Underground Space Technology
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Module No # 05
Lecture No # 24
Opening in Laminated Rocks - 01

Hello everyone, in the previous class we discussed about the stress distribution, in case of the multiple openings, so today, we will learn about the openings in laminated rocks. So, first let us understand what do we mean by laminated rocks, and then we will see that, how we can analyze the openings in such type of rocks? So, here is a picture that gives us the idea about the laminated rocks, so, basically laminated rocks are composed of succession of the parallel layers, whose thickness is a small as compared to span of the opening.

So, you can see here in this figure that, there are various layers which are formed, so, we call these as, the laminated rocks.

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Now, these layers if you just take a close look, these layers are either unbounded, or the bond strength between them is very small, as compared to the tensile strength of the rock. So, just one hammer is there, so, this is just kept to give you the idea, the relative dimension of the layer thickness and its length. So, basically these layers, say, it is the one layer, this is second layer, third layer.

So, in between these 2 successive layers, here you see, so the bond strength between them is very small, or sometimes they may be unbounded. So, when we say that, it is very small, so we compare it with the tensile strength of the rock, so it is smaller than the tensile strength of the rock.

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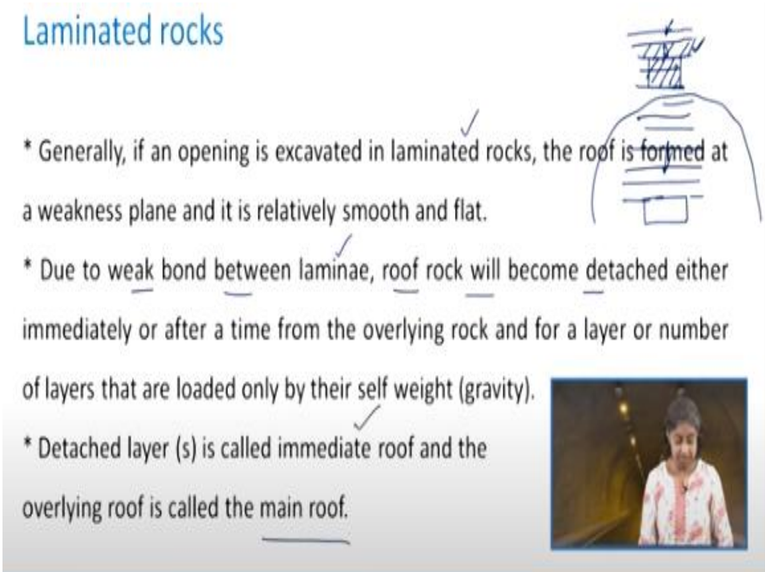

Now, if an opening is excavated, in such type of rocks, which we are calling as laminated rocks. Generally, the roof is formed at a weakness plane, and it is relatively smooth, and flat because of the nature, that there is either no binding between the 2 consecutive layers, or it has very low strength. So, due to the weak bonding between the laminae a roof, rock will become detached either immediately, or after some time from the overlying rock, and for a layer or number of layers, which are loaded only by the self-weight or the gravity.

See, what happens is, let us say that there are these are the laminated layers, now you make an opening here so what happens is this roof. So, there the layers are there, so the moment this opening is made, what can happen is that the roof can become, that is this roof is, what roof is this layer, so it may get detached from the overlying layers immediately or with respect to some time. Another property that can be there is, that these layers, they will be loaded only by gravity that is only because of their self-weight.

Now, the detached layers, they are called as immediate roof, and the overlying roof is called as the main roof. So, you see that, let us say if this is the opening, and you have these layers here, so the layer which is detached from the overlying rock, that we will call as the immediate roof. Now, there can be only one layer in the immediate roof, or there can be more number of layers in the immediate roof.

So, you see that the rock mass is huge, so there can be n number of layers, so it may happen that only 2 or three layers are getting detached from the overlying rock. So, in that case, only 2 or 3 layers will form the immediate roof and the overlying roof will be called as the main roof.

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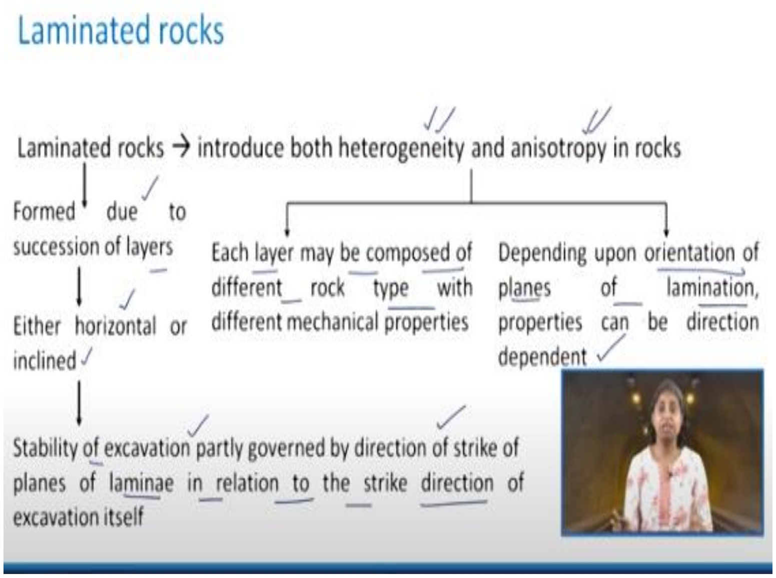

Now, in case of the laminated rock, there is going to be the heterogeneity, and also the anisotropic situation, these laminated rocks they are formed due to succession of layers. These layers can be either, horizontal or it can be inclined, and therefore the stability of the excavation, will partly govern by the direction of a strike of the planes of laminae in relation to the strike direction of the excavation itself.

When we talk in terms of the heterogeneity, and anisotropy in rock, so that comes as 2 situations. First situation, that can contribute towards this may be that each layer may be composed of the different rock types, with different mechanical properties because this, layers they are forming because of the geological processes. So, they can be of different rock types depending upon the deposition over so many years.

Another way, that heterogeneity or anisotropy can be introduced, that will depend upon the orientation of the planes of lamination, because in that case the property can be different in different direction or these can be direction-dependent.

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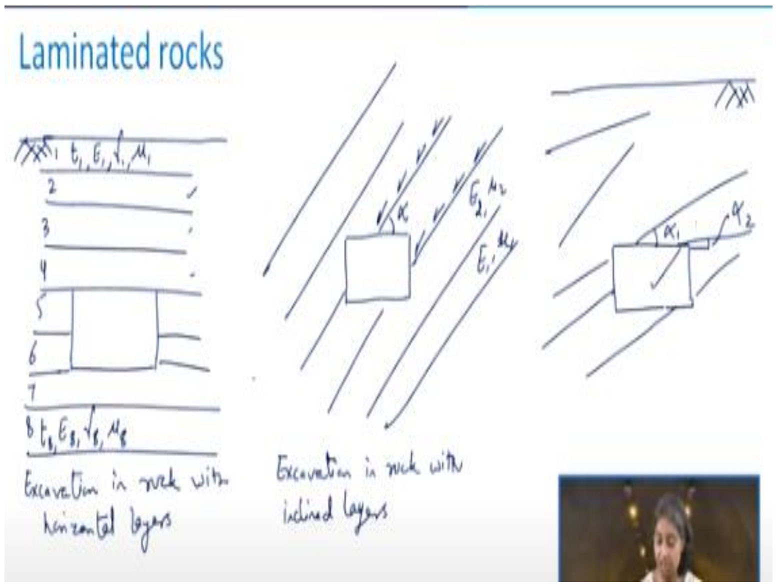

So, let us take a look that how various types of the laminated rocks and the opening look like. So, you can have the excavation in rock with the horizontal layers, ok, so, let us say that there are few horizontal layers, and then maybe you have the excavation like this. So, you have here the ground surface and various rock layers are there maybe 1, 2, 3, 4, 5, 6, 7, and so on, there can be any number.

Now, their properties can be different, their thickness can be different their elastic modulus can be different, unit weight can be different, and poisons ratio also can be different, for all of these layers. So, let us say for this one maybe it is t_8, E_8, γ_8 and μ_8. So, this we call, as the excavation in the rock with horizontal layers, you can have excavation in rock with inclined layers. So, say let us say that this is what is, the excavation and you have the, this type of rock layers may be.

So, in this case again you can have the different properties, so maybe say this is E_1, μ_1, say this is E_2, μ_2, and so on. And, the orientation of the joint is, say alpha so what kind of situation this kind of sliding. So, this is excavation in rock with inclined layers, or you can have a random situation. Let us say that, here it is your ground surface, and then you have an opening here, say so maybe you can have different types of let us say the joint planes, may be here this is say α_1 and this is say α_2.

So, this type of situation can also be there in the field, so what we are going to see is that different types of situations are there. And, of course, as I mentioned then the stability of the excavation, becomes the function of the orientation of these joint planes with respect to the orientation of the excavation itself.

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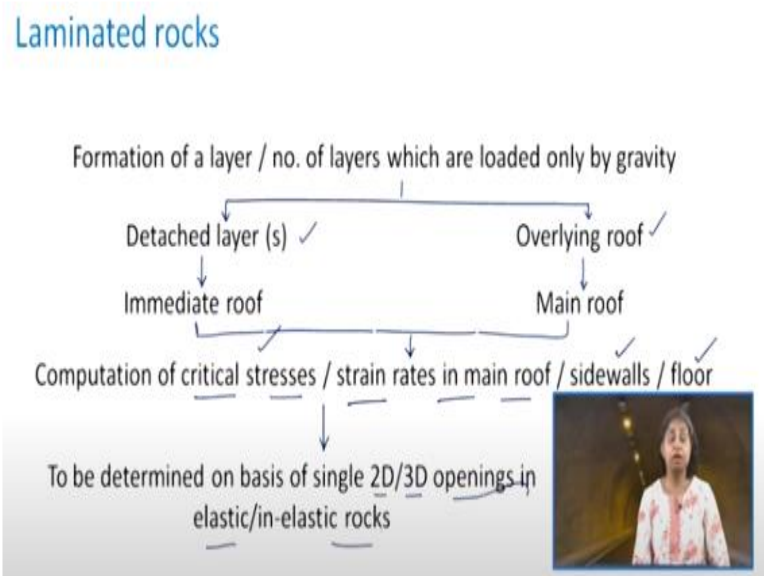

Now, what we have is the formation of a layer or there can be number of layers, which are loaded only by gravity. Now, there is going to be 2 situations, we can have the detached layers or we can have the overlying roof. Further, this detached layer is called as the immediate roof, and overlying roof is called as main roof. I explained you that, what exactly do we mean by detached layer and the overlying roof.

Now, for both of these the computation of the critical stresses, or strain rates in main roof or side walls or floor of the excavation becomes the concern for us. So, when we say that we are trying to analyze the opening in the laminated rock. So, basically, we want to compute the critical stresses and these strain rates in main roof side walls and floor. So, this is to be determined on the basis of single 2D or 3D openings in elastic or inelastic rocks.

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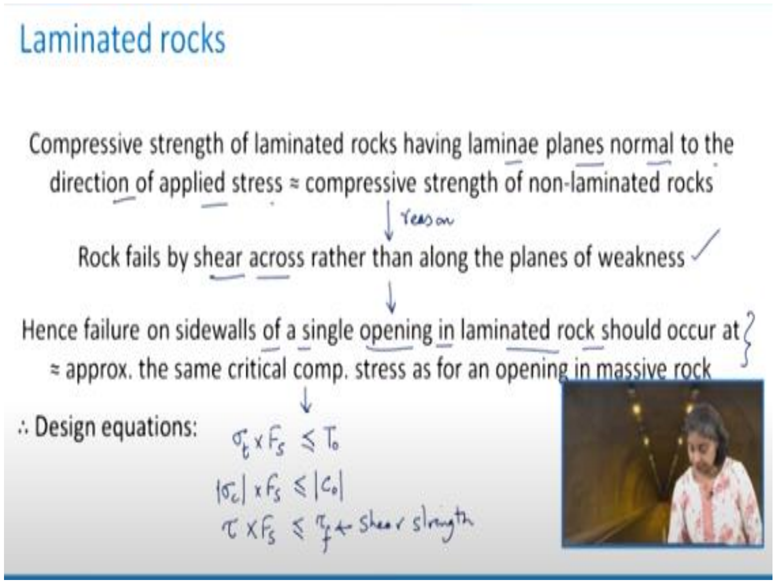

Now, the compressive strength of the laminated rocks having laminae planes normal to the direction of the applied stress, they are approximately equal to the compressive strength of non-laminated rocks. The reason for this is that, the rock fails by shear across, rather than along the plane of weakness. And, therefore the failure on the side walls of a single opening in laminated rock, it should occur at approximately at the same critical compressive stress for an opening in the massive rock, kindly keep this in mind.

And, therefore we can have the design equations as:

$\sigma_t \times F_s \leq T_0$

$|\sigma_c| \times F_s \leq |C_0|$

$\tau \times F_s \leq \tau_f$, that is the shear strength.

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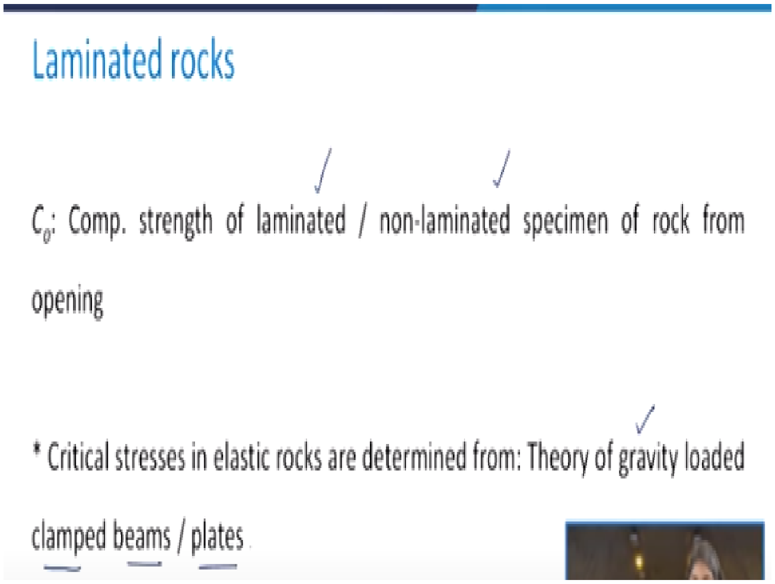

Now, this C_0 is the compressive strength of laminated, or non-laminated specimen, of the rock from the opening. How to determine, then the critical stresses in elastic rocks, these are done using the theory of gravity loaded clamped beams or plates. Now, which situation it is going to be beams, which situation it is going to be plates, that we will learn in a while.

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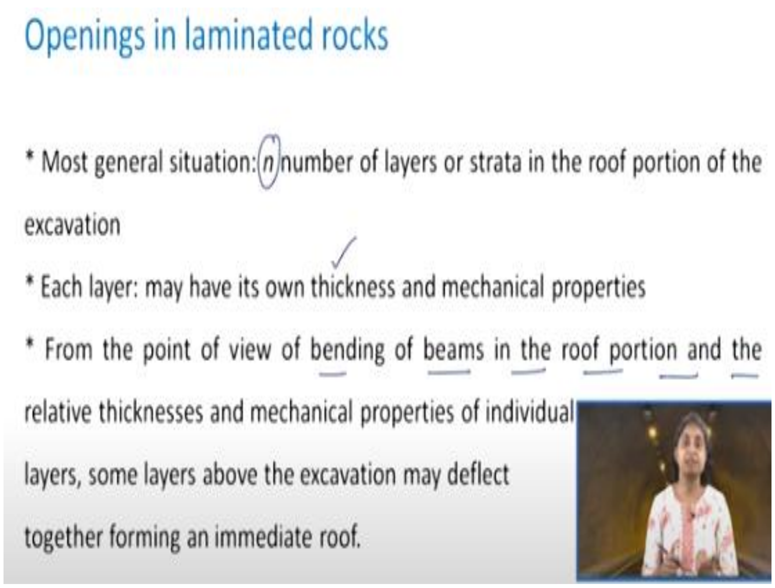

So, in the most general situation, you can have n number of layers, or these strata in the roof portion of the excavation. Now, each of these layers they may have their own thickness, and mechanical properties. So, as I showed you earlier with the help of a figure that, their modulus of elasticity, their poisons ratio, they can be different from the neighboring layers. Or they can have their own thickness, it is not necessary that all the layers they have the same thickness.

Now, from the point of view of, bending of beams in the roof portion and the relative, thicknesses and mechanical properties of the individual layer. What will happen are, that few layers above the excavation immediately above the excavation, they may deflect together forming an immediate roof. The moment they deflect, they are going to have the separation from the overlying layers in the roof, and those we are calling as the main roof.

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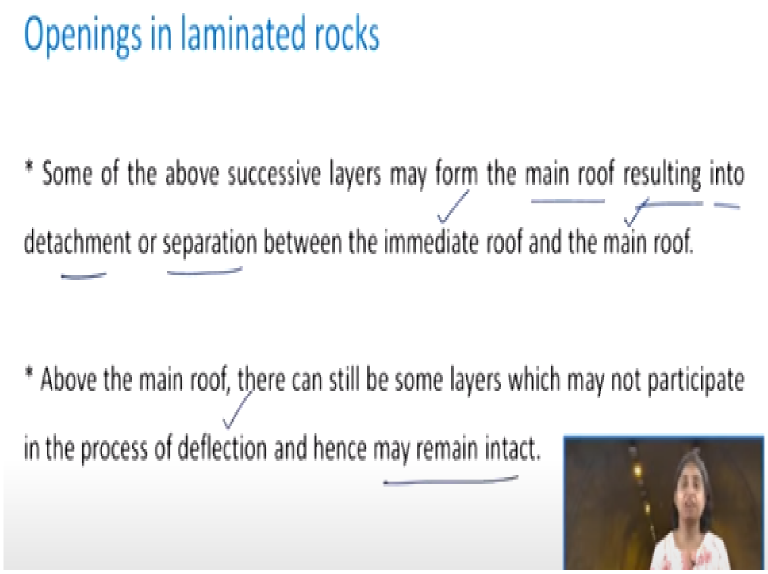

Now, this some of the above successive layers, they may form the main roof resulting into the detachment, or separation between the immediate and the main roof. Above the main roof, again there can be still some layers, which may not participate in the process of deflection, and therefore, they will be remaining intact.

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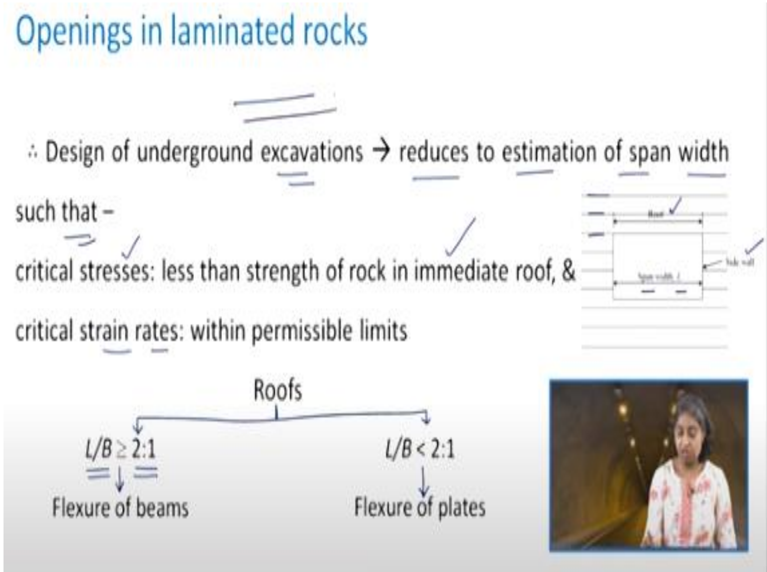

So, the design of underground excavations in the laminated rocks, it reduces to the estimation of span width. Such that critical stresses, they are less than the strength of the rock in the immediate roof, and also the critical strain rates, they are within the permissible limits. See here, I have shown you an excavation which has the span width of l, and the length of the opening which is in the direction perpendicular to the plane of screen, that I am representing as B.

So, in this case this portion is called as roof, and this portion is the side wall. So, here we are taking a particular case where all the layers are horizontal, so, only the horizontal layers that we are talking about. Now, if you have in the roof, 2 conditions can be there when you have this $L/B \geq 2:1$, it will be treated by the theories related to flexure of beam. But in case, if you have $L/B < 2:1$, we need to use the flexure of plates.

So, this is how depending upon the size and the length of the excavation, or the width and the length of the excavation, we need to decide whether there is going to be the applicability of the beam theory, or plate theory while analyzing the openings in such laminated rocks.

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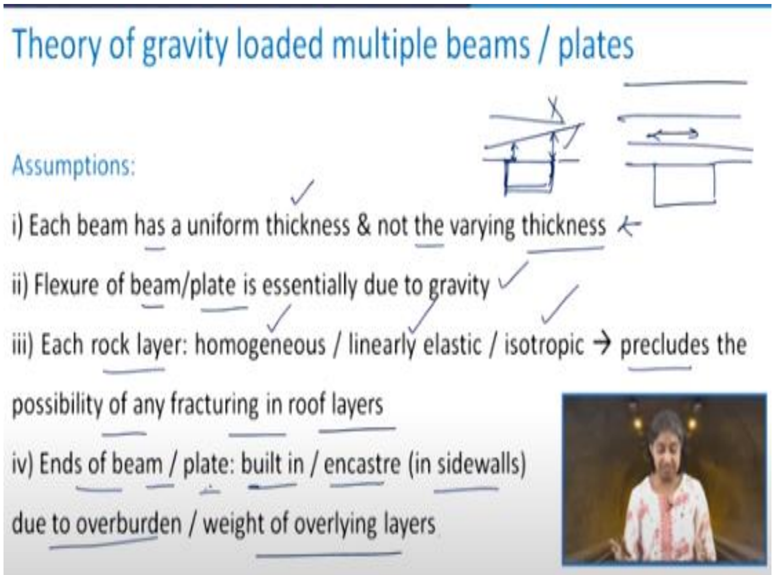

As I mentioned to you that, we are going to use the theory of gravity-loaded multiple beams, or plates, how should we decide whether it is beam or plate, that also I discussed with you, it will depend upon the size of the excavation. Some of the assumptions, which are involved with respect to this theory, these include that each beam has a uniform thickness and not the varying thickness.

This means that, say this is the excavation and we have the thickness, that means that throughout the extent of the opening the thickness of any layer is going to be uniform. You cannot have a situation like, this if this is the opening let us say, so basically you cannot have the situation like this. So, in this case you see the thickness of this layer is this much here, and it is varying and became more here, so, this is really not allowed so this is the first assumption, which is there.

Then, flexure of the beam, or the plate is essentially due to gravity, that means that, there are no external load, but only the self-weight, with which that these beams, or the plates, they are going to deflect. Now, each rock layer was considered to be homogeneous, linearly elastic and isotropic, and therefore, it precludes the possibility of any fracturing in roof layers. Although, later on many research workers, they have talked about the fracturing in roof layer, but that is the matter of the research.

So, to start with we are going to consider one of the very very simple case, where we are considering each rock layer to be homogeneous linearly elastic and isotropic. And, hence there is not going to be any possibility of any kind of fracturing in roof layers.

Then, the fourth assumption involves the ends of the beam or the plate, and it assumes that, these ends are built-in or encastre, in the side walls because of the overburden or weight of the overlying layer. So, basically, we are going to consider them as, the fixed end beams or fixed end plates while making the analysis.

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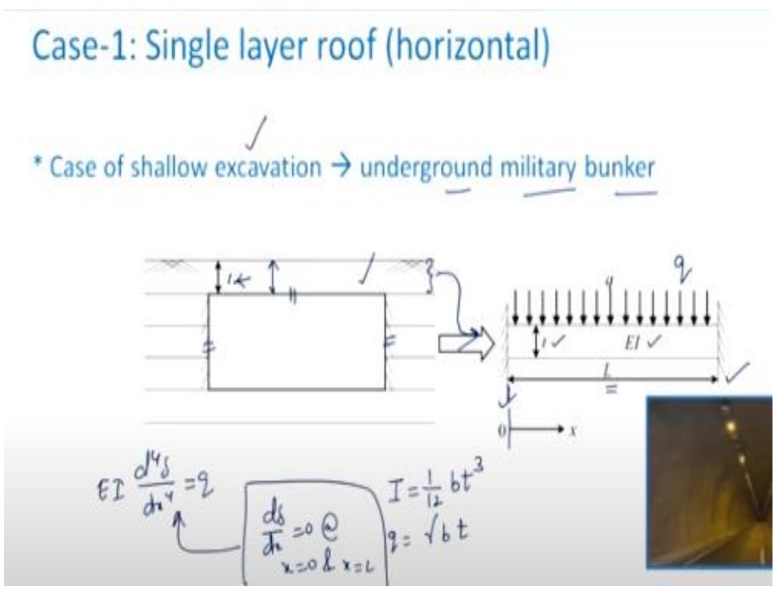

So, we take few cases, so to start with we take the most, simple case that is, when you have the single layer roof, and it is horizontal. So, you see that how it looks like, how the geometry looks like, we have this as the excavation, this is the side wall, and this is the roof, and there is only one layer here, single layer and it has a thickness t. Now, this represents the case of the shallow excavation, and this situation can be there in case of the underground military bunker.

So, how are we going to represent it, when we have to analyze it? So, you see that I take this layer here in this particular form. So, this is the layer, which has the opening width as the length of this beam, it has the thickness t, it has the flexural rigidity of EI, and it is subjected to a uniformly distributed load that is q. Now, what will be the magnitude of this q, that will be the weight of this beam, because that is what is the assumption, which is involved that every layer is deflecting under gravity only, that is under its weight.

Now, if you take 0 from here and you measure x from this end of the beam, you all know that, how to analyze the fixed end beam subjected to uniformly distributed load. So, I am not going into the details of that, let us try to take a look that how the deflections and other aspects are going to be. So, in this case, see

$$I = \frac{1}{12} bt^3$$

b is the dimension in the direction perpendicular to the plane of screen, and what is q,

$$q = \gamma bt$$

So, we have this equation beam equation,

$$EI \frac{\partial^4 \delta}{\partial x^4} = q$$

and then you can apply the appropriate boundary conditions. Such as, delta is going to be 0 at x = 0 and at x = l, and the slope of deflected shape of the beam will also be equal to 0, that is

$$\frac{\partial \delta}{\partial x} = 0 \quad @\, x = 0 \;\&\; x = L$$

So, using this boundary condition, if you just solve this equation.

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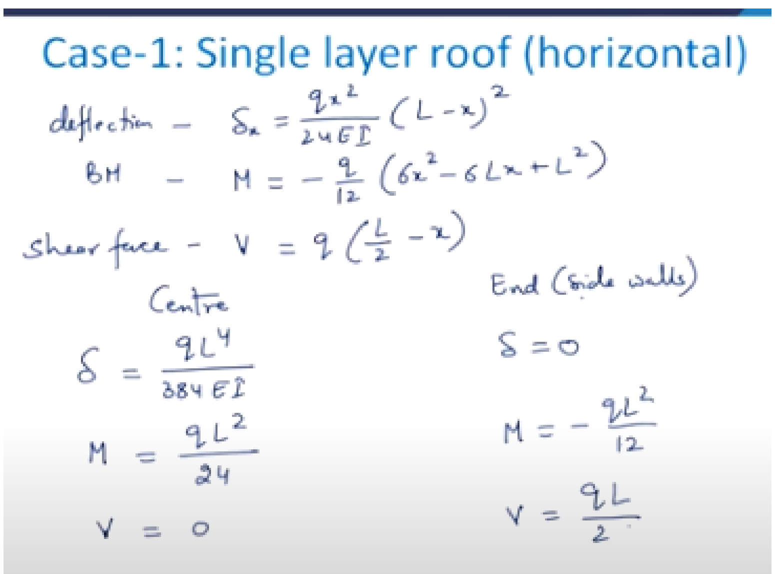

What we get is the deflection as,

$$\delta_x = \frac{qx^2}{24EI}(L-x)^2,$$

then we have the bending moment, that is

$$M = -\frac{q}{12}\left(6x^2 - 6Lx + L^2\right)$$

then we have the shear force which is represented by say V. So, that is going to be

$$V = q\left(\frac{L}{2} - x\right)$$

So, if we take the 2 locations that is 1, at the center of the beam, and another at the end which is the side walls.

So, at the center what delta is going to be, so at the center, your x is going to be l by 2, so this will give you

$$\delta = \frac{qL^4}{384\,EI},$$

and in this case,

$$\delta = 0,$$

because that is what the boundary condition is and the bending moment is going to be

$M = \frac{qL^2}{24}$, and in this case at the end it is going to be

$$M = -\frac{qL^2}{12}.$$

Similarly, the shear force at the center is going to be V = 0, and at the side wall it is going to be

$$V = \frac{qL}{2}$$

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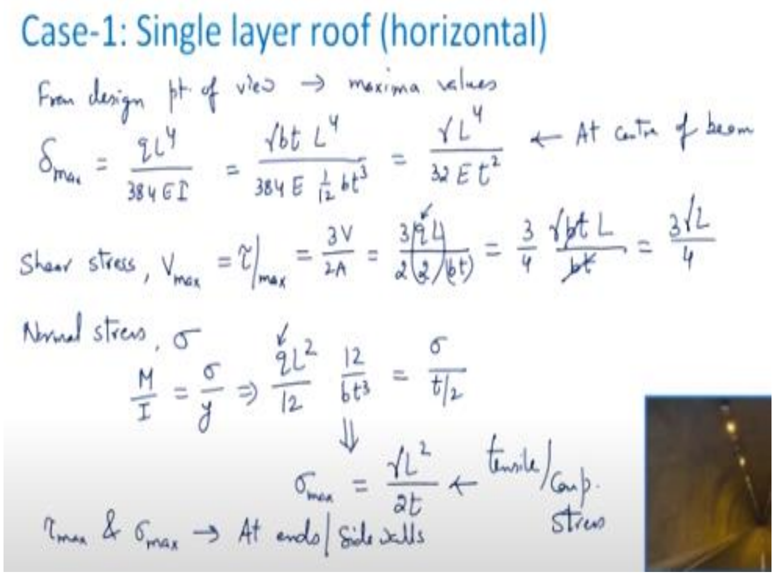

Now, from the design point of view we have to look at the maxima value right. So, from the design point of view we should be worried about the maxima values. So, what we have here as the

$$\delta_{max} = \frac{qL^4}{384\ EI} = \frac{\gamma bt\, L^4}{384\ E\ \frac{1}{12} bt^3}$$

So, what we are going to get is $\frac{\gamma L^4}{32\ E\, t^2}$, and where this is going to be, this is going to be at the center of the beam. Likewise, we can have the shear stress, which I will write as v max and this will be or I write it like this. So, that is going to be if you go through the theory of elasticity, basically, it is

$$V_{max} = \tau_{max} = \frac{3V}{2A} = \frac{3}{2}\ \frac{qL}{2\ (bt)}$$

So, I just substitute it here and then a is going to be your b, into t, so this is going to be again substitute this q as γbt. So, that is $\frac{3}{4}\ \frac{\gamma btL}{bt}$, so this will get canceled, so, ultimately you will get $\frac{3\gamma L}{4}$. Now, coming to the normal stress, so, we have say, I will marking it as sigma. So, what I have is $\frac{M}{I} = \frac{\sigma}{y}$, this you know from the theory of elasticity only.

So, the bending moment is $\frac{qL^2}{12}$, and then you have $\frac{12}{bt^3}$, which is I, so here you have $\frac{12}{bt^3}$ and, in this case, the neutral axis that is going to be $\frac{t}{2}$. So, from here if you just substitute q as $\gamma bt$and try to do the, or try to simplify this, what you are going to get is

$$\sigma_{max} = \frac{\gamma L^2}{2t}.$$

So, this can be tensile or it can be compressive, so, basically this τ_{max} and, your σ_{max} they will be occurring at the ends or the side walls.

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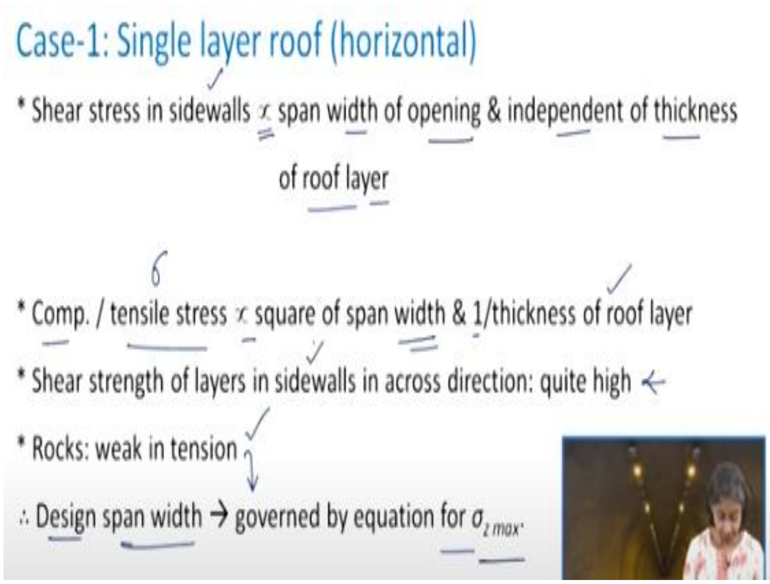

So, if we just try, to take a look here, that the shear stress in the side walls is proportional to the span width of the opening, and it is independent of the thickness of the roof layer. And, the compressive or the tensile stress, which is the σ value, that is the, that is proportional to the square of the span width and inversely proportional to thickness of the roof layer.

So, the shear strength of layers in the side walls in across direction, they are quite high, rocks being weak in tension. Ultimately, what we come here, to the design of the span width, and that would be governed by the equation for $\sigma_{z_{max}}$. So, how do we determine this?

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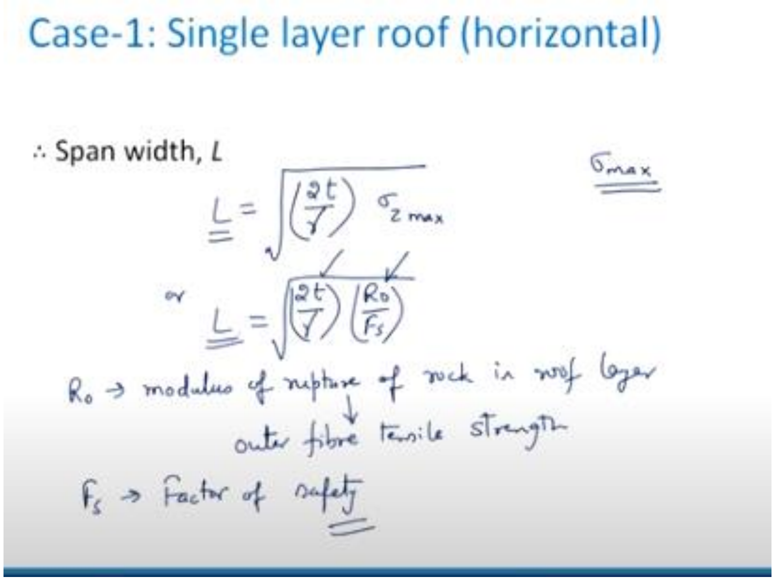

So, this span width is going to be say l, that is going to be

$$L = \sqrt{\left(\frac{2t}{\gamma}\right)\sigma_{z_{max}}}$$

you remember we obtained the expression for sigma max? So, from there I will find out what is the expression for l, or we can write this

$$L = \sqrt{\left(\frac{2t}{\gamma}\right)\left(\frac{R_0}{F_s}\right)}$$

Fs is the factor of safety, and R_0 is the modulus of rupture of the rock in roof layer.

So, this is given by the outer fiber tensile strength, and of course, factor of safety is represented by Fs. So, here what we can do is we can consider various values of $\frac{R_0}{F_s}$ and, then we can try plotting the variation between the roof thickness, that is t and the span width, which is l, but then we have to choose the appropriate scale.

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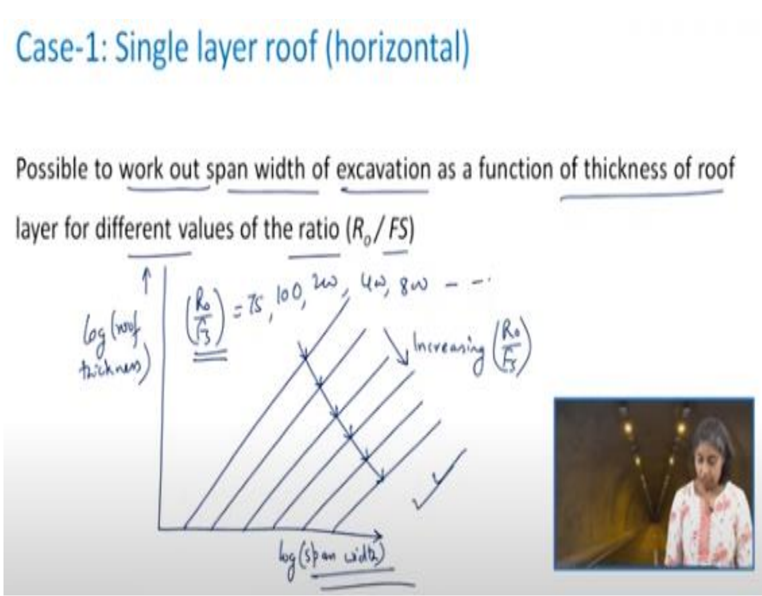

So, what we do is, we work out the span width of the excavation, as a function of thickness of roof for different values of this ratio $\frac{R_0}{F_S}$. And, how do we plot it see we take here as log of span width on x axis, and on y axis, we take log of roof thickness, and then we get this kind of set of lines. So, in this direction it is increasing value of r naught upon fs. So, maybe you can have this $\frac{R_0}{F_S}$ you can have maybe 75, 100, say 200, 400, 800, and so on.

So, likewise you can develop this kind of chart, so in case, if you know that what is the roof thickness, and corresponding to any value of $\frac{R_0}{F_S}$, you can use this chart to find out what is the span width. So, this is how the analysis of the single layer roof is carried out. So, here we discussed very simple situation where you had only one layer in the roof, which was participating in the deflection.

But then, you can have, more number of layers as I mentioned that you can have in general n number of layers. So, we will continue our discussion in the next class with the situation where you have 2 layers, which are participating or which are, the part of the immediate roof. And then, we will extend that to the most general case where you can have n number of layers. Please keep, that in mind that, we will focus only on the horizontal layers for the time being. And then once we are done with this analysis portion, then maybe we can take a look at some cases related to the inclined layers as well, thank you very much.