant evaporation of A naught into e t, where e t is the rate of evaporation in depth units, typically will talk about millimeters and converted into meters, because this would been millions square meters or square meters etcetera. It is the area unit. Then, what is happening beyond this time, this part you just look at this, we want to get the area corresponding to an average storage S t plus S t plus 1 divided by 2. So, this is the average storage storage in period t.

That is you started with S t, just look at this, this is the period t, your initial storage was S t; you ended up with the storage of S t plus 1. So, in this time period, the average storage is S t plus S t plus 1 divided by 2; corresponding to this S t plus S t plus 1 divided by 2, what is the area? It will be a, which is the slope multiplied by that storage. So, a multiply by the storage here will give the surface area at any given point. So, S t plus S t plus 1 divided by 2 is storage here multiplied by that slope, you will get the surface area. So, a into S t plus S t plus 1 divided by 2 is the actual surface area, corresponding to the average storage in period t; that we multiply by e t, which is the rate of evaporation to get the total evaporation loss E t. So, this is the total evaporation loss.

The first term corresponds to the evaporation loss associated with the dead storage, which corresponds to an A naught, area of A naught. The second term here corresponds to the average active storage or average live storage namely S t plus S t plus 1 divided by 2; and the area corresponding to that is a into S t plus S t plus 1 divided by 2 into the rate of evaporation; and therefore, this is the additional loss, evaporation loss.

So, this is what you get in volume units, remember that; this small a is that I am showing here is area per unit active storage above A naught. That means, A naught is the area corresponding to the dead storage and from this point onwards we have put, we have fit a straight line for the area capacity relationship.

And therefore a, which is the slope of the line is area per unit has units of area per unit active storage, which means meter square per meter cube or something like that. Now, we use this, now in our storage continuity equation, so what we did essentially here is?

That we express the evaporation loss as a function of the storage, this we will use now in the continuity equation.

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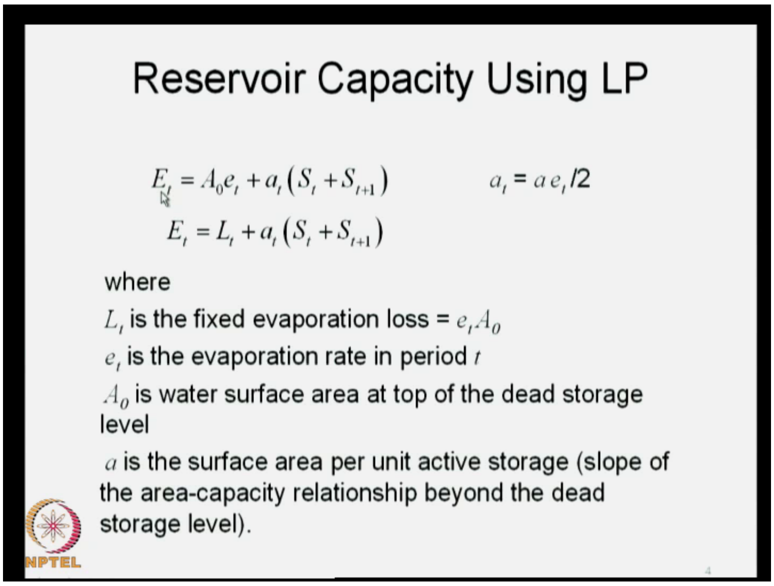

So, we will write this as E t is equal to A naught e t; and then, I will introduce for a into e t divided by 2; a e t by 2, I call it as a t some constant a t, which in this from time period to time period, because e t changes from time period to time period. So, I will write it as a t is equal to a into e t by 2; and therefore, I write E t as A naught e t plus a t into S t plus S t plus 1.

And A naught e t, I will write it as L t that is fixed evaporation loss, I will write it as e t into A naught and therefore, this term is L t plus, I will written this as it is, a t into S t plus S t plus 1. Now, this is the total evaporation loss in time period t, we will substitute this in the storage continuity equation.

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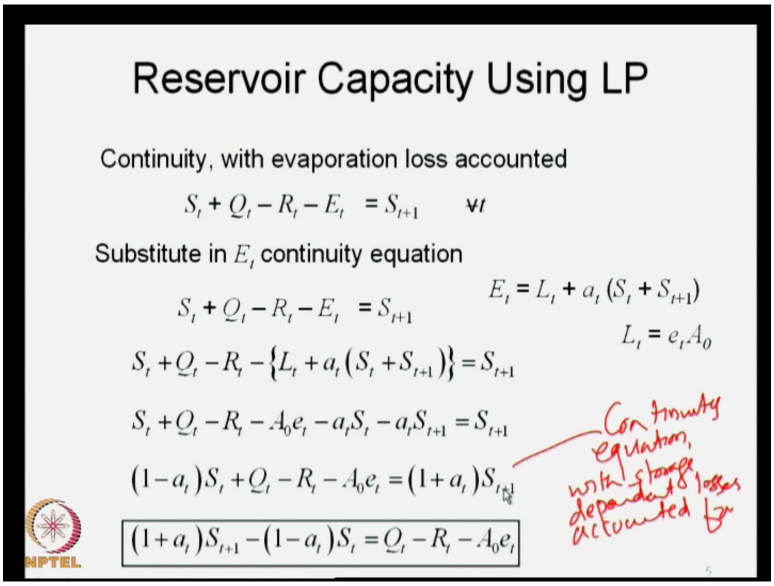

So, the storage continuity equation was S t plus Q t; that is the initial storage plus the inflow minus the release minus the total losses E t is equal to the end of the period storage S t plus 1; the evaporation loss E t we just wrote it as L t, which is e t into A naught plus a t, which has A naught e t by 2 into S t plus S t plus 1. So, this is what we substitute here.

So, in this equation, we write S t as it is, Q t as it is, R t as it is for E t, I will write it as L t plus a t into S t plus S t plus 1. And this should be equal to S t plus 1. We simplify this, and then get it as 1 minus a t into S t plus Q t minus R t minus A naught into e t is equal to 1 plus a t S t plus 1. Now, this is the continuity equation

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With losses accounted for; that is which storage dependent losses.

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Accounted for and putting all the decision variables on one side, on the left side, you can also write this as 1 plus a t into S t plus 1, 1 minus a t into S t is equal to Q t minus R t minus A naught into E t, just notice that Q t is given R t is that particular demand pattern that you want to me therefore, this is known. A naught is the area of water spread, which corresponding to the dead storage which is known; e t is the rate of evaporation that is known. So, the all the terms on the right hand side are known, S t is the decision variable and similarly, S t plus 1 becomes decision variable, a t can be determine from data and therefore, the left hand side has announce S t, right hand side everything is known.

Now, this is the set of constraint that we write for all t, now this notation, you must be aware now that we write it as for all t. So, this constraint set, we write for all t, which means that if you are if you are time periods or monthly time periods, then you will have 12 such constraints, because t will go from 1 to 12 and S 13 that is the last time step, you set into S 1, which will be also equal to the storage at the beginning of the first time period, will come to that as a we progress.

So, now onwards we will be using this form of continuity, where we are specifying the storage dependent losses. Now, in this, with this storage continuity equation then, we will reformulate our original problem, our original problem was if you recall, we wanted to find out the minimum value, minimum storage subject to the continuity and S t less than or equal to K and etcetera. So, only the storage continuity will change now, and incorporate the losses as storage dependent losses; and therefore, will replace the continuity, original continuity with this continuity now.

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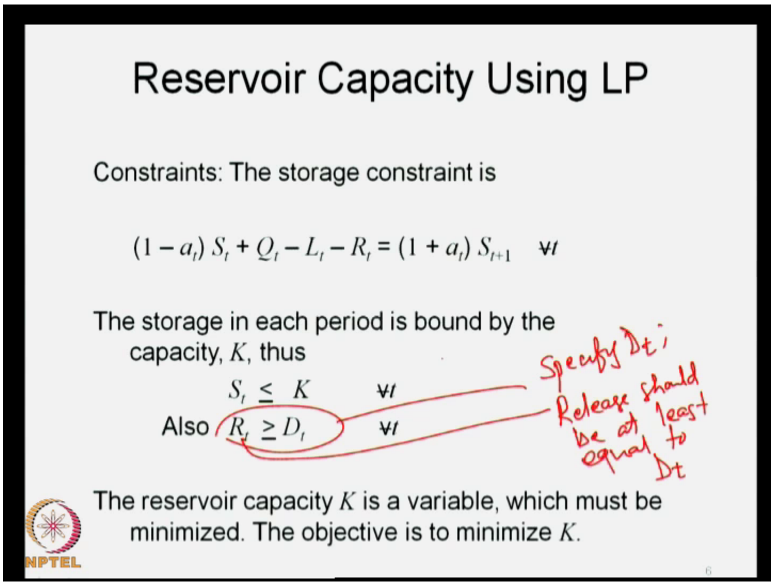

And Right therefore, the storage constraint, we write it as 1 minus a t S t plus Q t minus L t minus R t is equal to 1 plus a t into S t plus 1, where L t is defined as e t into A naught, and this figure should give you all the details, A naught is that particular surface area corresponding to the dead storage, a is the slope of the area capacity relationship approximated as the straight line beyond the dead storage, and S t plus S t plus 1 they are the storage is at the beginning and the end of the time period t. So, we write this constraint and S t is less than or equal to K and R t greater than or equal to D t. Now, this is in various forms, we use this; that is we specify D t and say that by release should be greater than D t, release should be at least equal to D t.

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So, when we have the demands specify, we can write as R t greater than or equal to D t, and sometimes we may write R t as D t itself here anyway these (( )) will come to later, but right now, you understand that we have replies the storage continuity by this particular equation expression here, which accounts for storage dependent losses.

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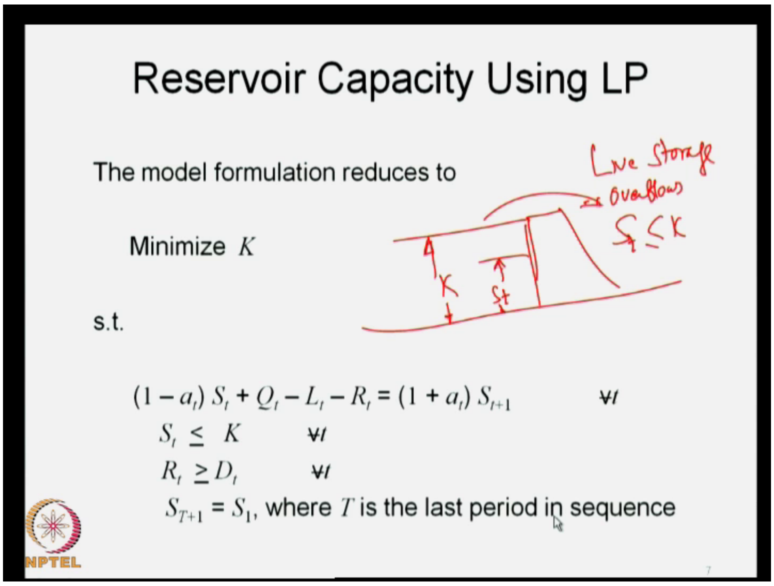

Now, let us see, how we state the problem and therefore, we state the problem as minimize K, which means we are looking at that particular storage capacity K, minimum storage capacity K, subject to the storage continuity equation, which account for the evaporation losses written for all time periods t, S t is less than or equal to K, because the storage must be less than or equal to the capacity.

Do not miss the point here; that this is the capacity K, and we are talking about live storage only; and therefore, your S t at any point, which is a actual storage must be less than or equal to K, what happens if S t goes above K the overflows occur. So, the overflows occur whenever K is less than S t, because you want to restrict S t up to this point anything beyond that will go as overflows and sometimes depending on your formulations, overflows can be absorbed into the release R t itself.

So, in this form now we are not accounted for overflows separately; R t itself accounts for the overflows, because of this constraint namely S t is less than or equal to K. You are restricting the storage to the capacity K, and anything in access will be filled in R t. So, this is the general formulation. Now, from the original formulation we have just change the storage continuity to account for the storage dependent losses; and then we will solve this. In addition, we put S T plus 1 is equal to S 1, where T is the last period in the sequence.

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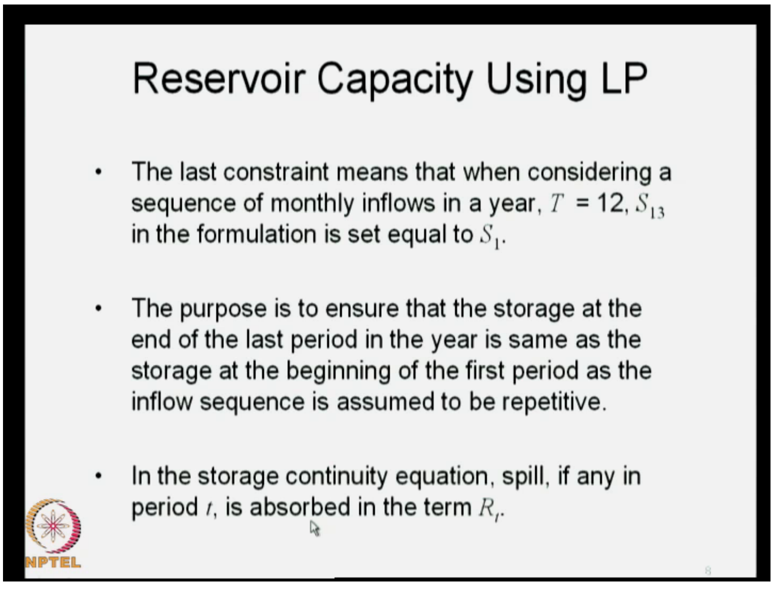

Let say that you are talking about monthly sequences, which means that you will have 12 time periods, in which case capital T is 12, T is the last time period. So, capital T is 12. And therefore, S 13 we set it as S 1, let say you are talking about 10 day time periods, which means that you will have, let say 36 time periods in a year, then S 37 will be equal to S 1. So, the last time period the storage at the end of the last time period must be equal to the storage at the beginning of the first period; that is what it in includes. Now, as I said the spill if any is absorbed in this term R t here.

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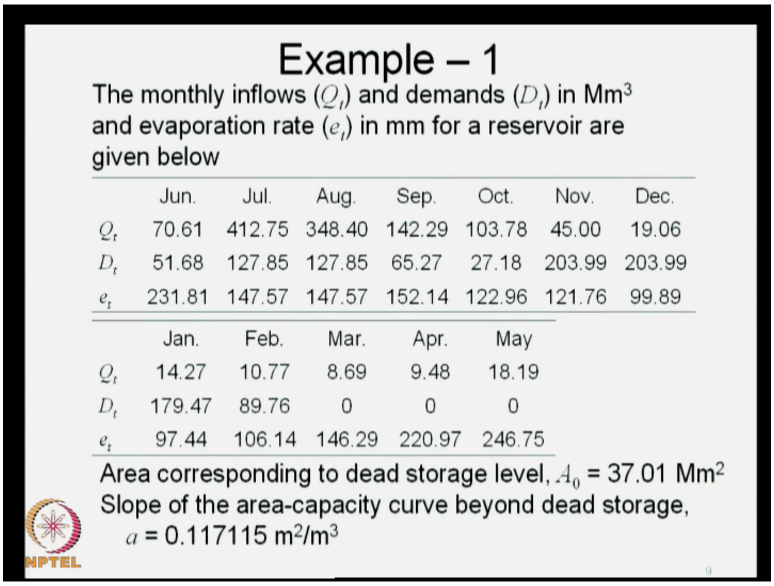

Example – 1

The monthly inflows (Q_t) and demands (D_t) in Mm^3 and evaporation rate (e_t) in mm for a reservoir are given below

	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Q_t	70.61	412.75	348.40	142.29	103.78	45.00	19.06
D_t	51.68	127.85	127.85	65.27	27.18	203.99	203.99
e_t	231.81	147.57	147.57	152.14	122.96	121.76	99.89

	Jan.	Feb.	Mar.	Apr.	May
Q_t	14.27	10.77	8.69	9.48	18.19
D_t	179.47	89.76	0	0	0
e_t	97.44	106.14	146.29	220.97	246.75

Area corresponding to dead storage level, A_0 = 37.01 Mm^2

Slope of the area-capacity curve beyond dead storage, a = 0.117115 m^2/m^3

Let us take an example now. Now, this is where specifying the monthly flows. So, the inflows are given, the demands are given D t, Q t and D t are in million cubic meters. These two are in million cubic meters, and the rate of evaporation e t is in millimeters. So, these are millimeters. So, like this from June to May all these values are given. So this is the data now, this data is typically available at any reservoir side, this is the rate of evaporation, remember this is typically major at the reservoir side by weather stations and may be by pane evaporation and so on.

So, you will have estimates of the rate of evaporation, Q t is the flow which is majored at the reservoir, and D t is the demands that you have estimated that need to be met by this reservoir; for such a combination, we need to find the minimum storage that is necessary to meet this demand demand pattern for this given inflows, when these rates of evaporation occur; that is the problem now.

We also have the area capacity relationship from which we know the area corresponding to the dead storage level, which is in our notation A naught. So, this A naught is known and also this slope a is known. So, those values are necessary. So, we know A naught as 37.01 million meter square, and slope of the area capacity curve beyond the dead storage in our notation, it is small a is 0.117115 meter square per meter cube, this is the slope therefore, it is in meter square per meter cube. So, all the data is given.

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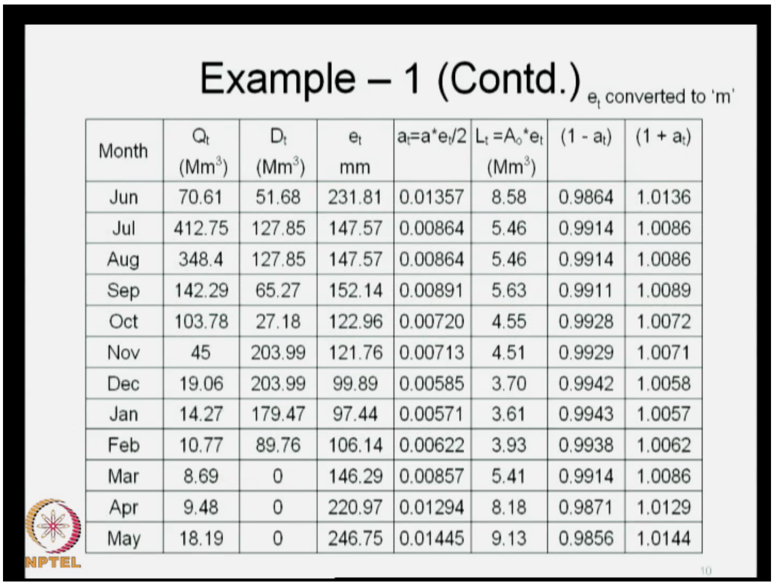

Month	Q_t (Mm^3)	D_t (Mm^3)	e_t mm	$a_t = a*e_t/2$	$L_t = A_o*e_t$ (Mm^3)	(1 - a_t)	(1 + a_t)
Jun	70.61	51.68	231.81	0.01357	8.58	0.9864	1.0136
Jul	412.75	127.85	147.57	0.00864	5.46	0.9914	1.0086
Aug	348.4	127.85	147.57	0.00864	5.46	0.9914	1.0086
Sep	142.29	65.27	152.14	0.00891	5.63	0.9911	1.0089
Oct	103.78	27.18	122.96	0.00720	4.55	0.9928	1.0072
Nov	45	203.99	121.76	0.00713	4.51	0.9929	1.0071
Dec	19.06	203.99	99.89	0.00585	3.70	0.9942	1.0058
Jan	14.27	179.47	97.44	0.00571	3.61	0.9943	1.0057
Feb	10.77	89.76	106.14	0.00622	3.93	0.9938	1.0062
Mar	8.69	0	146.29	0.00857	5.41	0.9914	1.0086
Apr	9.48	0	220.97	0.01294	8.18	0.9871	1.0129
May	18.19	0	246.75	0.01445	9.13	0.9856	1.0144

Now, we will formulate the continuity equation. To form the continuity equation what are the terms necessary, we need a t here; and therefore, we have to estimate a t.

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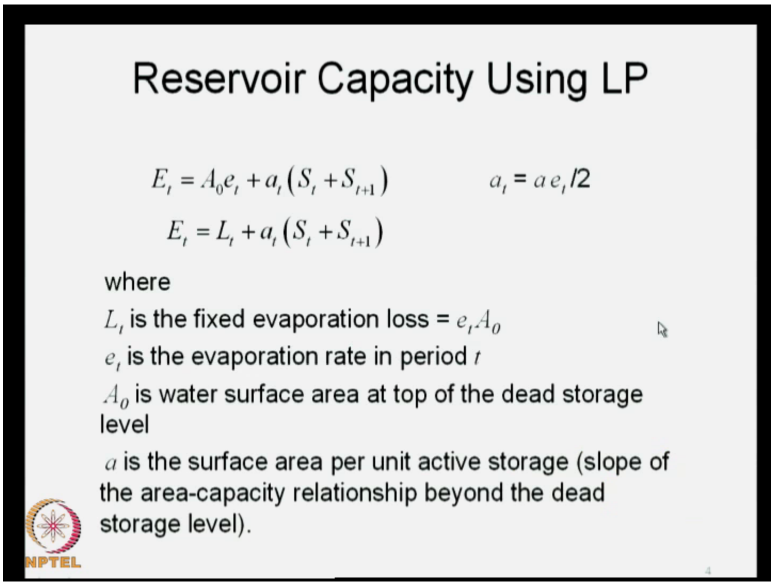

What is a t? a t we defined as, a into e t by 2, and a is the slope of a area capacity relationship beyond the dead storage, e t is the rate of evaporation therefore, you can get a t, because e t changes from one time period to time period. Although, a remains constant, a t changes from one time period to time period.

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Example – 1 (Contd.) e_t converted to 'm'

Month	Q_t (Mm^3)	D_t (Mm^3)	e_t mm	$a_t=a*e_t/2$	$L_t=A_o*e_t$ (Mm^3)	$(1 - a_t)$	$(1 + a_t)$
Jun	70.61	51.68	231.81	0.01357	8.58	0.9864	1.0136
Jul	412.75	127.85	147.57	0.00864	5.46	0.9914	1.0086
Aug	348.4	127.85	147.57	0.00864	5.46	0.9914	1.0086
Sep	142.29	65.27	152.14	0.00891	5.63	0.9911	1.0089
Oct	103.78	27.18	122.96	0.00720	4.55	0.9928	1.0072
Nov	45	203.99	121.76	0.00713	4.51	0.9929	1.0071
Dec	19.06	203.99	99.89	0.00585	3.70	0.9942	1.0058
Jan	14.27	179.47	97.44	0.00571	3.61	0.9943	1.0057
Feb	10.77	89.76	106.14	0.00622	3.93	0.9938	1.0062
Mar	8.69	0	146.29	0.00857	5.41	0.9914	1.0086
Apr	9.48	0	220.97	0.01294	8.18	0.9871	1.0129
May	18.19	0	246.75	0.01445	9.13	0.9856	1.0144

So, we will get a t, Q t is given, now this is given, this is given, this is given, this is what we compute, we convert this e t in millimeters into meters, because everything is in meters, million cubic meters and so on. So, we convert e t into meters, which means that this should be actually a t is equal to a into this e t divided by 2, divided by 1000. So, that is how we calculate a t, then you calculate L t, which is A naught into e t, you have A naught here and you have the e t here, corresponding to different time periods.

So, e t is given, so A naught into e t you get L t. This will be in million cubic meters, because A naught is in million square meters and e t is in million, e t e t is in meters convert e t into meters, and then calculate this, and then you need 1 minus a t and 1 plus a t. So, once you calculate a t, you get 1 minus a t for example, this will be 1 minus 0.01357 that will be this ,and 1 plus a t is 1.0136 and so on. So, given a t you can calculate these. So, like this for every month, June to May you have 1 minus a t and 1 plus a t and therefore, we should able to now write the continuity equation for all the time periods. Q t is given, D t is given, E t is given, and we have now calculated 1 minus a t and 1 plus a t.

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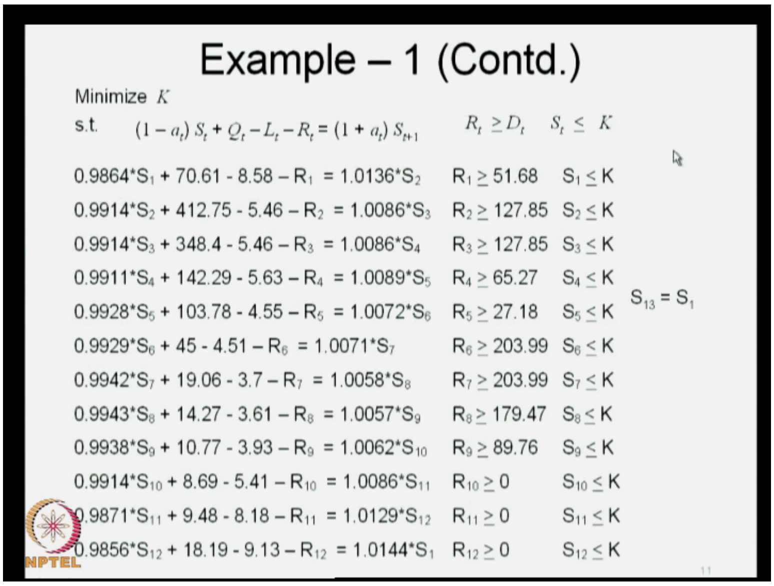

So, we will write the continuity equations, in the long form this is how it looks. So, our model is minimize K subject to 1 minus a t into S t plus Q t minus L t minus R t is equal to 1 plus a t into S t plus 1; and R t is greater than or equal to D t; and S t is less than or equal to K; and S 3 S 13 is equal to S 1, this is the additional constraint to make sure that the storage at the end of the last time period is equal to storage at the beginning of the first time period.

So, we start filling this 1 minus a t, which is 0.9864 into S 1. I am writing it for t is equal to 1 first plus Q 1 which is 70.61 minus L 1 which is 8.58 minus R 1 which is the decision variable is equal to 1 plus a t which have calculated 1.0136 into S t plus 1, which is S 2; like this, we write for all the time periods 1, 2, 3 etcetera 12, when you come to 12, this will become S 13, but S 13 we replies it as S 1 here, and that is what I am writing here, S 13 will be equal to S 1.

Then R t is greater than or equal to D t. We write it as R 1 greater than or equal to this D 1; R 2 greater than or equal to this D 2 etcetera. Like, this we write and then, this set of constraints is written like this, there are 12 sets like this, then S t is less than or equal to K S 1 to S 12 all less than or equal to K. So, this is the complete statement of the model of course, there is the non negativity, the set of non negativity constraints namely S t is greater than or equal to 0; and R t greater than or equal to 0; that is assumed in any LP problem formulation unless otherwise state here.

So, this is the linear programming problem, how many decision variables are there, S t is the decision variable, R t is the decision variable, and therefore, you have 12 decision variables corresponding to S t; and 12 decision variables corresponding to R t; how many constraint are there, there are 12 constraints here, 12 constraints here, and 12 constraints here, which means 36 constraints apart from the non negativity constraints. So, 36 constraints and 24 decision variables, we solve using any LP software this particular problem, and then look at the solution.

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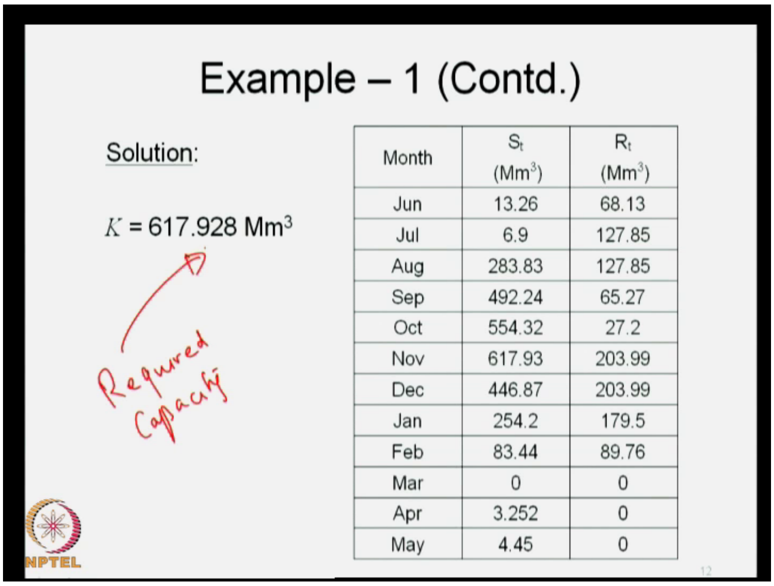

Month	S_t (Mm^3)	R_t (Mm^3)
Jun	13.26	68.13
Jul	6.9	127.85
Aug	283.83	127.85
Sep	492.24	65.27
Oct	554.32	27.2
Nov	617.93	203.99
Dec	446.87	203.99
Jan	254.2	179.5
Feb	83.44	89.76
Mar	0	0
Apr	3.252	0
May	4.45	0

So, when you solve this, you get the solution like this, S t and R t and the associated K value. So, this is the storage capacity, required capacity.

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Our objective function was this 617.93 that is minimize K. So, we are getting minimum required capacity is 617.928 million cubic meters, associated with that we also get the storages 13.26, 6.9 etcetera up to 4.45; and the associated releases 68.13 and so on. Remember that every time period, you should be able to meet, if you pick up any points here, you should be able to meet the associated storage continuity equation; for example, you pick up time period t is equal to l, starting with this plus the inflow during time period 1 minus release during the time period 1 etcetera.

If you substitute this, you should be able to get S 2 which is 6.9 and so on. You can verify this, once the solution is feasible, then all the constraints are satisfied and therefore, that conditions will be satisfied automatically. The point you have to note now is that you got storage of 617.928 million cubic meters, when you accounted for storage dependent losses. So, you have the evaporation rates, and then you accounted for storage dependent losses.

Let us see now, what happens, if the storage dependent losses are not accounted for, which means that we are saying that the losses are negligible. If the losses are negligible, then we write the storage continuity equation as S t plus Q t minus R t is equal to S t plus 1. Assuming that the losses are all neglected, then what would you expect, let say that you solve the minimization problem again; that means, the linear programming problem, where you are looking looking for the minimum required storage to meet that particular demands.

If you neglect losses would you get a higher storage or a lower storage, your inflows are the same; your demands are the same; and you are ignoring the losses, when you are accounting for the losses actually, what is happening, the flow that is coming which adds to the storage, you are taking out something, which means that some amount of water is not available for you to meet the demands.

And therefore, when you account for the losses, you will end up getting a higher storage requirement, because the flows or the amount of water available during a particular time period is smaller by the amount that has been lost into lost as losses, evaporation losses specifically. And therefore, when you ignore the losses, you will get a lower storage required, because with the lower storage, you will be able to meet the demand, because you have ignored the losses. So, let us see what happens, if you ignore the losses.

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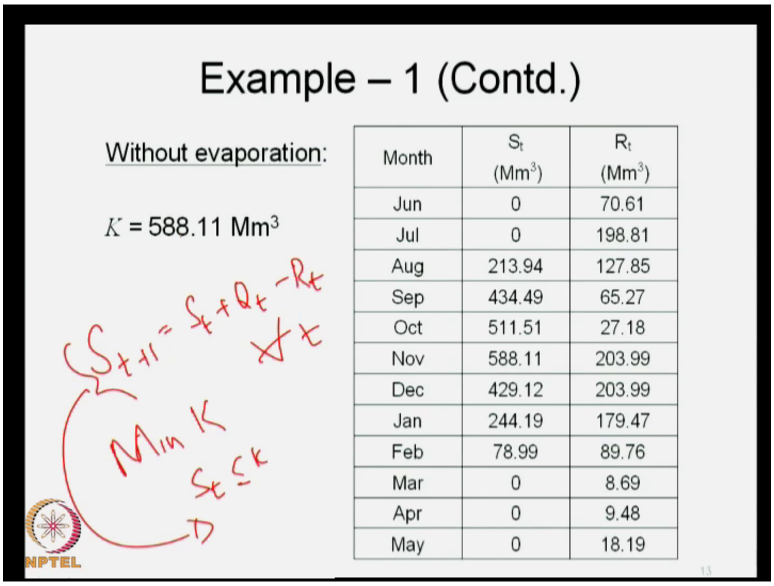

So, without evaporation now, I resolve the problem, which means essentially what I would (( )) means that I will write S t plus 1 is equal to S t plus Q t minus R t; that is all for all t, I will write, and then we are looking at the objective function of minimize K with all other constraints; that is S t is less than or equal to K and so on. So, all of this will write only the storage continuity equation, I will change ignoring the losses, with losses, we get about 617 million cubic meters, without losses you will get about 588.11cubic meters, which is much 588.11 I am sorry 588.11 million cubic meters, which is smaller than the 617.93 million cubic meters, you must always remember this that if you ignore the losses, you will get a lower storage, but physically if you construct this storage, you will not be able to meet those demands, because there are losses. So, you have to account for losses, and then you will get a higher required storage.

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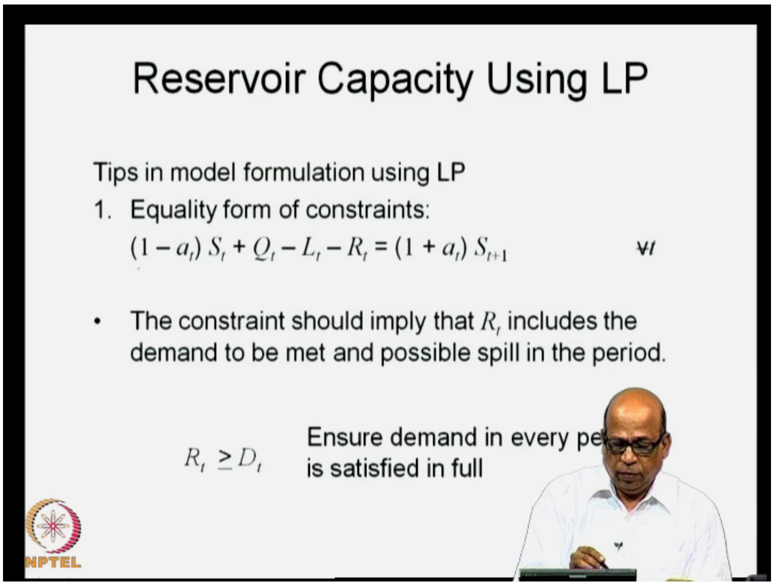

Now, there are some certain points here; that you should note, one is that we wrote the equality in the the storage continuity in the equality form 1 minus a t into S t plus Q t etcetera. We wrote it for all t. So, if you look at this set of constraints, all of these are equality constraints, in the linear programming algorithm, in the simplex algorithm, recall that whenever you have an equality constraint, what do we do, we put an artificial variable; and therefore, associated with all of these equality constraints you may have artificial variables, and the equality constraint must to be met exactly, because if you have less than or equal to constraint, you have certain flexibility that the left hand side can be less than the right hand side; and therefore, you have flexibility.

If you have greater than or equal to you still have flexibility, because you can make the value to be greater than any any value, greater than the right hand side value; and therefore, you have the flexibility whereas, when you put the equality constraint in optimization problems in general, it creates some problem when you have large number of equality constraints like like this, because the left hand side has to be exactly equal to the right hand side for all these constraints; and sometime and in fact, often times you get infeasibility when you have large number of equality constraints, typically in reservoir operation problems or reservoir sizing problems, when you have continuity in the equality form unless you account for the spills explicitly, and make this exactly equal to the right hand side in some sense, you will almost always end up in getting a in feasible solution.

And therefore, we need to rewrite the equality constraints in some other form, which is more friendly for the linear programming. Let us see, what we do that, what we do here, so you must understand that a large number of equality constraints in linear programming will cause problems; and therefore, we need to account, we need to rewrite the equality constraints in some form, which is, which will make the linear programming give us feasible solutions for the same type of constraints.

So, what we do there is in your earlier formulation, we said that R t will be equal to D t; although, the problem that I solved, I put R t greater than or equal to D t, because I wanted a feasible solution, but in general what we say, we specify the demand and say that R t must be equal to D t; and therefore, in the continuity equation, I would written R t, what we do is we put this condition, but put extra additional additional constraints saying that R t should be greater than or equal to D t.

And therefore, suddenly I mean, incorporating flexibility here; that means, instead of saying R t to be exactly equal to D t, I am saying R t should be greater than or equal to D t, I specify the demand and say that release must be greater than or equal to demand therefore, you are incorporating the flexibility here. So, this is one way of doing. So, this will ensure the demand in every period is satisfy to the fullest and any additional amount that you have will be put in to R t here. So, at least you are able to meet the demand and any additional amount will be put in to the term R t here; that is how you avoid the problems created by a set of equality constraints.

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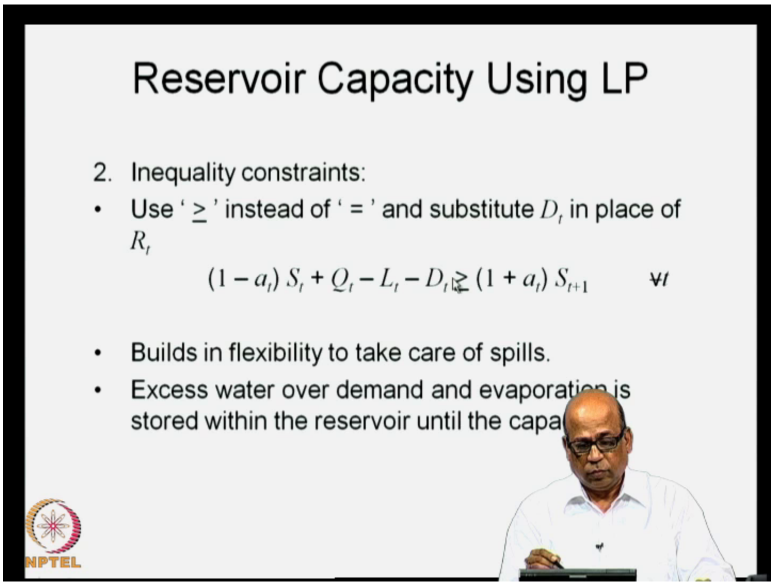

Then the other one is that you put D t, which is a demand, but make the left hand side greater than or equal to the right hand side, instead of saying exactly equal to the right hand side, you make it greater than or equal to the right hand side. What does that mean, you started with let say that you assume, you understand this way, you started with the particular storage, added the inflow to that took out some amount D t and accounted for the losses. So, you end up with one end of the period storage that is the left hand side.

Now, this end of the storage, I will make it greater than or equal to the actual end of the period, actual beginning of the period storage for the next time period. Suddenly, we are putting some flexibility into the mathematical statement of that; what happens to the additional amount, the additional amount will be accounted for as a spill. So, this can be greater than or equal to the right hand side, this is what we are stating mathematically; although, physically we will see that the additional amount will come as spill. I will tell you, what what that spill will be.

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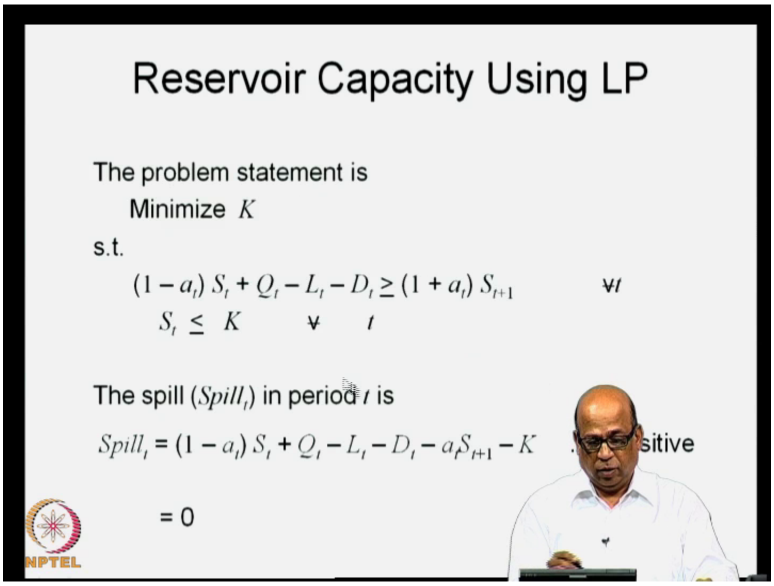

So, we will write this problem in using this formulation of the constraints as minimize K; that will remain as it is, this is the objective function, subject to 1 minus a t plus Q t minus L t minus D t greater than or equal to 1 plus a t into S t plus 1.So, instead of equality constraint here, I making it as greater than or equal to and S t less than or equal to K.

In the solution, then the mass balance will be met only by accounting for spill. So, if you look at the solution and then do the mass balance; that is S 1 plus Q 1 minus all the losses etcetera must be equal to S 2. If you do that mass balance, it can be satisfied only after you account for the spill and that spill will be equal to whatever is the left hand side minus the right hand side. So, this is the difference minus K, which is the capacity. If it is positive, which means, what that the additional amount; if it is greater than the capacity, then it will be accounted for as the spill; otherwise 0. So, this is how you can meet the storage continuity equation.

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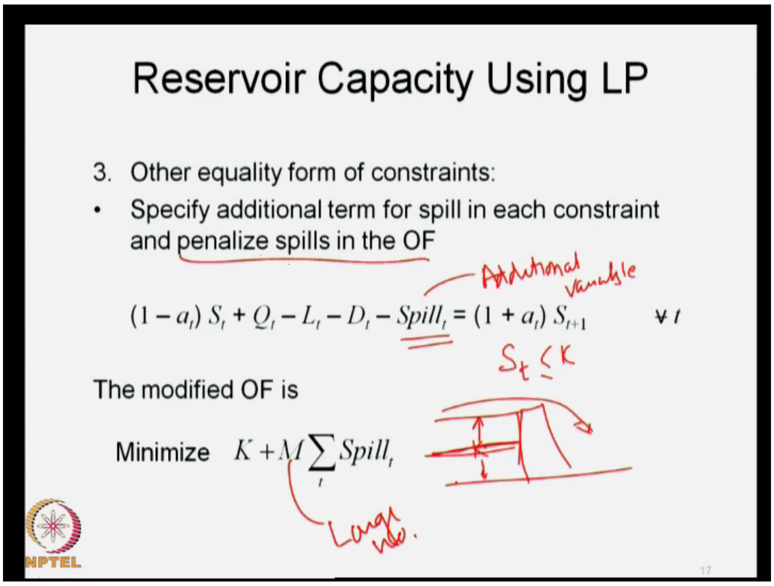

We also have another way of doing it, we can specify spill explicitly; this is the overflow. So, typically what happens is as I said your reservoir capacity comes up to K and then additional amount, we put it as spill. So, we are putting another variable (( )) spill as on additional variable.

(No audio from 43:48 to 43:54)

Now, if you try and I encourage you to try this, this expression for your continuity equation, without modifying the objective function, I will tell you, why this modification is necessary, let say that you try this form of the continuity equation, what we are saying here is; that you meet the demand exactly, L t is known; Q t is known; S t is the decision variable; S t plus 1 is the decision variable; a t is known.

And therefore, 1 plus a t and 1 minus a t are known; and we are saying spill t, we are also putting S t is less than or equal to K t and all that all other constraints are remaining as they are. So, S t is less than or equal to k. So, from this what we are saying is that the storage must be limited to k; however, I am also putting a spill t here, if you do not modify the objective function, what will happen is, even when storage is less than K, let say that your storage was here, even then the spill t may have a non negative value, non zero value in fact, because there is no other mechanism by which the linear programming algorithm knows that it should not give a non zero value to spill t, when storage is less than K.

To make sure that the spill t, which is actually the overflow will occur only when absolutely necessary, which means that only when the storage has reached the maximum storage, only then the spill should occur to ensure that what we do is, we say minimize K, which was our original objective function plus I will put an arbitrary large number, and then penalize the spill t.

So, what happens even if you put any value to spill t, because of this large number, the objective function values suddenly shoots up and therefore, there is a penalty associated with it, because we are talking about minimization objective function. So, we are actually penalizing the spills in the objective function by putting a large number M and, because is a minimization problem any value other than 0 to spill will shoots up the objective function and which is penalize actually, because you are looking for a minimization problem. So, this is another way of handling this.

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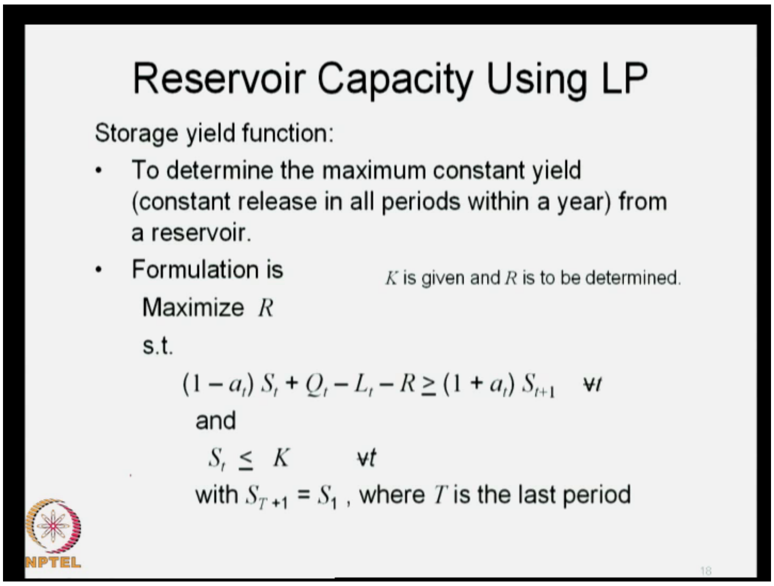

So, we know now that the storage continuity equation, which the evaporation losses can be handled by several means, one is simply put the equality constraints, but make R t greater than or equal to D t; and another is make the left hand side greater than or equal to the right hand side and so on. So, there are three ways of doing it, there is also yet another way by using the integer variable and so on, but right now you just be aware that the storage continuity equation and in fact, several other constraints that we may get in more complex problems, where you have a large number of equality constraints for example, you may want to include the soil moisture balance, where the soil moisture at the end of the time period must be equal to all the continuity mass balance etcetera that takes place during the time period and therefore, you will have large number of equality constraints.

Whenever, you have such large number of equality constraints, the linear programming generally gives infeasible solution, because the values have to be met exactly and these have to be, these values must be consistent across different time periods in an exacts sense, the left hand side has to be exactly equal to the right hand side; and therefore, large number of equality constraints must be in general avoided in the linear programming problem, and this is these are some of the ways by which you can avoid the large number of equality constraints.

Now, what we will do is. So, for what we did, if that we are looking for a minimum capacity of the reservoir; that is minimum required capacity of a reservoir to meet a pre specified demand pattern that is D t you have specified, demand during time period one, time period two, etcetera, you have specified that for a given sequence of inflows Q 1, Q 2, Q 3 etcetera Q t is known for a given rate of evaporation e t e 1 e 2 e 3 etcetera these are rates of evaporation. So, these are known; for a given area capacity relationship, you know the contour levels, as the storage increase is how the area changes you know that through area capacity relationship. So, for all of these given data you are looking for the minimum capacity of the reservoir to meet this demand patterns. So, this was our problem, this was the capacity determination problem.

Now, we will pose the problem slightly in a different way, let say that you have a given reservoir size or you are examining a given reservoir size, you want to ask the question, what is the maximum demand, constant demand? That you can meet from this given reservoir for a given sequence of inflows and for associated losses and so on. So, from the problem of capacity determination, we graduate a step further, and then ask for what is the minimum? What is the maximum demand, constant demand? That you can meet from the reservoir for a given sequence of inflow, which means your capacity is fixed, your flows are fixed, and you are now asking the question, what will we the maximum demand that you can meet from maximum constant demand; that you can meet from such a reservoir.

Now, this maximum constant demand that you can meet over all periods; that means, R 1 equal to R 2 equal to R 3 etcetera or if you want D 1 equal to D 2 equal to D 3 etcetera the demands are all same across the time periods. And we are looking for the maximum demand that can be met such maximum demand from a given reservoir, such maximum constant demand that can be met from a given reservoir is called as a yield of the reservoir.

So, we now talk about storage yield functions; that means, for a given storage the hydrologist fixed anyway, we are talk that I mean by inflow pattern is fixed, we are talking about for a given storage, what is the maximum constant demand that you can meet. Let us pose this problem and look at, how we determine the storage yield function? So, the storage yield function will give us corresponding to a given storage, what is the maximum constant demand that you can meet across the time period; that is what is called as the storage yield function.

Now, once we are aware of the linear programming formulations for the reservoir sizing, you can play around with several of these aspects, you change the objective function to reflect what you you need in terms of the problem statement and so on. You can also play around with the constraints and and so on. So, you can in the broad framework that we talked about just now in the specific framework, you can incorporate large number of flexibilities to answer specific questions. So, we will use the similar formulation in terms of the constraints, the constraints remain the same, but will change the objective function now. In the earlier formulation, we looked at minimization of K a; that is the active storage minimization of K a subject to capacity; that is the storage continuity and so on. We are now looking for maximization of a constant demand or constant release.

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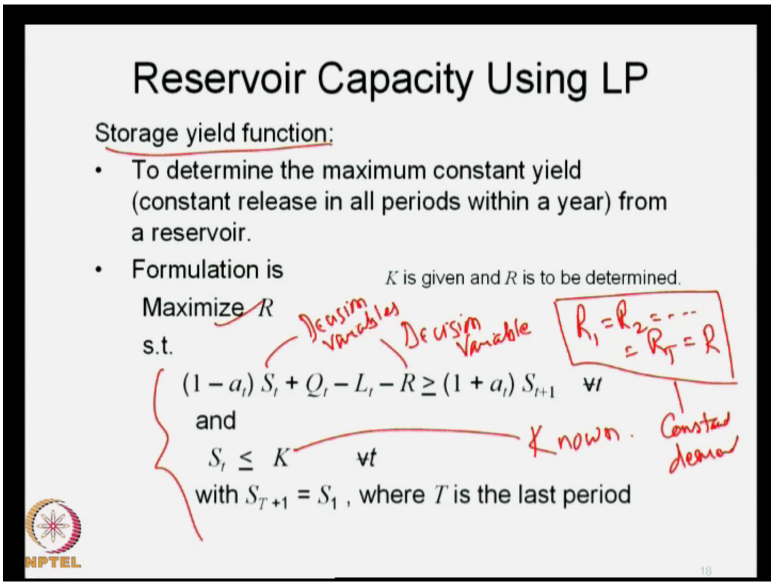

So, we will write, this as Maximize R. So, from R t now I have gone to R, which is the constant release in all periods within a year subject to the continuity. So, the continuity remains the same S t less than or equal to K remains same; S t plus 1 is equal to S 1; this is again the same same constraint, as we wrote there except that the K is known now.

In the earlier case demands were known, now demands are not known. So, this is the decision variable. And S t and S t plus 1 these are also decision variables. So, look at this formulation, we are looking at maximize R, remember I am not writing R t here, because R1 is equal to R 2 is equal to R 3 etcetera will be the same as R. So, I will write Maximize R subject to the storage continuity equation, in the continuity equation, I will write this not as R t, but as R, because I am saying R 1 is equal to R 2 etcetera R t capital T is equal to R; these are a constant demand.

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And we write all other constraints. So, the formulation simply becomes maximize R subject to this and we are doing this for a known storage, known active storage. So, this is how we obtain the storage yield function, then what we do, we solve this for a given storage, we get a particular R, we increase the storage, we get another R, as we increase the storage obviously, your yield should increase. So, keep on increasing the storage and then generate the trade off, not really the trade off, generate the relationship between storage and the yield. So, this is what we do in formulating the storage yield function.

So, essentially will maximize R subject to the continuity equation and all other constraints are there.

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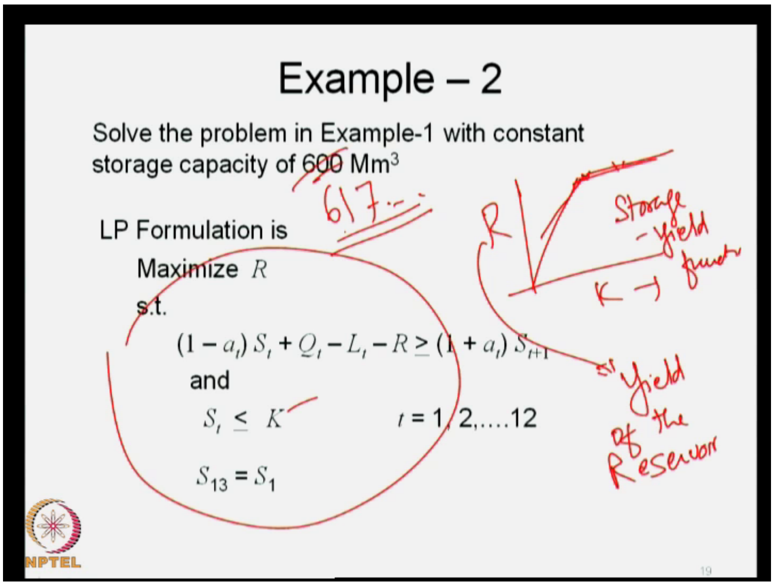

So, the LP formulation, we got earlier constant storage capacity of 600 million cubic meters; that was in the last example where you got 617.928 will use the same data and then look at this was 617.something will use the same data, and then look at the Maximize, Maximization of R. So, we we write this, and specify K for various values, not necessarily this value ,what we generally do is that we start with some small value of K look at the associated yield increase the K the yield increases; keep increasing the K in certain intervals; the yield keeps on increasing, but beyond the certain point, the yield becomes constant, because that will determine the inflow limitation; that situation will determine the inflow limitation, as I keep saying that the total demand that you can meet in a deterministic formulation cannot be more than what is available.

So, you have Q t some of Q t and minus the losses. So, what is available to (( )) is simply Q t minus losses; the total demand that you can meet irrespective of the size of the reservoir, irrespective of how high you build the reservoir is only limited to what is available to you, namely the sigma of Q t minus the losses and therefore, beyond the certain point, even if you increase the storage; the yield remains the same.

So, this is what we will examine, start with some amount of some minor some storage look at the R that is the maximum R that is all that for a given K, and then look at the R, increase K, look at the R again etcetera like this for various values of K you solve this problem, and then you get a relationship something like this. You you may have K here, you may have R here, and for given K, it may increase, and then it may start becoming increase, it may become constant at this point. And in fact, this will be from 0. So, this will be the storage yield function, this constant maximum release is called as the yield of the reservoir. So, we will solve this example in the next class, and then see how we formulate the storage yield function.

So, essentially then in today's lecture, we looked at the LP formulation that we had introduced in the previous lecture, we looked at how you account for the storage dependent evaporation losses. So, starting with the continuity equation that we write without the evaporation, we include the evaporation as storage dependent losses for which you need the area capacity relationship, how the area that is the water spread area varies with the capacity; now, by approximating the area capacity relationship beyond the dead storage with the with the line, with the state line. You can formulate this in the linear programming form.

And we have seen, how to rewrite the storage continuity equation accounting for the storage dependent evaporation losses, and then we use that in our LP formulation to get the minimum active storage, then we compare the minimum active storage that we obtain with the minimum with the evaporation losses accounted and without the evaporation losses accounted, remember that if you do not account for the evaporation losses, the storage requirement will be always smaller.

And therefore, but it will not be accurate that towards end of the lecture. I have just introduced the concept of storage yield function, where to meet a constant release or the constant demand for a given pattern of inflows for a known storage capacity is what will be interested in. So, this we discussion will continue in the next class, specifically solving the example that example of the formulation; that I just mention and generate a storage yield function for a given site, storage yield function will be dependent on the hydrology, which is a Q t; as well as on the demand pattern. Thank you for your attention.