Rock Engineering
Prof. Priti Maheshwari
Department of Civil Engineering
Indian Institute of Technology – Roorkee

Lecture - 57
Ultimate Bearing Capacity Using Bell's Approach

Hello, everyone. In the previous class, we started our discussion on foundations on weak rocks. I gave you the idea that what all can be the problems associated with the foundations, when they are founded on the rocks. So, today, we will learn about the determination of ultimate bearing capacity of the foundations and we will be using Bell's approach for this purpose. All of you are aware of the fact that if the footing is to be founded on the soil, then 2 criteria should be met in order to have the safe bearing capacity of the foundation.

What are those 2 criteria? The first one is that the footing should be safe in shear and the second one is that the settlement of the footing should be within the permissible limit. Similar is the situation in case, the foundation is to be placed or rested on the rock or rock mass. So, let us try to learn about this Bell's approach first and then we will proceed further with other methods in order to obtain the safe bearing pressure of footings, resting on rock or rock masses.

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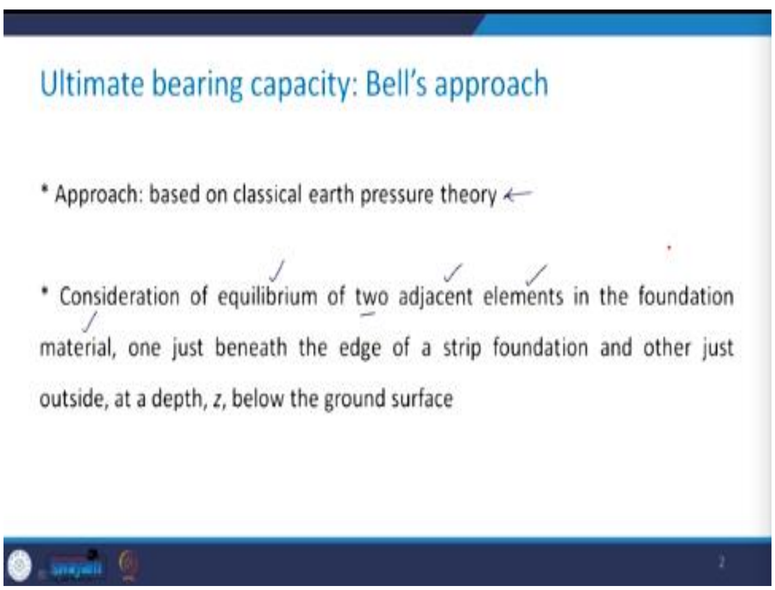

So, this Bell's approach is based on classical earth pressure theory. You all must be aware of the active earth pressure and the passive earth pressure. So, this theory or this approach is based upon all those concepts. So, here there is going to be the consideration of equilibrium of 2 adjacent elements in the foundation material. And one element would be taken just beneath the edge of a strip foundation and another one would be taken just outside it and these elements would be taken at a depth z, below the ground surface where the footing would be placed.

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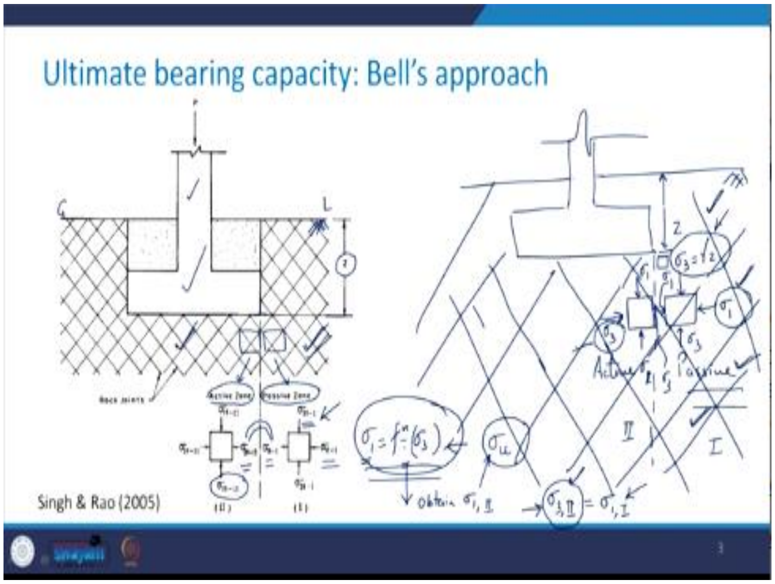

Take a look at this particular figure. So, here this is the footing which is placed at a depth of z, below the ground surface. So, this is your ground level and from there, at a depth z below, you have placed the footing. So, as I mentioned that we are going to consider 2 elements. One is just beneath the footing here and one is just outside it and these have been shown here like this. So, the zone which is immediately below the footing, that is this zone, we call as active zone.

And the zone which is outside this which is this portion, this is going to be the passive zone. So, just let us see that what exactly is this Bell's approach? So, I just redraw this footing here, say, this is what is your footing and it is there at a depth of z and these are the zones. So, this is here, you have the active zone and here, you have the passive zone and these are all rock mass like this. Now, as I mentioned that you take a element here and take one element here just outside.

So, what will happen? This is just below the footing. So, what is going to be value of σ_3 here? That is going to be your γ times z where gamma (γ) is the unit weight. And see although for clarity, I am drawing it little bit below, but this element is just adjacent at this particular level only. And here, we have σ_1. Likewise, here you will have σ_3 and for this element, you will have σ_1.

Now, depending upon what is this rock material, which constitutive law that it is following, you will always have a relationship between σ_1 and σ_3. Now, you know the value of $\sigma 3$in this zone and using this expression, you will be able to get the value of σ_1 in this passive zone. Now, come to the active zone. Again, here in this case, you will have σ_1 and you will have here as σ_3. So, this is σ_1 and here you will have σ_3.

So, what Bell's approach says that σ_1 of the passive zone becomes equal to σ_3 of the active zone. So, σ_3 of the passive zone is known to me. I can find out σ_1 of the passive zone. Now, this σ 1 will become equal to $\sigma_3$of the active zone. So, this $\sigma_3$of the active zone now would be known. So, here let us say this is zone I and this is zone II that I am representing.

So, what will happen is $\sigma_3$of zone II will become equal to σ_1 of zone I. So, this we have already obtained. Therefore, we know this. Now, after knowing this and we know that this rock mass is following this expression that is σ_1 is some function of σ_3, then here we know σ_3 and using this expression, we can obtain σ 1 in the second zone that is the active zone.

Now, this σ_1 in the active zone is going to give us the ultimate bearing capacity of the foundation σ u. So, that is what the concept of Bell's approach. Take a look once again. In the passive zone, we know what is σ_3. Say, in this case, it is γz. We can find out σ_1 of the zone. Now, σ_3 of the active zone becomes equal to σ_1 of the passive zone.

And using this relationship between σ_1 and σ_3, we can find out σ_1 in the active zone and that is what is going to give me the ultimate bearing capacity of this strip footing.

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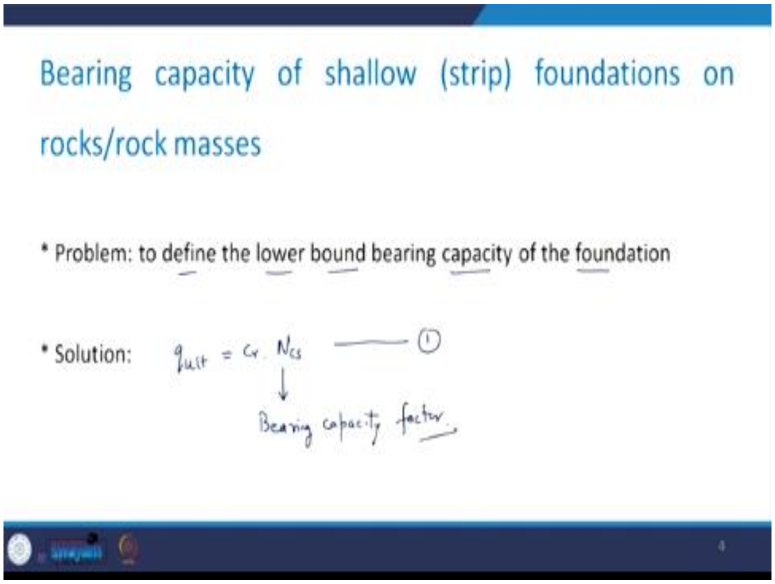

So, the problem is to define the lower bound bearing capacity of the foundation and the solution can be obtained in this particular fashion that is

$$q_{ult} = c_r N_{cs}$$

This is going to be equation number 1. Where this N_{cs} is the bearing capacity factor.

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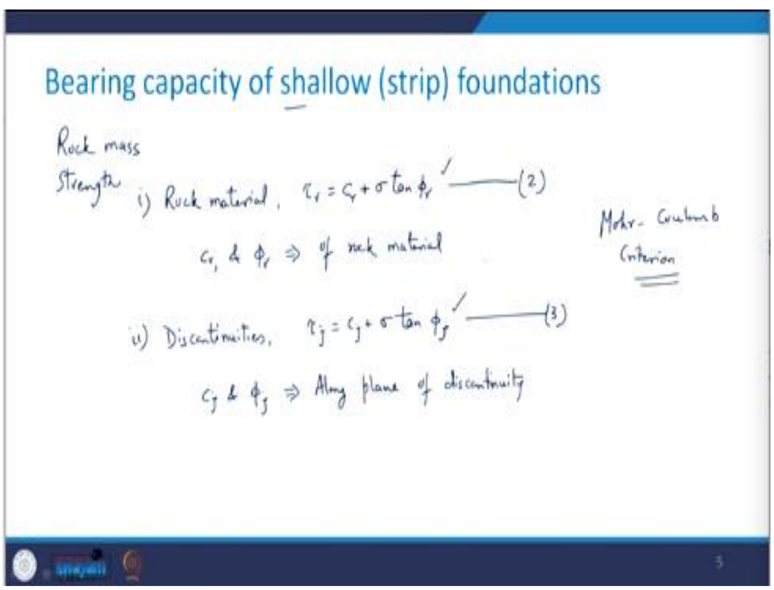

Now, how to get the strength? So, because here, we are dealing with the rock mass, so, we will have to get the strength in terms of the rock material and the discontinuities. So, let us say that we take a very simple example and use this Bell's approach and try to get bearing capacity of this shallow or this strip foundation. So, in the in case of this strength of the rock material, I assumed that it is following the Mohr-Coulomb failure criterion and therefore, we are going to get this expression, equation number 2.

$$\tau_r = c_r + \sigma \tan \phi_r$$

Where, what is this c_r and ϕ_r these are of the rock material. Then the second component of this rock mass is the discontinuity and for that, I can again write

$$\tau_j = c_j + \sigma \tan \phi_j$$

where this c_j and ϕ_j, they are the properties along the plane of discontinuity. So, this is how we can find out the strength. Again here, an assumption is involved that these are following Mohr-Coulomb criteria. Otherwise, these equations 2 and 3, they will not be valid.

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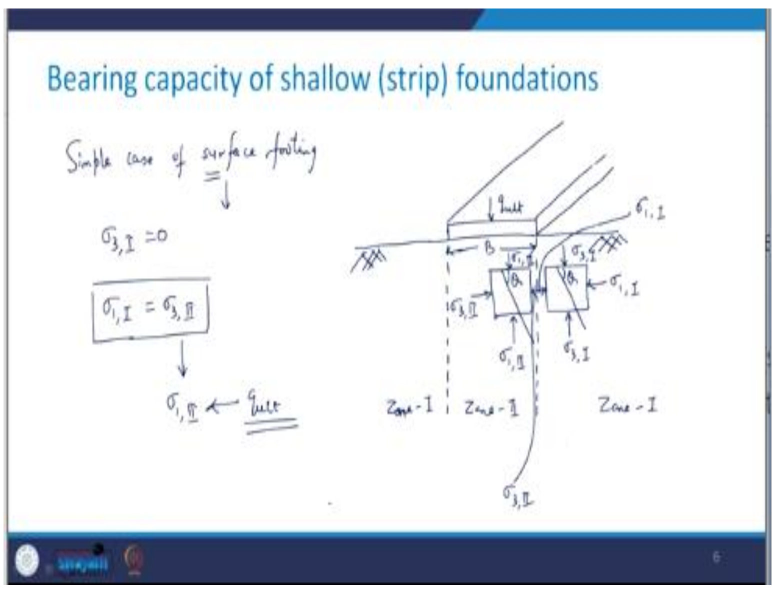

So, let us now take a very, very simple case of a strip foundation, which is resting on surface only. Let us take the case of surface footing. So, I take a simple case of the surface footing. So, say here, you have the ground surface. And since, I am considering a theoretical case of a surface footing. So, this is how the footing is going to look like. And we need to find out the q_{ult} of it. And here, like in case of the concept with respect to Bell's approach, I explained you.

So, I will have these 2 zones here. The weight of this footing is say, B. I take the 2 elements. One is just outside and one is in this zone, which is like this. I am showing that little bit bigger element but these are the small elements. For the clarity, I am showing drawing it little bigger.

Now, in case if you have the discontinuity like this, which is making an angle of say, theta, now, this is here σ_3 of this zone one and since, this is the surface footing.

So, this σ_3 of zone I is going to be equal to 0, because there is no overburden and here, we will have σ_1 in the zone I. So basically, this is what is my zone I and here, this is zone II and obviously this one will also be zone I. So, here, we have σ_3 of 1 and in this direction here, this is going to be σ_1 of the zone I. Now, come to this zone II. So again, here we have this discontinuity which is at an angle of theta (θ).

So, in this case here, we have σ 1 of zone II. Let me write it with more clarity here, σ_1 of zone II and here we will have σ_3 of zone II. Similarly, here also, we will have $_{=3}$ of zone II. So, as per the Bell's approach, what it is going to be that σ 1 of zone I would be equal to σ_3 of zone II. And from here, I will obtain σ_1 of zone II and this is what is going to be the value of the ultimate bearing capacity. Let us see how we can do that.

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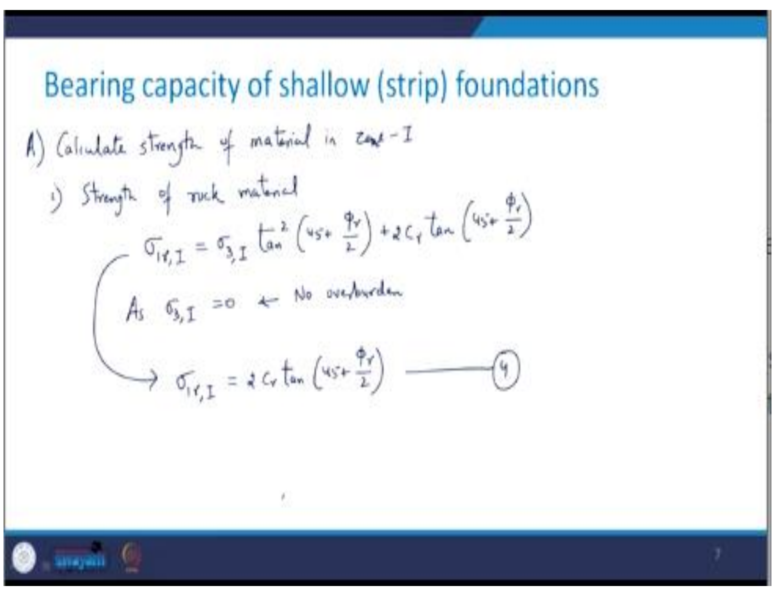

So, first, we will calculate the strength of materials in zone I. So, I write it as part A that is calculate strength of material in zone I. Now, the first one is going to be the strength of the rock material because it is the rock mass. So, rock material and the discontinuity both will come into picture. So, first, we deal with the strength of the rock material and this is what that we are going to get σ_1 r in zone I. This is going to be

$$\sigma_{1r,\mathrm{I}}=\sigma_{3,\mathrm{I}} \tan\left(45+\frac{\phi_r}{2}\right)^2+2c_r \tan\left(45+\frac{\phi_r}{2}\right)$$

Now, in this case, your $\sigma_3$in the zone I is going to be 0 as there is no overburden. So, what this equation will give us is

$$\sigma_{1r,\mathrm{I}}=2c_r \tan\left(45+\frac{\phi_r}{2}\right)$$

This equation, I will make it as equation number 4. Now, this was about the strength of the rock material. Now, let us try to find out the strength along the discontinuity plane.

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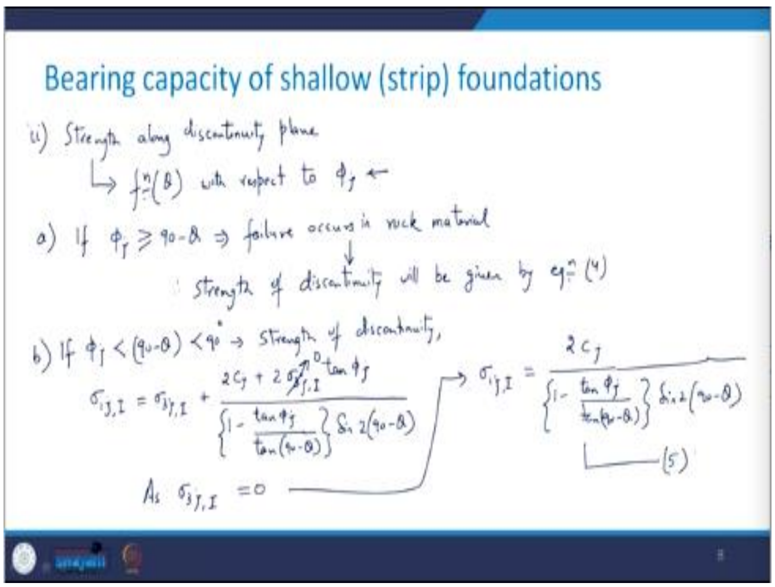

So, I will have the second part of it that is strength along the discontinuity plane. This is going to be the function of θ with respect to ϕ_j. You recall, we discussed about the single plane of weakness theory. So, that is what I am going to consider here once again. So, let us say that if ϕ_j is greater than or equal to (90-θ), what does this signify is that the failure occurs in the raw material only.

Please recall our discussion with respect to the theory related to single plane of weakness. There, I explained you that when the failure will occur in rock materials and when the failure will be there along the discontinuity. So, in this case, when the failure occurs in the raw material, the strength of the discontinuity will be given by equation number 4 that we derived just now. In case you have ϕ_j, if this ϕ_j is less than (90-θ) and less than 90 degree.

Then in this case the strength of discontinuity will be given by

$$\sigma_{1j,\mathrm{I}} = \sigma_{3j,\mathrm{I}} + \frac{2c_j + 2\sigma_{3j,\mathrm{I}} \tan\phi_j}{\left\{1 - \frac{\tan\phi_j}{\tan(90-\theta)}\right\} \sin 2(90-\theta)}$$

This expression, we have already derived when we were discussing about the single plane of weakness theory. There, it was in terms of beta and here, we are considering the angle theta, but it is one in the same thing.

Now, in this case, since σ_{3j} in the first zone is equal to 0, so, this is going to give us σ_{1j} in zone I as

$$\sigma_{1j,\mathrm{I}} = \frac{2c_j}{\left\{1 - \frac{\tan\phi_j}{\tan(90-\theta)}\right\} \sin 2(90-\theta)}$$

This is how that we are going to get because this term will become equal to 0. So, I will mark this equation as equation number 5. Now, this is what was there in zone I.

So, now, let us see that what will happen to zone II, but before that here, we are considering only one plane of weakness, but in case of the rock mass, you can have more number of discontinuity planes. So, how to tackle such situation?

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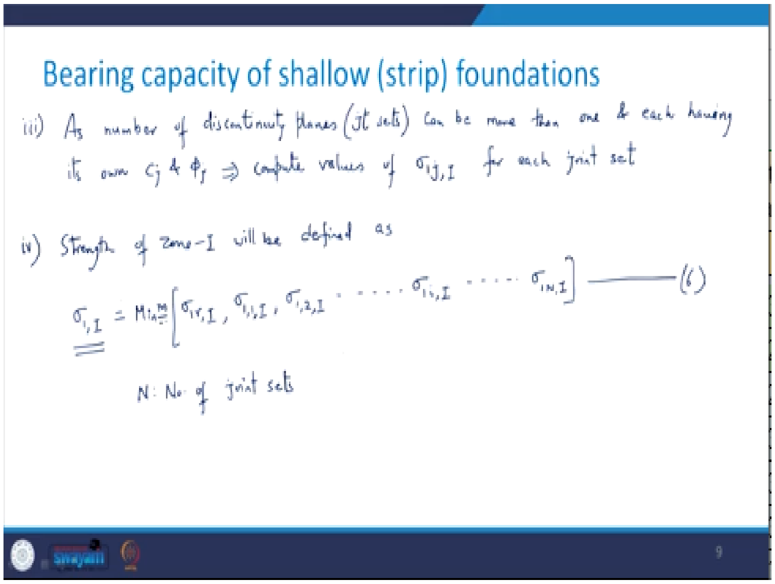

So, I have a third case here that as the number of discontinuity planes or maybe, you can say that these are the joint sets can be more than one and each may have their own c_j and ϕ_j. So, each having its own c_j and ϕ_j. Then what we will do? In this case, what we are going to do is, we will compute the values of σ_{1j} in zone I for each joint set. And then the 4th step is going to be that the strength of zone I will be defined as σ_1 of zone I; will be minimum of σ_1 or of zone I that means the intact rock.

Then σ 1 of say first join set in the zone I, σ 1 of the second joint set in zone I and so on, σ1 of ith joint set in zone I and so on, σ 1 of the nth joint set in zone I. So, whichever is going to give you the minimum value that is going to be defined as the strength in the zone I where this capital N is giving as the number of the joint sets. So, I will make this equation as equation number 6. Now, this was about the strength of zone I and this is how using equation number 6, we can determine what about in zone II.

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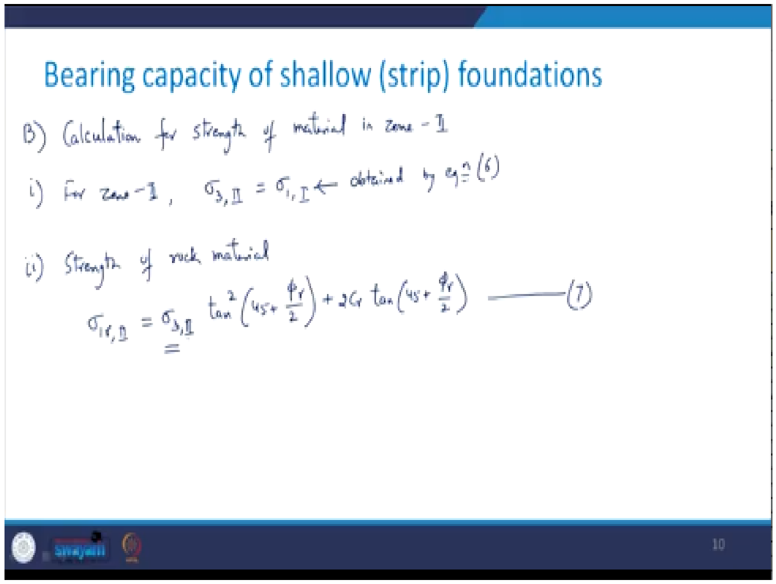

So, the next segment comprise of the calculation for the strength of material in zone II. Now, again, we will say that we will have the strength of the rock material and the strength of the discontinuities. So, in case of this zone II, σ_3 of zone II is going to be equal to σ_1 of zone I as per the Bell's approach and this one, we have obtained already by equation number 6. So, this is known to us now. So, let us go to the next step and try to get the strength of the rock material in zone II.

So, what we are going to get is σ_{1r} of zone II will be equal to

$$\sigma_{1r,\mathrm{II}}=\sigma_{3,\mathrm{II}} \tan\left(45+\frac{\phi_r}{2}\right)^2+2c_r \tan\left(45+\frac{\phi_r}{2}\right)$$

Make it equation number 7. So, here $\sigma_3$is no more 0 as it was there in case of the zone I. Now, let us try to get the strength of the discontinuities in the second zone. So, we have the third step for the evaluation of strength of discontinuity.

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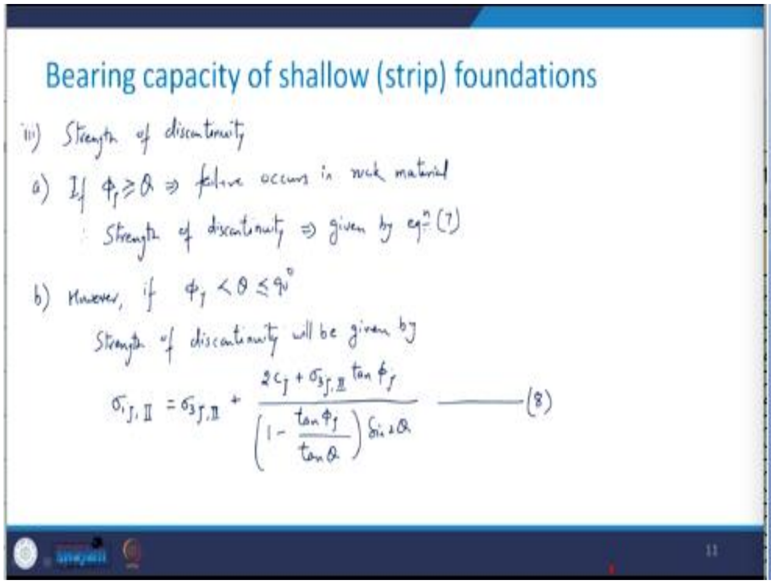

Now, in this case again, if this phi j is greater than or equal to theta, then failure occurs in the rock material and therefore, the strength of the discontinuity will be given by the equation number 7 which we just derived. However, if you have phi j less than theta less than or equal to 90 degree, then in this case, your strength of the discontinuity will be given by σ_{1j} of the second zone, will be equal to

$$\sigma_{1j,\mathrm{II}}=\sigma_{3j,\mathrm{II}}+\frac{2c_j+2\sigma_{3j,\mathrm{II}}\tan\phi_j}{\left\{1-\frac{\tan\phi_j}{\tan\theta}\right\}\sin 2\theta}$$

You need to be careful when to put theta and when to put 90 minus theta. If you recall the single plane of weakness theory, the beta angle, we took from the major principal axis direction. So, that you need to keep in mind when you are applying it here.

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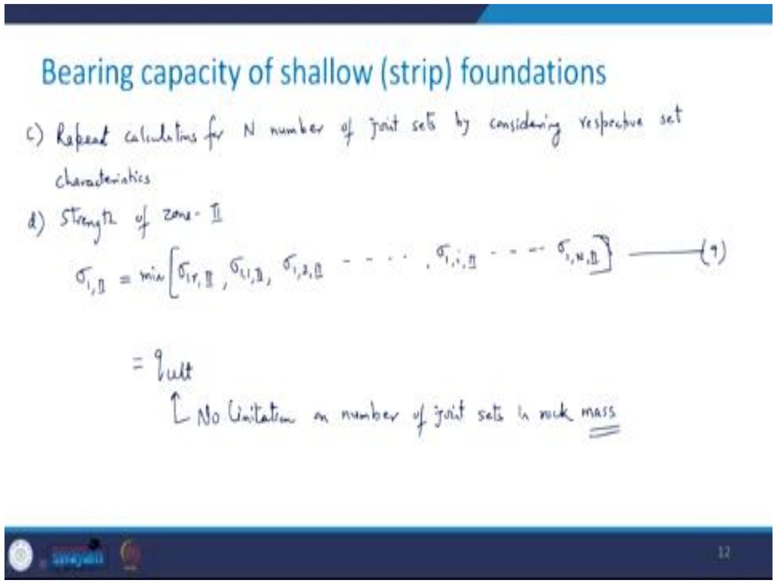

Then what we need to do as a third step in this sequence is, we need to repeat these calculations for N number of joint sets by considering the respective set characteristics and ultimately, we will find out the strength of zone II that is going to be σ 1, II will be equal to minimum of σ_{1r} of zone II, σ1 of the first set of the zone II, then σ1 of the second set of the zone II and likewise, σ1 of the i^{th} set of zone II and so on, σ1 of the nth set of zone II. Make this equation as equation number 9.

And this is what is going to give us q ultimate and here, you can see that there is no limitation on the number of joint sets in the rock mass. So, we can have as many number of joints set as it is there in case of the rock mass. So, this is how you will get the strength of zone II.

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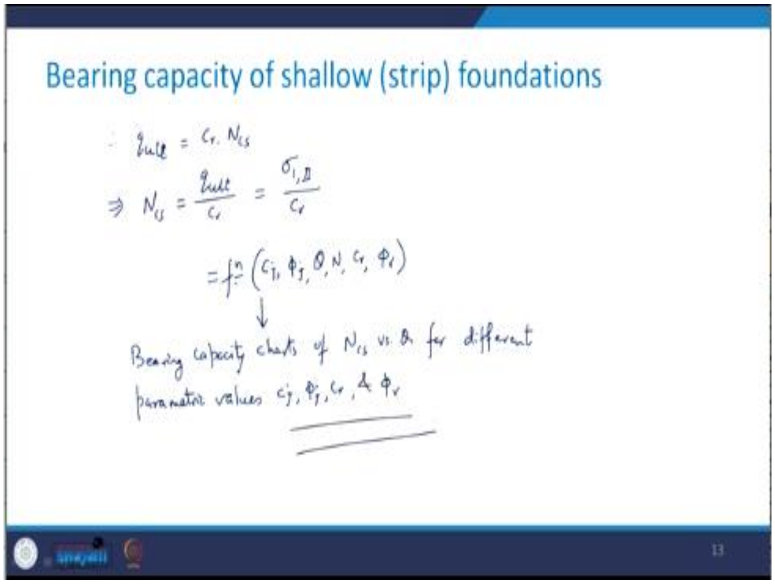

So, therefore, our $q_{ult} = c_r N_{cs}$ that was the first equation that we wrote. So, from here, this N cs is going to be $\frac{q_{ult}}{c_r}$ and this is going to be σ1 of zone II divided by c_r and this is the function of

$$N_{cs} = f(c_j, \phi_j, \theta, N, c_r, \phi_r)$$

So, from here, the bearing capacity charts of N_{cs} Vs θ for different parametric values of c_j, ϕ_j, c_r and ϕ_r can be obtained. And once you have these available readily available charts with you, you can directly use this expression in order to obtain the q ultimate.

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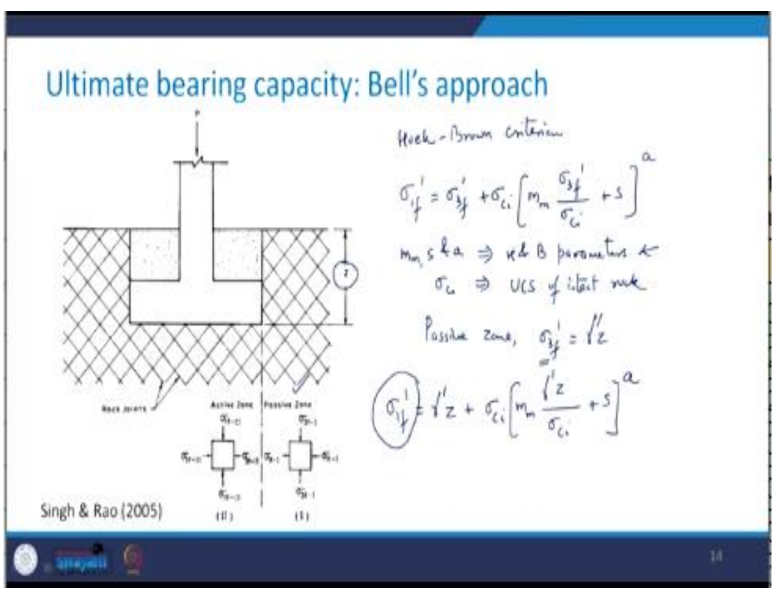

Now, let us take another example. Till now, the rock mass was following the Mohr-Coulomb failure criteria but let us say that it follows the Hoek-Brown criteria, then what you will do? And in this case, say, it has this overburden, which will be equal to gamma into z. So, what we have here is

$$\sigma'_{1f} = \sigma'_{3f} + \sigma_{ci}\left(m_m \frac{\sigma'_{1f}}{\sigma_{ci}} + s\right)^a$$

This is what is the Hoek-Brown criterion where this m_m, s and a, these are Hoek-Brown parameters.

And σ_{ci} is what? It is the UCS of the intact rock. So, in this passive zone, what we are going to get is, σ_{3f} prime will be equal to γ times z or since, we are writing in terms of the effective stresses say, I will write it as γ'. So, σ_{1f} prime is going to be

$$\sigma'_{1f} = \gamma' z + \sigma_{ci}\left(m_m \frac{\gamma' z}{\sigma_{ci}} + s\right)^a$$

See, the parameters of Hoek-Brown parameters, they will be known to me and knowing this overburden.

And therefore, σ_3f prime, from here, I can determine this in the passive zone that is σ 1 f in the passive zone.

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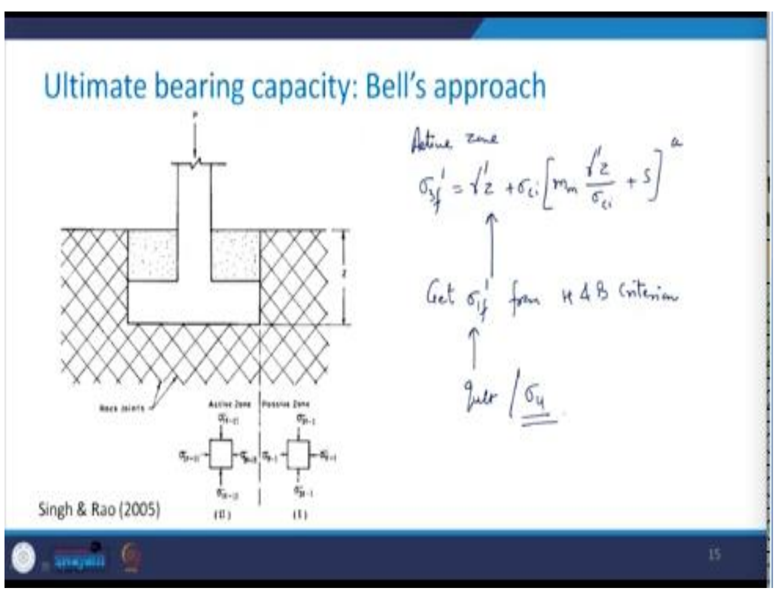

Now, what will happen in case of the active zone? Your σ_{3f} prime would be

$$\sigma'_{3f} = \gamma' z + \sigma_{ci}\left(m_m \frac{\gamma' z}{\sigma_{ci}} + s\right)^a$$

Now, from here, we can obtain the σ_{1f} prime again using the Hoek-Brown criterion and this is what is going to give us the q ultimate or σ u that the ultimate bearing capacity.

So, this is how using the Bell's approach. You can find out the ultimate bearing capacity of the foundation. Here, we took a case of the strip foundation and I tried to show you with the help of Mohr-Coulomb criterion and Hoek-Brown criterion that how the procedure can be adopted. So, there are various other methods that we can find out the bearing capacity of the shallow foundations. So, we will take up some of these methods in the next class. Thank you so much.