

Water Resources Systems
Modeling Techniques and Analysis
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Lecture No. # 18

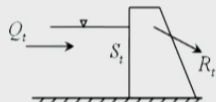
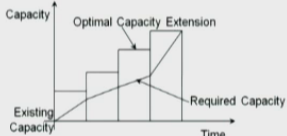
Simulation: Introduction to Multi-objective planning

Good morning, and welcome to this the lecture number eighteen of the course Water Resource Systems, Modeling Techniques and Analysis. In the previous class, in the last lecture that is, we continued our earlier discussion on dynamic programming problems, and we specifically dealt with three different problems over the last two classes, **we have been** last two lectures, we have been talking about this.

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Summary of the previous lecture

- Dynamic programming:
 - Reservoir operation problem
- Capacity expansion problem
- Shortest route problem

numerical example, a simple numerical example containing only four time steps, and demonstrated how we use the dynamic programming for this classical problem in water resources systems **systems**.

Then, we also dealt with the capacity expansion problem. Where the problem is, given an existing capacity, how do we plan for the expansion of the capacity in future, in discrete time steps. Let say 4 time steps of 5 years each, where we are specifying the minimum required capacity, at the end of each of the time step here. **And this will be the...** This is the optimal capacity expansion, so that at the end of the time horizon, you meet the required capacity at that point in time. So, while this was your specified expansion of the capacity, this is the optimal capacity expansion.

And we formulated this also has the dynamic programming problem, and use the backward recursion; starting with the last time step, you came up to the **existing time**, the existing capacity of the present time step. This is the problem that we solved, and then towards the end of the last lecture, I introduce the shortest root problem, although we did not solve this problem completely, I left it to the students to solve the problem completely; though complete solution is available in the ppt slides that I will be providing along with the lecture.

So, the problem here is that starting with the source node, **source node**; you want to reach the destination node in the shortest route. This is the classical shortest root problem, which we can also formulate using the dynamic programming. We define each of these as stages. So stage 1, stage 2, stage 3, etcetera. And then we formulated this as the forward recursion problem where at any given stage, we ask the question, if we are in this state, let us say I am in node B 2, from which node in the previous stage, I should have come to reach that point, such that the total distance from the source up to that point is the minimum.

For the first stage, it is the trivial exercise; when you come to second stage, you ask the question if you are in C 2, whether you should have come from B 1 or B 2 or B 3, such that the total distance covered from A to that point is a minimum; like this, we keep on moving, until we move up to the destination. And in the dynamic programming problem, once we solve for the last stage, once we obtain the solution for the last stage, you trace

back the solutions and get solution at each stage. So, this is what we did in dynamic programming.

So, essentially at the at this point, we will conclude our discussion on dynamic programming. So, we have learned, we have studied earlier the classical optimization techniques using the calculus, and then we introduce the linear programming, and then we introduce dynamic programming. So, for that I am being, we will stop an optimization; we will revisit optimization using more advanced and more recent algorithms towards the end of the course. Such as the genetic algorithms, biologically inspired algorithms for optimization etcetera, I will just give a brief introduction to such algorithms perhaps towards the end of the course.

So, right now, we will **we will** pause for a while on optimization, and then start moving towards other topics. There is the very powerful technique that we use in water resource systems analysis, which is the simulation. Much unlike optimization, in any of the optimization techniques, remember you are looking for either a maximum value of a function or a minimum value of a function and so on, subject to a large number of constrains. So, you form such problems as mathematical statements, and then you are depending on the way you formulate, you can either use linear programming or dynamic programming or non-linear programming etcetera, depending on the structure of the problem itself.

However, when you in when you are looking at large water resource systems, let say that there are several reservoirs, several user points, several irrigation districts that you want to supply water, several municipal and industrial supply you needs and so on; when you are looking at large water resource systems, and when you are looking at complex water resource systems, in the sense that even if it is a single reservoir, you are looking at conflicting objectives, you may be, you may need the water for irrigation, hydro power flood control and so on; in such situations, optimization becomes slightly cumbersome, and also the single point optimal solutions that we look **look** for in the optimal optimization problems, may not be really useful for planning decisions for making real time operational decisions and so on.

In such situations, simulation is a very powerful tool. So, we use simulation especially, when there are large water resource systems, there are complex water resource systems,

and then you want to look at various alternatives. If you recall, what we did in optimization is that in a straight jacketed fashion, we formulated the problem, and then arrived at one optimal solution. And perhaps in a linear programming, you could do some sensitivity analysis, and then see how the **how the** objective function is sensitive corresponding to changes in several of these parameters that we considered; for example, the right hand side of the constraints and so on.

However, when you are dealing with complex water resource systems, you would like to examine several alternatives. And simulation is the powerful tool for such exercises. We will now introduce simulation, the more **more** involved **(())** of simulation, we will study through applications later on, but right now, just I will give a just a broad flower of what we mean by simulation in water resource systems.

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Simulation

Necessity :

- Modeling of complex water resource systems.
- Screening of large number of alternatives.
- Detailing to any desired level

Features of Simulation:

- No optimal solution.
- Specified operating policy; Detailing to a degree possible.
- Even very large simulation programs with ease on computers

The diagram shows a flowchart of a water resource system. At the top, a catchment area is depicted with a reservoir. Below it, a box labeled 'M & I' (Municipal and Industrial) is connected to a central box 'R' (Reservoir). From 'R', arrows point to 'Flood control' and 'Recreation'. Below 'R' is a box 'PH' (Public Health). To the right, an arrow points from 'R' to 'IRR' (Irrigation). A red circle highlights the 'IRR' box and the arrow connecting it to 'R'. The NPTEL logo is in the bottom left corner.

So, we will start with the new topic now. This is simulation; we have formally closed the discussion on optimization. So, we have starting with simulation now. As I just mentioned, let us say that you have a complex water resource system, a large catchment in which you are planning for construction of several reservoir storages, may be also the lift irrigation schemes, and then you may want to supply the water for municipal and industrial water use and so on.

You may also consider inter basin transfer. Let say that there is another basins somewhere here adjacent to that. And then you may be interested in supplying water

from this basin to from a particular node to another basin. So, inter basin transfer of water. So, these are all essentially large scale, and complex water resource systems where, not only the hydrologic features, but there are a large number of other features that you need to build in. For example, that will be environmental features that you need to build in ecological features, socio economic features that you need to build in and so on.

So, whenever you are looking at modeling of large water resource systems, you need to bring in large number of complexities and therefore, large amount of sophistication. In terms in as much as the models reflect, reality to the best extent possible. In such situations, optimization is not always the best tool. You need to look at several alternatives with a great degree of detail and in such situations, we use simulation. So, screening of large number of alternatives is one of the major motivation for simulation, then detailing to any desired level; I will explain it with respect to one simple example here. Let us say that you have a reservoir, and then you are using this reservoir for municipal and industrial water supply, this (()) has to be on this type.

Municipal and industrial water supply as well as for irrigation; let say that this point is for irrigation, and you also have for hydropower. So, you are using this water for municipal and industrial water supply for hydro power generation, and for irrigation. Perhaps from the power house, the water that is left out of the power house also comes to the irrigation. On the upstream side, you have this reservoir water; therefore, you may be using that reservoir also for recreation, let say the that for boating purposes, for fishing purposes and so on. In in western countries it is very common, although in India it is not so common, but it will eventually pick up, because you have a large reservoir there, it essentially it opens up other avenues of revenue; for example, recreation and so on.

Then you also have flood control as one of the purposes, which means that if there is the flood wave you will use this reservoir, as a flood buffer buffer storage for observing the flood storage. Now, this is the broad level of description of the system. Any component here, will have its own details. Let us say that, if you want to optimize this system; what you would have done, you would lumped all of these and then called it as municipal and industrial demands at a particular time period. Similarly, irrigation demands at a particular time period, let us say 10 day time period, monthly time period and so on.

Similarly, power house demands you would have put it as, certain amount of water to be released for a given storage level in every time period.

And then, you would have put it in to a optimization problem to obtain the release policies. However, you look at what is happening at the irrigation reservoir, the irrigation demand point. At the irrigation demand points, you may have several crops and then there is the soil measure that is involved - the soil measure is continuously changing that is also a rainfall that is occurring in the irrigation districts or irrigated area and that is the vapor transportation that is taking place, you would like to put all of these components in this irrigation, irrigated area when you are modeling for water resource system.

Similarly at the power house, you want to relate the power generated as a function of the storage which is continuously changing, because of the release that you are making as well as, because of the inflow that is coming. And the storage the head, that is head required for power house, as well as the release required for generating a certain amount of power, they are all related in a non-linear fashion. And you would like to include such relationships. Similarly, at the flood control, you may perhaps want to use the routing procedure, flood routing procedure, both the channel flood routing as well as the reservoir routing to examine what happens with respect to the flood.

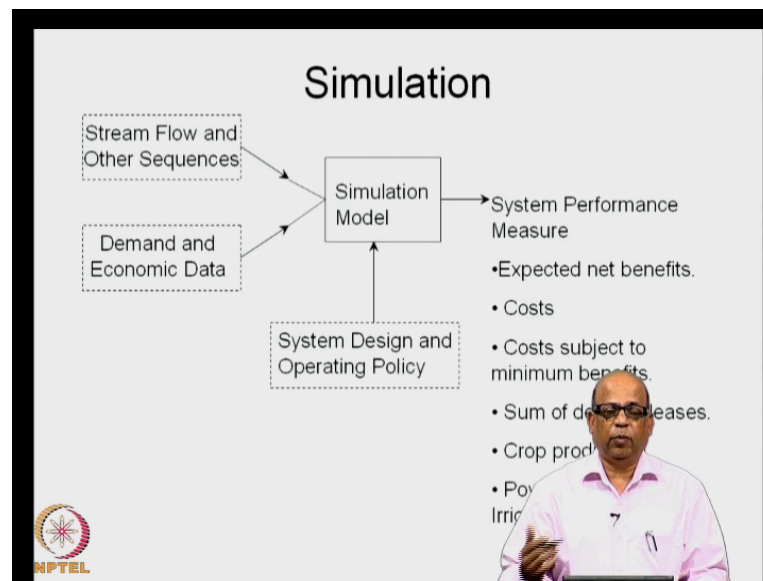
So, all of these complexities in great detail you want it put in to a mathematical model. In such situations, simulation becomes extremely handy tool. So, detailing to any degree desired - any desirable degree, you can include in simulation, because you are not putting it in a hard optimization type of problem, mathematical problem. So, you are simply trying to mimic, the way the system behaves for given input and for given system design. So in the simulation, we are not looking for optimal solution straight away; we are simply looking at how the system behaves for a given set of conditions that is all.

We are putting all the details, and then saying that the system will behave or perform in certain **certain** of fashion; if the inputs are like this, and if our operating policy is like this and so on. So, that you can examine several alternatives; we specifying the operating policy, if you are talking about reservoir systems, you says that if my reservoir storage is so much, the typical rule curves is what we use for reservoir operation, we adapt the rule curves and then operate the policy. Now, operate the reservoirs. And therefore, the operating policy is pre specified. And as I said detailing is possible, and even very large

simulation package, simulation programs for example, large systems like Narmada river reservoir system, the (()) reservoir system, the gang basin simulation model and so on.

So, large reservoir systems, large river basins with several complexities can be very elegantly formulated as simulation problems, and can be very easily handled with the available computers today.

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And therefore, simulation as the first step of screening out the alternatives is a very powerful tool. And we will see, what we do in simulation through a simple example subsequently. So, let us say that in **in** a water resource system, you may have stream flow and other sequences; for example, the demand sequences may be known, the rainfall rainfall sequences may be known, the how the soil moisture behaves what are the cropping patterns and so on, all of these are known. Then, you also have demand an economic data; for example, if you are looking at a water resource system which essentially caters for irrigation.

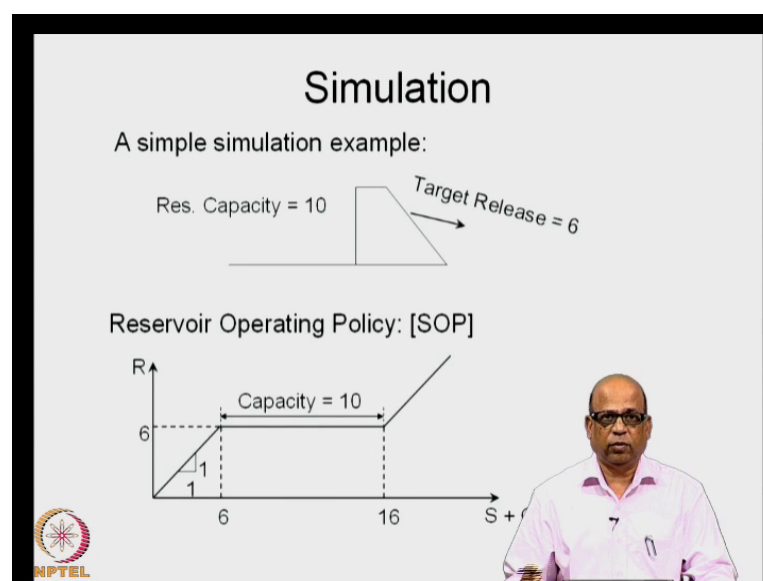
And then, you are looking at the socio economic states of the districts, irrigation districts in which the water is being supplied, and that data can come as input to the simulation model. So, all of these extraneous considerations of water resources is planning will come as input to the simulation model. To the simulation model, you also specify the system design and operating policy. For example, you may say that I may build a certain capacity of reservoir - capacity of storage, and I may build a certain capacity of lift

irrigation, and I may build a certain canal network etcetera. So, that is the system design that goes in to this, that is the hydraulic design or hydrological design that (()).

In addition, we may also have operating policy. So, all of these come in to inputs as system model; and then you get the outputs in terms of the actual amount of water that is available made available from period to period and so on. From which you can get the system performance measure. Typically, in terms of how many time periods the system was not satisfactory. **System performance was not satisfactory**; in terms of its ability to meet the demands that we specified earlier. And similarly, we can look at the net benefits cost and some of the (()) is and so on. So, once you build a good simulation model with all the details incorporated in to it. You can examine for several different types of inputs here, you can examine how the system performs.

And you can also play around with the system design and operating policy. Once a simulation model is built correctly to reflect a real features of the system - features of the real system, then you can play around with all of these inputs, and then obtain the associated performances of the system. So, this is what we typically do in simulation. When we come to perhaps large scale reservoir system etcetera which I will deal with in the towards the end of this course, you will appreciate more the use of simulation, we saw we optimization.

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We will take a very simple example, class room example to drive home the point of what we mean by simulation. Let us say that, you have a reservoir capacity of 10 units. And there is a inflow that is specified, there is the target release of 6 which means that you want to maintain a release of 6 units every time period. We use what is called as the standard operating policy. This SOP stands for standard operating policy which means what? It does not look at what is likely to happen in other time periods; it will simply look at this current time period, by time period remember we mean is there let us say 10 day interval, 1 month interval, seasonal interval and so on. For which we are making the decisions.

It says, if the amount of water available is adequate to meet the demand, you meet the demand to the full extent. If the water available is less than the total demand, you release all the amount that is available, to meet the demand to the best extent possible. Which means, let us say that 6 is the, my target release; every time period I have to meet an amount of 6 units. If the water available which is S plus Q the storage plus the inflow, during that time period is less than 6 units, here if S plus Q is less than 6 units, you release all the amount of water available. So, this release will be equal to what is available?

If S plus Q which is the amount of water available is more than 6 units, you release 6 units and start building the storage. So, it was the target is 6, you have S plus Q getting inflow here, and there was the initial storage, if the storage plus the inflow is less than 6 units, you empty the reservoir. So, in this period here, during this situation the reservoir will be always empty after the release. Once your storage plus inflow exceeds 6 units, then you are releasing 6 units, you keep releasing only 6 units, and the remaining things you start building up the storage.

So, this phase here is the filling phase. That is during this period, this phase where the S plus Q is between 6 and 16 units the reservoir starts filling. So, you are continuously making the release of 6, and also you are building up the storage. Once, the storage reaches full level which is capacity of 10 units; once it reaches 10 units at this point; what you do is you allow for the spill as well as the 6 units, which means the release will keep on increasing like this; that is a 17, then release will be 6 plus 1 unit; that means, you are maintaining the storage at 10, and then releasing whatever is excess.

So, this is the place where the spill occurs. So, this is the reservoir, empty place, empty phase; this is the filling phase and this is the spill phase - spill or overflow phase. Now, this simple operation, we will try to simulate now. How do we simulate? If you have a sequence of inflows given; let us say that, we have 10 time periods or 12 time periods etcetera. For which the inflows are known, and you have the capacity that is known, and you have the target release, that is also known. Let us try to simulate such a simple system. And for this example, we will ignore losses; how to account for your evaporation and seepage losses etcetera which are storage dependent losses, we will cover you may be in a next class (()), where we are talking about reservoir systems specifically.

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Simulation given Target Release = 6

Period t	Initial Storage S_t	Inflow Q_t	Release R_t	Final Storage S_{t+1}	Deficit
1	9	12	11	10	0
2	10	11	11	10	0
3	10	12	12	10	0
4	10	5	6	9	0
5	9	4	6	7	0
6	7	2	6	3	0
7	3	1	4	0	2
8	0	3	3	0	3
9	0	5	5	0	1
10	0	8	6	2	0
...
20	1	11	6	6	0

Total Deficit = 18
 Total Inflow = 5, 4, 5, 7, 8, 2, 2, 1, 7, 11
 Total Release = 11 to 20

Here, I am talking about simulation in a general sense. So, this can be done with a very simple spread sheet type of problem program like, Microsoft excel or (()). You have 20 time periods let us say, t is equal to 1, 2, 3, etcetera up to 20 time periods. We start with an initial storage, the inflow sequence is known. So, the inflow sequence is given; and target release is given. So, target release is 6. And we will compute the deficit. So, our interest is how the system performs in terms of the deficit releases. So, this is the very simple excel of a problem where you start with the initial storage, you add the inflow this becomes the total water available 12 plus 9 is 21, and because your capacity is 10 units, you release 11 units; now, this includes the overflows.

So, you will release 11 units and make the reserve final reservoir storage as 10. How do we get the final reservoir storage. $9 + 12 - 11 = 10$. Remember this should not be more than 10, because 10 is your capacity. And therefore, deficit is zero. Whenever the release is more than the target release, the deficit would be zero. We calculate the deficit only when the release is less than 6. Then this storage, final storage becomes a initial storage for the next time period, there you have the inflow 11; so, $10 + 11 - 11 = 10$ again you release a 11, you make the storage to be 10, this 10 comes here, again the deficit is 0, because the release is more than 10. Then this 10 comes here, as the initial storage, and then this is the inflow that is given $12 + 10 = 22$, you maintain the final storage to be 10 therefore, release 12 again deficit zero and so on.

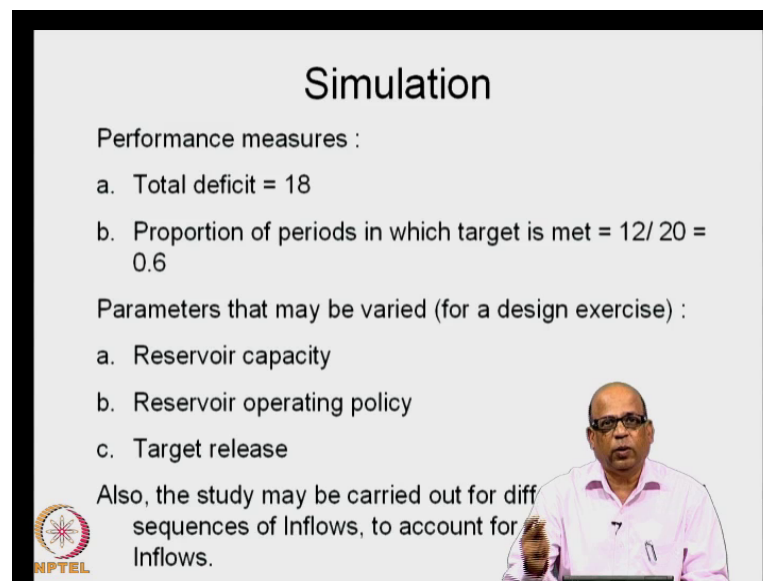
Like this you keep on going, then the inflow starts decreasing. Now, this is the excess flow period, and this is the deficit flow period. Let say what happens in the next time period. All though you started with that full storage, your release your inflow is 5 and therefore, you are still able to maintain a release of 6, final storage will be 9; like this, you keep on going you maintain 6 as long as your availability is more than 6. So, $7 + 2 = 9$ is 6.

Here you end up with the storage of 3. And that 3 becomes the initial storage for the next time period, the inflow is small $3 + 1 = 4$ and therefore, you release all the 4 units. So, whatever is storage plus inflow you release, because your policy is to meet the demand to the best extent possible. Of course, this is not a good policy, but this is the standard policy, we will see how the system performs in this. You released all the 4 and therefore, you ended with up the 0 storage; and this 0 storage comes as initial storage, and then you release all the 3 again 0, again you release all the 5 again 0; now, again the inflow picks up 8, but you do not release 8 now, you release only 6, because 6 is the target storage.

Like this, you keep on simulating until all the time periods, there these are the other flows up to 10, I have shown here, 11 to 20 these are the flows, you please complete this as a exercise and then compute the total deficit. So, total deficit that you get out of 20 time periods is 18, 18 units of deficit occurs, if you operate the system like this. So, I will write here, because in your screen you may not get this. So, let may write here, total deficit is 18, and then Q 11 to Q 20 are given here; you may refer to the ppt, they are 5, 4, 5, 7, 8, 2, 2, 1, 7, 11. So, these are the flows, you can just refer to the slides, and then complete this; when you complete this, you get the total deficit to be 18 units.

Now, the question that you can ask is, suppose I want to reduce this 18 units. And bring it down to let us say 7 units or 10 units etcetera. What are all the controls at I have here, I can change the operating policy altogether. So, we can examine using different operating policies. You can also examine by increasing the storage capacity. Let us say that I had a reservoir capacity of 10, if I make it 15 what happens to the deficit storage and so on. So, you can once you have such a simple structure available with you, you can examine various implications of both the design as well as the policy.

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Simulation

Performance measures :

- a. Total deficit = 18
- b. Proportion of periods in which target is met = $12/20 = 0.6$

Parameters that may be varied (for a design exercise) :

- a. Reservoir capacity
- b. Reservoir operating policy
- c. Target release

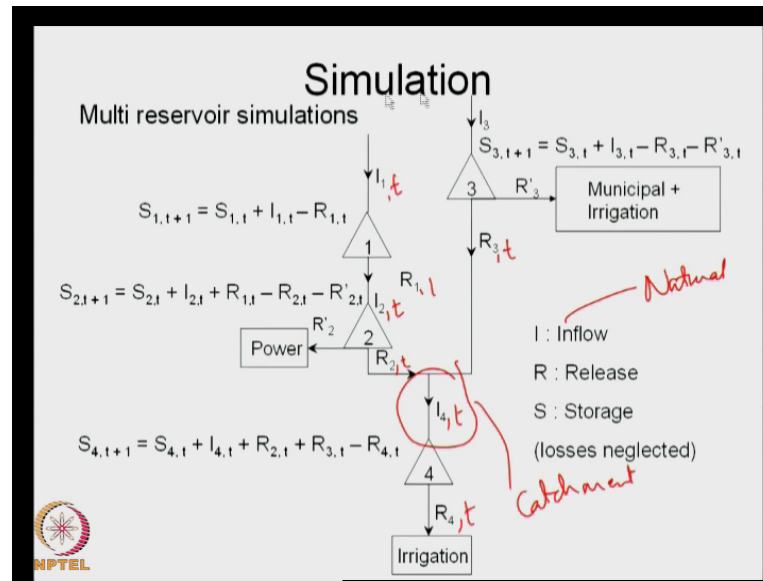
Also, the study may be carried out for different sequences of Inflows, to account for Inflows.

So, in this particular case, you got the total deficit to be 18, you can say that the proportion of periods in which the target is totally met is 12 out of 20; that means, out of 20 time periods, you could meet the release target in 12 number of the periods, therefore it is point 6. You can vary the reservoir capacity, you can varies the reservoir operating policy, you can also vary the target release; instead of 6 if I make 5, can I get a better performance in terms of the proportion, and in terms of (()) the definition. So, like this you can play around with the design parameters, the operating pair of parameters, you can play around with the demands, you can play around with the the policies of how you operate based on the storage and so on.

Once you have a structure of a simulation available with you. Also you know, you use one sequence of Inflows here. Typically in simulation, what we do is, we we use the synthetic generation techniques to simulate several sequences of inflows, and then read

on the simulation model for examine the performance of the system. When I cover some case studies this point to be discussed in more details. So, right now you understand that the simulation is simply reproducing, how the system is likely to behave, how the system behaves in fact, for given set of inputs, for given set given system design and for specified operating for this. So, this is what we do in simulation.

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For example, if you are looking at from the single reservoir system, let say that we go to a complex multi reservoir system; how do we simulate this. Let say that you have a reservoir here 1, another reservoir here 2, third reservoir 3 and forth reservoir 4 here. The reservoir 1 is almost like a balancing reservoir which simply stores a water, and then religious to reservoir 2 which uses the water for power generation, and it also release an amount of water to reservoir 4. Reservoir 3 use this some water for municipal and irrigation demand, and release this for reservoir 4. Now, the reservoir 4 is the major reservoir here, which takes inputs from all the upstream reservoirs, and then uses the water primarily for irrigation.

There are several such cases in our country where one of the major reservoir, major reservoirs receives input essentially from the upstream reservoirs - the upstream reservoirs have their own minor demands, but essentially they at to feed in to the major reservoir here. Let say you want to simulate this, we denote it by the denote the inflow by I, release by R and storage by S and for that I am being we will neglect the losses.

What we do in such situation, we start with the upstream of reservoirs, 1 and 3 are upstream reservoirs here, and they do not have any input from any other upstream reservoirs, you may still have some other upstream reservoirs feeding into these and so on.

Start with the upstream reservoirs. Look at the storage continuity there. Essentially mass balance for that time being we are neglecting the losses. So, if I write the storage continuity at reservoir 1, this across time periods, t denotes the time period, this is the initial storage at the reservoir 1 at the time period t , that is the storage at the beginning of time period t at reservoir 1. So, that is what I represent it as $S_{1,t}$ plus the inflow that comes in due to natural flows from its catchment. So, this is the natural inflow that comes in, $I_{2,t}$ reservoir 1 in time period t minus the release that you are making in time period t at the reservoir 1, that solve. So, this defines your $S_{1,t}$ reservoir continuity. Of course, we put the constrain that the storage should be less than the capacity at this point.

This release that you make here $R_{1,t}$, here there is no other demand. So, simply you are making the release to make the requirement at 2 un subsequently at 4. This release that comes here, adds $I_{2,t}$ the natural inflow that is coming here $I_{2,t}$. So, $I_{2,t}$ is the natural inflow, because of the catchment between the reservoir 2 and reservoir 1. The $R_{1,t}$ which you have made as release here, adds to the reservoir 2, we can add $R_{1,t}$, $R_{3,t}$ etcetera. $R_{2,t}$ and so on, to indicate that they are correspond to periods $I_{1,t}$ and so on. So, when you write the continuity equation for reservoir 2 them, you will account for everything that is getting added, and detect whatever you are talking out from the reservoir.

So, you will write this as the storage in reservoir 2 at the beginning of time period t plus $I_{2,t}$ is equal to what, he was the initial storage which is storage at reservoir 2, at the beginning of time period t less than the natural inflow $I_{2,t}$ that has contributed to the storage plus the upstream release $R_{1,t}$ that has added to this storage minus you would have taken $R_{2,t}$ to supply to reservoir 4 minus $R_{2,t}$ dash t which you have taken for power which you may go as some other flow, it is not available for $I_{4,t}$.

So, this is how you write the continuity equation. Then you come to reservoir 3, again reservoir 3 will be similar to reservoir 1, except that you also have a municipal and irrigation component. So, you have $S_{3,t}$ plus $I_{3,t}$ is equal to $S_{3,t}$ plus $I_{3,t}$ minus $R_{3,t}$ which we have talking out for reservoir 4 minus $R_{3,t}$ dash t which is the municipal and

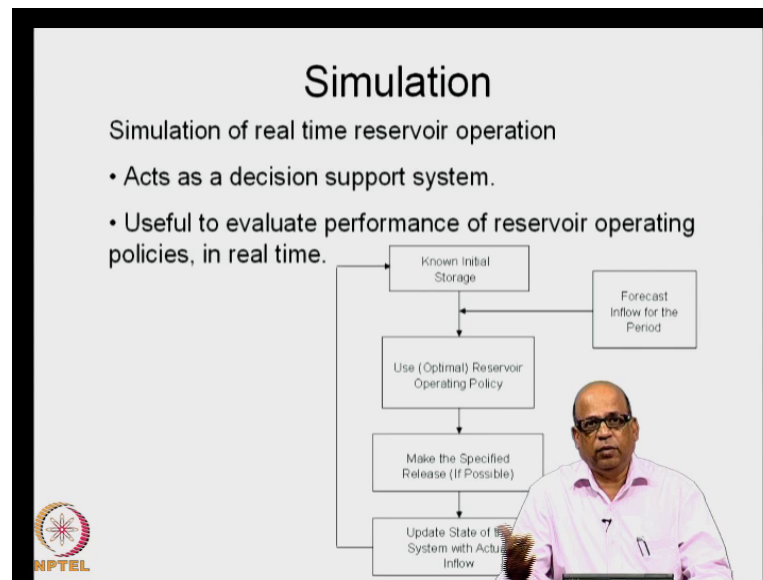
irrigation demand. So, and then you come to I 4, the reservoir 4, again you write the continuity equation just accounting for what is coming in to the particular reservoir, and what is being taken out from the particular reservoir.

So, you write $S_4 t + 1$ is equal to $S_4 t$ that is the initial **initial** storage plus what has been contributed by the natural inflow here. So, the **the** I is the inflow which is natural in flow. What I mean by that is natural inflow is that the reservoir has its own catchment and the catchment provides the natural inflow plus you have the regulated religious coming from the upstream reservoirs namely $R_2 t$ and $R_3 t$ in this case, you add those 2 and then you are talking out $R_4 t$ here. So, these are the continuity equations that you **you** write. And this is the way you simulate. In simulation what do you do, you specify let say at the time period t is equal to 1, at the beginning of time period t is equal to 1, you specify all the storages. And then the inflow sequence is will be known from the historical data. So, I all the inflows are known here.

Then, you specify an operating policy. Let say that this reservoir will only operate based on it so one storage and the inflow, and then it makes the release and so on; you specify the operating policy like you did in the previous example, at every reservoir you specify the operating policy based on the target religious that are required, and then operate these using the continuity equations that you have here. So, what we did earlier for a single reservoir system same thing can be extended by adding several other reservoirs which their own continuity equations, and their own specified operating policies, their own specified storage requirements, power requirements, irrigation requirements and, **and** so on.

So, this is the way, you **you** can write a simulation problem, the I would encourage you to take up a example which I will given in tutorial, and then work it out in the any of the spread sheets. Like a microsoft excel **(())**, this can be done very easily.

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Now, another application of simulation is that we would like to see how the system performed in real time operation. That means, we are not actually physically operating, but we want to examine how the system is like you to operate in real time. In real time what is the issue - that you know the storage at the beginning of the time period, and you perhaps have a cost of how the flow is likely to be in the next time period. So, you know the storage at the beginning of the time period which is major. So, you are sure of the storage, based on this storage and based on the forecasted flow, you want to operate the system now. And then keep doing this over a period of time.

So, you operate the system; let me explain in to this. You have the initial storage now at the beginning of time period t , and then you have the forecast for the current time period. Let say that you are standing at the beginning of the month June, you have you know the storage at the beginning of the time period June, and you have the forecast of the inflow for the time period June, depending on this forecast and the known storage, you make the release using some policy. It may be optimal policy or it may be just a specified standard of operating policy or you may specify your $(())$ and then used at policy which specifies for a given storage, for a given inflow what should be the release.

So, you make the specified release if possible. By I say if possible, it is because the policy may be such that for this combination of storage and inflow the release that is being specified may not be feasible. So, if it is feasible, you make the policy release or

make the release to the best extent possible. And then, you applied the state of the system which is the storage at the end of the time period which becomes the beginning of the storage time period storage for the next time period and so on; so, this we continue.

But when we are updating the storage state of the system, you use the actual inflow, because at the end of the time period, you know the actual inflow. So, use the actual inflow and update the storage state of the system, and then go back to the time period, next time period and then redo the computation, like this you keep on doing it, until the end of the time arisen. This is how you examine the operation of the system in a real time. So, this is called as the real time operation which will be based on the forecast of the inflows. And you simulation becomes extremel important, and extremely useful then we have forecast available or a forecast in model is available, and then we want to use the forecast in model for a given reservoir operating policies.

So, the reservoir operating **operating** policy, we could have determined based on an optimization problem. So, you have the policy ready, you want to examine how this policy performs in real time, and that is what you do to simulation. We will considered several applications of simulation later on this just an introduction to simulation **alright**. So, we completed optimization techniques namely the linear programming and the dynamic programming. We also gave up, we also discussed briefly about simulation what we mean by simulation. In fact, we do what are called as Monte carol simulations, we generate the sequence of inflows, we choose the probability distributions of the inflows, and then generate not one sequence per several sequence is of typically 50 years and 100 years etcetera, and then use these sequences in the reservoir operating models - a simulation models.

So, we use also the Monte carol simulation techniques in water resources system which will seen the applications. Now, the last part of the techniques that we are using is the multi objective planning. So, we had the classical optimization techniques, we had the **the** linear programming, the dynamic programming and then we also know in the simulation, but the last part is multi objective planning. Now, this is not. So, much of technique as in as much as linear programming is a technique or dynamic programming is a technique and so on. But, this is a way of handling several objectives in the planning perfective. Typically, when we are dealing with water resources systems, we have not one objective function, in **in** all the classical optimization problems that I delete with

earlier, we simply stated maximize that is equal to 3×1 plus 5×2 and so on. But when we dealing with real water resources systems, you will not have one objective, you will have several objectives.


Meeting the irrigation demands to the best extent possible may be one of the objectives. Meeting the environmental requirements may be another objective, meeting hydro power requirements may be another requirement, another objective and so on. Often these objectives will be conflict with each other. For example, you consider irrigation and flood control; what does flood control require - flood control requires that the storage be kept as low as possible. So, that the flood that is coming in can be observed in the reservoir; whereas, irrigation requires that you store as much as water as possible; so that I will be able to use the water for irrigation over the next time period.

The hydro power requires that the storage be kept at high level. So, that I will have the head as well as the amount of water available for operation; hydro power and irrigation make conflict to each other, because both of them or competing for the same amount of water available. Environmental considerations may require water for down stream water quality control which will conflict with irrigation, which will conflict with hydro power and so on. So, typically in a large water resources systems, there are there is not one objective, one straight forward, the state (()) kind of approach that is possible, you will have to consider several players each with each with their one objective, and these objective will be conflicting with each other.

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Multi-objective Planning

- Water resource planning is a complex and interdisciplinary problem in which we may have to consider multiple objectives.
- Some objectives may conflict each other.
For example, a reservoir project intended to satisfy irrigation, hydropower and recreation.
- The concept of noninferior (or Pareto optimal) solutions is basic to the mathematical framework for multi-objective planning.




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And therefore, we need to go for what is called as multi objective planning in water resources. And there are very nice techniques available for multi **multi** objective planning. We will just go through some of the techniques. So, because of conflicts, because of objectives conflict with each other; we will not be looking at a single optimal solution, instead what we will be looking at is a best compromise solution. And this is where we introduced the concept of noninferior or pareto optimal solutions.

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Multi-objective Planning

- A noninferior solution is one in which no increase in any objective is possible without simultaneous decrease in at least one of the other objectives.
- No optimal solution to a multi-objective problem.
- Determine the noninferior set and get the best solution out of this. (best compromise solution or the perfect solution)



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So, the pareto optimal solution, we will understand it like this. Let us say there are 2 objectives; you cannot increase one objective, as the value of one objective function without decreasing the other objective function. Let us say that you are talking about irrigation and hydro power. The pareto optimal range - solution range is that particular range of solutions where any increase in one of the objectives, let us say the irrigation objective, is not possible without decreasing the values of the other objective function in **in** that case, it is the hydro power. So, this range is called as the pareto optimal range or the no inferior range of solutions.

And this is the very important concept when we are talking about multi objective optimization. So, first we have to identify the noninferior range. Remember multi objective optimization, multi objective planning will not have a single optimal solutions. You need to generate a range of solutions, and you need to generate trades of between different objectives; that means, if I bring down one of the objectives how the other objective is likely to increases. So, this is the type of questions that we ask in multi objective optimization.

So, there is no optimal solution to multi objective problem; we determine the no inferior set and get the best solution out of this, it is called as the best compromise solution or in some terms it is called as the perfect solution; it is not an optimal solution, it is not a mathematically optimal solution. It is just called as the best solution or best compromise solution or in fact, it is also called as the perfect solution

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Multi-objective Planning

- Consider a problem in which two objectives z_1 and z_2 are to be maximized.
- Let both be functions of decision variable x .
- Solutions with $x < x_1$ and $x > x_2$ can be eliminated.
- The range $x_1 \leq x \leq x_2$ is the noninferior range.
- In this range, it is not possible to increase the value of one OF without decreasing the other.

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Let us say that, you have 2 objective functions; both are to be maximized. You have z_1 and z_2 both are the functions of the same decision variable x and z_1 is like this, and z_2 is like this. For any value less than x_1 , for any value of x less than x_1 . You can keep on increasing z_1 without affecting the value of z_2 ; z_2 is not even existent below x_1 . Therefore, this range can be left out for identifying noninferior solutions. So, this does not come under noninferior solution. We look at this range now. Beyond x_2 neither z_1 nor z_2 can be increased. So, x_2 beyond x_2 you can leave out, what with left is x_1 and x_2 .

So, the range between x_1 and x_2 you see here. In this range, you can keep on increasing z_2 - the objective function z_2 , but as you keep increasing z_2 , z_1 is start coming down up to z_2 reaching its maximum value. So, this is the range that provides the range of noninferior solution; noninferior or pareto optimal solutions. So, we need to focus on this particular range when we are dealing with multi objective optimization.

So, the first thing that we do is to identify the noninferior range, and then look at all the possible solutions here; for example, what you may be asking is that if I increase z_2 to certain degree how much is z_1 coming down, and what is my acceptable solution. I may place a lower bound on the z_1 and starting z_2 or I may put wattage is some both z_1 and z_2 , and then look at best compromise solution or I may put a target for both these

objective functions and try to minimize the (()) in the targets and so on. So, there are several ways of doing this.

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Multi-objective Planning

- Let X be a vector of decision variables,
 $X = (x_1, x_2, x_3, \dots, x_n)$
- $Z_j(x), j = 1, 2, \dots, p$ denote p objectives, each of which is to be maximized.
- The multi-objective ~~function~~ Problem is written as

Maximize $[Z_1(x), Z_2(x), \dots, Z_p(x)]$

s.t.

$$g_i(x) \leq b_i \quad i = 1, 2, \dots, m.$$

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So, what a noninferior solution, this also called as the efficiency frontier here.

(No audio from 50:40 to 50:49)

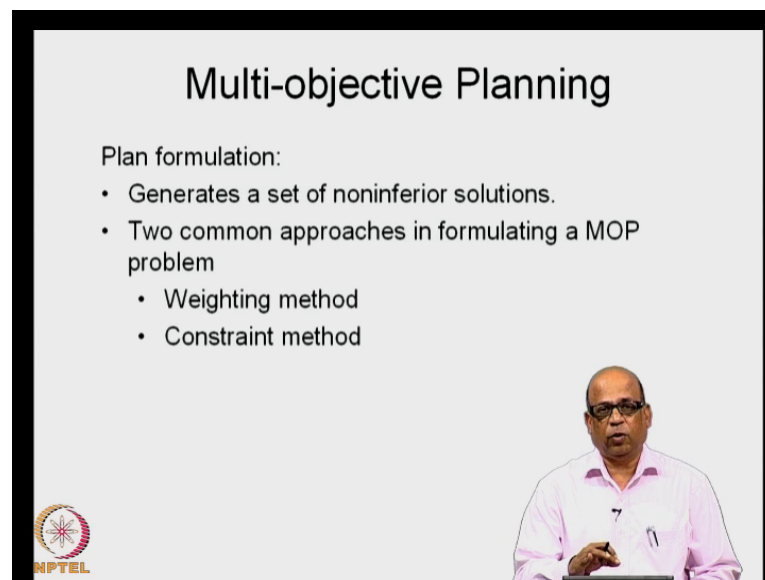
Does is, it provides a trade of between the objective function z_1 , and z_2 - objective z_1 and z_2 in the case of 2 objectives. As you can see here, as z_1 increases, z_2 decreases and similarly as z_2 increases, z_1 decrease is and so on. So, let us say that you have p such of (()), p objectives for the water resource problem each of which is to be maximized. Let us say that you want to maximize hydro power, you want to maximize copied, because of irrigation, you want to maximize the flood control storage, you want to also maximize the irrigation benefits and so on.

So, you may have several such objective functions each of which is to be optimized, each of which is to be maximized in fact. So, we denote this as z_j of x , where x is the vector of decision variables x_1, x_2 etcetera. Like I showed in the previous case, you may have this is a single variable problem, but you may have several such variables. So, x is the vector of decision variables. Then we write the multi objective problem, as we will write this as multi objective problem, we write this as maximize $z_1(x), z_2(x)$, etcetera $z_p(x)$

which means all of these are objectives functions we want to maximize subject to the set of constraints g_j of x less than or equal to b .

So, you may have a m constraints. Remember this sets of constraints that we are writing, we will be common to any of these objectives functions, because these are system generated constraints which means that, let us say that you may have a certain constraint on the line availability, you may have certain constraints on the type of tracks to be ground, you may have certain constraints on the hydro power maximum capacity and so on. So, these are all system constraints. And no matter what is your objective function, all these constraints must be satisfied.

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The slide is titled "Multi-objective Planning". It contains the following text:

Plan formulation:

- Generates a set of noninferior solutions.
- Two common approaches in formulating a MOP problem
 - Weighting method
 - Constraint method

In the bottom right corner of the slide, there is a small video inset showing a man in a light pink shirt and glasses speaking. In the bottom left corner, there is a logo for NPTEL (National Programme on Technology Enhanced Learning).

So, we do what is call we use this multi objective planning for what are called as the plan formulations. You generate a set of noninferior solutions by using one of the approach is, and the most common reused approach is or the weighting approach, and the constraint approach. In weighting method what we do is let us say, you have 2 or 3 different objective functions. Based on our judgment or based on involvement of the so called stakeholders, we assign it is to each of them objective function. Let us say between drinking water and irrigation you are talking about.

Drinking water has to get a higher priority especially in country like ours, where the drinking water supply has to take a higher priority. So, we attach a must higher weight age to the drinking water supply or the municipal and industrial supply compare to

irrigation, but when we are talking about it trade of between irrigation and hydro power, you may assign a higher weight age to irrigation and a lower weight age to hydro power.

So, like this when you have different objective functions, you assign weights w_1 , w_2 , w_3 etcetera, to each of the objective functions z_1 , z_2 , z_3 etcetera. And then, you solve the optimization problem. That means, there where p objective function, you now assign its, and convert that in to one single objective function, and you have the set of constraints, and you solve the objective optimization problem. In the constraint method what we do is we maximize, in the in the maximization problem, we maximize one of the objective functions. Subject to a lower limit on all the other objective functions, which means that we place a constraint on all the other objective functions and maximize the **the** maximize one of the objective functions.

So, these are the 2 most popularly used, most commonly used methods of a methods in multi objective planning, we continue with discussion in the next class and perhaps look at problems associated numerical examples associated with both of these methods. So, especially, essentially in a today's class we covered simulation, and we delete with a simple problem of reservoir operations, simulation of reservoir operations. And we also mentioned that is simulation is the powerful technique, especially when we want to include large number of details, and we want to examine several alternatives in complex water resources systems. Where optimization will not become a good tool to use. In fact, in many of the water resources systems, we first build water resources systems analysis problems, we first build a good simulation problem.

Simulation models always give a good insight into how the systems in behavior. And then using the **using the** knowledge that you gain of the system out of the simulations; perhaps we can look at the optimization solutions. So, simulation is always a good tool to use for complex water resources systems. Then, we went on to introduce the multi objective problem, essentially in water resources systems you have not one objective, but several objectives of an conflicting with each other. And we introduce the concept of noninferior solution or the pareto optimal solution. And in the next class, we will start with the weighting method and the constraint method of solving the multi objective problems, thank you for your attention.

