Water Resources Systems

Modeling Techniques and Analysis

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Lecture No #19

Multi-objective Planning

Good morning and welcome to this the lecture number 19 of the course Water Resources Systems Modeling Techniques and Analysis. In the last lecture, we essentially concluded our discussion on the systems techniques, by introducing simulation; essentially, I talked about three different problems, where simulation can be used; one is to simulate the reservoir operation under a specified operating policy, and we took a very small a numerical example through which we showed, how we simulate the reservoir operation using the Standard Operating Policy, the so-called Standard Operating Policy; where the policy specifies that, you meet the demand to the best extent possible in each of the time periods; that means, simply look at the total water available, S plus Q the storage plus the inflow during that time period, compare it to the demand, if the total amount of water available is more than the demand, meet the complete demand; and if S plus Q is less than the demand, you empty the reservoir; that means, supply all the amount of water that is available to meet the demand to the best extent possible. So, this was the simple policy SOP, it is called as Standard Operating Policy.

And we saw, how we simulate the reservoir operation using the standard operating policy, it was the simple numerical example; when I deal with applications perhaps I will be able to show some more complex systems through which, we will simulate using the Standard Operating Policy. Then we also looked at multi-reservoir systems; I I just introduced, how we simulate the multi-reservoir systems; we look at continuity at each of the reservoirs, by accounting for, not only the natural flow that comes from the catchment of the reservoir itself, but also the control flows from the upstream reservoirs, upstream control structures that contribute to the inflow at this particular reservoir.

So, like this from one one reservoir to another reservoir in a downstream direction we move; and then account for all the water flows in a systematic manner and that is how we simulate the operation. There can be any number of details that can be added to this, for for example, if you are looking at system simulation for hydro power generation; you can add all the hydro power details to that, and relate the hydro power generated with respect to the release that you are making, and with respect to the storage which governs the head that is available in the next 1 or 2 lectures, I will deal with this particular problem of simulation, specifically for hydro power.

Then you may talk about simulation specifically for irrigation, where you will also look at the soil moisture balance from time period to time period, how the root growths are growing, crop root growths are growing, and how the uptake of water takes place and that you relate it with the actual water, utilized by the crops, and then you come upstream, and then look at, what is the water available at the canal level, and then you relate it with reservoir operation. So, the entire thing from the reservoir up to the crop level, you integrate in a simulation.

Then we also talked about simulation of real time reservoir operation; in real time, the issue of operation is standing at the end of a particular time period, you would like to decide, how much to be released from the reservoir during the next time period, and typically these time periods if you are operating for irrigation purposes, these time periods can be 10 day periods, 15 day periods and so on. So, you need to look ahead in time of what is likely to be the inflow to the reservoir, and what will be the likely rainfall in the command area, and then decide on your reservoir operation in real time, as you are progressing in real time.

Now, this is typically done by using a inflow forecasting model; starting with the given storage, you know what is the likely inflow that is, likely to come to the reservoir in the next time period, of let us say 10 day period; and then based on this information, you make the release; and then at the end of next time period, you know exactly, what has been the release; what has been the inflow, update the state of the system with the actual inflow, and then make your release decision again. So, this is the real time reservoir operation; again, when I deal with applications, I will demonstrate how we do this. So, that completed essentially the systems techniques that we wanted to touch upon.

We dealt with optimization techniques using the calculus, both constrained as well as unconstrained optimization; then we dealt with some dealt with in some detail the linear programming technique, then we went on to discuss the dynamic programming, then for the last lecture, I also introduced simulation; irrespective of the technique that we use, the water resource systems, problems, as I mentioned in the previous lecture, are characterized by not one objective but several objectives.

So, we started introducing multi-objective planning, where you are looking simultaneously at several of the objectives and you know, typical water resource systems, we will always involve several stakeholders with conflicting objectives, and that is where we start looking at multi-objective planning, multi-objective operations or in general, multi-objective optimization. So, this is what we talked about in the last lecture; we will continue that discussion now.

So as I mentioned, you will have several objective, let us say you are looking at hydro power; hydro power is one objective, irrigation is another objective; you are drawing from the same amount of water both for hydro power, as well as for irrigation, and therefore if you supply more to hydro power, irrigation will suffer; if you supply more to irrigation, hydro power will suffer; and therefore, the objectives will be conflicting with each other; you have to make a decision in the phase of conflicting objectives.

Similarly, you look at flood control as one of the objectives; flood control requires that your reservoir levels be low; so that you can absorb the flood waters into the storage, whereas both irrigation as well as hydro power require that your storage be as high as possible; so that, you can use the water during the non inflow periods typically, in monsoon countries during the non monsoon periods, you would like to use the water that is stored during the monsoon periods. So, during the monsoon periods, you accumulate the storage; use that storage during the non monsoon periods;

And therefore, these objectives will be conflicting; a hydro power, irrigation, flood control will be conflicting with each other; like this, then you also add environmental objectives; environmental objectives typically, require that you release as much water downstream as possible to maintain the ecology and environmental integrity, downstream of the reservoir; however, your developmental objectives, for example, hydro power and irrigation, municipal and industrial supply etcetera, require that you supply the water to those purposes and not for downstream, and therefore the objectives will be conflicting with each other; like this as you keep on adding more and more complexities, more and more stakeholders into this planning problem or operational problem, the objectives - the number of objectives increases, also the degree of conflict among the objectives also increases, and this is where we start looking at multi-objective planning or multi-objective optimization and so on.

So, we will not be then interested in a single valued optimal solution; in the sense that we talked about solutions of linear programming, dynamic programming classical optimization techniques etcetera, where we may we would be interested, typically in what is the optimal solution; what is the single optimal solution to that entire problem; as you start including more and more details, as you start including more and more objectives into the planning problem, you have to start looking at the tradeoffs, what happens if I want to maximize one particular objective function, what happens to the other objective function.

Therefore, we start looking at tradeoffs, and like I introduced in the last lecture, we will be looking at the non-inferior solutions, set of non-inferior solutions, which is also called as set of Pareto optimal solutions. So, we are not looking exactly at one optimal solution, but you will be looking at tradeoffs between several objectives, and we would be looking at screening among several possible solutions. So, that is what we do in the… We will introduce how we do that in the present lecture. So, there are two ways of handling this, like I said, you have p number of objectives Z 1, Z 2, Z z p etcetera.

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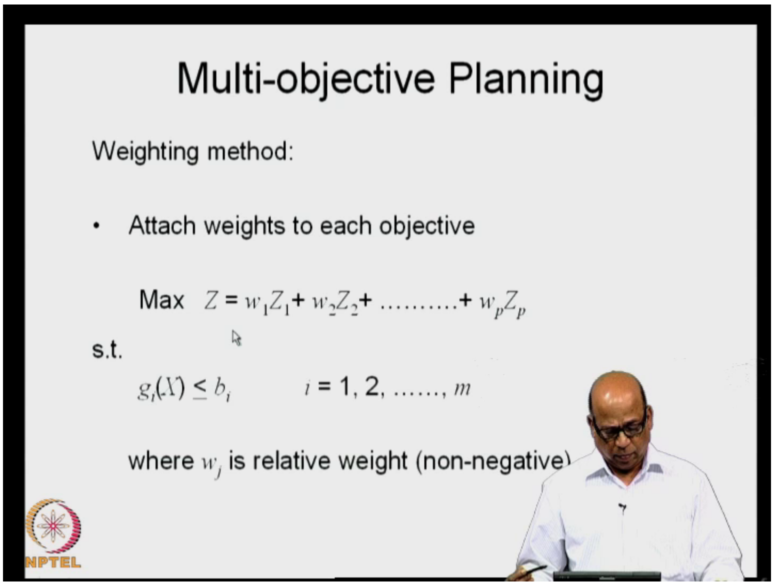

The first method is the weighting method; that means essentially sorry we assign weights to each of the objective functions Z1, Z2, Z3 etcetera are all objective functions to be maximized; let us say Z1 corresponds to irrigation, Z2 corresponds to flood control, Z3 corresponds to hydro power and so on. Like this, if corresponding to each of the purpose for which you are using the water, you may have objectives defined; then you have a set of constraints g i of X less than or equal to b i, and X is the vector of decision variables here; you may have m number of constraints; remember these m number of constraints will remain the same for each of the objectives, because these are physically defined constraints, for for example, they may deal with land availability, they may deal with water availability in a particular time, they may deal with minimum required release downstream and so on.

So, all of these are physically defined constraints that need to be satisfied always, irrespective of the objective function; recall what we did in the linear programming, relate this with what we did with the in the graphical method of solution; the first thing we do is look at the feasible space; now the feasible space is essentially defined by the constraint space, the set of constraints; it has nothing to do with the objective, simply you define the feasible space based on the constraints. So, the constraints have nothing to do with the objective, you may have your own operational objectives, planning objectives etcetera, but the set of constraints is defined by physical features of the system or planning requirements or operational requirements and so on; for example, you may set a higher release as a constraint based on the canal capacity; lower release as a constraint based on the downstream requirement and so on. So, these will always remain the same, irrespective of whether you are maximizing your hydro power, maximizing irrigation and so on.

So, these constraints are independent of these objectives, and they will always remain the same, irrespective of how many objectives that you consider. So, Z1, Z2, Z 3 etcetera, these are all objective functions to be maximized. The first way of handling this, one of the ways of handling multi-objective problems is to assign weights w1, w2 etcetera; assign weights to each of the objective functions; let us say you are looking at drinking water supply, irrigation, hydro power, flood control and so on. So, you may assign a very high weightage to the drinking water supply objective. The next level weightage to, let us say hydro power, next level to irrigation and so on. So, you may assign weights, based on the importance of the objective function as far as that particular system is concerned; obviously, these weights will be based on some judgment about the system.

So, these are typically decided based on what kind of priorities that you would like to give to different objectives, like I said drinking water may get always the highest priority, and therefore you may assign a very high weight to drinking water, such that the other objectives will be satisfied only after the first objective is fully satisfied and so on. So, you assign the weights, and then solve the solve the optimization problem, let us say Z1, Z2, etcetera, these are the objectives; all of which will be functions of the decision variable X, decision vector X, Z1 is the function of X, Z 2 is the function of X and so on. And therefore, this optimization problem can be solved, by solving for X - decision vector X; now the weights w j, these are actually the relative weights, they they should be non-negative, we assign non-negative weights and write this as a maximization problem.

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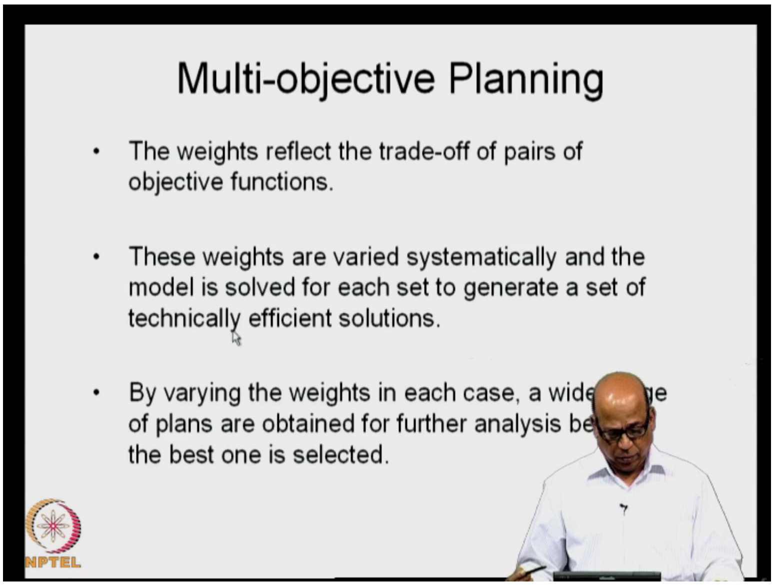

The weights actually reflect a tradeoff between different objectives; that means, your preference is to different objectives, objective functions is actually decided by the weights; typically you know in multi-objective problems, what we will do is we will not aim at one single solution; we will be looking at if I want to increase one of the objective functions, how much should I decrease the other objective function. And therefore, we will be typically looking at the tradeoff between two and more objective functions, and this we do by varying the weights, let us say that you assigned certain set of weights, and then keep varying these weights, and then see how the tradeoff is between different objective functions.

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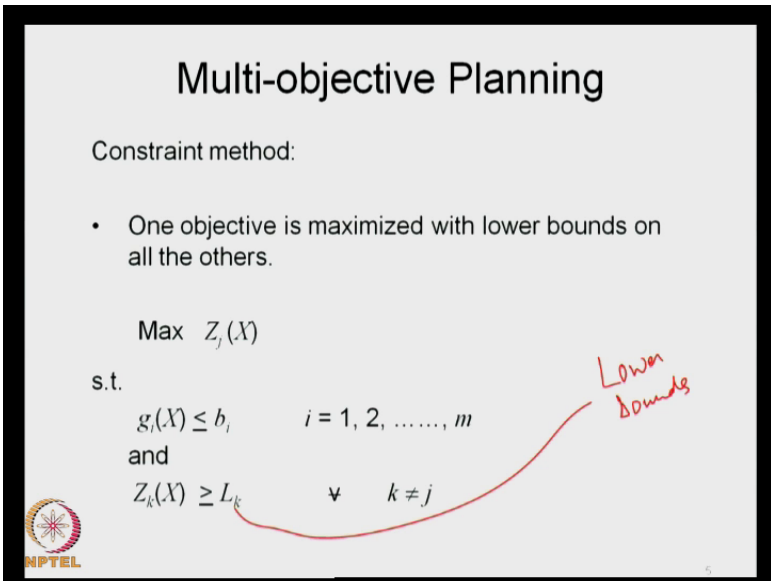

We will do that through a simple example; the other method is the constraint method, let us say you had p objective functions in the weighted weighting method, what did we do? We assign weights to each of the p objective functions; in the constraint method, what we do is, we pick up one of the objective functions and maximize that objective function, while setting lower bounds to all the remaining objective functions. Let us say maximize Z j of X, one of the objective functions, out of the p objective functions, we maximize one of them, and set lower bounds in a maximization problem, set lower bounds to all the other objective functions.

By stating Z k of X greater than or equal to L k, if this problem has a feasible solution, then all of these objective functions will be at… Will take on values at least equal to L k for Z k; so the Z k objective function will take a value at least equal to L k, and then subject to this condition, we will maximize this particular objective function. So, if you have two objective functions, let us say drinking water and irrigation; I may say that you maximize the drinking water objective function subject to the irrigation objective function meeting at least a particular level, in terms of crop yield or in terms of the economic returns and so on. So, like this we set lower limits on other objective functions, k not equal to j; and we maximize one of the objective functions, this is the constraint method.

Again, here in the constraint method also, we can play around and generate several sets of solutions, what do I mean by that, is every time you change this j, j eth objective function; for example, first you maximize drinking water subject to irrigation being at a certain level, then you try to maximize the irrigation objective function, subject to drinking water being at a certain level and so on. So like this, you generate tradeoffs between several objective functions; so that, you have a hand full of solutions to play around, and then decide what you would like to do in the planning scenario.

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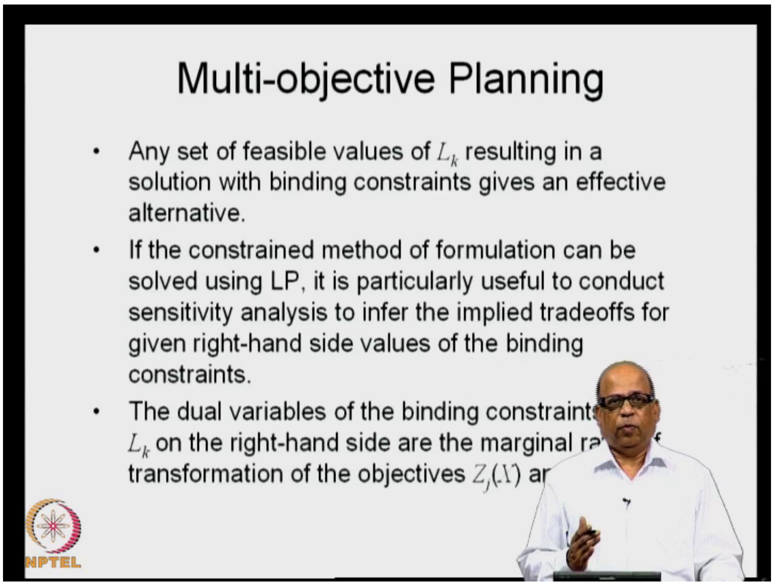

Now remember, I introduced what are called as the non-inferior solutions or the Pareto optimal solutions and so on. So, in the constraint method here, when you solve this objective, as I solve this problem, in the optimal solution, if these constraints are binding, what I mean by binding, you recall that a constraint is binding in the optimal solution; if the left hand side of the constraint is equal to the right hand side in a greater than or equal to, or in a less than or equal to constraint; that means, it has been fully satisfied, exactly at that point it has been satisfied. So, there is no slack available for that particular constraint in a less than or equal to constraint. So, then we call it as a binding constraint.

So, you can keep playing around with this L k values, generate more and more solutions associated with these L k values, all those solutions which have Z k of X greater than or equal to L k constraints as binding in the optimal solution; these will define non-inferior solutions, these will define a set of non-inferior solutions. Now if your objective functions and the constraints are such that the resulting problem including your Z k constraints is in fact, a linear programming problem; then you solve using the linear programming algorithm, and look at the Z k greater than or equal to L k constraints, if they are all binding constraints, then you are generating a set of non-inferior solutions.

The dual variables associated with these constraints Z k greater than or equal to L k, can then be use to look at the sensitivity of the solutions, refer to my earlier lecture one of the earlier lectures, where we have introduced the sensitivity of the objective function, with respect to the right hand side values. So, we use that dual variables associated with these particular constraints, and then check the sensitivity; these are some of the details, we will look at when we look at some applications. So, typically we will be interested in the tradeoff between Z j and Z k. So, we can use the dual variables, and obtain the tradeoff or the rates of transformation, marginal rates of transformation between these two objective functions.

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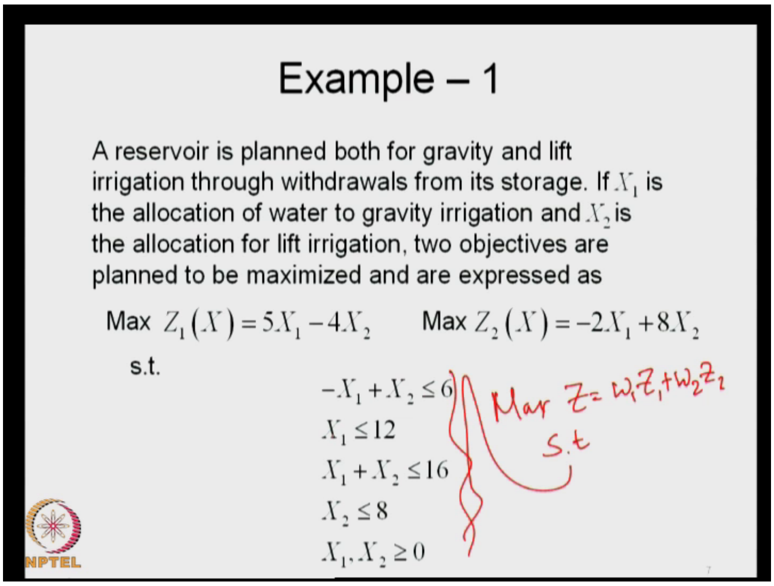

Let us look at one example now; we are looking at a reservoir, which is serving irrigation both gravity irrigation and lift irrigation; through withdrawals from its storage; remember that, lift irrigation requires pumping and gravity irrigation is simply let let the water through the canals. So, you have two different irrigations schemes, there supplying water for two different irrigation districts, two different irrigation area; lift irrigation supplies to certain area, a gravity irrigation supplies to certain other area; now X 1 is the allocation of water to gravity irrigation, and X 2 is the allocation for lift irrigation.

Let us say that we then formulate two objective functions, maximize Z 1 of X and maximize Z 2 of X like this; there are two objective functions. Now these may be related to the returns that you get out of the water allocations, these may be related to the crop yield, these may be related to the power that is consumed and so on. So, if you are looking at power that is consumed, you have to convert it into the costs that are associated with it. So, we will write this as maximize Z 1 of X, and maximize Z 2 of X; remember there is a negative term here 5X 1 minus 4X 2, and also here minus 2 X 1 plus 8X 2.

And these are the set of constraints; like I said these constraints are actually, typically defined by the physical constraints arising out of the problem, and we will assume that, these are the constraints that are defined by the particular physical system. So, you have two objectives both of which have to be maximized, let us first solve it by using the weighting method; if you look at the constraint set here, you also look at the objective functions, and the constraints sets, the constraints sets are all linear functions of the two decision variables X 1 and X 2, and based on that you define your feasible space.

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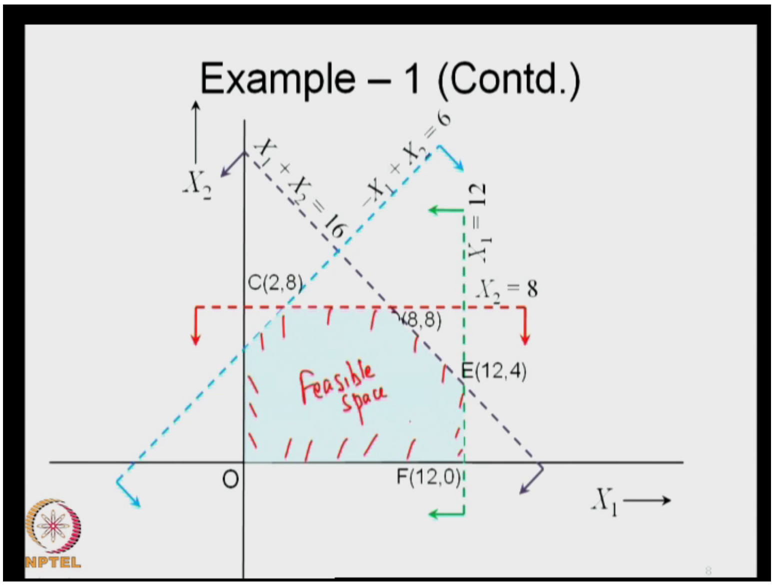

So this becomes the feasible space. (No audio from 23:50 to 24:03) So, you have the corner points, this is (0, 0), this is (12, 0), this is (12, 4), (8, 8) and (2, 8). Now, within the feasible space, now you need to look at the optimal solution, because there are two objective functions now.

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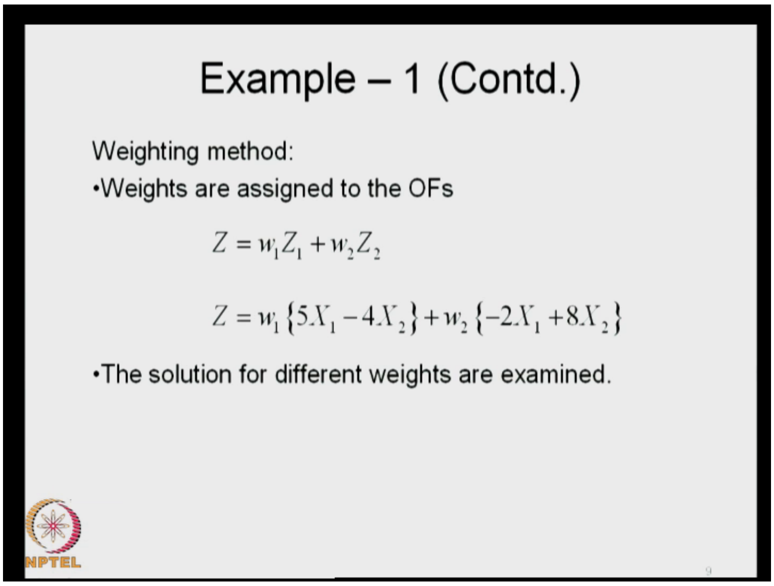

What we will do is we will assign weights, w 1 Z 1, and w 2 Z 2; so, this is what we do. We write a new objective Z as equal to w 1, Z 1 plus w 2, Z 2 and Z 1 is 5X 1 minus 4X 2, Z 2 is minus 2X 1 plus 8X 2. We can keep on generating different solutions by assigning different weights w 1 and w 2. So, that is what we will do now.

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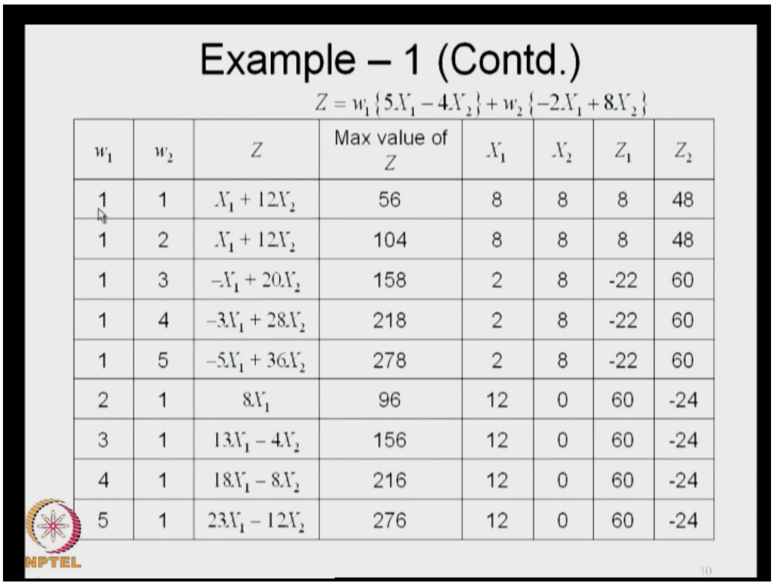

w_1	w_2	Z	Max value of Z	X_1	X_2	Z_1	Z_2
1	1	$X_1 + 12X_2$	56	8	8	8	48
1	2	$X_1 + 12X_2$	104	8	8	8	48
1	3	$-X_1 + 20X_2$	158	2	8	-22	60
1	4	$-3X_1 + 28X_2$	218	2	8	-22	60
1	5	$-5X_1 + 36X_2$	278	2	8	-22	60
2	1	$8X_1$	96	12	0	60	-24
3	1	$13X_1 - 4X_2$	156	12	0	60	-24
4	1	$18X_1 - 8X_2$	216	12	0	60	-24
5	1	$23X_1 - 12X_2$	276	12	0	60	-24

Let us say I assign equal weights, w 1 is equal to 1, w 2 is equal to 1; then my Z expression is this; w 1, 5 X 1 minus 4 X 2 plus w 2 etcetera. So, Z expression becomes X 1 plus 12 X 2 and you simplify that with w 1 equal to 1, w 2 equal to 1 and then you solve this problem. So, you are solving maximize Z is equal to w 1 X 1 plus Z is equal to w 1 Z 1 plus w 2 Z 2. This is the problem you are solving; subject to all these constraints, the set of constraints. Now when you solve that it is a two variable problem, so you can use your graphical method and solve; when you solve this, you will get the solution X 1 is equal to 8, X 2 is equal to 8, Z 1 is equal to 8, Z 2 is equal to 48, with maximum value of Z 1, Z is 56.

So, essentially what I did is, I solve this problem using the graphical method using Z as w 1 Z 1 plus w 2 Z 2, w 1 was 1 here, w 2 was 1 here, and that is a problem, that I have solved. I get the value 56 for Z, and (( )) 8 48, when I change returning the weight w 1 the same, I keep increasing w 2 now; remember, what is the Pareto optimal solution? Pareto optimal solution or the non-inferior solution is that particular solution, at which you cannot increase the value of one of the objective functions without decreasing a… Without simultaneously decreasing the value of the other objective function.

So, as we increased w 2, you have to looking at what is happening Z (( )). So, you are saying that my importance to w objective number 2 is (( )) So, 2 must remain at the previous level or it should increased as your w 2 increases, objective 2 value increases, but simultaneously the Z 1 cannot increase; so, the Z 1 will either decrease, or will it will remain the same; beyond a certain point the objective function Z 1 and Z 2 not be further increased, they will all remain at the same point, because otherwise, the solution starts becoming infeasible. So, if you have a feasible solution, then it will converge to certain values as you start increasing one of the weights, keeping the other weights as same; then what we will do is, we will retain w 2 at 1 and start increasing w 1.

So, w 1 starts now in this particular case, for the type of problem that we have it leads the same solution 60 minus 24 for all of this; however, you look at what has happening here, you got as (( )) one of the solutions (8, 8) was here. So, you are moving from as you change your weights, you are moving from one corner to another. So, the optimal solution was (8, 8) for w 1 equal to 1, w 2 equal to 1; when you change the objective function value the weightage to 1 and 2, you still are at (8, 8); when you are changing it to 1 and 3, you went 2 and 8; so, you went to 2 and 8; from this point, you move to this point; like this you move from one point to another point, then you go to (12,0) and so on. So, this generates a set of solutions for Z 1 and Z 2 as you change the weights.

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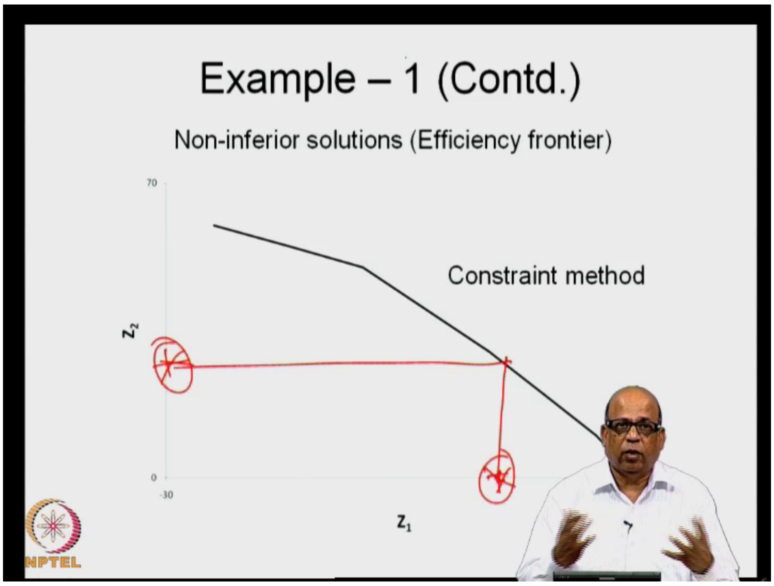

We now plot the Non-inferior solutions. So, we pick up Z 1 and Z 2, and then simply plot Z 1 versus Z 2. Now, this is called as the Efficiency frontier; we are dealing only with two objective functions; when you have more number of objective functions, you can pick up sets of two objective functions, and then plot them like this, let us say Z 1 versus Z 3, Z 1 versus Z 4 and so on, when you have number of objective functions. As you can see here, for any given value of Z 1, you can read the corresponding value of Z 2, also as Z 1 increases, Z 2 has to come down, that is how you get non-inferior solutions, efficiency frontiers; now this will give you a flexibility of planning for your either the operation in the in this particular case, how much to be release for a lift irrigation, how much to be release for your gravity irrigation and so on.

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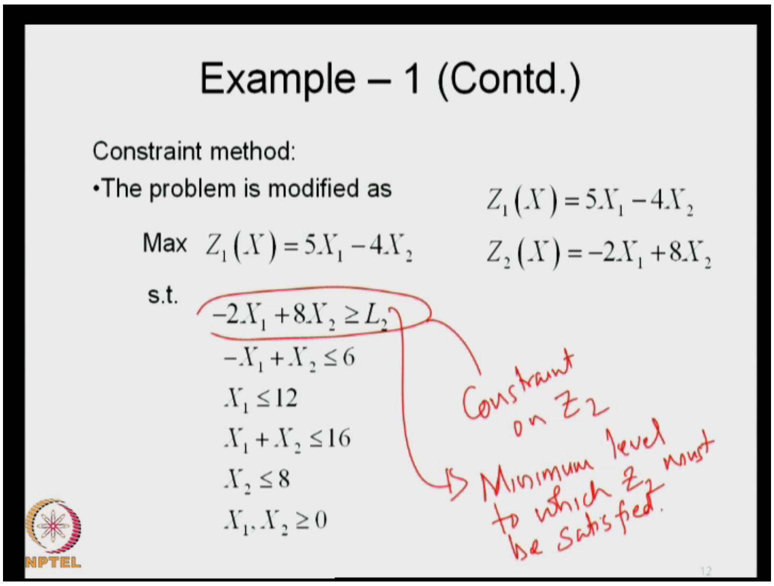

We will now do the same problem, we will carry out with…we will solve the same problem using the constraint method. In the constraint method, we pick up one of the objective functions, maximize that in maximization problem, and we set minimum levels for the other constraints, other objective functions. So, in this particular case, we had two objective functions; we will pick up Z 1 X first; now Z 1 X and Z 2 X are the two different objective functions; we will pick up the first objective function Z 1 X will maximize that, and we will put a constraint on the second objective function. So, this is the constraint on the second objective function, and these are the lower levels, (No audio from 30:52 to 31:00) which Z 2 must be satisfied; (No audio from 31:05 to 31:12) physically, what does it mean? Let us say that, in in our example, Z 1 X in some sense related with gravity irrigation, and Z 2 X dealt with the lift irrigation.

So, we are saying that, my lift irrigation objective function must be at least L 2, and then given that it is at least L 2, you then maximize the gravity irrigation; let us say drinking water, you want to meet the drinking water quantity at least to a given level L 2, and then you maximize irrigation like this. So, this constraint has to be always met, then subject to the condition, that it has it is always met, you maximize the other objective function. So, this is what we do in the constraint method; again here, it is not one solution that is important, we play around with L 2, start increasing L 2 starting with the minimum value and then see how the other objective function behaves; so that is what we will do; now any optimal solution for assumed value of L 2 is a non-inferior solution, if the L 2 constraint is binding like I mentioned earlier.

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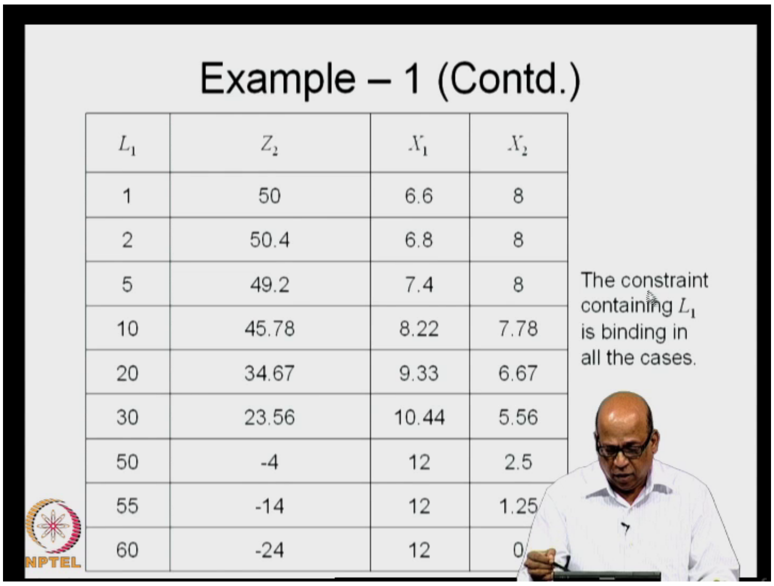

So, we will vary L2 and get the associated values of Z 1, remember we are maximizing Z 1 subject to the condition, that the Z 2 is at least equal to 1, for L 2 equal to 1, Z 2 is at least equal to 2, Z 2 is at least equal to 5 and so on. So we are setting a lower bound on Z 2, and then maximizing Z 1. So, this is the type of tradeoff that we generate. So, and in this particular case, the constraint containing L 2 is binding on all the cases, which means what? When we obtain the optimal solution Z 2 was equal to 1 here, Z 2 was equal to 2, Z 2 was equal to 5, etcetera, that is when we say it is the constraint is binding and because the constraint is binding this set in fact, reflects the non-inferior set of solutions, again because the there are only two variables involved only two objectives and so on.

You can solve this using a graphical method, otherwise you use any classical software that is available for solving a linear programming, hopefully I will introduce one of the simple softwares, when I am dealing with applications, but solving a L p, you are now well versed with it, you can either write your own program or use any classical software, Matlab has a routine for linear as well as non-linear optimization, you can use that and solve these problems.

So, you will get a tradeoff, why I keep on to a calling it as a tradeoff is because as your Z 2 is increasing Z 1 has to come down. So, this is how you generate a tradeoff between Z

2 and Z 1, this I am saying Z 2, because of the constraint that I have put here, and because it becomes binding it becomes equal, instead of greater than or equal to in the binding situation it becomes equal and therefore, Z 2 will be equal to this value; and then we generate the non-inferior solutions all I am doing is, I am plotting Z 1 versus Z 2 here. So Z 1 on this axis versus Z 2 So, this is the set of non-inferior solutions. This also called as Efficiency frontier. This also called as Pareto optimal frontier; Pareto was the author, who has introduced this Pareto was the researcher, who has introduced this concept of Pareto optimal solutions.

So far in the constraint method, you get this Pareto optimal solution; what did we do here, we set a minimum level of meeting the second objective, and then maximize the other objective - first objective, we will reverse this now, and see what happens that is we will set a minimum level for objective number 1, and maximize the objective number 2. So, while we reverse that, and say I want to maximize the second objective, subject to a minimum level of the first objective. So, I am putting a constraint on the first objective, everything else remains the same, I will generate solutions for Z 2 now; now this is again for all the solutions the constraint, containing the objective function number 1 was binding and therefore, Z 1 is equal to 1, Z 2 is equal to 2, Z 2 is equal to 5 etcetera.

So, the this is how you generate another set of non-inferior solutions. So, you get a solution like this, non-inferior solutions, again this looks the similar to what we obtained earlier, but there will be of course, it will not be identical, because your your way of handling has changed now, it will not be identical, because you are setting constraints on Z 2, and maximizing Z 1 here.

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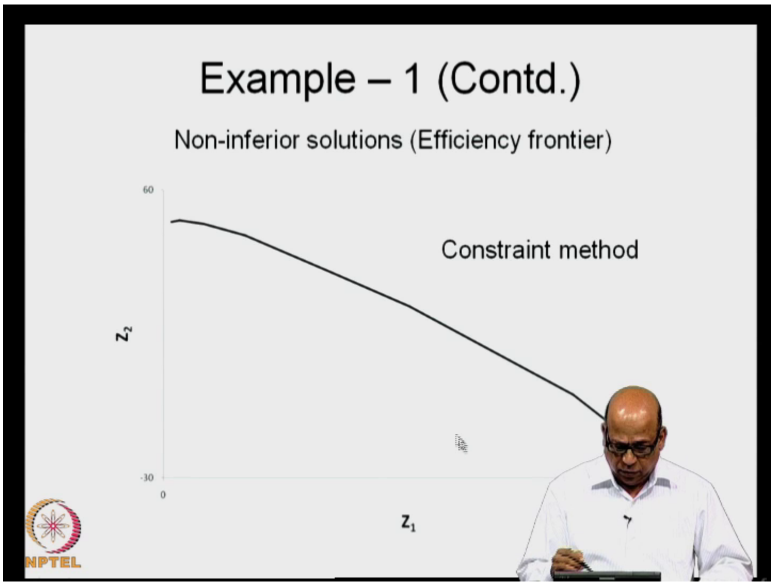

So, this is the solution that you get; like this, when you have several objective functions, now how do we use this any of this non non-inferior solutions; you would like to use, let us say that I want to examine, what happens if my Z 1 is at this level, and then I will get associated value of Z 2. So, knowing what Z 1 indicates physically and Z 2 indicates physically for example, in our case it may indicate lift irrigation versus gravity irrigation.

So, knowing that you can decide at what level you like to operate this, now both these will be, all of these will be non-inferior solutions, which means they are equally acceptable; the acceptability is the different criterion; in fact you can screen several such… You can generate several such non-inferior solutions and then, screen out depending on your priority, depending on acceptability and and so on.

So, right now our aim was to generate a large number of non-inferior sets of solutions to a given problem containing several objective functions, and that is what we have done here; after this is done you can have several screening alternatives, screening of alternatives for example, we are we're dealing with not 2, if we are dealing with not 2, but let us say 10 objective functions; and then, the non-inferior sets of solutions will be quiet large, and then out of these large non-inferior sets of solutions, you may screen out some of them depending on several other criteria, which I will just briefly mentioned towards the end of this.

So, we dealt with two methods: one is weighting method, and another is constraint method, and generated a Efficiency frontier or Non-inferior solutions frontier or Pareto optimal frontier and so on, which essentially gives a tradeoff between different objectives as one objective comes down by a certain amount, what is the sacrifice that you have, what is the gain in the other objective function like this. So, these are the type of questions that we answer using the non-inferior frontier.

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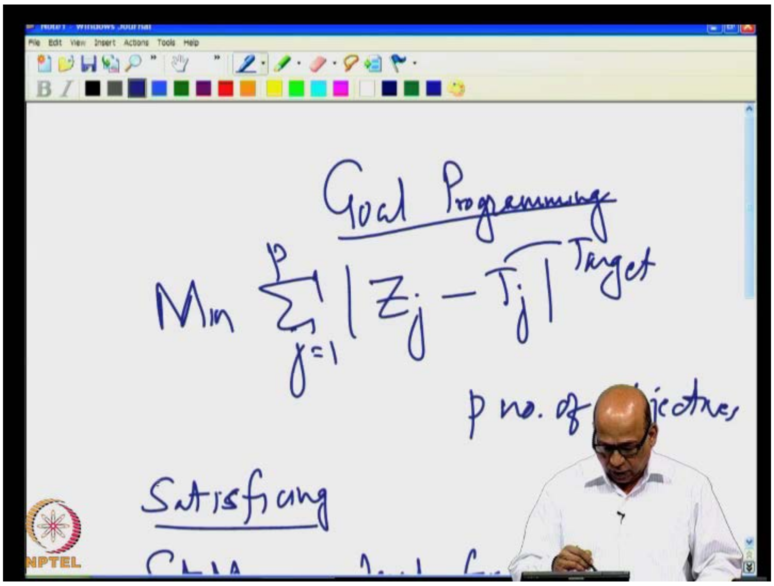

Now, there are other ways of screening out the alternatives for example, we may have the goal programming; in goal programming, what we do is that we set targets to each of the objective functions, for example, there are p objective functions, and you may set goals of attainment of each of the programming, each of the objective functions. So, we may say Z j is my objective function, I may set a target of T j for meeting this particular objective function, and the absolute value of the difference between Z j and T j for example, you may be talking about the amount of water release for irrigation, gravity irrigation that may be your Z j, then you may say that my target for that annual amount of water release for irrigation is T j, which means, you would like to meet this always, your goal is to meet this particular target, and Z j which is the actual objective value that you attained minus T j, which was the target, this is your target, the absolute difference, you would like to minimize over all js.

So, you may want to minimize the absolute difference, the sum of absolute differences over all the objective functions, there are p objective functions, p number of objectives; I will subject to all the constraints that you had originally. So, this is what we do in goal programming; now there is also something called as satisficing, these are just the methods that you must be aware of as existing satisficing. Now in this, what we do is that we set minimum levels (No audio from 41:33 to 41:38) for each objective function (No audio from 41:42 to 41:47) that is (No audio from 41:50 to 42:00) similar to what we did here, here we were setting it as a target to be achieved, but what we do in this case is in satisfying, let us say there are p objective functions.

I will say that Z 1 should be at least this much, Z 2 should be at least this much and so on; and generate, solutions non-inferior solutions, those solutions which do not satisfy this minimum level will be screened out; that means, you will not consider those solutions; this is what is called as satisficing, and typically this satisfying we used when there are lot number of stakeholders, each one expressing his or her own minimum level of achieving a particular goal. So, this is what is called as satisficing.

Then we also have what is called as a Lexicography. In which, we essentially rank the objectives; (No audio from 43:00 to 43:06) that means, I will say rank number is my drinking water supply objective, rank number two is irrigation objective and so on. So we rank these things and then pick up the particular solutions, these are all ways of screening out different solutions, once you obtain the non-inferior solutions, anyway these when you are doing slightly higher level applications, all of these will become important just keep them in mind that such methods also exist. So, you generate set of non-inferior solutions either using Weighting method or the Constraint method or even goal programming method etcetera; and then use screening alternatives to screen out several of these solutions.

So, essentially in multi-objective planning or multi-objective optimization, you have a number of different objectives, all of which may either to be maximized or minimized and so on. Many of them will be conflicting with each other, you generate non-inferior set of solutions, where you cannot increase one of the objective functions in a maximization problem without simultaneous decrease in at least one of the other objective functions in, i am sorry if you have p number of objective functions, you cannot increase one of the objective functions, anyone of the objective functions without a simultaneous decrease in other objective functions. These are called as the non-inferior sets of solutions.

So, with this now, we will close on techniques that are available; initially I talked about the optimization techniques, and then also I just now introduced the multi-objective planning or multi-objective optimization. In terms of the mathematical technique, the multi-objective optimization is not a new technique, essentially it is a way of generating several different solutions, non-inferior types of solutions; from this point onwards we will go into actual applications of the systems techniques that you have learned for water resources systems problems.

So, so far I did not deal so extensively with the water resources systems problems, from now onwards, we will start with water resource systems problems; we will begin with the classical reservoir systems problems, and the first level of problems that we will deal with or with respect to deterministic inputs or they are the deterministic problems; by deterministic problems I mean, the set of inflows will be known, and there is no randomness associated with it, there is no uncertainty associated with these kinds of systems.

So, given a set of inputs given a type of size of the reservoir, how do we operate the reservoir that is one type of problem or given the set of inputs, what is the minimum size of the reservoir that is necessary for meeting a given set of demands. So that is no uncertainty associated with it; simply, you are given a sequence of inflows, and may be you are give specified size of the dam or size of the reservoir, you are ask to operate the reservoir. So, everything is deterministic in that sense.

So, we will start with reservoir systems with deterministic inputs; as the introduction to reservoir systems, we must know some basic features of reservoirs, which perhaps some of you would have learnt in your water resources engineering course or in basic hydrology course; it depends on a where it is cover, but let us just go through it, when I mean, when I refer to a reservoir, we refer to a dam and upstream of the dam, there is the storage that is created. So, by reservoir I mean, the storage that is created, because of construction of a dam, a barrage or any other structure, even a (( )) and so on.

So, whenever you have a physical structure obstructing the flow, you create storage upstream of that, storage behind that structure, and this is the storage that we will be referring to whenever, we are talking about reservoir. So, let us look at different zones of this reservoir; typically when I, when you are talking about a reservoir, you may use the reservoir water for various purposes for example, you may use the reservoir storage for meeting irrigation demands for generating hydro power, and for you may use the reservoir space or reservoir storage for accommodating the flood waters and so on. So, you may use the reservoir waters for reservoir storage for different purposes, these different purposes can be often conflicting; they are in fact, often conflicting with each other. So, let us look at, different types of storage that we talk about.

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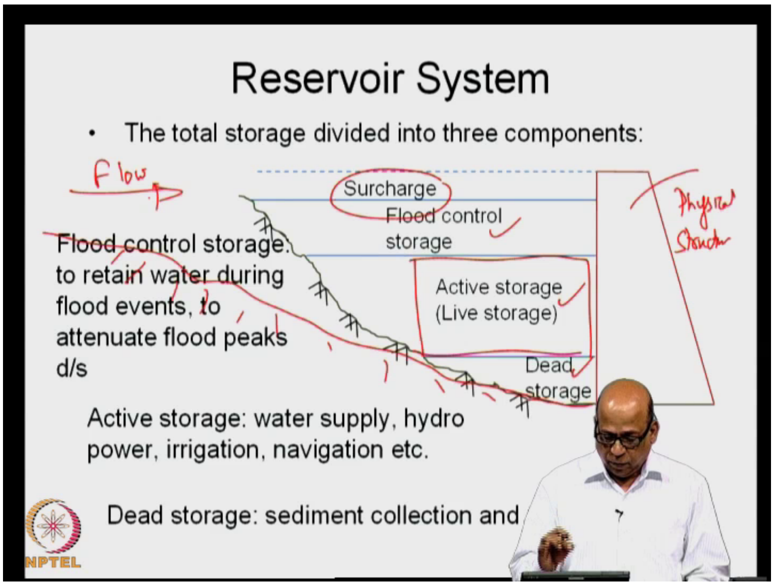

So, this is the dam that I was mentioning. So, this is the physical structure that we have constructed to (No audio from 48:49 to 48:54) and your flow is in this direction, so because of this obstruction to the flow, the flow builds up here; and you has different zones for operation, let us look at this different zones; we provide what is called as the dead storage, which means the water in this storage, we do not use, we do not extract the water, now this dead storage is essentially provided to accommodate for the (( )) or the sediment that comes in.

So, the dead storage is mostly for sediment collection and perhaps if you are wants to use this storage for recreation purposes, you may want to have a very high level of storage below which you are not using it. In fact, in some cases you also use it to build head for the hydro power. So, the dead storage is that particular storage, which from which you do not withdraw water. So, this is always there, this amount of water is always there that is what is called as dead storage.

Then, there is a zone called active storage. The active storage in this zone, you use the water for supply of water to various purposes for example, you may supply the water to municipal and industrial purposes, you may supply water to generate hydro power, you may supply water for irrigation and so on. So, this is the zone from which water will be physically withdrawn. So, these are called as the active storage or the live storage. Then, you have flood control storage, now the flood control storage is the storage that you provide as the buffer to absorb the flood waters; now in many cases you can combine the active storage with the flood control storage, by making use of this amount also during the non floods seasons, we will come to all those details later in a applications; however, the flood control storage is that particular storage that you provide to absorb the floods, to attenuate flood peaks downstream.

So, this is what we do; in the absence of flood control storage, what would have happened that you may be at the top of the reservoir and then, if the flood wave comes the entire flood wave will be passed downstream, but by absorbing the flood storage, flood control, by absorbing the flood waters into this flood control storage, you have actually attenuated the flood what would have happened downstream of this reservoir, because of that flood you have reduced that effect, attenuated the flood peaks that would have occurred downstream. So, this is the purpose of the flood control storage.

So, you have essentially three different zones, which are useful; one is the dead storage which is useful for absorbing, the sediment low, and also for providing head for the hydro power as well as for providing the storage for recreation purposes; then you have the live storage, from which the water will be actually withdrawn, and then you have the flood control storage phase, you will absorb the floods, and this is the ground level here; this is like the exaggerated ground level, but typically, it goes smoothly like this.

So, this will be the ground level; behind the behind the dam; over and above that you may also have certain surcharge surcharge storage; in systems techniques that we will be dealing with now, we will be typically concerned with flood control, active storage and dead storage. So, we will deal with these three different storages and specifically, we will be dealing mostly with active storage or the live storage.

Within this frame work now, we are now talking about reservoir systems with deterministic inflows; that means, the flows sequences are all known, given; now these would be typically historical absorbed sequences, let us say you have last 30 years of data on the flow that has occur, if you have a reservoir that is already in place, you would have measure the flows, which is which are coming into the reservoir, and call them as inflows to the reservoir. So, the sequence of flows is known, and that is why we call them as deterministic problems, where there is no uncertainty associated with the inputs or the physical features of the system, water resource system.

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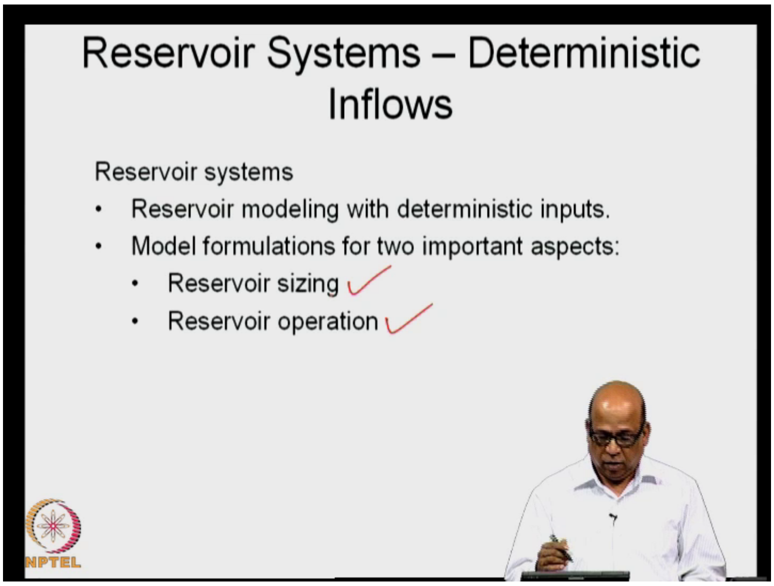

So, we will start with a Reservoir system with deterministic inputs; in this frame work we will deal with two specific problems: one is Reservoir sizing, another is Reservoir operation. So, we will start with the single reservoir none inputs, and then we will talk about two different problems; one is how much of storage should be provided, and second is given a reservoir of known storage capacity, how do we operate this to meet a set of given demands. So, the reservoir sizing problem deals with obtaining a minimum reservoir storage for a given sequence of inflows to meet a given sequence of demands.

The reservoir operation problem on the other hand deals with the operation of the reservoir, from time period to time period to meet a certain (( )) for a given sequence of (( )). So, this is the deterministic problem and within this deterministic problem, we will introduce both these problems in the coming lectures. So, essentially in a today's lecture, we continued our last our discussion in the last lecture on multi-objective optimization, we introduced the weighting method and the constraint method to generate the non-inferior set of solutions or the Pareto optimal set of solutions.

And then, we solve two simple examples, in fact, we solved one example with both the methods; using the weighting method as well as in the constraint method; in the weighting method, you recall that we assign weights to different objective functions, and then solve the problem and by paying around in the weights, by varying these weights systematically, you generate a large set of solutions, large set of non-inferior sets of solutions, and then you generate what is called as the efficiency frontier or the Pareto optimal frontier; in the constraint method, we pick up the one of the objective functions and maximize that objective functions in the maximization problem subject to constraints placed on the other objective functions.

So, you typically place Z j of X greater than or equal to L j, or Z k of X greater than or equal to L k in a maximization problem, which means that we are saying that all the other objective functions must be met with a certain minimum level, and then you maximize this particular objective function. So, that is what we do in constraint method; then towards, and I also mentioned that, you can use goal programming, you can use satisfying, you can use lexicography for multi-objective problems, essentially to screen out, out of these virtually infinite number of non-inferior solutions, you screen out some of the solutions, so that you have a handful of solutions, from which you can implement for your planning and operational decisions.

Towards the end of the class, I have just introduced, what we mean by reservoir systems with deterministic inputs. So, I have just introduced a the different zones in a reservoir typically, the dead storage, the live storage, and the flood control storage. And then which is subject to deterministic inputs, the known sequence of inflows in deterministic problem. The sequence of inflows is known and the sequence of demands is known and within this broad frame work of problems.

We will introduce two typical problems; one is of reservoirs sizing, where we are looking at the minimum capacity of the reservoir that is required to meet a set of demands for a given set of inflows; and the other one is reservoir operation problem; where will be interested in, how do we operate a given reservoir with an known capacity, with an known sequence of flows, with an known sequence of demands, how do we operate this reservoir from time period to time period. So, this discussion we will continue in the next class. Thank you for your attention.