Stochastic Hydrology

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Module No # 05

Lecture No # 22

Markov Chains – I

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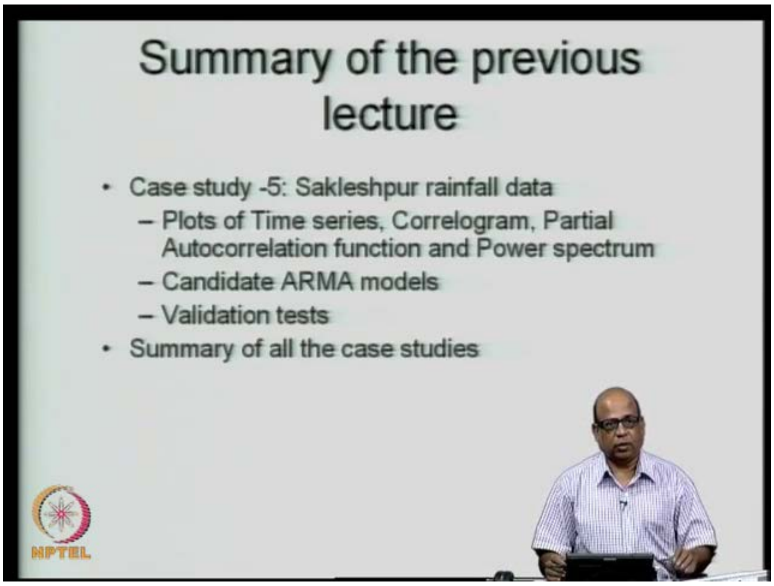

Good morning and welcome to this. So, the lecture number twenty two of the course stochastic hydrology. In the last lecture we concluded our discussion on time series analysis. Essentially in the last lecture we discussed another case study that of Sakleshpur annual rainfall data where we considered the time series plot, the correlogram plot, the partial autocorrelation function and the power spectrum.

We formulated candidate ARMA models for this data. For each of the candidate models we computed the mean square error as well as the likelihood values. And then we chose models corresponding to the maximum likelihood as well as minimum mean square error. The maximum likelihood values will choose a model which is which can be used for long term synthetic generation of data. Whereas, we use the minimum mean square error criteria for one time step ahead forecasting and typically for flow forecasting, we use such models.

Then, we also discussed for the case study the validation test. Typically the validation test that we carry out on the residual series are first to make sure that the residual series has a 0 mean and next we look for any significant periodicities. The residual series should not have any periodicities and the third test that we tell you is to make sure that the residual series that we get is in fact, uncorrelated or it constitutes a white noise.

Then, we also had an overview of all the case studies that we considered namely the Bangalore daily data of rainfall and then the Bangalore monthly data of rainfall, annual data of rainfall. Then we looked up for the case study of Kaveri river flows. We considered the monthly stream flows and distinguished the monthly stream flow correlograms, spectral density etc with the daily rainfall correlograms, spectral density and so on.

Then we also considered a western river US river, same monthly stream flow time series we considered and then we went on to consider the Sakleshpur annual rainfall data. As I mentioned in the last lecture towards the end of the last lecture, we now proceed to another topic. So, we formally close the discussion on time series analysis.

We proceed to another topic this is called as the Markov chains and I will give you some introduction about Markov chains and then we will see in today’s class some applications of the Markov chains in hydrologic problems.

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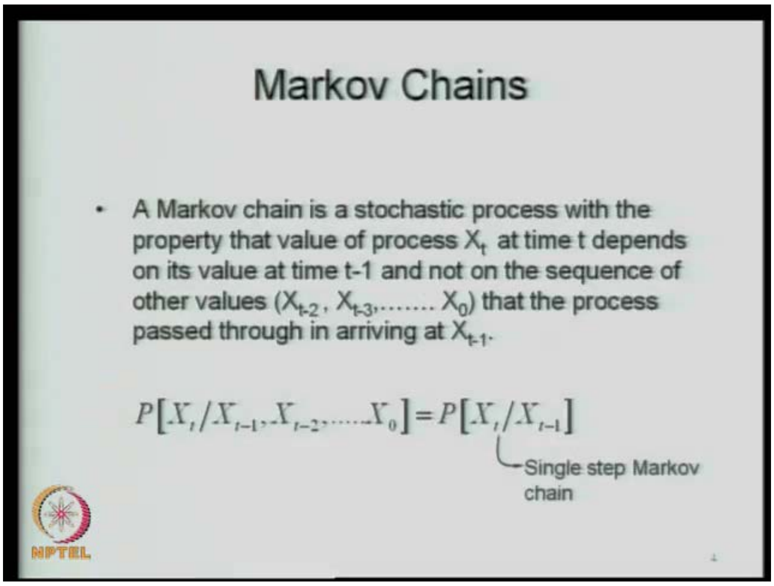

What do we mean by Markov chains? In fact, in one of our earlier lectures; I had introduced a model based on the Markov processes. Now, we will go to the more general Markov chain application. We formally define Markov chain as follows. A Markov chain is a stochastic process with the property that the value of the process X t at time t depends on its value at time t minus 1 and not on the sequence of other values X t minus 2 X t minus 3 etcetera X 0 that the process pass through in arriving at X t minus one.

In other words, we write this as probability that X t given X t minus 1 X t minus 2 etc X 1 and X 0 is equal to probability that X t given X t minus 1. What does this mean? This means that given the entire history of the process from X 0 to the previous time period X t minus 1, the conditional probability X t given the entire history can be equated to probability of X t given just its immediate previous value X t minus 1.

In other words, the memory of the process is only restricted to the previous time period, probability of X t given X t minus 1 determines the state to which the process goes from t minus 1 to t. In other words, all other states in the previous periods that it pass through to arrive at X t minus 1 do not contribute to the state in X t. Only X t minus 1 contributes to the state X t in time period t.

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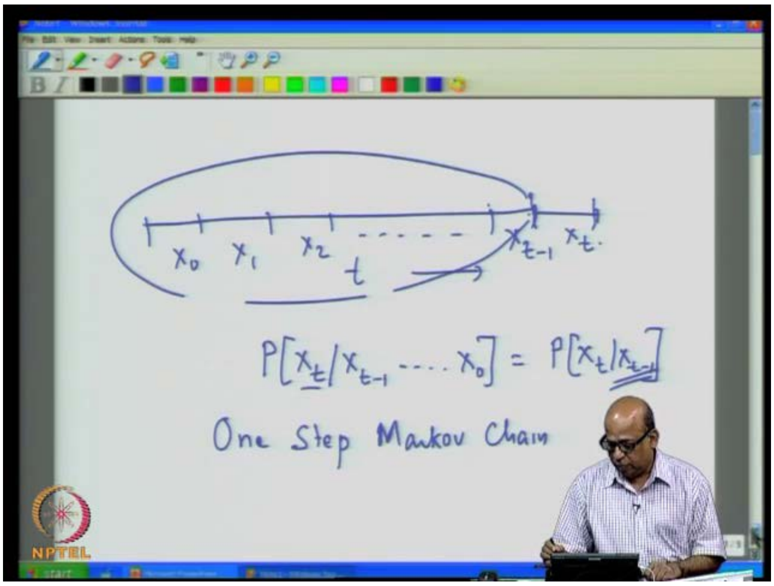

Look at the example of let us say the monthly stream flow. Let us say we are looking at monthly stream flows and then we have the stream flow sequence like this. This is the time scale t and then we have X 0, we are progressing in this time X 1 X 2 etc and we have come to X t minus 1 and then we are looking at X t.

So, let us say these are monthly stream flows and you have constituted a time series of monthly stream flows and then we are at a particular time X t minus 1. We are looking at what is a probability that the process goes into a particular state. I will define what is a state? It takes on a particular value in time period t given the entire history up to this point.

Now, this if we can write it as probability of X t given X t minus 1 etc X naught, if this can be written as simply probability that X t given, X t minus 1. Then this constitutes a Markov chain and in hydrology there are a large number of applications of the Markov chains specifically when we are talking about the stream flow sequence, the rainfall during certain time intervals, the reservoir water levels, the ground water levels which constitute a time series like this. Often we use the assumption of the time series constituting a Markov chain and we will now see in this lecture how we analyze such a Markov chains, such time series which we will analyze using the assumption of Markov chains.

Now, because the probability of X t depends only on the previous time period; such a change is called as one time step, one step Markov chain or a single step Markov chain. You may also have processes where it depends not only on the previous time period t minus one, but, perhaps on two times steps, previous two the correct time period. That is X t may depend on X t minus 1 as well as on X t minus 2. X t may depend on X t minus 1 X t minus 2 and X t minus 3.

Let's say you are looking at hourly rainfall and then you want to decide on the state of whether the rainfall is 0 or non 0 during a particular hour. Now, this may depend on what has happened during the previous hour, what has happened in the hour previous to that and so on. So, it may depend not on time step, but, on several time steps and then we define the n time step Markov chain n step Markov chain. Now, because we are able to write this expression probability that X t given X t minus 1 etc up to X naught is equal to probability of X t given X t minus 1.

We will be dealing with these probabilities now. Probability of X t given X t minus 1 and the specific cases that we are dealing with in Markov chain are that the process goes into certain finite number of states, finite or accountably infinite number of states. What do I mean by states? For example we may have the rainy day and non rainy day as 2 states which indicates the weather. So, we may say X t is equal to 0 if it is a non rainy day, X t is equal to 1 if it is a rainy day. So, X t takes on two states, two possible states.

Similarly, if you are looking at reservoir levels, we may say that the level is between 0 to 25 percent that state number 1, 25 to 50 percent state number 2, 50 to 75 percent and so on. Like this we may discretize the given variable and then define the process to go into one of the states. Similarly, if you are looking at monthly stream flows we may have a range of stream flows that are possible for the monthly stream flows. Let us say 0 to four hundred is the total range of 0 to four hundred million cubic meters is a total range of flows that you are considering. In that we may define certain discrete states into which the monthly stream flow goes.

For example we may say 0 to 100 we call it as state number 100 to 200 we call it as state number 2. So, whenever the monthly stream flow occupies, takes a value between 0 to 100 we call it as state number 1 and so on. So, like this we define the states of the process, states into which the process goes in and these states can be different during different time periods although for most applications; we define the states uniformly all across the time periods.

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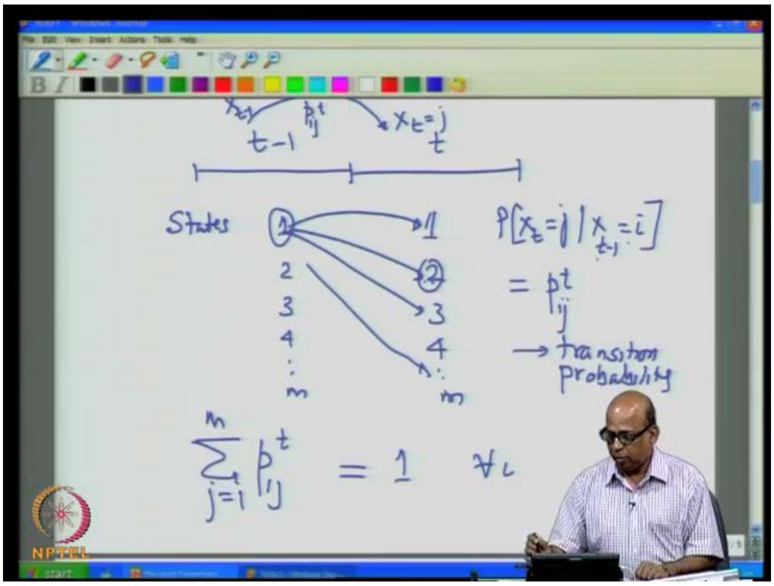

Now, let us understand what we mean by the states and what we mean by the transitions. Let us say this is time period t minus 1 and this is time period t and now we define the states as 1 2 3 4 etc m number of states we define. That means, the process in time period t minus 1 can be in any of these m number of states.

Like I said, if we are looking at dry day and wet day in terms of a whether it is a non rainy day or a rainy day then we will have two states. So, the weather can be either in state number one which may be dry day, state number two which may be a wet day. Or if you are looking at the reservoir storage as a as a Markov chain then in time period t minus 1, it may be either it may be in one of these several states 0 to 25 percent of the total storages as state number 1, 0 to that is 25percent to 50 percent as class state number 2 and so on.

So, we define a priory the states that the process can occupy. So, in time period t minus 1, it can occupy any of these m number of states. Exactly one of these m number of states it can occupy. Then in time period t, again it can go into any of these m number of states. Now, starting with a particular state let say at t minus 1 let say we were in state 1. The process was in state 1, it can go to state number 1 state number 2 state number 3 and so, on state number m. The probability that starting with a particular state X t is equal to j given X t is equal to I. Let us say I write like this where I mean X t is equal to I as the process is in state I in time period t minus 1, this is time period t minus 1 let me rewrite that.

So, this is t minus one. So, what I am writing is probability that in time period t, it goes into a particular class interval given that in the previous time period t minus 1, it was in a class interval I. So, staring with a particular class interval I in time period t minus 1, it can go to either class interval 1, class interval 2, class interval 3 etc to any of this class intervals m and it has to go to at least one. It has to go into exactly one class interval in next time period t by class interval I mean the state state of the system.

This probability that probability that it goes into a particular class interval j s for example, two given that in the previous time period it was in a particular class interval I let say 1 is denoted as P I j t. Let me write that more clearly. This is P I j t and that is called as the transition probability P I j t and this is called as the transition probability. So, the transition probability indicates, it gives the transition it gives the probability that starting with the class interval I in time period t minus 1 the process transits into class interval j or the state j in time period t and we denote this as P I j t.

As you can see here, starting with a particular class interval I in t minus 1 it must go into 1 of them 1 of the class intervals 1 2 3 4 etc up to m. What does that mean? It means that the probabilities, probability of going into class interval one starting with one, plus probability of going into class interval two starting with one, probability of going into class interval three staring with 1 etc.

When you add it up this must sum up to one because it has to go into one of these class intervals one of these states. So, that we write it more formally as summation of we are looking at j is equal to 1 to m. So, j is equal to 1 to m where j refers to the time period t and I refers to the time period t minus 1 P I j t must be equal to 1. So, we have two conceptions; one is the transition probabilities. The transition probabilities indicate the probability of the transition from t minus 1 to t. So, what do I mean by a transition? That it was in a particular process particular state and then it transits from X t minus 1, it transits to X t or X t minus 1 is equal to I it transits to X t is equal to j and with a probability of P I j t. This is the transition probability.

Then, because it has to go into one of the classes, one of the states starting with a given state, the sum of the transition probabilities for any given I must be equal to 1 over all j. So, that is a second concept that we got.

So, we write this as sum of P I j t s from j is equal to 1 to m must be equal to 1 for all I. That is for each of these I, this condition has to be satisfied and therefore, associated with each of these class intervals I we have the P I j t s defined from time period t minus 1 to t. Keep relating this with the hydrologic process. We are talking about let us say the stream flow during the time period t minus 1 and the stream flow during the time period t and typically this t can be a month duration.

So, we may be talking about the month June month flow and the July month flow and June month flow can take on any state between 1 to let us say 5 1 to 3 4 5 and then when I say it has taken a state of 1, we may have a discrete class interval of stream flows associated with that let us say 0 to 100.

Similarly, class interval 2 may have 100 to 200. So, whenever the stream flow is between 0 to 100 I say that it has fallen into class interval 100 or it has taken the value of state 100 and then starting with a particular class interval in t minus 1 it transits into a given class interval j in time period t and this transition is governed by the probabilities the transition probabilities P I j t.

So, associated with every time period, associated with every class interval I we have the P I j t s available to us. How we compute is a different story. We will see how we compute this transition probability later on, but, now we formulate what is called as the transition probability matrix.

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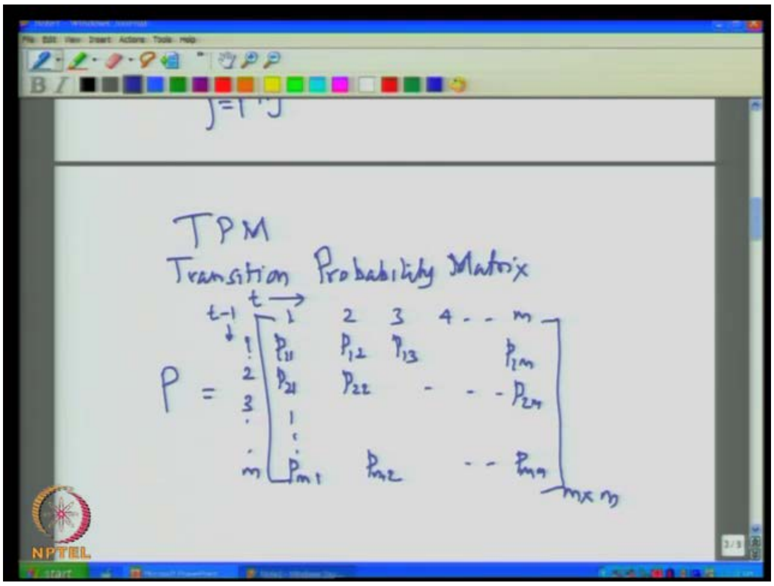

So, we write now the transition probability matrix TPM. This is transition probability matrix and we denote this as capital P and we write this as let us say we are talking about t minus 1 here and the states are 1 2 3 etc m, m number of states and t on this direction we have again 1 2 3 4 etc up to m.

So, this will be P 1 1, P 1 2, P 1 3 etc that is starting with class interval 1 in time period t minus 1. The probability that it goes into class interval 1 in time period t in the next time period t is given by P 1 1. Probability that it goes into class interval 2 in the next time period t starting with the class interval 1 in time period t minus 1 is t 1 2 and so on. So, this will be P 1 m that is starting with the class interval 1 in time period t minus 1 it goes into class interval m in time period t is P 1 m. I am using the class interval and the states analogously.

So, whenever I say class interval and I mean it is a state state of a system. So, similarly, this will be P 2 1, P 2 2 and so on P 2 m like this. So, we have P m 1 P m 2 etc P m m. So, this is a m by m matrix where m is the number of states that it can take in time period t minus 1 as well as time period t.

So, this we will explain again using more formal notations here. So, this is what we call it as single time step, single step Markov chain and then we have a discrete number of states possible in each of the time period t t minus 1 t minus 2 and so on.

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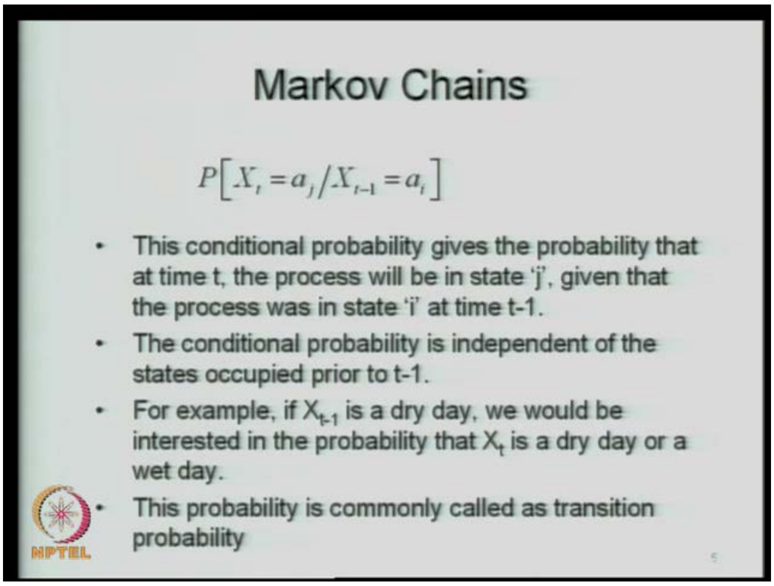

So, we then write the conditional probability. Let us say that our notation is X t is equal to a j indicates that X t goes into state j which can be represented by the value a j. Now, when I talk about the applications it becomes clear what I mean by that.

Let us say state 1 in the case of monthly stream flows state 1 is defined by the stream flow having any value between 0 and hundred and we may represent that particular class with a particular value of the stream flow. So, we may write it as either X t is equal to j indicating that X t belongs to the class j or we may write it as X t is equal to a j saying that takes on a particular value.

But essentially we understand that X t has gone into that particular state j. So, this conditional probability now, probability that X t is equal to a j given X t minus 1 is equal to a I, this conditional probability gives the probability that that time t, the process will be in state j given that the process was in state I at time t minus 1.

Keep relating this with the physical problem that we just mentioned that let say you were in a rainy day. What is the probability that the next day is a non rainy day? Or what is the probability that next day is also a rainy day? So, we are talking about two states here; rainy day is 1 state and non rainy day is another state. So, we may say I is equal to 1 I is equal to 2.

So, given that we were in a particular state a I; the probability that we go into a particular other state a j, into another particular state a j in the next time period. This is a conditional probability and this is also called as a transition probability. Ah The conditional probability is independent of the states occupied prior to t minus 1 and that is a requirement of the Markov chain. So, we are saying this is single state Markov chain. So, we are saying that probability that X t is equal to a j or X t is equal to j X t belonging to class j depends only on the state that it occupied in time period t minus 1 and not on any other states that it occupied previously previous to t minus one.

So, we are interested in for example, if X t minus 1 is a dry day, we would be interested in the probability that X t is a dry day or a wet day. Typically, this we use in the analysis of rainy days non rainy days etc. In the reservoir flows cases we may we may use this to develop stochastic optimization techniques where we consider the inflow data, inflow to the stream flow as a Markov chain and then discretize the inflow into several number of states and then identify which state or which class interval the inflow during a particular time period belongs and then what is a transition probability with which it transits into a given state in the next time period.

So, this conditional probability is called as the transition probability or transitional probability. Both of them are use either way called it as transition probability or the transitional probability and we denote this as P I j t.

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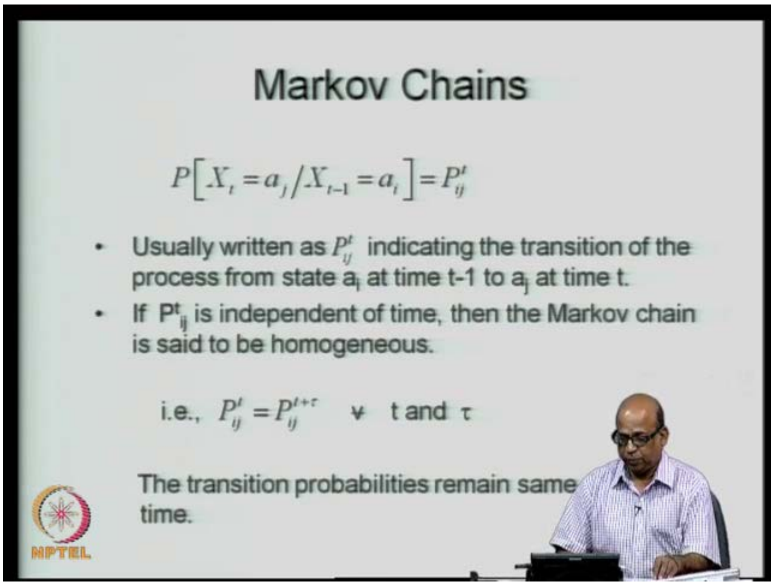

So, in in our notation this P I j t indicates that from t minus 1 we are considering the transition to time period t. So, this is this indicates the transition of the process from a state a I at time t minus 1 to state a j at time period t.

Now, we consider a special case where the P I j t is independent of the time. What I mean by that? That once we have the class intervals defined for t minus 1 and t the transition probability is defined for time period t minus 1 and t, the transition probabilities remain unchanged irrespective of which time intervals you are considering, which two time intervals you are considering.

So, we if we can approximate P I j t if we can state that P I j t is equal to P I j t plus tau for all t and tau; that means, irrespective of where the time series you are positioned; the transition probabilities remain the same. Then such a Markov chain is said to be homogeneous Markov chain and we will be dealing with a homogeneous Markov chains for the advantage that it provides, the homogeneous Markov chains provide.

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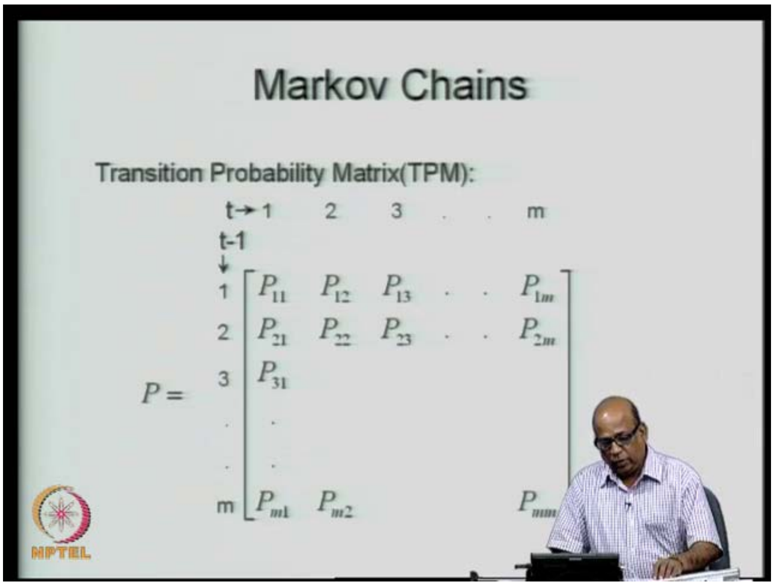

Let say now we write the transition probability matrix from time period t minus 1 to time period t. So, like I said starting with the class interval 1 the probability that it goes into class interval one in the next time period is P 1 1, starting with class interval 1 the probability that it goes into class interval 2 in the next time period is P 1 2 and so on.

So, you get P 1 m here and like this you write, starting with class interval m what is the probability that it goes into class interval 1 P m 1 to P m 2 m P m m. So, this is a m by m matrix and this is a called as a transition probability matrix t P m. Now, because starting with a particular class interval one. Let us say it has to go into one of these class intervals, the summation of all this probabilities must be equal to 1 as I just stated.

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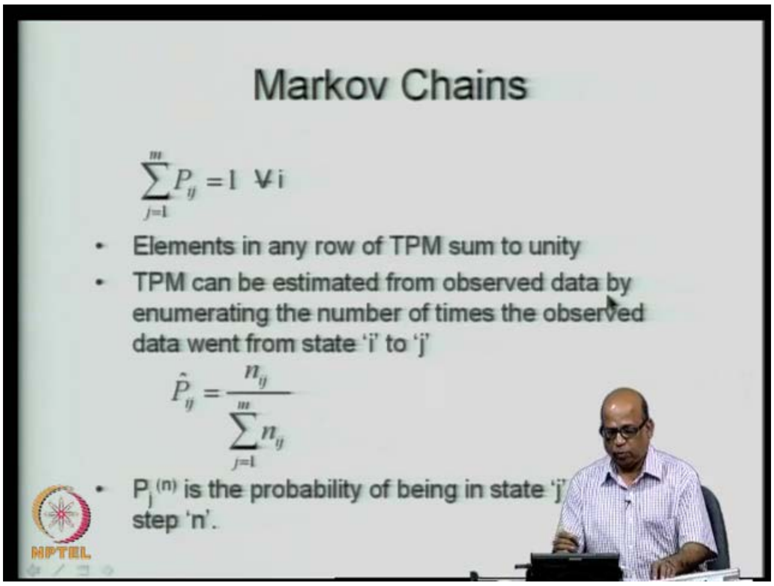

And this we write more formally as P I j summation j is equal to 1 to m is equal to 1 for all that means, for each of these terms the rows must add up to 1. Now, such matrices where the rows add up to 1 are called as stochastic matrices in the stochastic process theory. The element in any row of t P m sum to unity. So, this is one important feature. So, all these rows sum to unity.

Why does that happen? As I said, starting with a particular state it has to go into one of the states. So, the probability that it goes into 1 starting with 2 or it goes into 2 or it goes into 3 etc this must be 1. So, it has to go into 1 of the class intervals and therefore, the rows add up to 1. Now, how do we determine this transition probability matrix? Remember we are talking in this particular course with data analysis. That means, we have the observed data from the observed data we are developing all this analysis techniques. So, we must have the where with all or we must have the method by which we can estimate the transition probabilities.

So, let say we are considering the monthly stream flow time series. That means, for June month we have let say 50 years of data, July month we have 50 years of data, august and so on.

So, if we have 50 years of monthly stream flow data and we are talking about t as a month and we write t and t plus one side by side. Let us say June and July month side by side and we reckon the state to which the monthly stream flow went in the month t and the state to which it went in month t plus 1 and from the relative frequency approach, we estimate the transition probability.

So, this is written here as the estimate of the transition probability P I j I am removing P here because we are assuming that the transition probabilities remain the same across all the time is equal to n I j. That means, starting with class interval I in time period t it went into class interval j the number of times from class interval I in period t, it went into class interval j in period t plus 1 divided by the number of times it went into class interval I itself which is summation of j is equal to 1 to m.

So, this gives you an estimate of the transition probabilities. Let me explain that a bit more because in hydrology we will be dealing with this particular problem many times. How do we estimate the transition probabilities itself?

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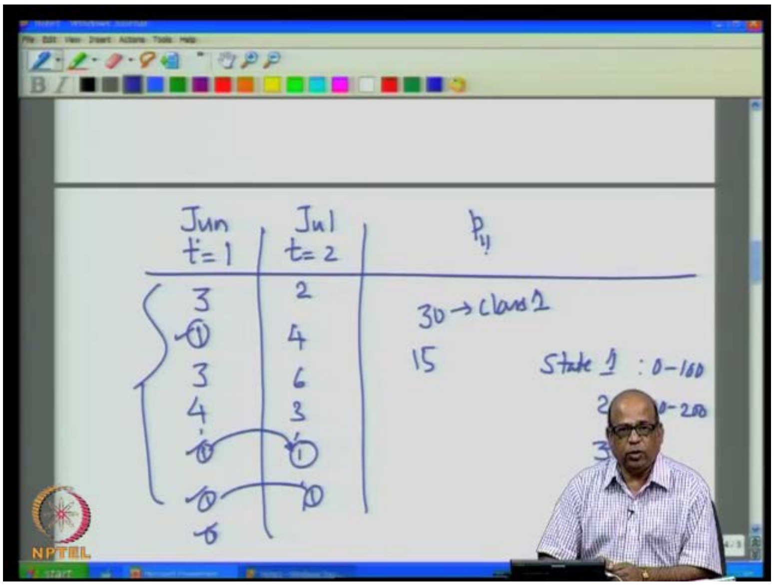

So, we will also do some case studies later on, but, let us examine how we estimate the class interval from, the transition probabilities from one time period to another time period. Let us say we are in t is equal to 1 and t is equal to 2. So, this is t is equal to 1 may be June month, t is equal to 2 may be July month. Because you know what is the state number 1 by our definition.

Let us say that we are talking about monthly stream flows and then say that state 1 is whenever it falls into 0 to 100, I call it as state one and state two when it falls between 100 and 200 and so on. Like this we would have defined states let say we define 10 states for the stream flows. Then on the observed data you will have the June month flows for so many years. Let us say you have 50 years of data. So, you have 50 values for June month 50 values for July month take the first value of June look at where does it fall? Let us say it fell between in the class state number three. So, we put it as state number 3 and then in the next time period in the next year in June month it fell in class interval one.

In the next year in June month it fell into class interval three again and four again and so on. Like this you identify the state into which the process went in the month of June. Similarly, you identify the state in which it went into in the month of July 2 4 6 3 and so on. Like this you identify corresponding to the data that you have you identify the state into which the process went in two adjacent time periods t is equal to 1 and t is equal to 2 in this particular case.

Now, let say I am interested in getting P 1 1 that is probability that starting with class interval one in June, it went into class interval one in July. What do we do? First of all you collect all those periods in which it went into class interval one. Let us say out of 50 times it went into class interval one about 30 times in July. So, in July month, in June month that is here in June month thirty it went into class interval 1 or the state one. So, it was in state 1, 30 numbers of times out of the total of 50 times.

Then, we look at every time it has crossed gone into class interval one which means out of these thirty times, how many times it went into class interval 1 in July again? So, we pick up those many numbers in the month of July in which it was in class interval 1 starting with class interval 1 in June month. Let us say this was 15. That mean, 15 number of times out of these 30 times that it went into class interval 1 it went into class interval one again in July month. So, what is the probability of going from one to one? It is 15 by 30 or 0.5

So, this is how we calculate the transition probabilities from the data. So, you have the data, you identify for t P t and t plus 1, you identify the state to which that particular data belongs. Depending on the definition of the state that you have done and then look at how many times it went into a particular class interval I and then starting with this particular class interval I, how many times it went into class interval a given class interval j in the next time period.

So, this is how we calculate the transition probabilities. Now, in this particular example what will happen is that between t is equal to 1 and t is equal to 2 you get a particular transition probability matrix. Then July to august you will get another transition probability matrix and so on.

So, strictly it is not a homogeneous transition probability matrix in this particular case. But, this is how we obtain the transition probability matrices from the data. Now, that is what we mean here. So, this is how we get estimate the transition probability matrix.

Now, we will look at another important concept. At any time step n, we will denote P j n as the probability of being in state j in times step n. So, this is a, we are talking about Markov chains. So, t plus 1 t plus 2 etc you have sequence of sequence of the process, sequence of the values of the process across time. So, at any time step n, P j n is the probability of being in particular state at that particular time n.

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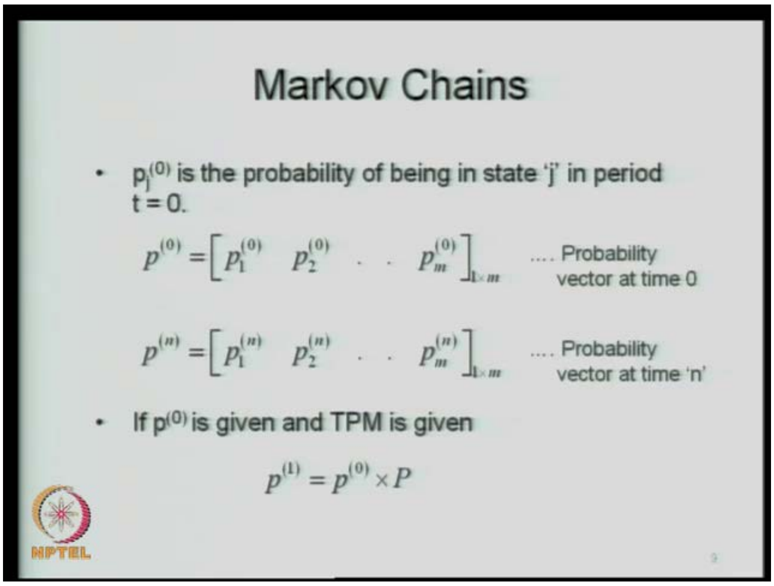

How do we get this? Which means that if we are starting with time period t is equal to 0, we write this as P j 0 is the probability of being in state j in period t is equal to 0. j is again a finite number of classes j is equal to 1 j is equal to 2 etc.

So, in time step 0, P j 0 is the probability of being in state j in period t is equal to 0. So, at period t is equal to 0 it can be either in class state 1 state 2 state 3 etc state n. So, this is how it is written P 1 0 which is in time step 0 it is in state 1. That is a probability of being in state 1 in time step 0, probability being in state 2 in time step 0 and so on. Probability of n being in state being in state m in times of 0.

So, this is a 1 by m row vector this is called as the probability vector at timer 0. Similarly, at any time step n I may be either in state 1 with a probability of P 1 n state 2 with the probability of P 2 n and so on. Probability of P m n I belong to state m the process belongs to state m the process belongs to state m. This is a 1 by m row vector. It is called as probability vector at time n.

Let say that we are given P0. That means, we know that at time t is equal to 0 the probability of being in any of the given states which means P 0 is given if P 0 is given I can obtain P 1 as P 0 into P what is this P? This P is the transition probability because I am in a particular class interval P 1 I am sorry I am in a particular class interval one with a probability of P 1.

So, the probability of being in class interval 1 in time period 0 is P 1. If your in class interval 1 in time period 0 the probability that you go into class interval 1 in time period 1 will be P 1 0 into P 1 1. That is a transition probability.

So, given any class interval, any state in time period 0 you go to any given state with the associated transition probabilities and therefore, the probability vector at time one can be related to probability vector at time 0 as P 1 is equal to P 0 into p.

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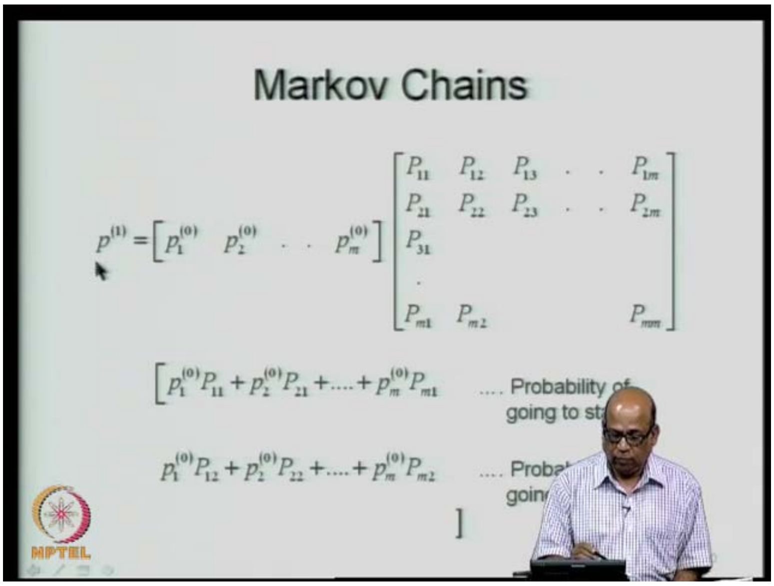

I will elaborate this with what what I mean by that is P 1 is given by P 1 0 P 2 0 etc this is the probability vector at time period 0. So, you are in class interval 1 in time period 0 with a probability of P 1 0 and so on multiplied by the transition probability matrix P 1 one P 1 2 P 1 3 etc P 1 m and this is a transition probability matrix. What does this give when we when when we multiply this P 1 0 into P 1 1?

Look at this term that is you were in class interval 1 in time period 0 with a probability of P 1 0. Once you are in class interval 1 the probability that you go into class interval 1 in the next time period is P 1 1. So, this gives you the total probability of going to state 1 starting with state 1 in time period 1 for the time period 0.

Similarly, if you are in class interval two, you you will be in class interval two in time period 0 with a probability of P 2 and if you are in class interval two you go to class interval one with a probability of P 2 1 and therefore, this gives you the probability of going into class interval 1 with starting with a class interval two and so on.

So, the first term here P 1 0 into P 1 1 P 2 0 into P 2 1 etc P m 0 into P m 1 this term here gives a probability of going to state 1 when you are given P 0. The second term here P 1 0 into P 1 2 P 2 0 into P 2 2 etc. Look at this terms starting with class interval 1, you are going to class interval 2, starting with class interval 2, you are going into class interval 2, starting with class interval m you are going into class interval 2.

So, this gives this term gives probability of going to state 2. So, in essence this multiplication gives P one; that means, in time step one what is a probability of going to class interval or state 1 probability of going to state 2 etc. This is again a row vector. So, we write P 1 as P 1 1 P 2 1 P m 1 etc 1 into m.

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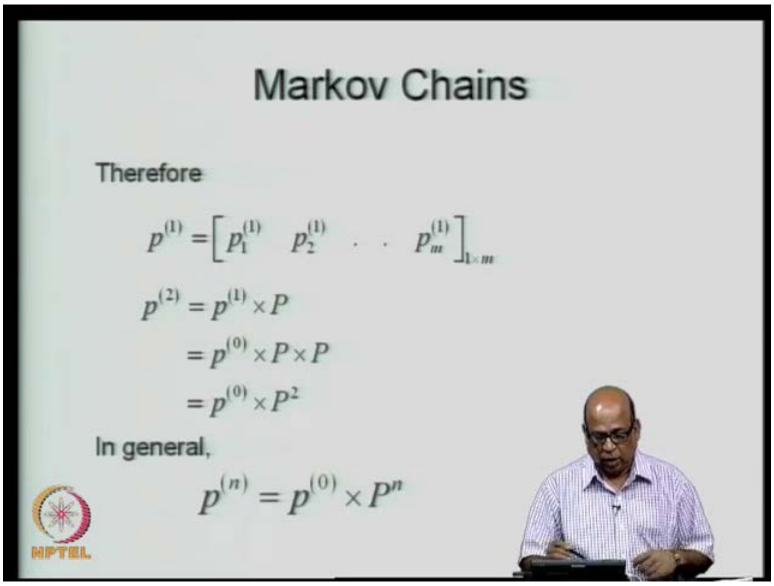

So, we obtained P 1 as P 0 into p; that means, the previous time period. We now the P 0 and multiplied it by the transition probability to get P 1. So, we have obtained P 1 now; that means the probability vector at time period one is known.

We will now use this probability vector to obtain the probability vector for the next time period. So, I will write P 2 as P 1 into p, but, what is 1 P 1 is P 0 into P and therefore, we write P 2 as P 0 into P square P is the transition probability matrix. So, given the probability vector at time 0, you can obtain the probability vector at time two and in general you can obtain the time probability vector in times n starting with the probability vector P 0 by P n is equal to P 0 into P to the power n where P is the transition probability matrix P 0 is the probability vector at time 0 and P n is the probability vector at time n.

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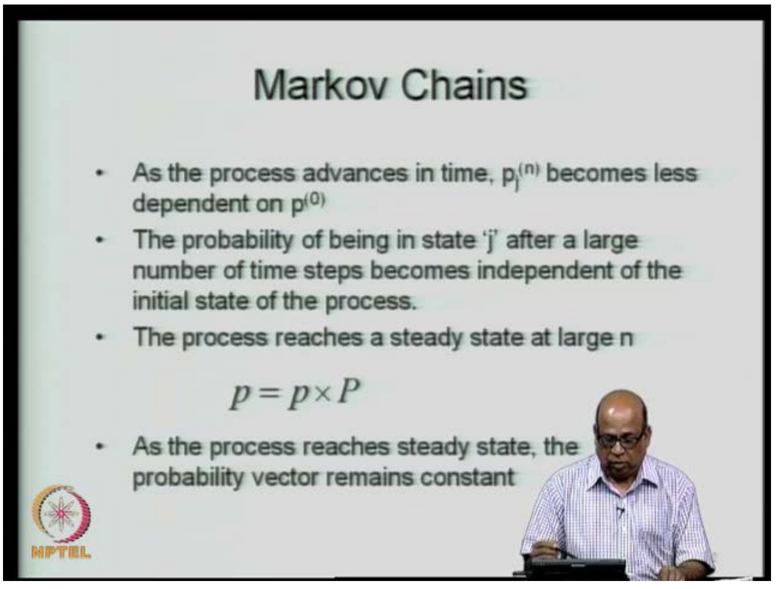

Now, as the time progresses you started with a particular initial state. That means, P 0 which is the probability of being in a particular state in time period 0 you know that then as time progresses at a far away time from time t is equal to 0 what will happen? The dependence of the process on initial state at time t is equal to 0, this starts weakening. So, it becomes less and less dependent as the time progresses.

Suppose, we are looking at a far away time in the sequence and then we are looking at the probability of the process being in a particular state at that far away time; that may become independent, totally independent of the state with which we started. So, the probability of being in a state j after a large number of time steps becomes independent of the initial state of the process and because of which we will be able to write this as;

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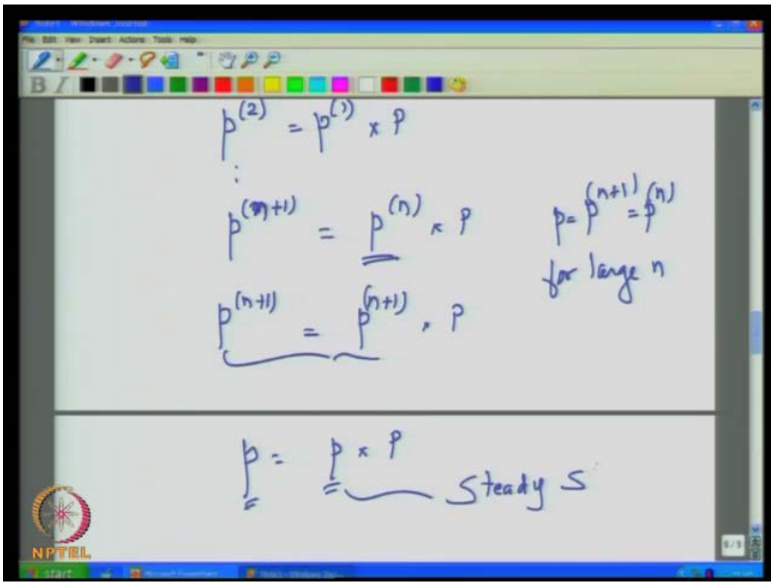

Let me write it here. We will be able to write, what we did write for P 1? We wrote it as we wrote P 1 as P 0 into P transition probability. Similarly, we wrote P 2 as P 1 into transition probability and therefore, we can write P m plus 1 r let say n plus 1 because m I am using it for state is equal to P n into P. That is we are relating it with the probability vector of the previous time period.

Now, as the time progresses the P n will not be much different from P n plus 1 or P n and P n plus 1 will converge because they are not dependent on the initial state. When that happens, we can write that as P n plus 1 is equal to P n itself and therefore, I will write this as P n into P and both these are equal. So, I can write this as P is equal to P n plus 1 is equal to P n for large n. I will not say n tending to infinity. We will say for large n that means as you progress well into the time, a stage will come for homogeneous Markov chains when the transition, when the probability vectors converge and therefore, we will be in a position to write this as P is equal to P into P where P is the probabilities when the convergence occurs and these are called as the steady state probabilities. This is called as steady state probabilities or we say that the Markov chain has reached a steady state. Now that is what is explained here.

So, the process reaches a steady state at large n and then we write it as P is equal to P into capital P. Capital P is the transition probability matrix and small P is a steady state probability vectors. So, as the process reaches a steady state, the probability vector remains constant. that is That is the idea there.

So, determination of the steady state probabilities is an important exercise that we do in Markov chain. That means, given that given a time series, given a process we should be able say the probability with which it goes into a particular state at a certain time period when the steady state is reached. So, that is we determine the steady state probabilities using this expression. In fact, with a computational power that is available these days, we simply do it with you know raising the matrices to several powers.

So, the transition probability matrix P to the power 2, P to the power 4, P to the power 8 etc. So, we are talking about n step transition probability matrices the transition probability matrix raise to the power n as n becomes larger and larger, the probability vector converges and that is what gives you the probability vector. We will see that through an example presently.

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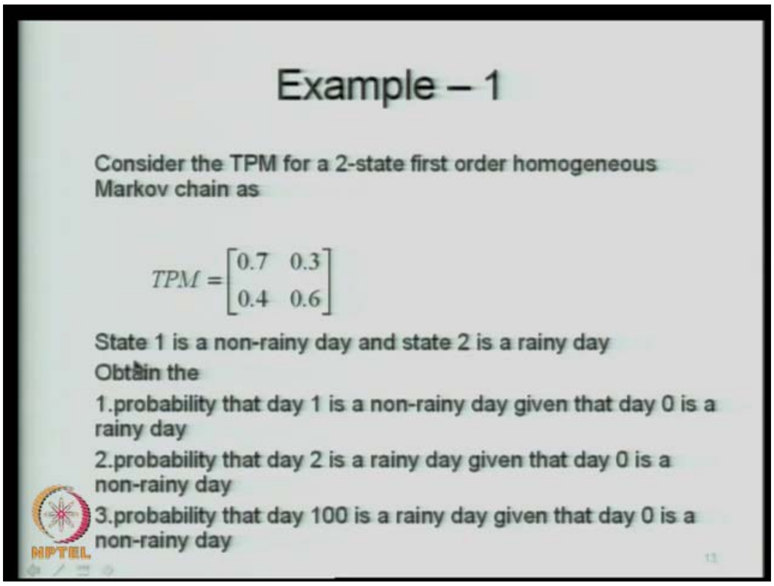

So, let us try to understand what we discuss. So, far with a small exercise let say we have two states one is a non rainy day and another is a rainy day. Typically this is the problem that we come across in modeling the weather. We say that the starting with a non rainy day what is the probability that it goes into a non rainy day? The probability that it goes into a rainy day and so on?

So, we define two states here state one is a non rainy day and state two is a rainy day. So, if we are looking at a particular time t and these are the two states this is non rainy day and this is a rainy day now starting with a non rainy day again here it is non rainy and rainy. So, starting with a non rainy day what is a probability that it goes into a non rainy day is given by this.

So, 0.7 is the probability with with which it goes into a non rainy day if the previous day was a non rainy day. So, that is a, and if the previous day was non rainy day, the next day will be a rainy day with a probability .3. So, this is how we interpret transition probability matrix.

So, we will first obtain the probability that day one is no rain day given that day 0 is a rainy day. Then we will see probability that day two is a rainy day given that day 0 is a non rainy day. Similarly, probability that day hundred is a rainy day given that day 0 is a non rainy day. So, these are the probabilities that we will see this.

Let us look at the first problem probability that day 1 is a non rainy day given that day 0 is a rainy day. What does this transition probability matrix give? It says state 1 which is a non rainy day. So, in day 0 we were in non rainy day, day 0 is a rainy day. So, here you are non rainy and rainy. So, day 0 is rainy. So, you are here now.

What is a probability that day 1 is a non rainy day? So, this is non rainy and this is rainy. So, starting with a rainy day, the probability that you go into a non rain day is .4. So, that is what we get here.

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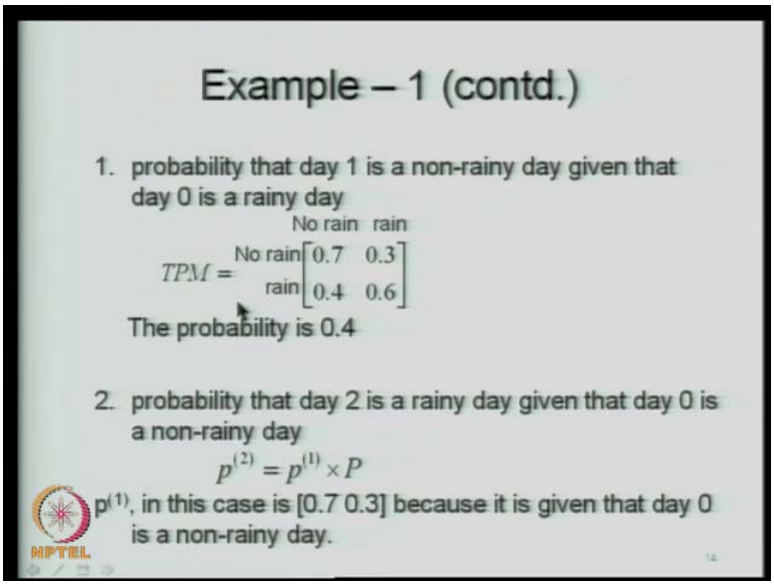

So, probability that day 1 is a non rainy day given that day 0 is a rainy day. So, no rain and rain this is no rain and rain. So, you are here it is a rainy day in 0 and you go into a non rainy as probability as .4. So, directly from the transition probability matrix you obtain this.

We will continue this a example in the next class and then we will also see how for this particular example we can determine the steady state probabilities. So, essentially what we have done in today's lecture is that we have introduced the concept of Markov chain and then looked at what is the transition probability and then formulated the transition probability matrix. From the transition probability matrix, we have formulated an expression to obtain the steady state probabilities. We say that the Markov chain has reach to the steady state when the probability vectors are no longer dependent on the initial state.

So, the probability vectors converged and by the steady state probabilities; we we will be able to say that at any given time step n in future, when the Markov chain has achieved the steady state at any given time step n, the probability that the process will be in a particular state will remain constant. Then, we had just seen a simple example by which we interpret the transition probability matrix for a non rainy and rainy as two states of the system given that the state was in non rainy. What is the probability that it goes into rainy and so on. So, these are the kind of kinds of problems that we will be examining typically in the Markov chains.

So, in the next lecture we will continue the discussion on the Markov chain. We will also look at some numerical example through which we estimate the transition probability matrix and then from the transition probability matrix, we will estimate the steady state probabilities and specifically for the case study of monthly stream flows in a particular location and so on.

So, thank you for our attention. We will meet again. .