Water Resources Systems

Modeling Techniques and Analysis

Prof. P.P.Mujumdar

Department of Civil Engineering

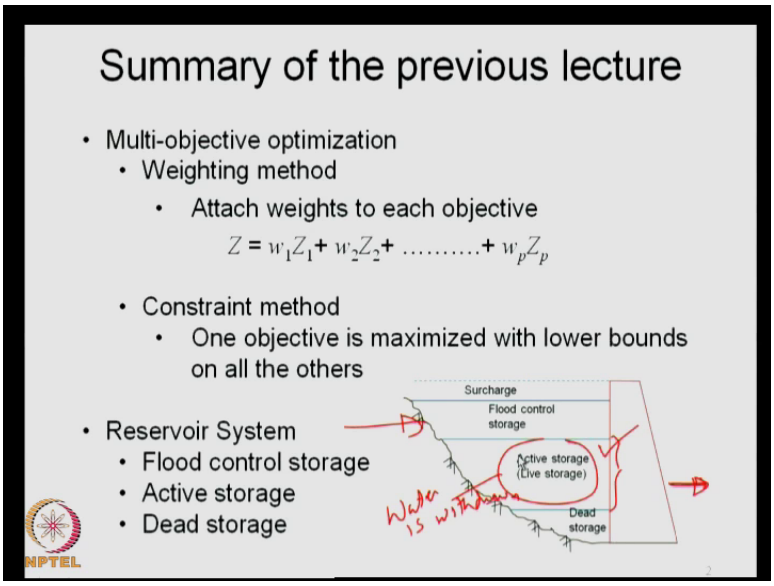
Indian Institute of Science, Bangalore

Lecture No. #20

Reservoir sizing

Good morning and welcome to this the lecture number 20 eth of the course Water Resource Systems Modeling Techniques and Analysis. So, in the previous lecture, I just towards the end of the lecture, I introduced the Reservoir System specifically, the storage zones that we have; we have the flood control storage, we have the live storage from where the water is withdrawn, and you also have the dead storage to accommodate the sewage load, sediment deposition and so on; before that we completed our earlier discussion on multi-objective optimization.

(Refer Slide Time: 00:52)



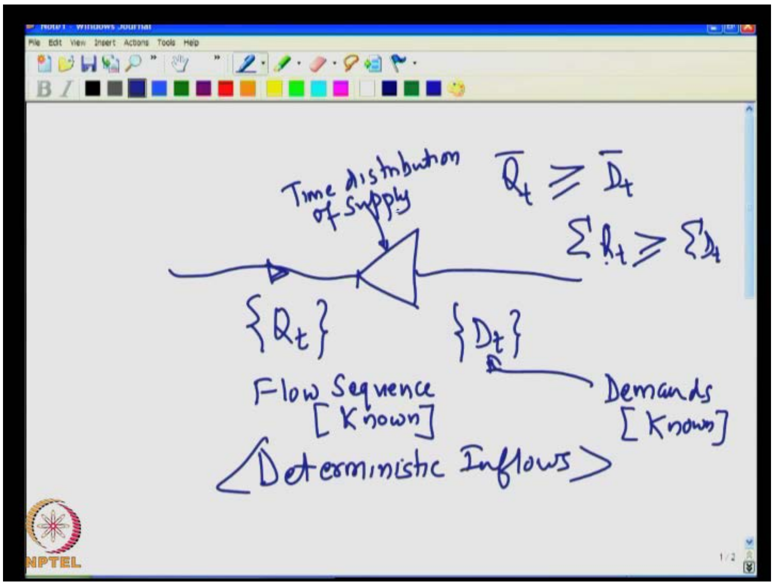
Recall that we talked about the weighting method, where we assign weights to individual objective functions Z 1, Z 2, etcetera up to Z p are the p objective functions. We assign weights to these, and generate non-inferior solutions or the Pareto optimal solutions. Now, the assignment of weights to the objective functions is the judgmental issue; and in fact, what we typically do is that we generate large number of solutions; in fact, we generate the say frontier, among different objectives for different sets of weights, and then you screen out the alternatives. That is what we typically do in multi-objective optimization with weighting method.

Then we also considered the constraint method, where we maximize one of the objectives in a maximization problem; we maximize one of the objectives subject to constraints placed on the other objectives. Let us say, you are maximizing Z j, one of the objective, you pick it up pick up, and then maximize that; subject to Z k greater than or equal to L k, some minimum value we assign to each of the constraints, and this is we are stating that the solutions are acceptable only if these objectives are met to that minimum (( )); and that through that method, we generate Pareto optimal solutions or the non-inferior solutions. We solved to… solved a simple example, by which these two (( )) in the last lecture.

Towards the end of the lecture, we started introducing the reservoir systems; you just recall that we had typically a flood control storage to accommodate the flood volumes, so that the downstream flood discharges are attenuated - the peak flood discharge downstream of this is attenuated. Then we have the active storage, from which we actually derive the water; so, this is the storage from which the water is withdrawn. And in most of the problems dealing with reservoir systems, we will be discussing the active storage zone; and dead storage as I just mentioned, is essentially to accommodate the sediment deposition, and also to provide storage for recreation and hydro power, providing head for hydro power and so on.

So now, we will continue the discussion on reservoir systems; as I mentioned in the last class, the first level of problem set we deal with in reservoir systems are the deterministic problems, where the sequence of inflows is known, and also the sequence of demand set you would like to meet in a many situations they are known, also there are variations or variance of this problem where you may want to maximize the demands and so on; so, we will come to that later; but in general, the there is no uncertainty associated with either the inflows or the demands or the reservoir capacity itself - reservoir storage itself. So, these are called as deterministic problems. So, the first level of deterministic problem that will consider is the one associated with determining the reservoir storage. The problem here is that you have a known sequence of flows, and then you would like to determine, what is that minimum storage capacity that is necessary to meet a certain demand?

(Refer Slide Time: 04:44)

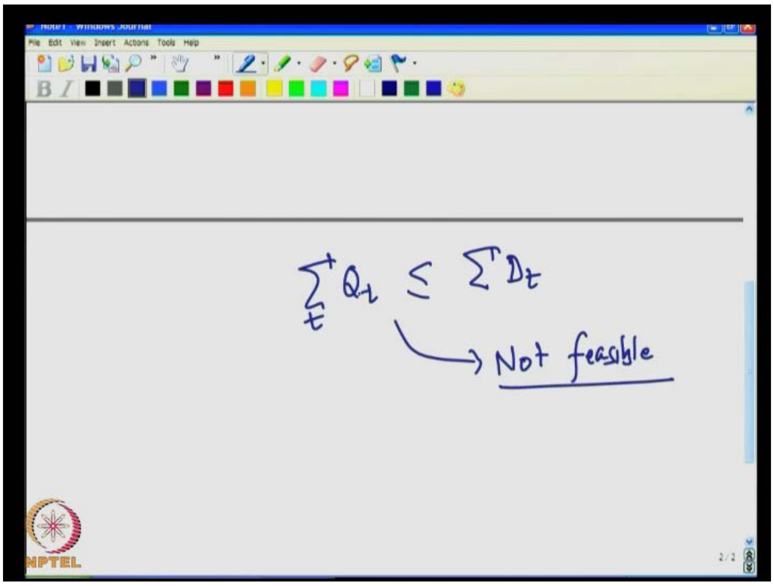


Let us say, this is the stream; and you have a site for the reservoir. The stream sequence Q t - this is the flow sequence, and this is known; and that is why we call it as deterministic inflows. (No audio from 05:14 to 05:25) So, you know the flow sequence, and you know, let us say that you would like to build this demand, this reservoir for beating a certain set of demands and these demands are also known; that is pre-specified. So, you ask the question to meet this set of demands, what is the minimum capacity that you need to provide; minimum reservoir capacity that you need to provide at this location, knowing the flow sequence and knowing the demand sequence. It is obvious that if your demands in all the periods are lower than the flows.

Let us say that Q t was always greater than or equal to D t for all t. In such a case, there is no need for a reservoir; simply you draw the water and supply the demands. The need for reservoir arises; mainly because you want redistribute the water among several periods. In certain periods, Q t was greater than or equal to D t, but in certain other periods, Q t is smaller less than or equal to D t; and therefore, you store the water in those periods in which Q t was in excess, that is the flow was in excess; that excess flow you store, and supply the water during the deficit periods. So, you are actually redistributing the flows among the time periods in a year, if you are talking about early operation. So, when you are redistributing these flows, you would look at the periods in which the maximum deficit occurs, and that maximum deficit is what you have to provide through storage. So, that is the principle on which we decide the storage reservoir capacity. So, you are looking for that minimum value of the reservoir storage that needs to be provided to meet these demands from a given supply; that is the problem of determining reservoir size.

There is one more issue here. Let us say that the sum of Q t or average of Q t over the time period is greater than or equal to average of demand or let us say, simply we will put sum of Q t is greater than is greater than or equal to sum of D t, as I said. Then you can this is supply is greater than the demand, therefore you will be able to meet the demands, by providing a certain capacity of the reservoir. So that, the time distribution is maintain. So, the reservoir essentially, ensures the time distribution of the flows; (No audio from 08:39 to 08:46) that is why you provide the reservoir. Let us say, your sum of Q t over all the time periods, let us say you are talking about monthly time periods over a year.

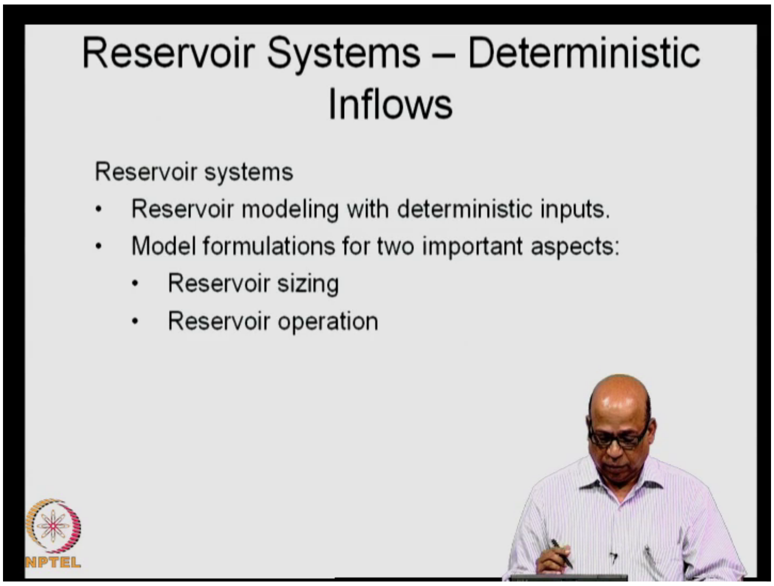
(Refer Slide Time: 08:59)



And sum of Q t over all t is less than or equal to sum of D t; that means, the demands - the total demands that you are talking about is less than is greater than the total supply. Then no matter what capacity of the reservoir you provide, you will not be able to meet these demands during all the time periods; so, this is not feasible. (No audio from 09:28

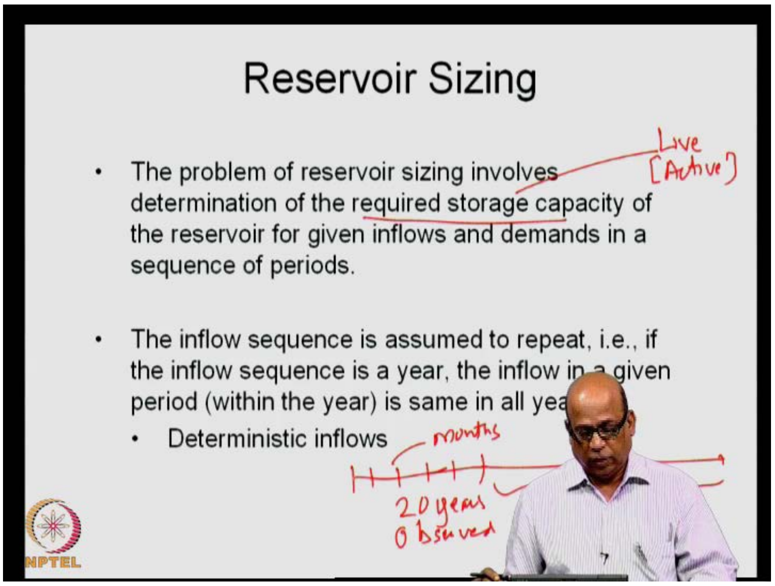
to 09:35) In a in a deterministic sense, where you are talking about deterministic flows and deterministic demands, so, your total supply, if it is less than the total demand, no matter what kind of capacity you provide, it will not be able to it will not be able to meet the demands as specified. So, these the things will keep in mind, when we go to reservoir systems.

(Refer Slide Time: 10:07)



So, the problem now is that we are talking about Reservoir sizing problem, where we would like to determine the minimum required capacity of a reservoir at a particular site, where the flow sequences are known, and you are specifying the demand sequences; to meet that particular demand sequences, given the flow sequences, what is the minimum capacity that we need to provide? And we are talking now about the live storage capacity. So, whenever, I am talking about reservoir sizing in the in the context of a deterministic flows, we are only specifying the live storage capacity.

(Refer Slide Time: 10:43)

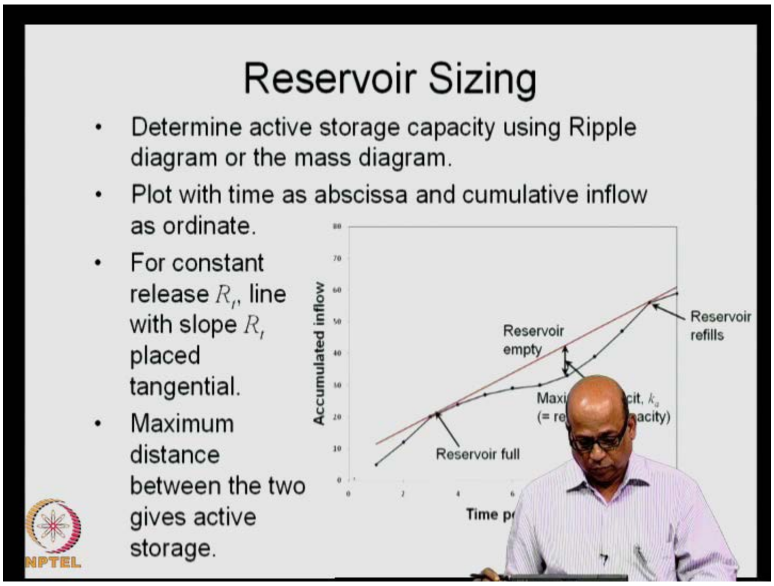


So, required storage capacity, this is actually the live storage capacity. We are talking about live storage here or the active storage capacity for given inflows and demand. Now, when in the deterministic inflow scenario, let say that you have 20 years data; assume that you have 20 years of inflow data or flow data; and you are doing it on a monthly time scale. So, this is 20 into12; so, these are months; so, 20 into12 values you have.

In the deterministic case, what we generally do is that we assume that this sequence itself repeats. So, these 20 years is observed data, and we say that this a sequence keeps on repeating, next 20 years, next 20 years etcetera. But typically what we do is, we solve this over one year period with average flows. So, we take let us say, June average over the 20 years and put it as June flows, July average over the 20 years and put it as July flows and so on. And construct a one year sequence, and then solve for that one year sequence, the minimum required capacity and so on.

So, typically the time periods that we consider need not be calendar years or calendar months; it need not be a calendar year, it can be let us say three seasons, you may have three seasons. So, month may a year may consider, may consists of three seasons or you may have 6 time periods, consecutively 6 years you may have; and so therefore, you may consider 6 time seasons periods or 6 years, every year two seasons; like this 12 periods you may have and so on. So, it the reservoir capacity determination actually, depends on the purpose for which purpose with which you would like to operate the reservoir. So, it may be, the time periods may be seasons, time periods may be years, time periods may be months, etcetera. So, you use the associated sequence of flows and the corresponding sequence of demands. And the principles of determining the size remain the same, as we will discuss presently.

(Refer Slide Time: 13:16)

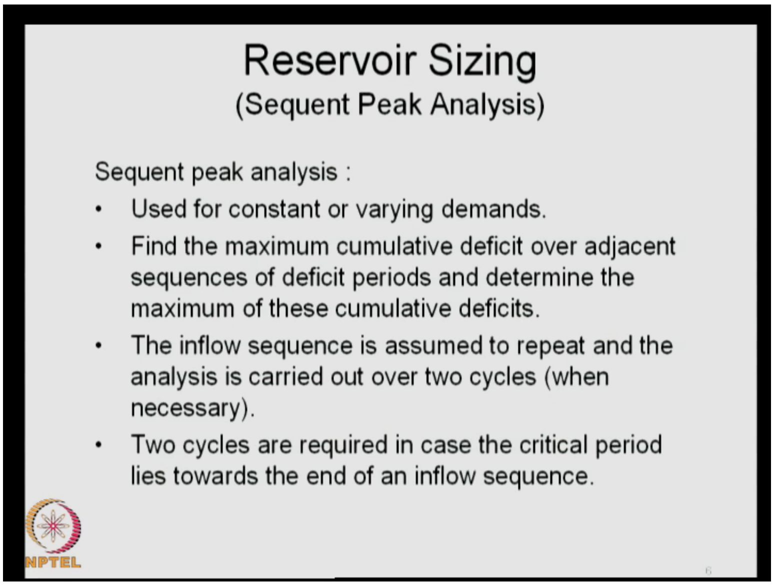


A classical approach, which is to be practice much before the computers came and so on; is the ripple or the mass diagram - ripples approach or the mass diagram approach, where you may have learnt in your basic hydrology or water resource course; this is as cumulative inflow will not do any exercise among on this. But you just, see how we determine using the mass diagram; you plot the cumulative inflows. So, this is the cumulative inflow diagram, this line that you are seeing is the cumulative inflow diagram. And then, you have a constant release R t, you place a line with the slope as R t. This is the constant release, it is not changing from time period to time period; and this is the cumulative inflow diagram; on that you place this constant release line, that is the slope of this line is R t, you place it just tangential to the maximum, the there is a cumulative inflow line.

And then wherever the maximum deficit occurs, that will give you the reservoir capacity; obviously, what we are talking about is, you are you keep on making the releases at the same rate, and then your reservoir is filling like this; so, the flow is coming like this; and then the maximum deficit is what you have to supply through the storage. So, the maximum deficit corresponding to this constant release line with the inflow line - cumulative inflow line, that is what we will give you the reservoir capacity; and this is the classical ripple diagram or the mass diagram; however, now with the advent of computers, and then very nice elegant algorithms etcetera. This is slightly outdated now; but this is the basis on which we start.

Obviously, here accounting for time varying demands or time varying releases, and also accounting for evaporation losses etcetera, becomes quiet involved. In fact, time varying demands we accommodate by taking the cumulative differences Q t minus R t in each time period; and then looking at the maximum deficit that occurs from the peak to the trap and so on. So, anyway we will not worry too much about this, this is just as a background you must know that, this is how the reservoir storage use to be computed before the computers came by simple graphical methods.

(Refer Slide Time: 16:08)



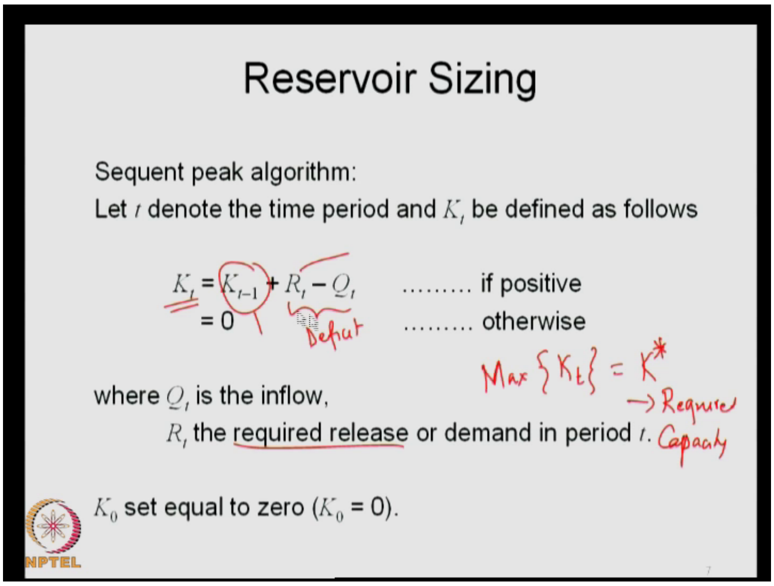
We will go into a simple analysis through which you can determine the reservoir capacity, and this is the Sequent Peak algorithm or the Sequent Peak Analysis. Now, essentially the principle remains the same; that is what is that you are looking at? You are looking at filling the maximum deficit that is likely to occur in a given sequence of flows to meet a given sequence of demand. So, essentially you keep looking at, how much is the deficit during this time period, how much is the deficit during the next time period, and so on. So, typically in a flow sequence, there will be certain critical period, in which continuously the deficits will be occurring; and that is the critical period which determines what is the maximum storage that is necessary, what is the storage that is necessary.

So, we capture in the sequent peak algorithm; we capture that critical period in the given sequence of flows and compute the deficit. Now, this critical sequence may occur towards the end of the sequence, critical period may occur at the end of the sequence. What I am mean by this is? That let us say, you have 5 years of monthly data, and based on this 5 years of monthly data, you would like to determine what is the reservoir capacity required. So, you have 60 months of data; now in the 60 months of data that you have, it may so happen that the last few months, let us say from 40 th 45 eth month to 60 eth month that may constitute a critical period.

In which case, it is better, in fact it is prescribed that you run this algorithm, which are presently discuss over 2 cycles. You had 5 years of data; you run that over 1 cycle, but you have still not reach the correct critical period, because it is occurring at the end of the sequence. You append the append the sequence again to this end of the first cycle and run it for 2 cycles, so that, you capture the critical period correctly; by critical period, I mean the period, in which the deficits keep on occurring and you are you want to capture the maximum deficit, and that maximum deficit in fact, corresponds to the required storage.

Keep in mind always that the storage is provided essentially to offset the deficits that are occurring. So, how much minimum storage that is required, that corresponds to the maximum deficit that occurs; and that is what we do in sequent peak algorithm. You capture the critical periods, in which continuously the deficits are occurring; you keep on accumulating the deficits, and that kind of that amount of storage is what you have to provide; that is the basis of the sequent peak algorithm.

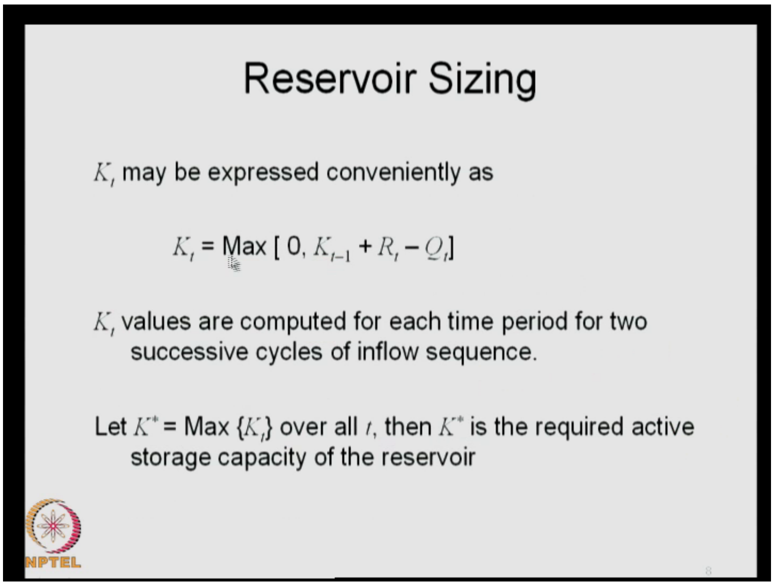
(Refer Slide Time: 19:10)



Let us write the sequent peak algorithm in a mathematical notation. So, what we do is, we define K t as K t minus 1, which was the required storage or required maximum required deficit, the deficit that occurred in time period t minus 1 plus R t, which is the required release, R t in this case is the required release; it is defined here; and Q t is the inflow. So, essentially what we are doing is, R t minus Q t if positive, this R t minus d Q t is the deficit; you wanted R t, but only Q t is available. So, R t minus Q t is the deficit. To that, we add whatever was the deficit earlier, and that we call it as K t. So, K t is K t minus 1 plus R t minus Q t if positive, why if positive? If let say Q t is very high compare to R t, and K t minus 1 plus R t minus Q t is negative; it means that the release R t can be met with available storage, and therefore you do not need any additional storage, that is the idea there. And therefore, if it is positive we put K t as K t minus 1 plus R t minus Q t; and if it is negative, we set it as 0.

Now, we begin the computations by K naught equal to 0. So, we set K naught equal to 0, and then we compute K 1, knowing R 1 knowing Q 1; then for the next time period, you put K 2 is equal to K 1, because K t minus 1, K 1 plus R 2 minus Q 2 and so on; like this; you keep on continuing. So, you get a sequence of K t, when we do this. Sometimes it will be 0, because Q t may be higher in that particular time period. You get a sequence of Q t, you pick up the maximum of the K t, and that is the required capacity, we put it as K star or required capacity. Now, in the form that I have expressed here, we are neglecting losses. So, we are not accommodating evaporation losses for reasons that I will tell presently. This expression here K t is equal to K t minus 1 plus R t minus Q t if positive, and 0 otherwise.

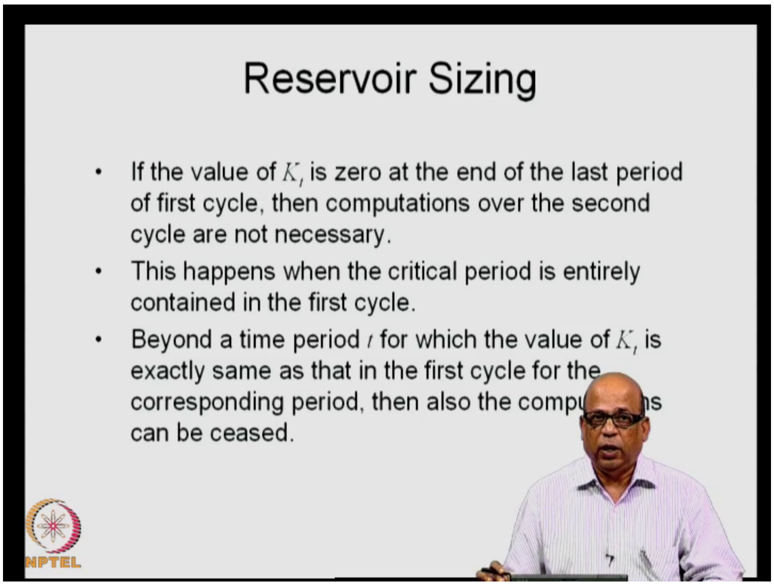
(Refer Slide Time: 21:54)



We write it in a simplify form a form, like K t is equal to maximum of 0 or K t minus 1 plus R t minus Q t, which means that if it is less than 0, this is negative, so you pick up this one. And as I said, you pick up the maximum among the K t values, and that gives you the capacity. Remember always that you are looking at the maximum deficit period, including the storage that is necessary during the previous period. So, with the storage that is already provided during the previous period, if you are able to meet the demand here, then it is not a deficit period.

And therefore you look at continuous deficits, which keep on adding to your storage requirements; and then pick up that particular value of K t, which is maximum among all these K t values that use thus computed; and that is in fact the minimum storage that you need to provide - minimum live storage. At this point, we are not worried about the dead storage or the flood control storage, they are different topics all together; we will not touch them at this point in time.

(Refer Slide Time: 23:09)

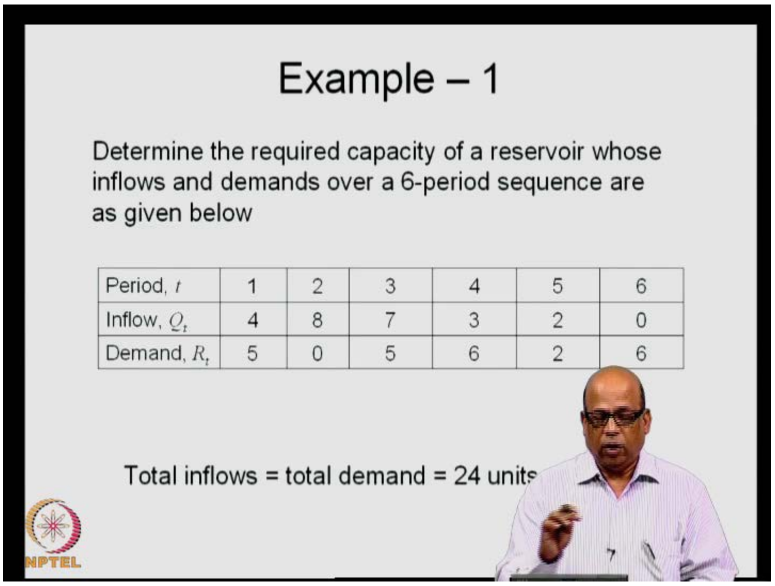


Now, let us say that you are doing the computations, you started with K 0 is equal to 0, and then computed for one cycle, and you ended up with let us say, you had 6 time periods, so at the end of 6 time period, you got K 6 as 0; then if you append this sequence again, what will happen the sequence will simply repeat; the computations will simply repeat; because you started with 0 - K 0 is equal to 0 and you ended up again at the last time period, you are again ended up with 0; therefore, thus whole sequence will repeat, if you repeat it for the second cycle. Then thus the computations for the second cycle are not necessary.

The computations for the second cycle are necessary, if at the end of the last time period in the sequence, you do not end up with the K t is equal to 0 for that particular time period. In which case, the critical period may occur towards the end and towards the beginning, and therefore you need to repeat it for the next cycle. So, a maximum of two cycles you repeat, and even when you are repeating for the second cycle, at any particular time period, if you get the same value of K t as you got in the first cycle for the same time period; then again the sequence will repeat. So, at that point, you can terminate the computations, as I will presently show in the example.

So, this is what we do in the sequent peak algorithm; simply start with k naught is equal to 0, every time you add the deficit that is likely to occur, and if you are able to meet the deficit with the storage that we already provided, there is no storage requirement for that time periods etcetera. Like this, every time period you compute the storage requirement, as a as a deficit that is occurring during that time period; accounting for the storage that is already provided; and then pick up the maximum of such K t values, and that is the minimum storage that you need to provide to meet the particular demand pattern that you have specified for the given inflow sequence.

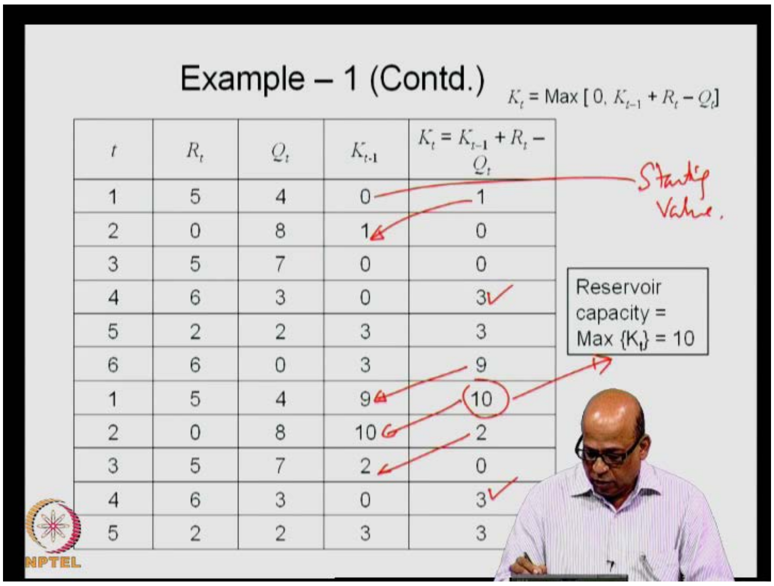
(Refer Slide Time: 25:17)



Let us take a simple example, to demonstrate this point; there are 6 time periods here; and there are these are the inflows 4, 8, 7 etcetera, in some consistent units, and these are the demands 5, 0, 5 and 6 etcetera. As you can see, if you did not have a reservoir, what will happen here is, there is a deficit here; you cannot meet 5 units of demand; whereas there is an excess here, but there is no demand. So, this would have just gone. There is an excess here, you met 5 that is the 2 unit, which would have just gone, if you did not have a reservoir. So, you essentially provide the storage to store this excess amount of water to meet the requirements during the deficit time periods; there is a deficit here; there is a deficit here etcetera.

So, meet that you use this excess amount of water to meet the deficits occurring in some other periods. That is the idea of storage; how much of storage is to be provided is what we will check now. In this particular case, always whenever you want to examine the reservoir size of the reservoir, first check the total inflow versus the total demands. If your total demands are higher than the total inflows, no matter what kind of capacity you provide, you will not be able to satisfy those demands. So, the total demands must be less than or equal to the total inflows. In this particular case, the total inflow is equal to total demand, as you can verify both are equal to 24 units. And therefore, all you are going to do is distribute the inflows and accommodate the storage, so that, you are able to meet the demands in all the time periods. That is the idea with which we carry out the Sequent peak algorithm.

(Refer Slide Time: 27:16)



So, this is typically done in a very simple tabular manner. Remember, we are neglecting the losses here, neglecting evaporation and c pad losses and so on, simply assuming that all the inflow is available to meet the demands. So, in the first time period, R t is 5, Q t is 4; this is the data that I am using R t is 5, Q t is 4; and we start with K t minus 1 is equal to 0, this is the starting value; and then we compute this term, K t is equal to K t minus 1 plus R t minus Q t, R t minus Q t. So, this is 5 minus 4; this is 1; and this K t minus 1, this K t becomes K t minus 1 for the next time period.

So, when you go to the next time period R t is 0, Q t is 8, K t minus 1 is 1, therefore 1 plus 0 minus 8 that is negative, and therefore this becomes 0; we are doing this, this computation now; this is 0; and this 0 becomes K t minus 1, R t minus Q t; this is 2; and again it is negative; and therefore it becomes 0; and this 0 becomes K t minus 1 here and so on. So, here you look at this computation, this is K t minus 1 which is 0; R t which is 3; minus Q t which is 3; and therefore this becomes 3. So, we are using this expression.

Like this, you keep on doing this until 6 - time periods 6; you had periods 6 periods in this example. So, you compute up to 6. At the end of the 6 time period, you look at K t. If this K t was 0 here, what you get here is 0, then what would have happen? For the next time period, K t minus 1, you would have put 0, and the same sequence here 5, 4, 0, 8 etcetera. So, the sequence would have repeated, and therefore you will terminate the computations here, if you get a 0 here, at the end of the 6 eth time period in this example.

But because it is not 0 here; it means that, it may mean that the critical time period is in fact, happening at the end of the sequence; and therefore you go to the next sequence. So, you again do this calculation for the second cycle. So, what do we do? We repeat the same flow same demands etcetera with this K t minus 1 now. And therefore, you get a value of 10 here; that means, you are putting this is 9 plus 5 minus 4 that will be 10, and this 10 comes here, you get a K t of 2, and this 2 comes here you get a K t of 0, etcetera.

You look at time period 4; you got a value of 3 here; at time period 4, you again got a value of 3. If you keep repeating, then from this point onwards, the computations will repeat, the calculations will repeat, because K t minus 1 will become 0 here. And therefore, these K t is that you get will keep on repeating; therefore you can stop at this point particular point. So, if you get the same K t value in the second cycle for a particular time period, as you got for the same time period in the cycle number 1, you stop the computations.

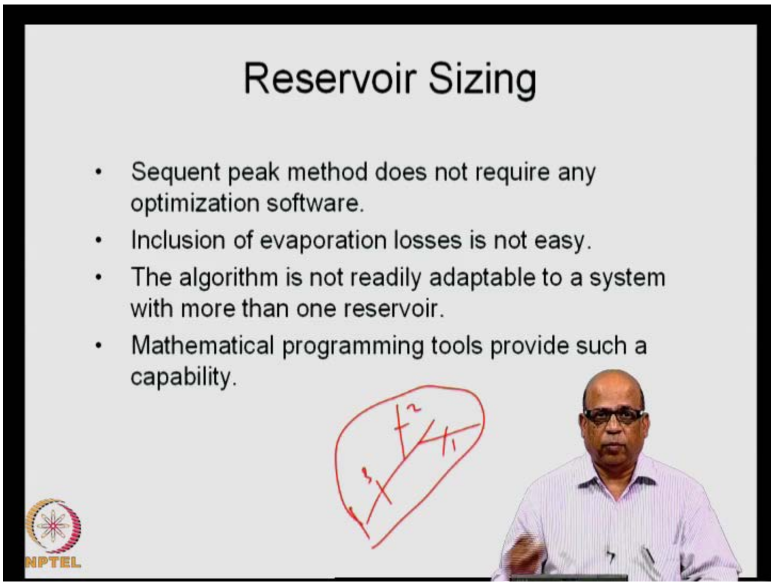
If you get a value of 0 for K t, at the end of the first cycle, you stop the computations. Then you look at all the K t value that you got, and pick up the maximum value that you got, and that is the reservoir capacity. So, in this particular case, the reservoir capacity is 10 units. So, essentially what did we do, we had the inflow sequence for some time periods; these are 6 time period; you had the associated demand sequences, you kept on calculate seeing what is the deficit set is occurring, and then in the critical period as you can see here, this is the critical period in fact, 3, 3, 9, 10, 2; these defines the critical period; and the maximum K t value actually occurs in the critical period.

So, just show that you do not miss the critical period, you typically carry out the computations in 2 cycle, for 2 cycles, over 2 cycles; and the computations may not be necessary for 2 cycles, if either at the end of the first cycle, you get a K t value of 0 or as you are going into the second cycle, a particular time period you get the same K t value as you got for the same time period in the first cycle; then you stop the computations. So, this is how the sequent peak algorithm works.

And then, you pick up the maximum of the K t values, this is in fact the minimum storage that is necessary, minimum live storage that is necessary, to meet the particular demand patterns - demand sequence for the given inflow sequence. So, in this particular case, you get a storage of 10 units. Very simple procedure, however, it is not really applicable or it is not so easy to apply, when you want to include the evaporation losses. Evaporation losses will be a function of the storage at a any given time period, because the storage will decide on the surface area of the water, and associated with the surface area thus evaporation losses will take place.

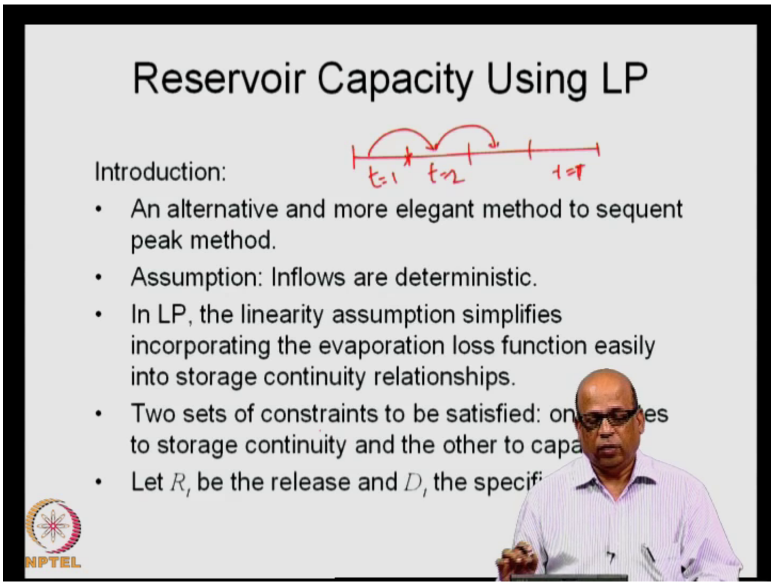
Therefore, for a realistic assessment of the storage that is necessary, you have to necessarily account for storage dependent losses, both evaporation as well as c pad losses. And that is not very straight forward in sequent peak algorithm, although I must mention here that Lele of… (No audio from 33:46 to 33:53) Let me write that in 1987, he has modified the sequent peak algorithm (No audio from 34:10 to 34:17) to provide for the evaporation losses. But it is again, as I mentioned, the mathematical programming tools or the optimization tools that we have learnt earlier like linear programming etcetera. They become very handy, when we want to include the evaporation losses.

(Refer Slide Time: 34:42)



Now, what we will do is, also there is another case where sequent peak algorithm is not readily adaptable. You talk about one single reservoir here, but in many cases, what happens is that you may be talking about reservoir capacities necessary in at several locations, like this location number 1, location number 2, location number 3 etcetera. Simultaneously, you would like to determine the reservoir capacities, such that some total demands are met including the demands at the individual reservoirs and so on. So, in such cases, sequent peak algorithm becomes quiet cumbersome and quiet in elegant and unwieldy, and therefore, we do not go with such algorithms. However, the as the background to determining the reservoir capacities, you must know the sequent peak algorithm as well as the mass curve.

(Refer Slide Time: 35:35)



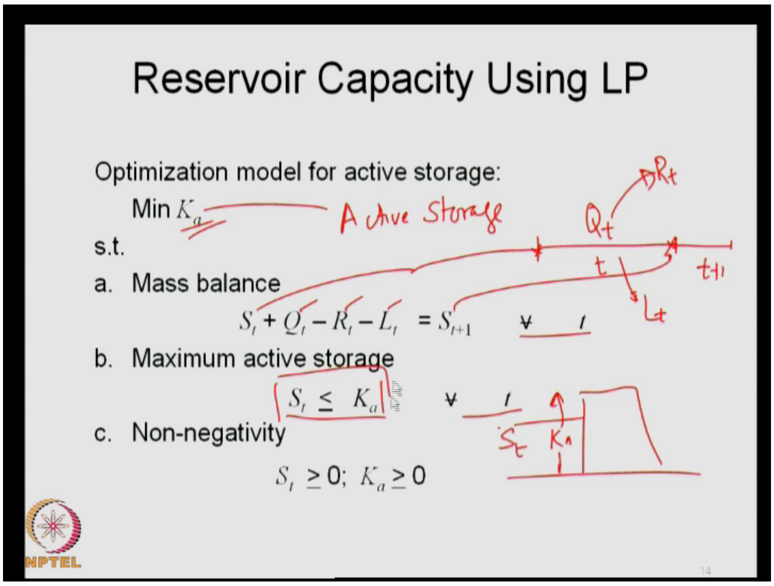
Now, we will go to sequent peak algorithm that is the reservoir capacity determination using linear programming. We have studied the linear programming earlier, and then now, what will do is, we will apply the linear programming for obtaining the minimum requires capacity to meet a given set of demands for a known sequence of inflows; that is a problem now. Whenever, we are talking about reservoir systems, the first constraint that needs to be met - first set of constraints that needs to be met is the one related with mass balance; that is you are talking about time periods like this.

Let us say that you have as in the previous example, you had 6 time periods, t is equal to 1, t is equal to 2, etcetera; t is equal to t - last time period, like this you had. So, the mass balance has to be met from one time period to another time period. So, we are writing the storage continuity relationship between one time period to another time period. Typically, starting with the known storage, if you have a given inflow and take out certain amount of release, and take out the losses, what will be the storage at this point? That will govern the storage continuity. You recall what we did in the dynamic programming exercise for reservoir operation. Similar thing is in fact, the exactly the same mass balance equation is what we use in determining the reservoir capacity.

So, whenever we are talking about reservoir systems, the storage continuity has to be satisfied from one time period to another time period. And this becomes the major constraint in the LP problem. So, in the LP problem, we pose the optimization as follows. We want to minimi we want that particular storage, which is the minimum storage, which satisfies all the continuity constraints, S t plus Q t minus R t, where R t is the demands, which are known; minus L t for example, the losses that are taking place should be equal to S t plus 1; that is the storage at the end of the time period. So, these sets of constraints to be satisfied for all the time periods, and then you are looking at that particular storage, which is the minimum storage.

We will write that in mathematical form, but you remember there will be two sets of constraints; one relates to the storage continuity, another relates to the maximum storage. So, you are looking for let us say, a storage of K, which is the minimum storage that is necessary; and at at no time, your storage should be more than that particular K. So, that is the maximum storage constraint that is the capacity. And then you are pre-specifying the demand patterns D t. So, this is the problem now. We will see, how we state the problem.

(Refer Slide Time: 39:02)



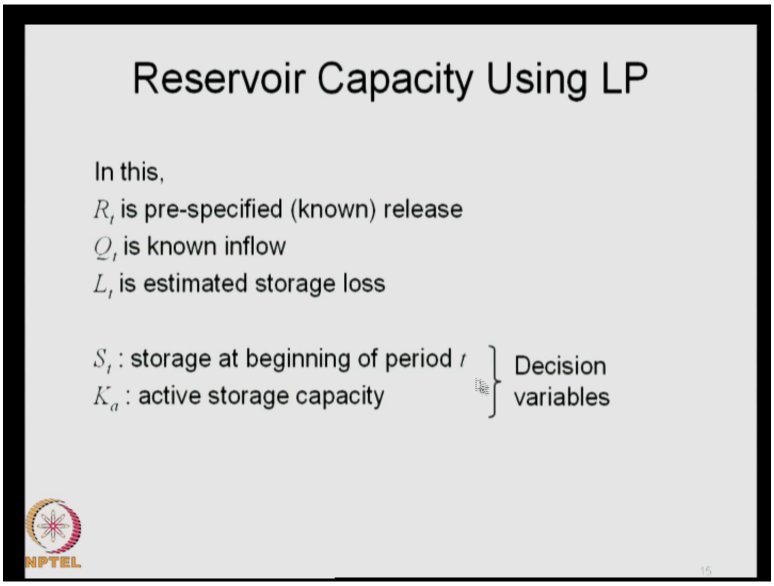
So, we are looking at the minimum value of K a, this is the active storage; subject to the mass balance which is S t; there are two consecutive time period t, t plus 1. So, S t is the storage at the beginning of this time period, Q t is the inflow that is coming during this time period, R t is the release that are gone out of this time period, and L t is the losses that are taken place in this time period. And therefore, you define S t plus 1, which is the storage at the time period, t plus 1. This is the simple mass balance; then at any time period your S t should be less than or equal to K.

Now, this is the storage that you are looking for; this is actually a decision variable, S t is also a decision variable; S t should be always less than or equal to K, this is the capacity. You are looking at active capacity therefore, the storage at any time period must be less than or equal to K a. This is for all t; this also for all t; then you have non-negativity; obviously, S t should be greater than or equal to 0, and K a should be greater than or equal to 0; Q t is known, R t is known, L t we will see how to specify or L t is also known, you assume that L t is known, these are the losses. S t is the decision variable therefore, S t plus 1 becomes a decision variable, K a is a decision variable.

So, for a given sequence of flows and for a given sequence of demands, you are determining that particular value of K, which is a which is the minimum storage that is necessary to meet this demand patterns. That is the idea here. And this constraint ensures that your storage is never above the capacity. So, this is the capacity K a, you are talking about. So, the storage that you are computing, S t should be always below K a, that is what this constraint says.

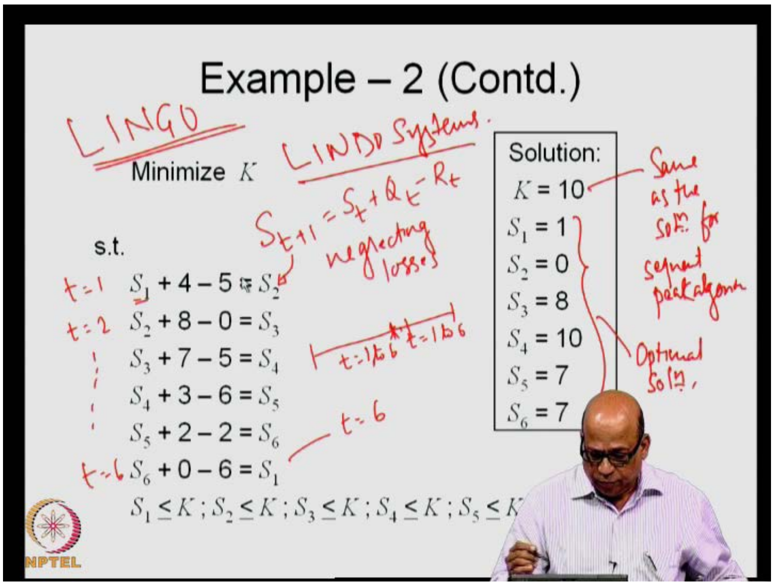
Now, this is the optimization problem that you write, and you can solve it using linear programming, because these sets of constraints are linear; for example, here you would have written S t minus Ka less than or equal to 0, which is the linear constraint; and therefore, you can use linear programming to solve this problem. Now, there are some nuances associated with the loss, because this loss here will be also a function of the storage. We will see in the subsequent portion of this lecture or perhaps the next lecture how we account for this L t as a storage dependent loss. For the time being, what will do is we will take up a simple example, neglecting these losses, and then see how we get this capacity K a.

(Refer Slide Time: 42:25)



Now, R t is the release, these are all the definitions, as I said S t and K a become the decision variables in all the LP problem. So, we will take the same example, as we did earlier using the sequent peak algorithm. So, we use the same example, we got a capacity of 10 units here. Let us take the same example, and again neglect the losses. So, these are the inflows, these are the demands as in example 1 using the sequent peak algorithm. We will write a linear programming problem here.

(Refer Slide Time: 43:00)



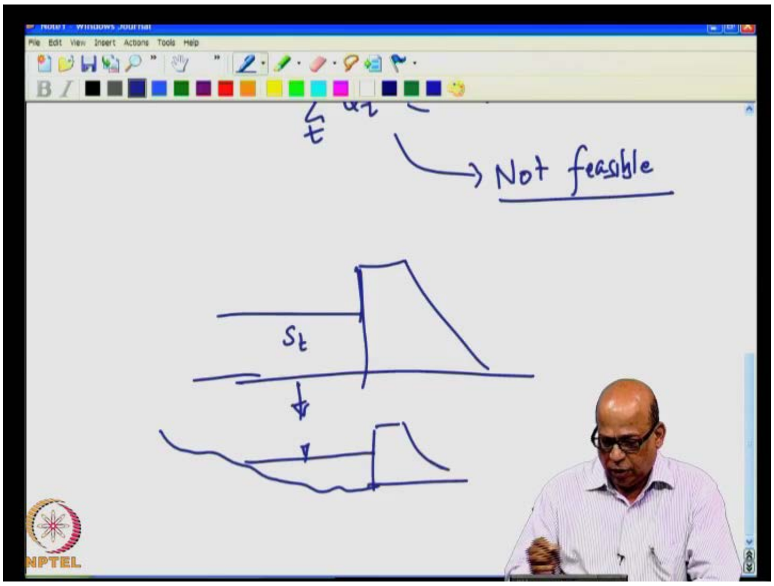
There are 6 time periods, we are looking at minimize K as objective function, subject to the continuity equations. So, S 2 is equal to S1 plus Q1 minus R1, Q1 is 4, R1 is 5. So, S1 plus 4 minus 5 should be equal to S2. Now, this set of constraints is essentially the mass balance, I am writing it as S t plus 1 is equal to S t plus Q t minus R t, neglecting losses. And therefore, when I am writing this for t is equal to 1, this becomes S2 should be equal to S1 plus Q1, which is 4 minus R1, which is 5 is equal to S2. Like this, you go S2, S3, S4, S5, S6, then when you are writing for t is equal to 6, the S7 becomes S1, because as I said you assume that the same sequences repeat. So, this is t is equal to 1 to 6, t is equal to 1 to 6, 1 to 6, t is equal to 1 to 6, etcetera.

So, at the end of 6 time period you are going into time period 1. So, the storage at the end of the time period 6 is equal to the storage at the beginning of the time period 1, which is S1. So, this is how you write the storage continuity equations, and always pick up the flows. So, this is for t is equal to 1, t is equal to 2 etcetera, t is equal to 6. So, 6 storage continuity equations by picking up the flows and the releases or the demands from the data that is given, and you write the conditions like this. Then you have the set of constraints S t is less than or equal to K. So, the storage at any time period must be less than or equal to the K that you are obtaining from the optimization problem. And that is what you do here; S1 is less than or equal to K, S 2 is less than or equal to K, etcetera.

This is the rather long way long hand way of writing this, a model; there are very simple ways of writing depending on the software that we use; we will worry about all those things later, but this is the complete model. There are 6 time periods, you write the continuity for the 6 time periods; and, write also the associated storage limitations, and then you are looking for minimize K. So, you are looking for that particular value of K, which meets this set of demands, for this given set of inflows, and meeting the reservoir continuity equations, and meeting the reservoir capacity constraints; this is the model; we solve this and you get the solution like this. So, you get K is equal to 10, which is the same as what you got for the sequent peak algorithm.

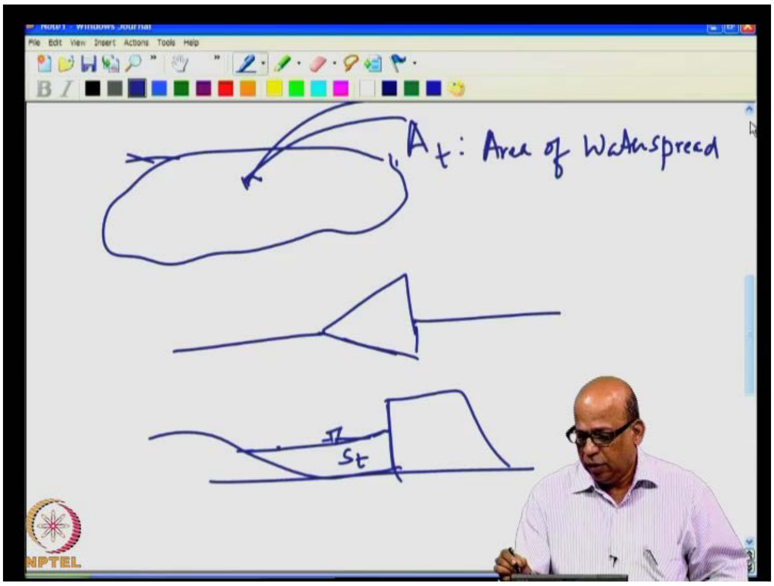
As the solution earlier, algorithm and these are also associated with a beginning storage of S1 is equal to 1, this comes as the solution; these are optimal solutions. And once, S1 gets fixed, all other storages also get fixed S 2, S 3, S 4 etcetera. So, this is the solution that you obtain, when you solve this example. How do we solve this example? You can use any of the available routines, for example, we use a routine called LINGO - Linear and General Optimization; this is a software. It is available for educational purposes, for free download from this from the LINDO Systems Softwares. And they it is a very simple software to use; in fact, Matlab also has softwares to solve optimization problems; you can use any of those, and then solve such problems. So, this is the solution now. Now, what did we do in this, we have ignore the losses, and wrote this continuities simple in simple fashion ignoring the losses. Now, we will graduate further, and start looking at inclusion of evaporation losses.

(Refer Slide Time: 48:13)



Now, the evaporation losses will depend on the storages. You understand this part correctly. Let us say that, in time periods t, this was your S t; now typically, the storage will have a contour level like this, and this is the reservoir storage. So, this is the water spread area. So, if you look from top, the water spread area, we look something like this.

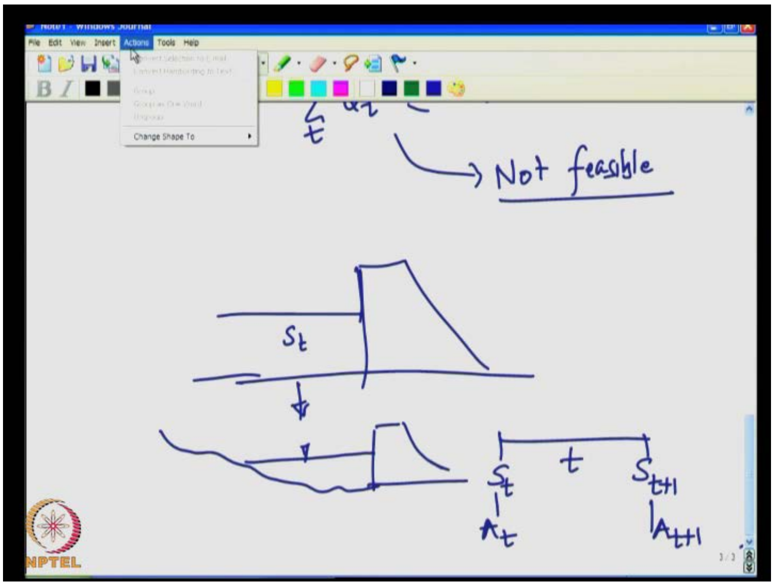
(Refer Slide Time: 48:51)



And now this will be the water spread area, and this continues as and this is in the plan section. So, you are looking at this area now - water spread area, this is the exaggerated point, and in the section you are looking at the storages like this, this is the storage value.

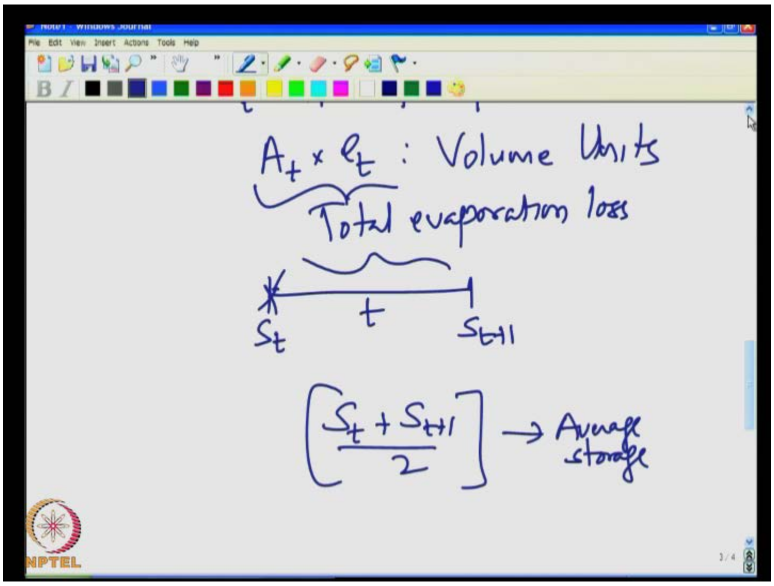
So, at any given time period t, let us say this was S t, this corresponds to this water spread area which may be this, this may be A t we will call it as A t, this is the area of water spread. And this area of water spread depends on the contour levels, at that point, like I showed here. So, it will depend on how the contours are changing. Let us say, St St becomes St plus 1 here, then the area of water spread will be corresponding to this. So, in the time period t, the storage changes from t St to S t plus 1.

(Refer Slide Time: 50:02)



Let us say, this is the time period t, you had a storage S t at the beginning of the time period that changes to S t plus 1 at the end of the time period; and therefore, the area changes from A t to A t plus 1, because as the storage changes the area of water spread changes.

(Refer Slide Time: 50:40)

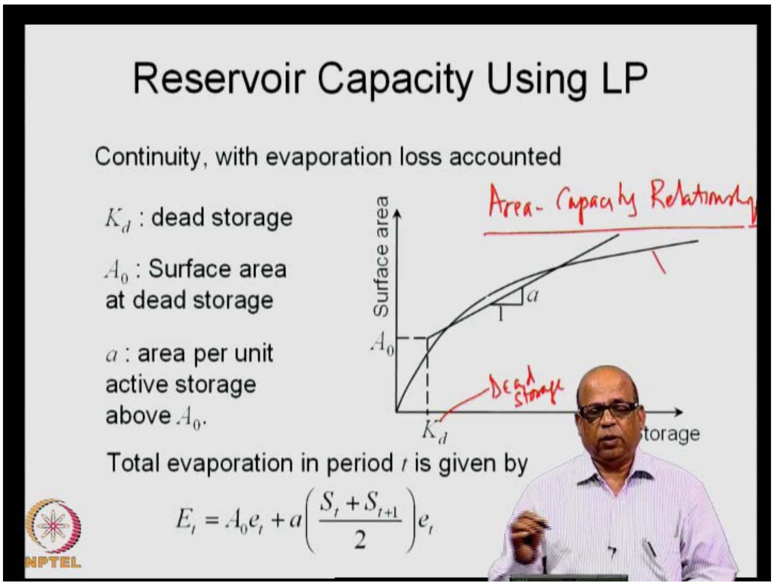


Now, the evaporation losses are dependent on the cross section area. So, what do we have? We have the rate of evaporation e t. Let us say, e t is the rate of evaporation; and this will be typically in depth units millimeters, and specifically we may have pan evaporation rates or actual majority evaporation rates etcetera. So, it is in depth units, when you multiply e t by A t, this you get it in volume units, and this is the total loss. However, what happens is, because t is an interval, you had one storage here, and another storage here.

So, to reckon the total losses that happen during this time period t, what we do is we take the average storage S t plus S t plus 1 divided by 2, this is the average storage during the time period t; during in this time period t; we calculate the area associated with this average storage, and that average area or the area associated with the average storage, we use on e t to compute the evaporation loss during the time period t. So, this is what we do in accounting for storage accounting for the losses as storage dependent losses.

So, essentially we look at the area associated with the average storage S t plus S t plus 1 divided by 2; and then use that area to get the total evaporation loss during the time period t. Now, when we are doing this, we use some simple methods of a accounting for this accounting for the evaporation losses, because we are interested also in making the relationships linear. So, at this point what I am discussing now is, in this type of LP model, how do we account for storage dependent evaporation losses is the question that I am discussing now.

(Refer Slide Time: 53:24)



For the storage dependent losses, you need the surface area corresponding to a given storage, and these are typically given by what are called as area capacity relationship, and these are available for a given side; that means as your storage grows as your storage increases, how the area surface area increases or how the surface area changes is provided by the area capacity relationships. Typically, the area capacity relationships are non-linear like this. So, this is the area capacity relationships. But for use in linear programming, we cannot have the non-linear relationships or it is slightly cumbersome to use the non-linear relationships, we can also have the piece-wise linear relationship and so on, but what we do is that we approximate this with the straight line, beyond the dead storage.

So, this is the dead storage, and the area capacity relationships beyond the dead storage level is approximated by a straight line, and then we convert that use this straight line relationship and express the evaporation losses as storage dependent losses. We will do this exercise in the next lecture. So, essentially in today’s lecture, what we did is, we started with the sequent peak algorithm for determining capacity of the reservoir, the minimum required capacity, and we are talking about the live capacity of the reservoir. To meet a certain demand sequence for a known inflow sequence, and in the sequent peak algorithm, essentially what we do is, we capture that critical time period in the sequence, and if it is not available in the first sequence, you do it two cycles; so that, you capture the critical period, and compute the deficit in Samson's and Reckon or a Reckon the reservoir capacity as the maximum of such deficits. However, inclusion of evaporation losses in the sequent peak algorithms and also accounting for several reservoirs, simultaneously will be cumbersome and therefore, we look at the optimization problems.

In the optimization problems, we seek solutions to the minimum required capacity K. So, that is why we wrote the objective function as minimize K, subject to the continuity equations that is S t plus 1 is equal to S t plus Q t minus R t, simple mass balance equation, neglecting the losses, and subject to the maximum storage constraint; that is S t should be less than or equal to the capacity itself. So, this becomes are very compact linear programming problem, and remember there R t is pre-specified. So, S t plus 1 is equal to S t plus Q t minus R t when we write, R t will be pre-specified as the demands. So, you want to meet the demands R t and then you look at that minimum capacity requirement.

We solve the example using a 6 period problem, in which the sequence of inflows are specified, sequence of inflow is specified as well as the sequence of demand is specified; we saw that you get the same capacity of 10 units. Then towards the end of the lecture, we are just talking about, how to incorporate storage dependent losses in the continuity. Now, the evaporation losses will depend on the water spread - area of water spread. And the area of water spread itself will be changing as the storage changes, because it depends on the contour levels - the ground levels.

So, you are actually looking at how much of area of water spread exist corresponding to a given storage, and this relationships is given by the storage area capacity relationship or area storage relationship. And we at any given time period t, we look at the average storage S t plus S t plus 1 divided by 2, and look at what is area of water spread associated with the average storage during this time period, and then use the rate of evaporation, and obtain the total loss as storage dependent loss. And we will do this exercise in the next lecture and rewrite the continuity equation by accounting for storage dependent losses, and reformulate the LP problem looking at the storage dependent losses. So, we will continue this discussion in the next lecture. Thank you for your attention.