Underground Space Technology
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Lecture – 43
Rock Mass Support Interaction Analysis: Influencing Factors, Introduction to Ladanyi's Elasto-Plastic Analysis of Tunnels

Hello everyone, in the previous class, we learnt about the Rock Mass Tunnel Support Interaction Concept, with the help of a 3D situation where the tunnel was being advanced. And we took a particular section if you recall the name that section as X-X, and then we tried to develop the ground response curve as well as the support reaction curve for that particular situation.

So, today we will learn about various factors which influence this interaction analysis and then we will start our discussion with Ladanyi's Elasto-Plastic Analysis of Tunnels. So, today, we will just discuss the various aspects related to that may be the material properties, geometry etcetera. And then, we will proceed with the numerical or the stress analysis in the next class.

So, as far as the factors influencing the interaction phenomenon are concerned, the first one includes the category related to geometry.

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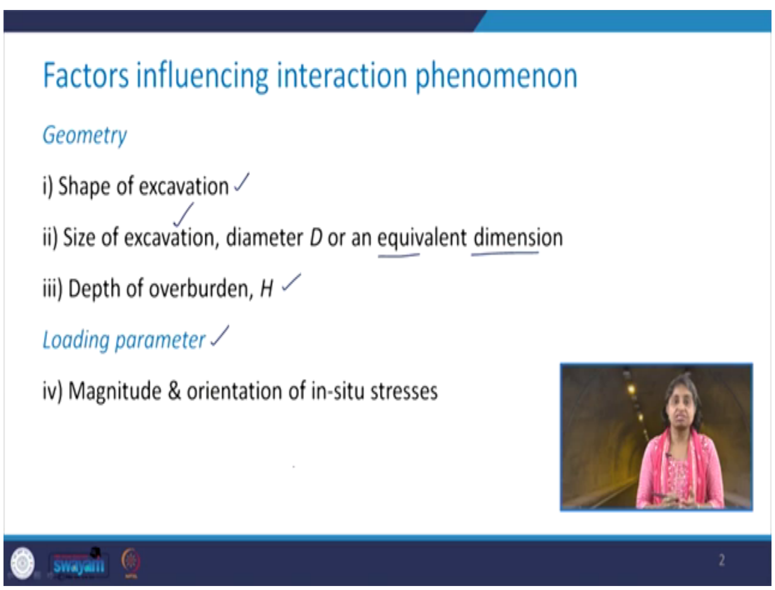
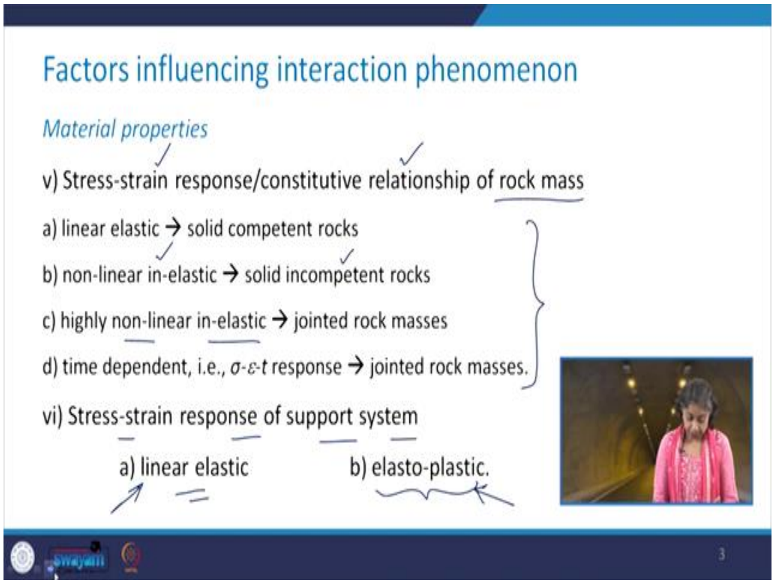


So, when we talk about the geometry, it is the shape of the excavation, size of excavation. When we say size of excavation means it can be it is diameter or an equivalent dimension of the excavation. Then comes the depth of overburden. By now, you are aware of all these terms. Then the second one is the loading parameter. As far as the loading parameter is concerned, it is the magnitude and the orientation of in-situ stresses which influence the interaction phenomenon. **(Refer Slide Time: 02:17)**



Coming to the material properties, so which is defined by the stress-strain response or constitutive relationship of the rock mass. This we have discussed earlier, but just for the sake of continuity and completeness, we will take this up once again. You know that solid competent rocks will behave in a linear elastic manner, that means their stress-strain response is going to be linear elastic. In case, if you have solid incompetent rocks, then the behaviour is going to be non-linear inelastic.

In case of the jointed rock masses, it can be highly non-linear inelastic or you may observe the time-dependent behaviour as well. That means it will not only be the stress-strain response, but we will say that it is stress-strain-time response because the stress-strain response is time-dependent. Then the stress-strain response of the support system, we saw that it can be linear elastic or it can be elasto-plastic.

So, if you recall our discussion of the previous class, I mentioned to you that rock mass tunnel support interaction has two components in the system. The first one is the excavation itself and the second one is the support system. So, as far as the excavation is concerned, it is the rock mass that will come into picture.

And the constitutive relationship of the rock mass will define it is characteristic, and for the support system, we will have either linearly elastic stress-strain response or the constitutive relationship or we will have elasto-plastic stress-strain response of the support system.

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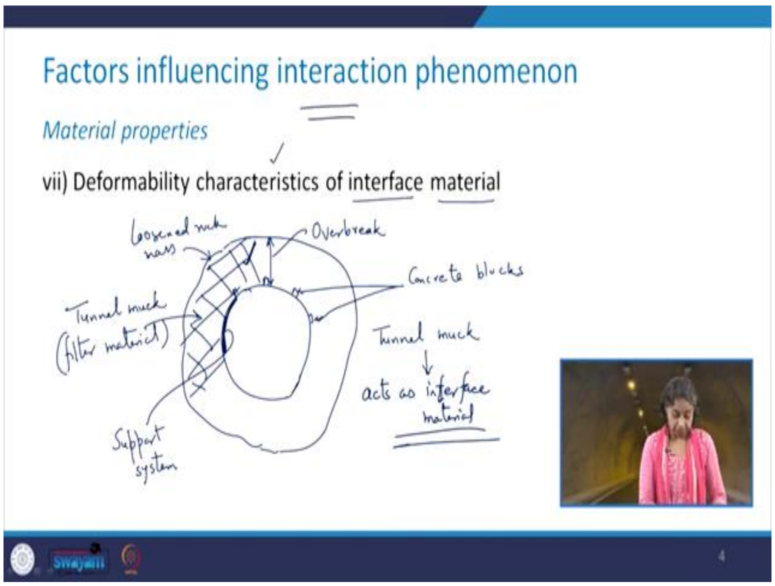

Coming to the another important material properties which influence the interaction phenomena that is the deformability characteristic of interface material. So, let us try to understand with the help of the figure that what exactly do we mean by this interface material? See, here we have the may be the excavation, and what will happen when we excavate? We have a zone of the loosened rock, so this is what is the loosened rock mass.

So, this portion we call as over break and here of course, we provide the support system. Of course, it will be throughout but then I am just showing it for a particular cross-section. So, here this is what is the support system, then we will have some kind of concrete blocks. Then we discuss about various support systems then you will realize that why do we provide these concrete blocks?

So, here this particular material in this loosened zone, this is what is called as tunnel muck, and this is also called as filter material. So, this tunnel muck basically it acts as the interface material.

So, the deformation characteristic of this tunnel muck material also influences the interaction phenomenon.

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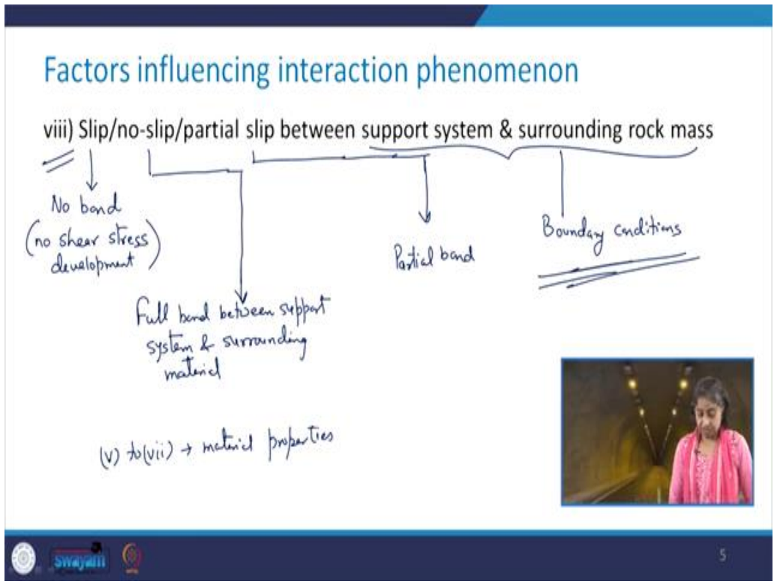

Coming to the next category, which is slip, no-slip or the partial slip between the support system and the surrounding mass. So, first let us understand that what exactly do we mean by slip, no-slip, and partial slip? So, when we talk about the slip condition, this includes that no bonding or in a sense, that we say that no shear stress development. When we talk about the no-slip condition, we have full bond between support system.

And the surrounding material and coming to this partial slip, as the name suggests obviously, there is going to be the partial bonding, so basically, these act as the boundary conditions. So, the property that we discussed earlier, which was fifth to seventh, they were the material properties. And this eighth one, here we talk in terms of the boundary conditions.

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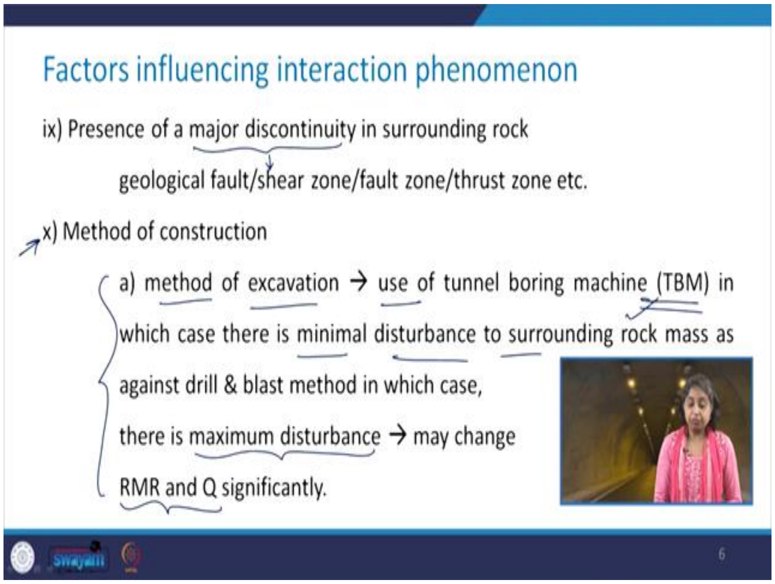

Presence of major discontinuity in the surrounding rock is also one of the major factors influencing the interaction phenomenon. Now, this major discontinuity can be a geological fault or the shear zone or a fault zone or thrust zone or things like some of these features. Then the next factor which influence the interaction phenomenon it is method of construction. So, first thing here comes as the method of excavation.

So, I have already discussed this with you that we can either use the tunnel boring machine or drill and blast method. So, depending upon which method of excavation that you are adopting. That also influences the interaction phenomenon. So basically, the use of TBM, where there is the minimal disturbance to the surrounding rock mass, as compared to the drill and blast method where there is maximum disturbance.

So, what happens when we go for the drill and blast method of excavation? There is going to be the maximum disturbance in the surrounding rock mass, and this may change RMR and Q of that rock mass significantly. So, these factors influence the interaction phenomenon.

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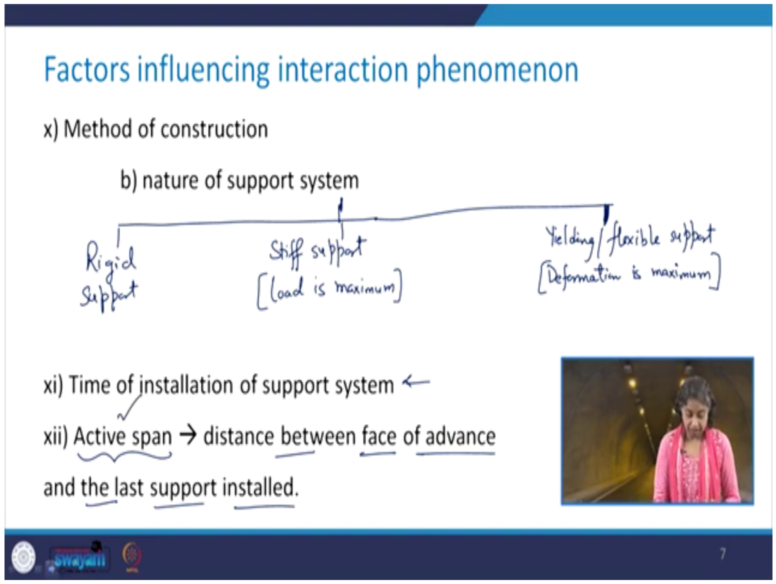

Then, as far as the method of construction is concerned, not only the method of excavation affects but nature of the support system also influences the interaction phenomenon. Let us see how? So, when we talk about the nature of the support system, we can have three categories. The first one may be the rigid support, the second one is stiff support. Here, in this case, load in the support system is going to be maximum.

And the third one is yielding or the flexible type of support, in this case we will have maximum deformation. Then the next factor is the time of installation of the support system. I have already mentioned to you that it should be done within stand-up time, and in case if the stand-up time is very large, it is not necessary that you wait till the end of the stand-up time, and just before that you install the support system.

The installation of the support system should be done as soon as possible after the excavation. Next factor is the active span which is the distance between phase of advance of the tunnel and the last support installed. So, if you just recall our discussion in the previous class, we saw that every time it was like some phase of advance or the distance by which the tunnel phase is being advanced in one go was there.

And then, we keep on installing the support system also simultaneously. So, the distance which remains between the face of advance of the tunnel and the last support which is installed that is what is called as the active span.

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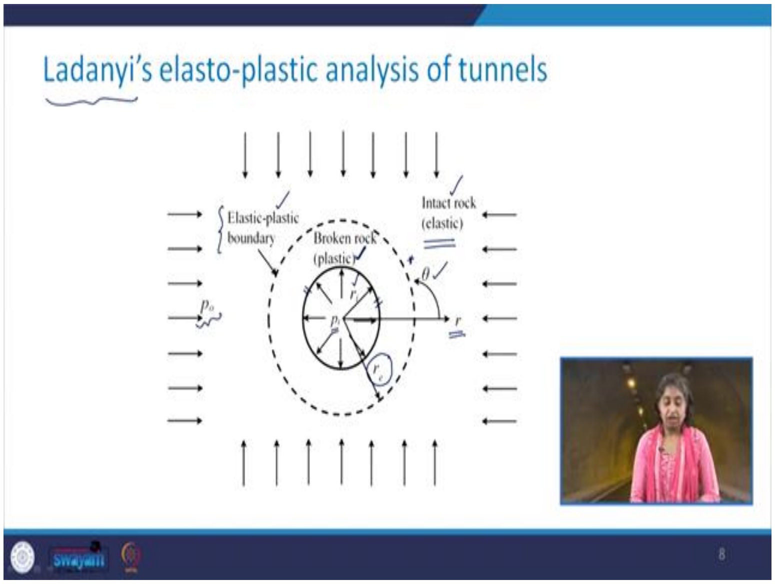

Now, this was all about the factors which influenced the interaction phenomenon. Let us start now our discussion on the elasto-plastic analysis of tunnels which was proposed by this author which is Ladanyi's, and therefore, it is also called as Ladanyi's elasto-plastic analysis of tunnels. So, we will learn about some of the introductory aspects related to this method of analysis, along with various assumptions and other parameters.

So, take a look here that we have the excavation with the radius r_i here so, this is the excavation. And it is the support system is installed so, this p_i is basically showing you the support pressure. Now, any point in the rock mass will be represented by r, theta where this theta will be measured in the anticlockwise direction, from this r axis which is kind of although it is radial axis. But here, we are measuring this theta in the anticlockwise direction from the horizontal axis. Then we have immediately in the vicinity of this excavation.

We have the broken rock which is in the plastic zone, and there we have a boundary called elasto-plastic boundary or elastic-plastic boundary, beyond which we have the intact rock which is again behaving as the elastic material. This whole system is subjected to hydrostatic state of stress, with the in-situ stress as p_0. The radius of the elastic-plastic boundary is represented by the term r_e.

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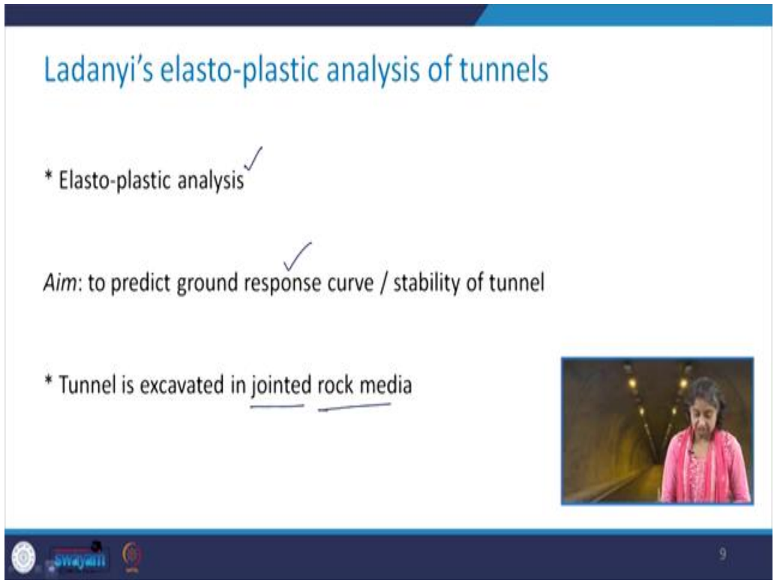

So, having known all these basic things, let us take a look that here we are going to discuss the elasto-plastic analysis. That means the material is going to behave not in elastic manner but elasto-plastic manner. The aim of this analysis is to predict the ground response curve or the stability of the tunnel. The tunnel is excavated in the jointed rock media. So, if you compare this with reference to the earlier analysis, which was elastic analysis that we discussed somewhat earlier.

There it was the rock that was assumed through which the tunnel was excavated. But here it is the jointed rock media.

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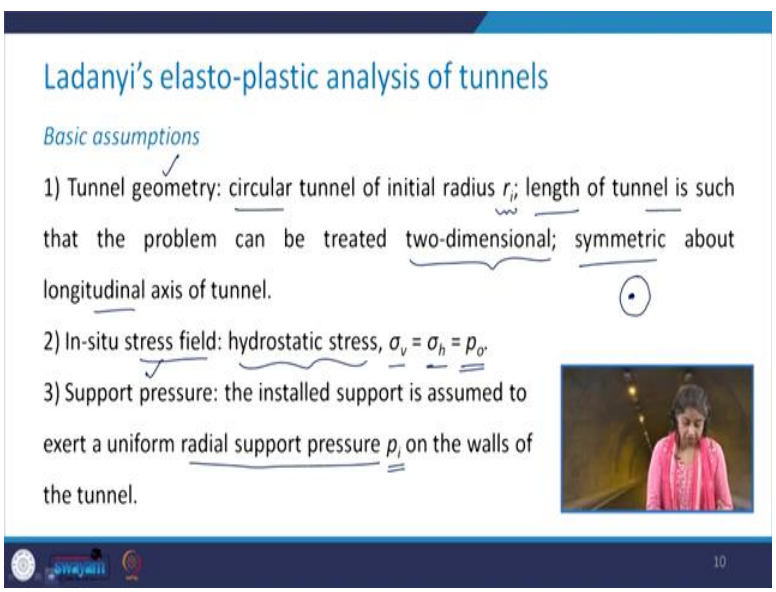

Coming to the basic assumptions of this analysis, so the assumptions related to tunnel geometry is that the tunnel is the circular tunnel having the initial radius as r_i. Length of the tunnel is such that the problem can be treated as two-dimensional and the tunnel is considered to be symmetric about the longitudinal axis of the tunnel. So, if the cross-section it is circular so, in the direction perpendicular to the plane of the screen.

That is going to be the longitudinal axis of the tunnel and it is considered to be symmetric about that. The in-situ field, stress field is considered to be the hydrostatic state of stress which is represented by $\sigma_v = \sigma_h = p_0$. The support pressure is assumed to exert the uniform radial support pressure p_i on the walls of the tunnel. So, this I explained you with the help of the figure on the previous slide.

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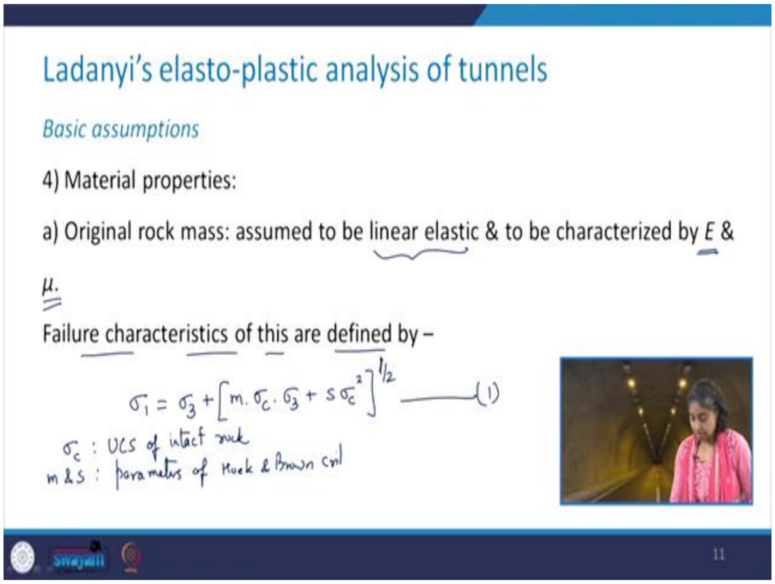

Coming to the material properties, so, the original rock mass is assumed to be linear elastic and therefore, it is characterized by E and μ, E is the modulus of elasticity, and μ is the Poisson's ratio. So, the failure characteristic of this original rock mass they are defined by this equation,

$$\sigma_1 = \sigma_3 + [m\sigma_c\sigma_3 + s{\sigma_c}^2]^{1/2}$$

Please make this equation as equation number 1.

Where this σ_c is UCS of the intact rock, m and s, these are the parameters of Hoek and Brown criteria.

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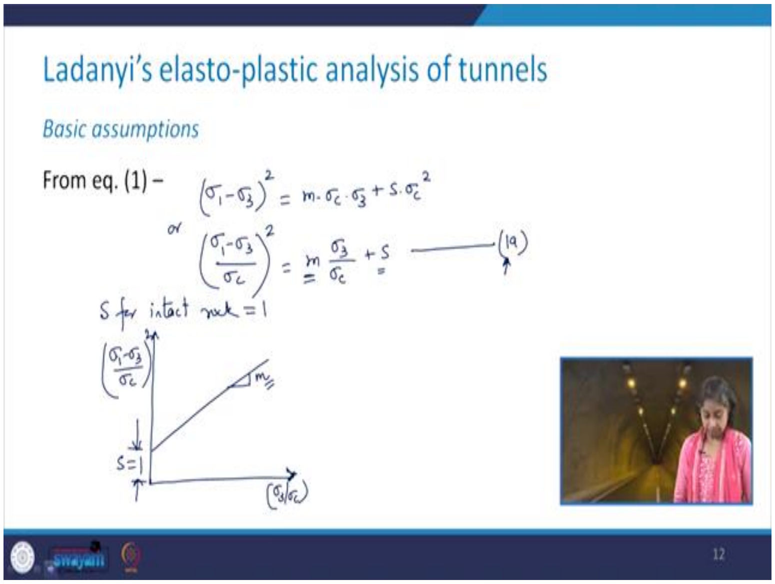

We can write like this:

$$(\sigma_1 - \sigma_3)^2 = m\sigma_c\sigma_3 + s{\sigma_c}^2$$

$$\left(\frac{\sigma_1 - \sigma_3}{\sigma_c}\right)^2 = m\frac{\sigma_3}{\sigma_c} + s$$

Let us make this as equation number 1a. Now, the question is what should be the value of m and s? So, you know that s for the intact rock is equal to 1.

So, how do we get the expression for m? So, just take a look here at the equation number 1 a, and if you plot the test data, on the space σ_3 upon σ_c and on y-axis, you have σ_1 minus σ_3 upon σ_c whole square, so y = mx plus c type of the expression so, this is how that you are going to get where this intercept is s = 1 and the slope of this is, m. So, this is how we can determine the value for? Maybe we can con find out the or we can care to conduct the triaxial test.

And we can find out that what will be the value of m, by plotting the data in σ_1 minus σ_3 upon σ_c whole square versus σ_3 upon σ_c.

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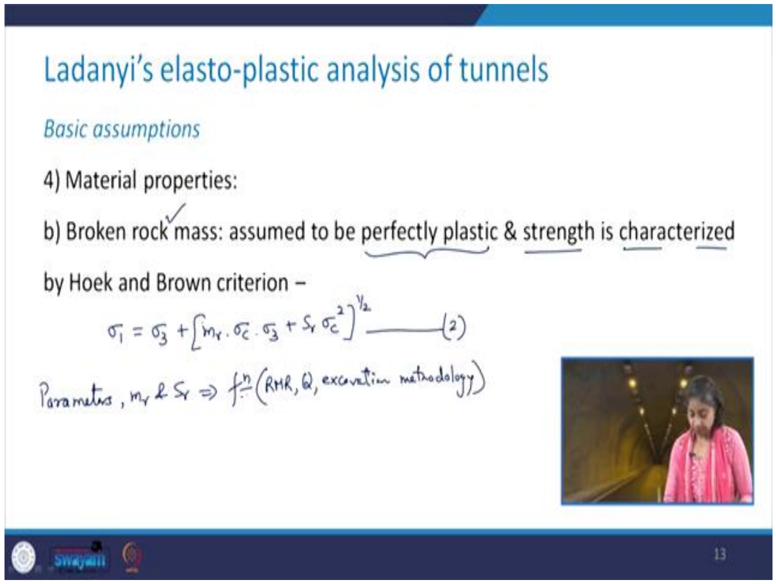

So, this was about the original rock mass, what about the broken rock mass? So, this is assumed to be perfectly plastic, and it is strength is characterized again by the Hoek and Brown criterion, which is given as:

$$\sigma_1 = \sigma_3 + [m_r \sigma_c \sigma_3 + s_r {\sigma_c}^2]^{1/2}$$

Make it equation number 2. So, here these parameters, m_r and s_r they are the function of RMR or Q or the excavation methodology.

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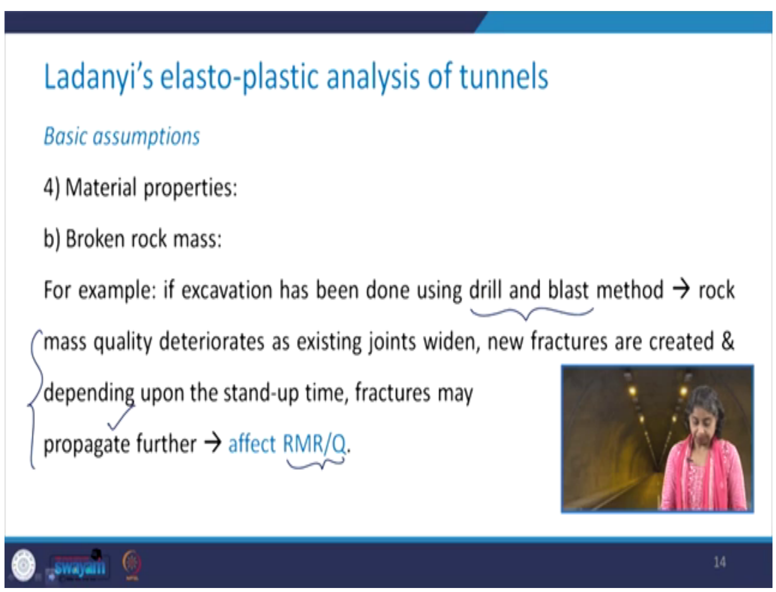

Coming to the some more discussion about this broken rock mass, so, in case, let us say if the excavation has been done using drill and blast method. So, the rock mass quality will deteriorate because what will happen to the existing joints? They will be widened, and the new fractures maybe they are created and depending upon the stand-up time, the fractures may also propagate further, so, all these things they affect RMR or Q.

So that is why here we write that these parameters of the rock mass these are not only the function of RMR and Q but also the excavation methodology. (Refer time: 23:13) If the excavation is done using multiple drift method, then the disturbance to rock mass is going to be minimal, and there may not be significant deterioration in the values of RMR and Q of the rock mass.

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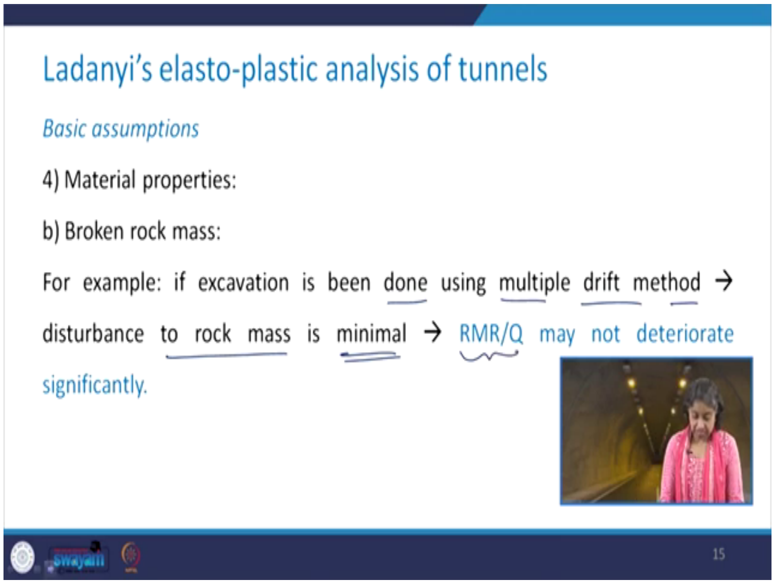

If the excavation is done using multiple drift method, then the disturbance to rock mass is going to be minimal, and there may not be significant deterioration in the values of RMR and Q of the rock mass.

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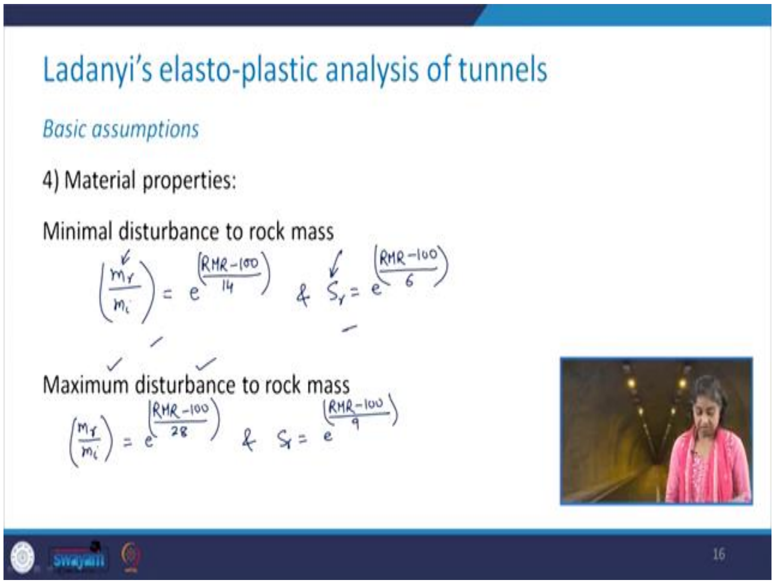

Just in case, if you have the minimal disturbance to the rock mass, let us see how these parameters can be determined? So, we have for this case m_r upon m_i, where r stands for the rock mass and i is for the original rock mass:

$$\frac{m_r}{m_i} = e^{\left(\frac{RMR-100}{14}\right)} \;\&\; s_r = e^{\left(\frac{RMR-100}{6}\right)}$$

And in case, if the excavation methodology is such that it is causing the maximum disturbance to the rock mass. In this case, the expressions are little bit different that is:

$$\frac{m_r}{m_i} = e^{\left(\frac{RMR-100}{28}\right)} \;\&\; s_r = e^{\left(\frac{RMR-100}{9}\right)}$$

So, we use these equations or these expressions to determine the value of mr and sr for the broken rock mass. Because for m_i, you can determine it by conducting the triaxial test in the lab, and s will be equal to 1 for intact rock.

So, having known the properties for or the material parameters for the original rock, we can determine the properties, corresponding properties for the broken rock mass.

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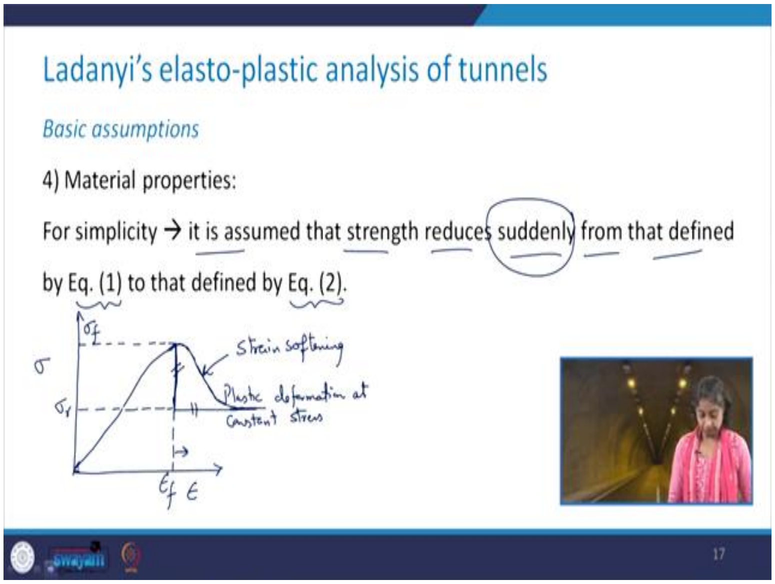

As far as the material properties are concerned for simplicity, we assume that the strength reduces suddenly from that which was defined by equation 1 to that defined by equation 2. Remember, equation 1 was for the original rock mass, and equation 2 was for the broken rock mass. So, what do we mean by this word suddenly here, take a look. We have on x-axis as the strain and here we have the stress.

So, we have the elasto-plastic type of behaviour of the rock mass. So, here we have the failure stress may be let us say σ_f. And then there is the sudden drop, and then we have this value as σ_r, which is for the rock mass. So, you see that here there is a sudden change in the strength. So, from the original one, this is just the sudden change. So basically, here and beyond this we have this as alpha f.

But then later on some of the research workers, they found out that this reduction is not this sudden but it is gradual. So, they came out with this type of gradual reduction so, this was called as strain softening. So basically, here you have this as the plastic deformation at the constant stress. So, what we are going to assume is that? It is the sudden reduction from the original rock mass to the broken rock mass.

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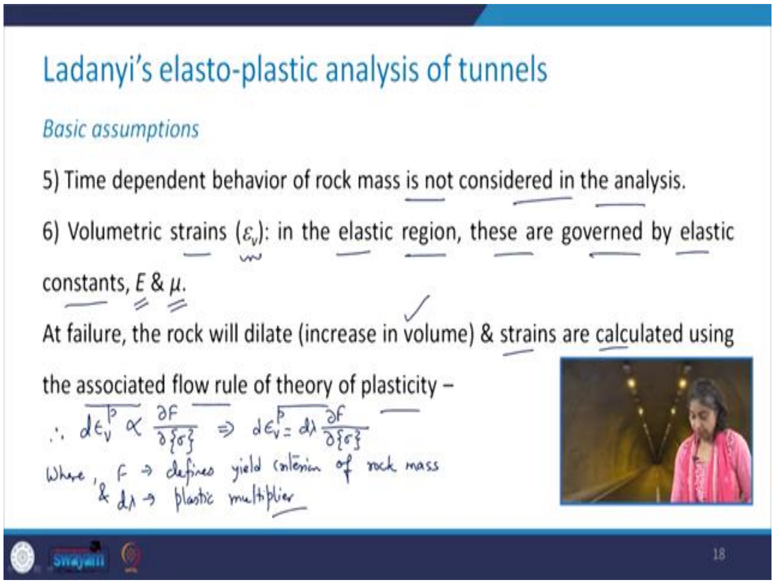

The time-dependent behaviour of the rock mass is not considered in the analysis that is another assumption. Then the volumetric strains which are represented by epsilon v so, in the elastic region, these are governed by the elastic constants obviously modulus of elasticity and the Poisson's ratio. But what happens at failure? At the failure, this rock will dilate. What do we mean by dilation of the rock? Means that it will increase in the volume.

And therefore, the strains are calculated using the associated flow rule of the theory of plasticity. Take a look how this is? So, we have here:

$$d \in_v{}^p \propto \frac{\partial F}{\partial\{\sigma\}}$$

$$d \in_v{}^p = d\lambda \frac{\partial F}{\partial\{\sigma\}}$$

Where this f defines the yield criterion of the rock mass and dλ is the plastic multiplier.

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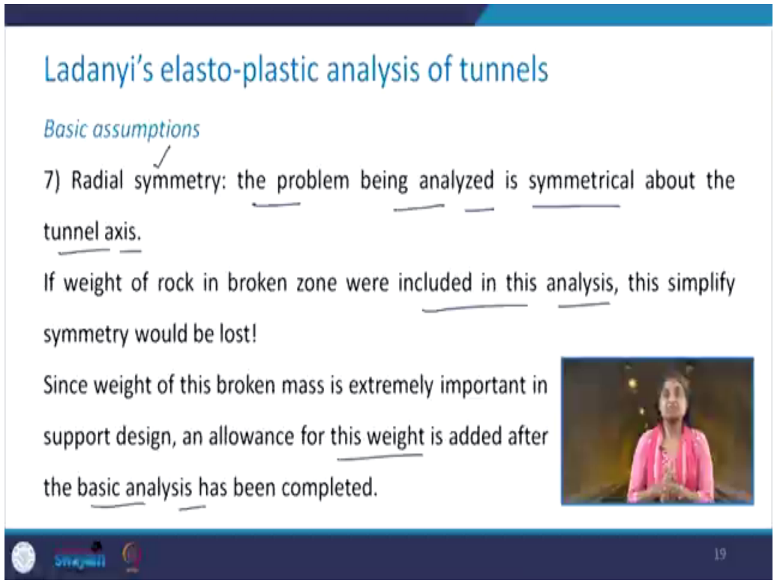

As far as the symmetry is concerned, there is the radial symmetry. So, the problem is being analyzed is considered to be symmetrical about the tunnel axis. The question is what will happen to the weight of the broken zone which are there in the roof portion? So, if we include these in the analysis, this simplified symmetry would be lost because ah the weight of the broken zone it may not be symmetric.

It will be there in the roof portion and not on the side walls. So, what we do is? We consider this weight after the completion of the basic analysis. You need to keep this in mind that the weight of the broken rock is extremely important in the support design and therefore, it cannot be ignored. So, what we do is? First, we carry out the analysis without the weight of this broken rock mass. And then later on once, the analysis is over then we take into account this particular weight by a means which we will be discussing when we discuss the numerical aspects related to Ladanyi's analysis.

So, this was all about the basic assumption and introduction related to Ladanyi's elasto-plastic analysis of tunnels. So, in the next class, we will learn about the analysis of stresses and the deformation. Thank you very much.