# Earthquake Geotechnical Engineering

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**Lecture 44**

## Slope Stability Analysis (Conti.)

I welcome you again in this NPTEL online course on Earthquake Geotechnical Engineering. And, we are at the lecture number 44 and we are going to discuss slope stability and retaining walls. So, we are discussing slope stability analysis under module 5. In this module 5, we already discussed slope stability analysis for static slope stability analysis over. Seismic slope stability analysis is pretty much over. What we are going to discuss in this lecture? The partly we covered in the last lecture Newmark sliding block analysis one part that is related to input motion will be discussed today. Then Makdisi seed analysis which is also for seismic slope stability analysis. And then we have stress deformation analysis which is under three categories, strain potential approach, stress reduction approach and non-linear analysis approach will be discussed. And finally, analysis of weakening instability.

And all these stuffs are taken from the Kramer's book. Coming to the input motion which is part of the from based on the Newmark sliding block analysis. The accuracy of the sliding block analysis will depend on the accuracy of the input motion applied to the inclined plane. If your input motion itself is not correct, then the analysis may not be the correct. Originally proposed the sliding block method assumes that potential failure mass to be raised. So, this was the one of the assumptions in which case the appropriate input motion would be ground motion at the level of the failure surface, but this is not the case. Actual slope however is not very rigid, they deform during earthquake shaking. So, that if we need to understand that. What happens? Dynamic response of this on the slope will depend on their geometry and stiffness and depend on the amplitude and frequency content of the motion input motion.

So, one side what is the geometry and stiffness of the slope, on another side what are the characteristics of your input motion based on the characteristics of your input motion like this dynamic response will come. For slopes which composed of very steep soils or slopes which are subjected to low frequency motion, this combination will produce long wavelength. So, this is explained here, I can go directly to the figure. So, in the first part of the figure, the wave which is coming is low frequency. Once you have the low frequency that it will be sparse, it is not very close like in the second case where you have a high frequency.

So, low frequency wave will produce long wavelength, wavelength is going to be higher. And the frequency is small because what is wavelength? Wavelength is the distance between two peak values, one cycle. So, for low frequency the wavelength will be large, while for high frequency component your wavelength is going to be small. So, as a result and in this case, this is your slope, this is your potential failure surface. In this potential failure surface, you will have the motion which is in phase. So, you have in phase, but when you have the short wavelength which is the case here, in this case the motion is not completely in phase rather the motion let us say like in between here almost at the top, first in this direction then the direction have changed. So, this becomes out of phase. So, that is the difference. So, this will depend whether in phase or out phase will depend on the frequency content as and then it will be the short wavelength or long wavelength. Lateral displacements in potential failure mass of slope in softer soils or the slopes subjected to higher frequency motion may be out of phase which we have already discussed.

When this occurs the initial forces at different points within the potential surface failure mass may be acting in opposite directions and the resultant initial force may be significantly smaller than that implied by the rigid block assumption. So, what you have here, the effects of slope response on the initial force acting on a potential failure mass can be computed using dynamic stress deformation analysis, using dynamic finite element analysis which is like in the next figure like here. So, this is what has been done. This is in fact actually a dam. It is because most of the analysis has been done and many of the things have come from the dam analysis.

So, in this case let us say it is appeared to be two dimensional. In this case you have finite element and finite elements are there and this is the pure potential failure surface. So, the horizontal component of the dynamic stresses which is acting on a potential failure surface are integrated over the failure surface to produce the time varying resultant force that act on the potential failure surface. So, you first find out the horizontal component of the dynamic stresses, the dynamic stresses which is acting along this plane in the failure surface and horizontal component of the dynamic and that is integrated from over the whole length of this failure surface from here to here. Then what you do, you divide this with the total mass, total we integrate from here to here for this horizontal component of dynamic stresses and then on the top of it you have the mass here.

So, when we divide the total force divided by total mass, then you find average acceleration of the potential failure surface. And this average acceleration of the potential failure surface is used and the evaluation of average acceleration for slope in embankment can be calculated by this way. And then once you have, then you find out ultimately what you get is time history of average obtained by dividing resultant force by mass of the potentially unstable soil. So, this was about from this input motions. Although this process which we developed originally for dams, the basic concept can be applied to any type of slope. Initially this process was developed for the dam. The average acceleration time history which may be greater or a smaller amplitude than the base acceleration time history. So, you find the average acceleration. If it is greater than the base acceleration time history, then depending on input motion and amplifying characteristics of the slope, then it will provide the most elastic input motion for sliding block analysis of the potential failure mass. So, here what is difference compared to what we discussed earlier for the Newmark sliding block analysis.

In that case, we define a yield acceleration Ay. Here rather than you know yield acceleration, yield rather than defining a yield acceleration, you find out what is called average acceleration time history. And how to find average acceleration time history? We already said using finite element analysis, you integrate over the potential failure surface, find the total force and divide by the mass on the top of it, this give you the average acceleration time history and this will be varying in the time because your input motion is varying. Then we compare this with the characteristic of input motion depending on input motion and the amplifying characteristics of the slope, then it will provide reliable to find out the stability of the slope. So, this was all about from Newmark sliding block analysis. Now, coming to the second part that is Makdisi seed analysis.

Makdisi and seed in 1978 use average acceleration which is computed by the process which is given by the Chopra in 1966 and sliding block analysis to compute earthquake induced permanent displacement of further dams and embankments. So, basically the Makdisi and seed analysis was developed for embankments and dams rather than only slopes. Using the simplifying assumption about the results of dynamic finite element and shear beam analysis of such a structure, a simplified processor for prediction of permanent displacement was developed by Makdisi and seed. And in this simplified process, the yield acceleration for a particular potential failure surface is computed using the dynamic yield strength that is what is dynamic yield strength? Considered to be 80 percent of the undrained strength of the soil. So, this is 80 percent of the undrained. So, once you have dynamic yield strength and then the Ay can be defined. So, in this analysis what is done? The dynamic response of the dam or embankment is accounted by an acceleration ratio and what is that acceleration ratio? It is defined here. And this acceleration ratio it varies with the depth, it is not constant with the depth.

For example, A max by average is the value at some depth z equal some depth which is let us say we have this value here, this is line here. You have the value of acceleration. Now what we do? Because this is not constant rather it will vary along this line. When the acceleration varies, we find out the average value of the maximum acceleration which is we find the average along this line. So, this is A max by average at z equal to some depth z.

What is A max z equal to 0? The value at the top. So, this ratio naturally will be 1 where z by h equal to 0. So, at the top this ratio will be 1. If you go at the bottom the fixed condition this ratio becomes minimum and here in this figure there is some range is given. So, this is average value and this is from using FEM method and the dotted line is average value.

So, dotted line can be used for the numerical analysis. So, using this way we can find this ratio of A max by average to A max at z equal to 0 at any depth z by h. The Makdisi further did the analysis by subjecting several real and hypothetical dams to several actual and synthetic ground motions which are scaled to represent different earthquake magnitude. Makdisi computed the variation of permanent displacement with A y by A max and magnitude. So, with the different magnitude as well as the variation A by A max the permanent displacement has been calculated.

What has been found? There is a scatter in the predicted displacement could be reduced by normalizing the displacement with respect to the peak base acceleration and the fundamental period of the dam. So, the scattering is removed and prediction of permanent displacement by the Makdisi is accomplished with the chart.  So, in this chart what has been done?  You have on x axis ratio of A y by A max where A y is yield acceleration A max is the maximum value of acceleration. So, naturally here when the permanent displacement develop permanent displacement will develop only when your A y is less than A max.  If your A y is greater than A max then there is no permanent displacement. So, that means only when this ratio is less than 1 then only permanent displacement will develop. So, A y by A max as low permanent displacement is going to be more when A y by A max becomes 1 then there is no displacement. So, because yield acceleration is more. In this case like when you have the ratio this A y by A max. So, what you see when A y by A max increases this ratio permanent displacement decreases number 1.

Here in this figure, you have 3 of region one for 6.5 magnitude earthquake another is 7.5 and third one is 8.25. And on the figure B is the same as a figure A only the difference that rather than this average value has been taken. So, here there was a region, but this region has been represented by 3 lines.  So, what we do using this figure for given value of A y by A max we find this ratio u into A max over t naught u into A max over t naught is find out. And because A max is known t naught is the time period of the dam. So, you can find the value of u your unknown is u in this case and so permanent displacement can be find out this way. Continually the Makdisi simplified process is widely used for estimation of permanent displacement in dams and embankment because the process is based on the dynamic response correction of dams and embankment its results must be interpreted when applied to slope.

Because this process is in general developed for dams and embankment rather than general slopes. A small example on using the Makdisi analysis the data is given assuming that a failure surface of the dam has the yield acceleration equal to 0.24 g.  So, the value of A y is given to you 0.24 g. To estimate the permanent displacement for an earthquake of magnitude 7.1 and for the value of A max equal to 0.442 g. So, A max is given A y is given. So, first thing is that find the ratio of A y by A max. A y by A max ratio comes out to 0.543 and the period fundamental period of the dam t naught will be can we find out using this natural frequency which is given to you. So, this is 2 pi w d. So, this comes out 0.336. So, what happen have for 7.1 magnitude earthquake and there is ratio by A y by A max equal to 0.543 this ratio u max u A max by t naught this ratio comes out to be 0.04 which is you can read from this chart. Then this ratio is 0.04 and once this ratio is known then using this ratio you can find the value of u as a 0.04 into A max into t naught which is comes out to be 5.68 centimeter.

So, this was one example how to calculate use the permanent displacement using this chart which is given by Makdisi. Now, some other researchers also did, but this we have discussed so far was related to like wherever you are like considering on the based on the reached condition and rather than like using the finite element analysis. But there is one method which was in static case also when we discuss the stress deformation analysis and the stress deformation analysis more accurate and this stress deformation analysis of system are usually done using dynamic finite element analysis like earlier for static case it was done by using static finite element analysis.

If in such analysis the seismically induced permanent strains in each element of the finite element mesh are integrated to obtain the permanent deformation of the slope. So, you find the strains in each element and then integrate and then find. Permanent strain within individual elements can be estimated in different ways. So, that is possible in when you carried out the finite element analysis. The strain potential and the stiffness reduction approach estimate permanent strains using laboratory test results to determine the stiffness of soils which are subjected to earthquake loading.

Non-linear analysis approaches use the non-linear inelastic stress strain behavior of the soil to compute the development of permanent strains throughout an earthquake. So, how this has been done? Here first approach is what we say the strain potential approach and in this case in landmark investigation of the slides that occurred one of the example is here. One of the example is related to upper and lower San Fernando Dam during the 1971 San Fernando earthquake. So, what happens during this 1971 San Fernando earthquake? There are two dams one is upper San Fernando Dam another is lower San Fernando Dam and both the dams get filled as we discussed earlier. Seed et al. in 1973 developed a procedure for estimating earthquake induced slope deformation from the results of linear or equivalent linear analysis. In this procedure the cyclic shear stresses are computed in each element of dynamic finite element analysis. For each element cyclic shear stress has been calculated and using the results of this cyclic laboratory test the computed cyclic shear stress are used to predict the strain potential which is expressed as a shear strain for each element. Then once strain shear strains are known then deformations are then estimated as the product of average shear strain potential along a vertical section through the slope multiplied by the height of that section, height multiplied by the strain that will give you the deformations. So, this way a method which is called strain potential method has been developed.

The method is implicitly assumed that the strain that develop in the field will be the same as those that developed in a similarly loaded laboratory test specimen. So, it is assumed the strain which is developing in the field and in the laboratory is same. So, it may not be the same but it is assumed here. And that the maximum shear stress act in the horizontal direction in all elements. So, considering the strain potential approach estimate only horizontal displacements because it is assumed that it acts along the horizontal direction. It will compute only horizontal displacement. Analysis which are based on the strain potential approach are clearly very approximate and their results should always be interpreted with the caution. Then there is another approach which is called stiffness reduction approach where the stiffness decreases. In this approach computed strain potentials are used to reduce the stiffness of soil and this is as illustrated. Earthquake, so here you have, so in this stiffness reduction approach you have shear stress versus strain. So, what you have in this approach that the value of like in this first case you have the shear modulus equal to g i initial shear modulus.

While in the second case the value of shear modulus is g f. Naturally the slope of the second line is less than the slope of the first line. So, in this case basically the final shear modulus g f which you have in this case will be smaller than g i. And once it is smaller what will happen the strain for the same level for the same level of stiffness strain will increase. So, for example, when shear stress is the peak value the strain level was gamma i and it is gamma f. So, in this case the strain potential is nothing but the difference between the two strains gamma f minus gamma i and gamma f will be quite more than the gamma i.

So, this will be the condition if provided your g f is less than g i. So, in this case in this approach computers are used earthquake induced slope displacements are then taken as this difference between the nodal point displacement and two static finite element is analysis one using the initial shear modeling another using the reduced shear modeling. And this technique can be used with linear as well as using nonlinear models. Unlike the strain potential approach the stiffness reduction approach can estimate vertical as well as horizontal movements. Strain potential approach only horizontal movement was expected because it is based on the horizontal displacement or horizontal stresses.

But in this case a stiffness reduction can use for both vertical as well as horizontal. It is very approximate procedure like the earlier one also strain potential approach was also approximate. Here it is subject to many of the limitations of the strain potential approach. A work energy principle can be used for reducing for more accuracy or for better results using the stiffness reduction approach. Now, continue with the stress deformation analysis if you need to carried out let us say non-linear analysis.

When we say non-linear analysis that means stress system relationship is not linear. So, permanent slope deformation can also be computed but in that case, you need to use finite element analysis and those analysis that employ the non-linear in elastic models. The performance of slopes has been analyzed with two-dimensional, three-dimensional finite element analysis using both cyclic stress strain models which is for example, Finn et al and advanced constitutive models for example, Prevost, 1981; Mizuno and Chen, 1982 and so many others are also there. The most common application of the analysis of the earthen dams some of the examples can be found by like for example, preversed et al or Elgamal et al in 1990. The accuracy of the non-linear finite element analysis depends primarily on the accuracy of the stress strain or constitutive models on which they are best.

Now, this was all what we have discussed was analysis related to inertial instability, but there could be case where we need to do the analysis for weakening instability and when we do the analysis of weakening instability that is the case where the soil loses its strength due to the liquefaction. So, then this is basically as we discussed in the beginning. Though a process of pore pressure generation or structural disturbance earthquake induced stresses and the strength can reduce the shear strength of the soil. If there is development of pore water pressure, then the shear strength of the soil can be reduced as we already discussed. As weakening instability can occur when the reduced strength drops between the static and dynamic shear stress induced in the slope.

And weakening instabilities are usually associated with liquefaction phenomena and can be divided into two main categories. One is called flow failures and another is called deformation failures. Flow failures the flow liquefaction and then you have the cyclic mobility. Flow failures occurs when the available shear strength becomes smaller than the static shear stress required to maintain equilibrium. So, this is basically and this is driven by static shear stresses as we discussed earlier and this will be like what we call the flow liquefaction.

And this when the flow liquefaction occurs, they can produce large deformation that occur quickly and without warming. While so, the analysis are two types one is analysis of stability, analysis of deformation required. Stability analysis is related to factor of safety, analysis of deformation is related to permanent deformation. There are a number of approaches like for deformation failures. This occurs when the shear strength of soil is reduced to point where it is temporarily exceeded by earthquake induced shear stresses. So, this strength have exceed shears the shear stresses which is generated due to earthquake exceed this strength and in that case deformation failure will occur. And the analysis which are many of like some of the analysis to carry out the weakening instability and in weakening with the deformation failure Hamada et al approach, Youd and Perkins approach, Bryne approach, then you have Baziar et al. approach So, many research are there that can be available and one of the references like further details you can go in the Kramer's book.

So, with this I stop it here. Thank you very much for your kind attention. With this lecture number 44, we are almost done with the seismic slope stability and we will discuss in the lecture number 45 one numerical examples also and then we will talk about the retaining wall. Thank you very much for your kind attention. Thank you.