Rock Engineering
Prof. Priti Maheshwari
Department of Civil Engineering
Indian Institute of Technology – Roorkee

Lecture - 54
Rock Slope Stability: Toppling Failure - 3

Hello, everyone. In the previous class, we were discussing about the toppling failure mode. As far as rock slope stability is concerned, we saw that how we can obtain the block geometry and then we started our discussion with block stability. Let us continue with the same discussion today and try to see that how we can find out whether the slope is safe against toppling failure or not.

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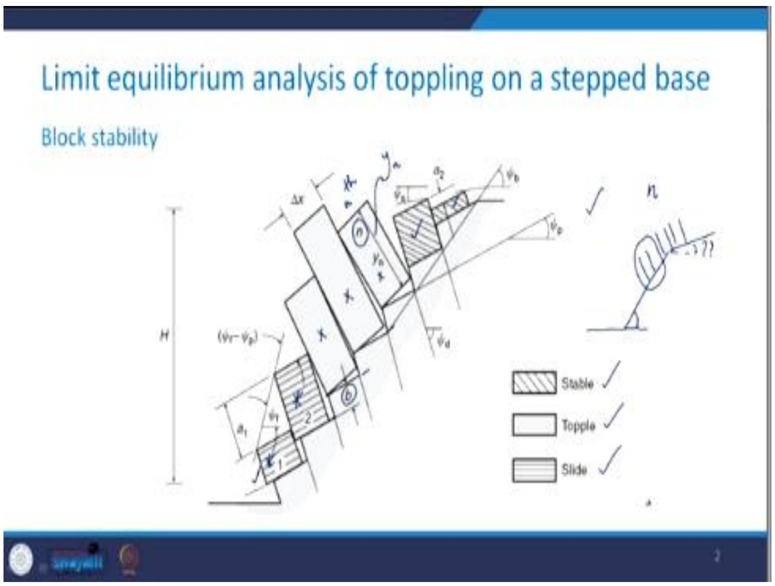

So, we saw this figure in the previous class, let us try to go through it once again and try to find out that what were the salient features as far as this figure is concerned. I mentioned to you that there are 3 sets of blocks which can be present. These can be stable; these blocks may topple or these may slide. As far as this figure is concerned, you can see that this block and this block, they are stable blocks.

However, this block, this block and this block, these 3 have the tendency to topple and the blocks, this one and this one, they are prone to sliding and it was assumed that the moment this block at the toe, it slides, the slope is considered to be unstable. This distance which has been shown here by *b* is constant for all the blocks. These were the assumptions which were involved and if you recall, we obtained the number of blocks that will be present in such type of rock slope and that we represented as n.

Here, the lower part of the slope has one slope, however, above the crest, the slope is something else. So, basically, it will look like this. Like, this has one value and the upper portion has the other value. Then any block which is say, n^{th} block here, this nth block can be above the crest that means in this zone, it can be just at the crest that is here or it can be below the crest which is like this.

So, depending upon where this nth block is located accordingly, you can obtain the block geometry as we have discussed in the previous class. Now, here you can see that for n^{th} block, we have taken the height of the block as y_n. Then now, what we will do is, we will try to have the forces which are acting on any typical n^{th} block, then we will take one n^{th} block which is having the tendency to topple and we will take another nth block which will have the tendency to slide.

And then we will try to get the force equilibrium equation and then try to see how this system can be analyzed.

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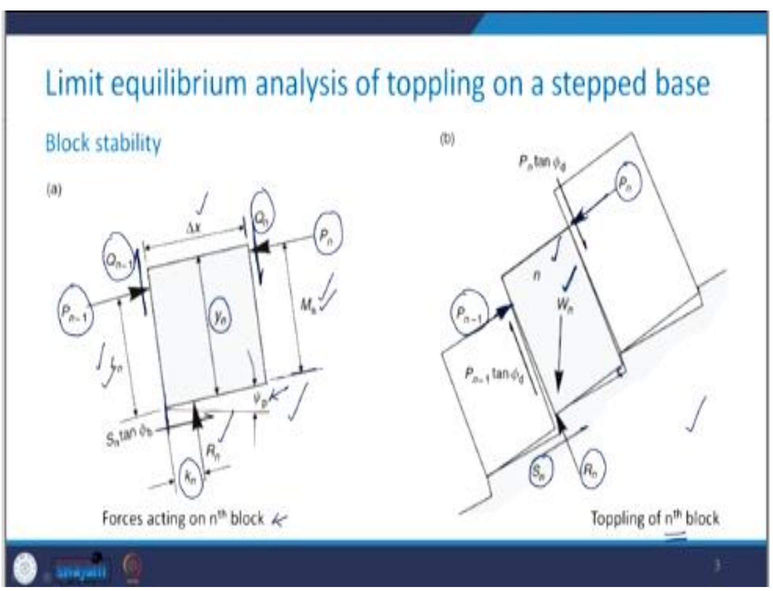

Take a look here. The first figure gives you the idea about the forces which are acting on the nth block. So, here you can see that these normal forces P_n and P_{n-1}, they are acting at a distance of M_n and L_n respectively from this base of the block. While these shear forces that is Q_n and Q_{n-1}, they are acting as shown in this figure in this particular manner. This width of the block is delta x and the height is y_n; this slope is given as ψ_p.

There is going to be another force, which is the normal to this block that is given by R_n and this is acting at a distance of K_n from this face of the block. Now, take a look at the second figure in this slide. This shows the toppling of the n^{th} block. So, you can see here that there is a gap between these 2 planes that shows that this nth block is going to topple. Now, here, you see that you have a force R_n and another force which is parallel to this base that is S_n.

Weight of the block is given by this W_n and these forces which are P_n and P_{n-1} acting in this particular plane at a particular location, which has been shown in this figure and given by M_n and L_n respectively.

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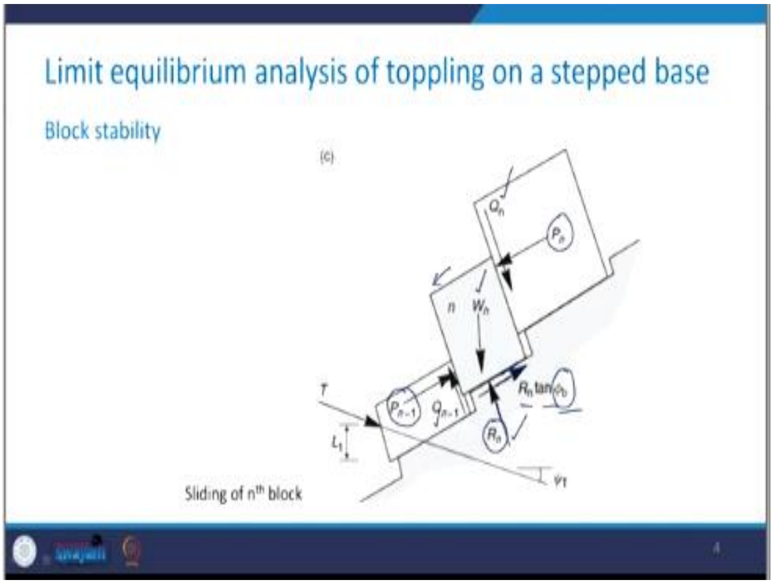

Now, if this nth block does not topple but it slides, then in that case, you see that there is no gap here in between these 2 planes and this block is going to only slide. Weight is again W_n. This normal force is R_n and base friction angle is ϕ_p and therefore, for this tangential component which will be perpendicular to this normal force that is going to be R_n tan ψ_p and the forces P_n and P_{n-1} are shown like this. Similarly, Q_n and Q_{n-1} which are the shear forces at that block in this direction has been shown here in this particular figure.

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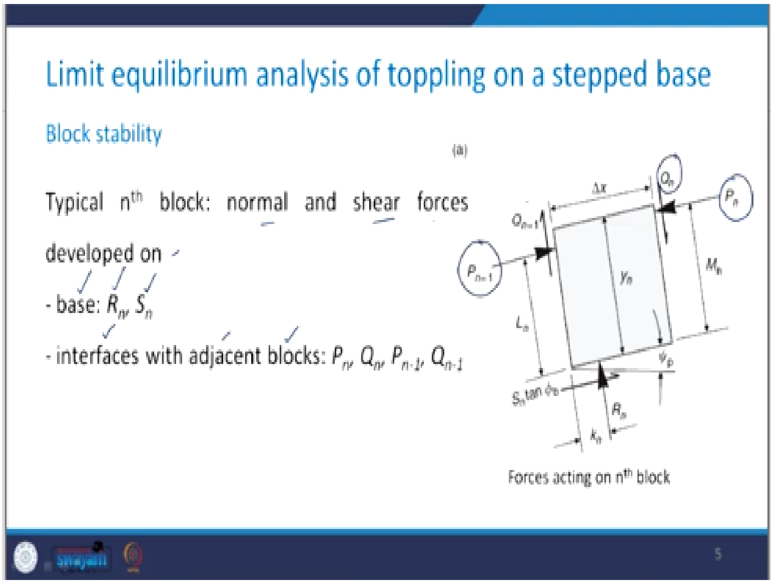

Now, let us consider a typical nth block. So, the normal and the shear forces that are developed on the base, I mentioned to you that these are R_n and S_n and at the interfaces with the adjacent blocks like has been shown here. Normal forces are P_n and P_{n-1} and the shear forces are Q_n and Q_{n-1}. So, these are the typical forces which are acting on the n^{th} block.

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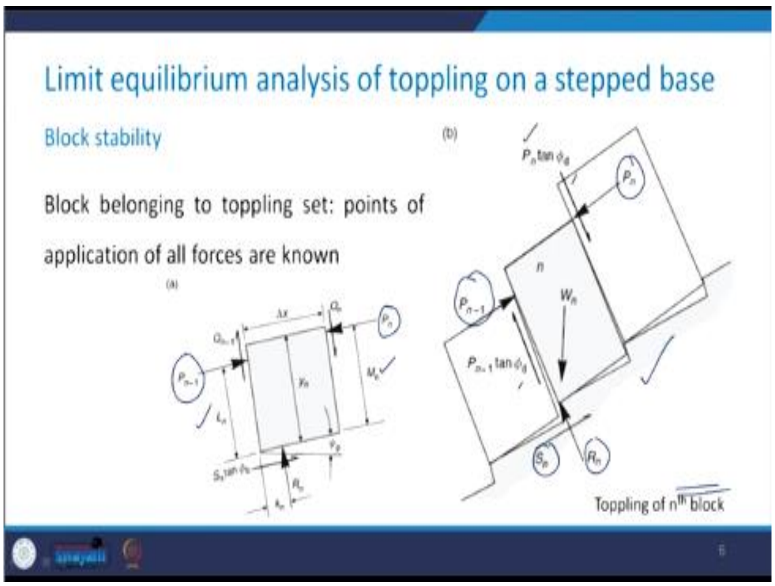

Now, let us first take a block which is belonging to the toppling set. So, in that case point of application of all the forces are known. Like here, you can see M_n and L_n, they are known where the P_n and P_{n-1} respectively, they are acting. So, this is the figure again showing the toppling of the nth block and then these forces are their R_n and S_n and P_n and P_{n-1} and here, you have Q_n and Q_{n-1} which have been represented in terms of P_n and ψ_d.

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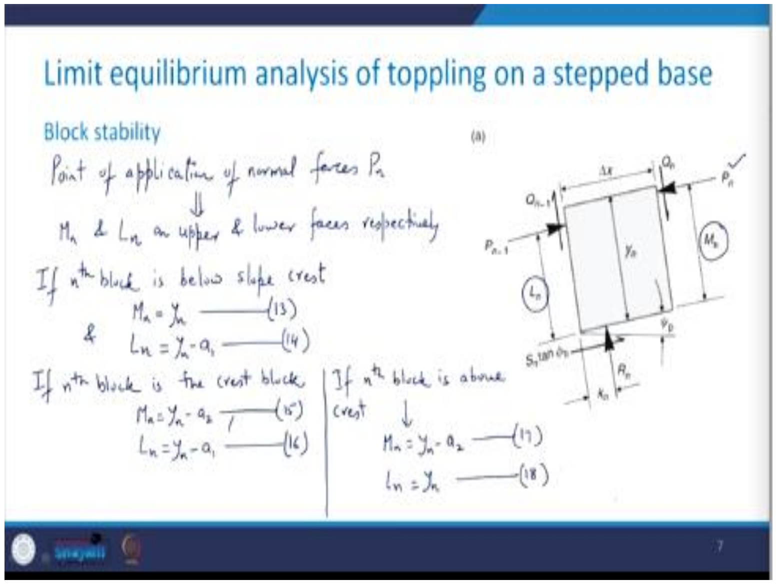

Now, in case of these toppling block, the points of application of the normal forces P_n, these are given as M_n and L_n; we have seen in the previous figure on the upper and lower faces respectively. See here, this P_n is acting at a height of M_n and $P_{n-1}$is acting at a height of L_n. Now, I take the 3 cases that is whether this n^{th} block is above the crest, at the crest or below the crest of the slope. So, there is going to be 3 cases.

If nth block is below the slope crest, what will happen in that case your$M_n = y_n$; make it equation number 13. Till equation 12, we have in previous class. And $L_n = y_{n-1}$that is equation number 14. Now, if this nth block is the crest block itself, then in that case M_n is going to be $(y_n - a_2)$. This will be equation number 15 and L_n will be $(y_n - a_1)$. This is equation number 16.

And then we will have the third case where you will have if this nth block is above crest, then in this case what will happen? This M_n will become equal to $(y_n - a_2)$ and L_n is equal to y_n. What are these a_1, a_2? Everything, you have with you and you can refer to the first figure of this lecture which I have already shown it to you.

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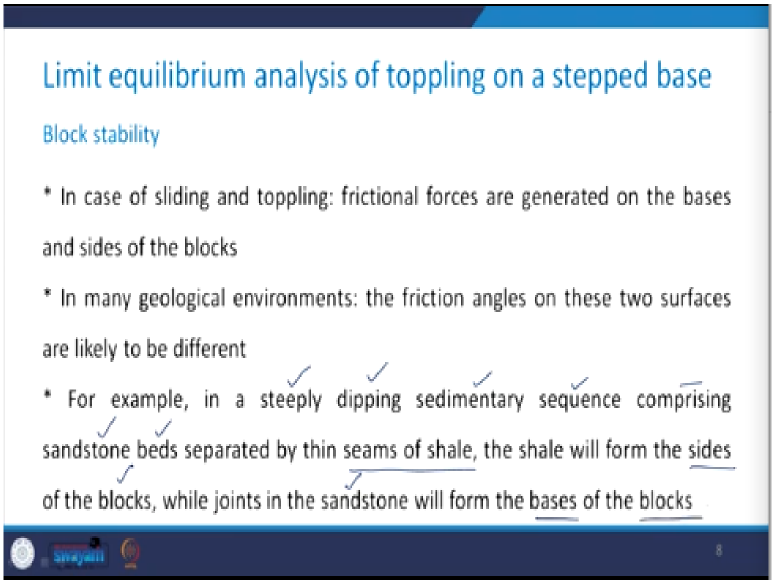

Now, in case of sliding and toppling, frictional forces are generated on the basis as well as on the sides of the blocks. In many geological environments, the friction angles on these 2 surfaces are going to be different. For example, if you take the steeply dipping sedimentary sequence, which is comprising of sandstone beds and separated by thin seams of shale, so, in this case, what will happen is that the shale will form the sides of the blocks while the joint in the sandstone will form the basis of the blocks.

And the moment the material at the base of the block and the side of the block is different, most likely the friction angles will also be different on these 2 surfaces.

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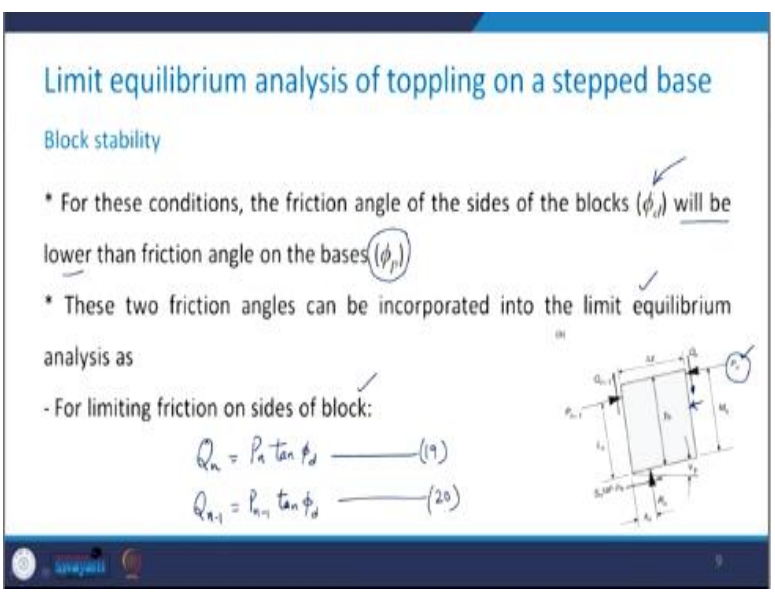

So far such type of conditions, the friction angle of the sides of the blocks is being represented by ϕ_d. This will be lower than the friction angle on the base which is ϕ_f. Now, these 2 friction angles can be incorporated into the limit equilibrium analysis as we are going to discuss. So, for the limiting friction on the sides of the block, we can write that $Q_n = P_n \tan \phi_d$

Take a look here. This is your P_n and here, the friction angle is ϕ_d. So, this $Q_n = P_n \tan \phi_d$

So, this equation will be 19. And similarly, you can write

$$Q_{n-1} = P_{n-1} \tan \phi_d$$

This will be equation number 20.

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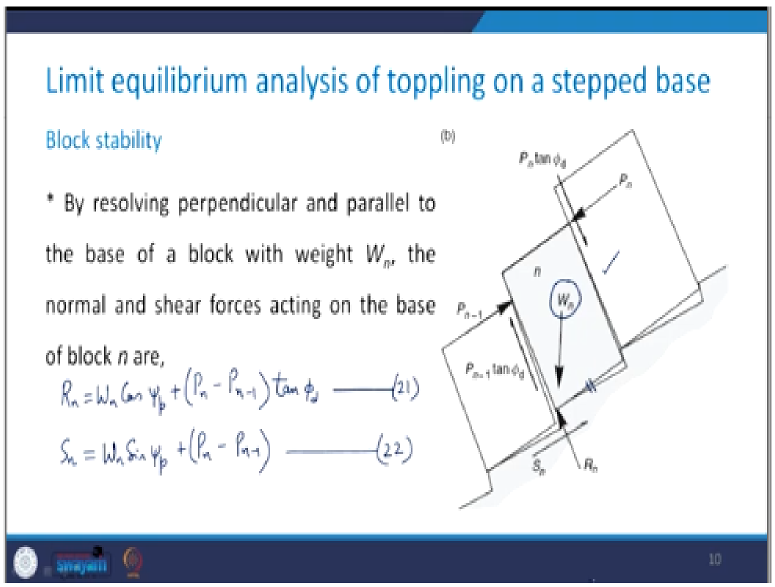

Now, by resolving the perpendicular and parallel to the base of a block with weight W_n, the normal and the shear force which will be acting at the base of the block, they can be obtained as

$$R_n = W_n \cos \psi_p + (P_n - P_{n-1}) \tan \phi_d$$

equation number 21. And then if you take the shear force, it will be.

$$S_n = W_n \sin \psi_p + (P_n - P_{n-1})$$

So, there is nothing difficult. Take a look at this figure and then get the force equilibrium in the direction perpendicular and parallel to the base of the block whose weight is given by W_n.

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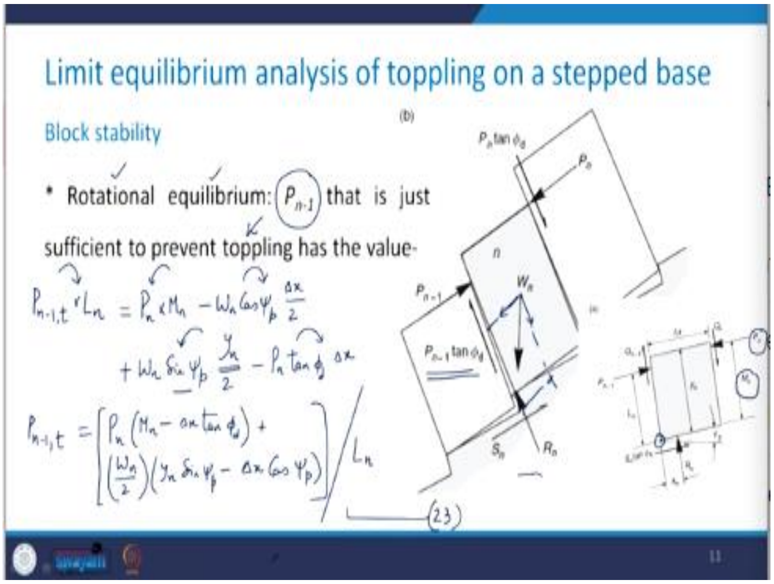

Then we take the rotational equilibrium and we try to find out this force P_{n-1} which is just sufficient to prevent toppling. How to do that? Take a look. So, you see that I take the rotation at this point. So, the first force is going to be $P_{n-1,t}$. Why I write *t* is because I am dealing with the toppling; this at what distance that it is acting L_n. So, this multiplied by L_n. And what is going to be its direction about this point?

This is going to be in this direction that is the clockwise direction. This is going to be equal to, see, this P_n is there and it is acting at this distance of M_n and when it creates the rotation, it is going to be $(P_n \times M_n)$and it will be in the anti clockwise direction. So, this is how we are going to write $(P_n \times M_n)$ and it is going to be in this direction. Then this weight component which is there, so, you will have these 2 weight components; one like this and another one like this.

So, the one component is going to give me that is ($W_n \cos \psi_p$) and where is this acting, this distance is Δx. So, this is going to be $\Delta x/_2$. So, this is going to be $\Delta x/_2$ and this is going to cause the moment in the clockwise direction and then you will have another component of *W* which is $W_n \sin \psi_p$ and this is acting at a height of this $y_n/2$. So, this is going to be $y_n/2$ and its direction will be this and then you will have the last component which is this $P_n \tan \phi_d$.

So, that would be $(P_n \tan\phi_d)\Delta x$.What about this$P_{n-1}\tan\phi_d$? Because it is passing from this, so, it will not have any kind of rotational moment there. So, this is what is going to be in this direction. So, you can write this $P_{n-1,t}$ is equal to

$$P_{n-1,t} = \left[p_n(M_n - \Delta x \tan\phi_d) + \left(\frac{W_n}{2}\right)(y_n \sin\psi_p - \Delta x \cos\psi_p)\right] \Big/ L_n$$

So, this whole thing in a box bracket and this divided by L_n. Make this equation as equation number 23. Take a note of these equation numbers because we will be referring these later when I will be summarizing the steps for the analysis in case of the toppling failure that we are taking up right now.

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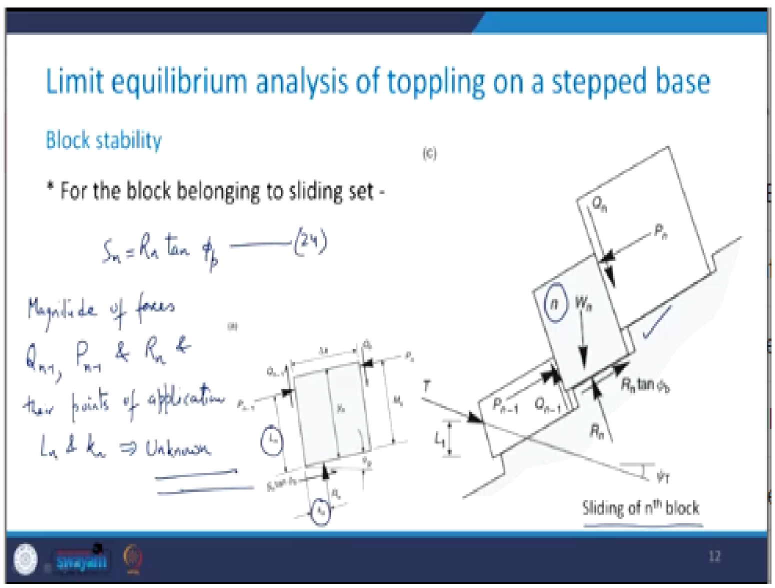

So, for the block which is belonging to the sliding set. So, till now, what we discussed about the toppling block? But then in case if the nth block is not toppling, but undergoing this sliding, then what can be the situation? So, here are the forces which have been shown in case of the sliding of the nth block. So, what we are going to get here as $S_n = R_n \tan\phi_p$that equation number 24 and the magnitude of the forces that is Q_{n-1}, P_{n-1} and R_n and their points of application which are L_n and K_n.

You can see here this is what is your K_n and these are L_n's. So, these are unknown. The question is if all these are unknown, how we will get the solution? So, this is a kind of indeterminate problem.

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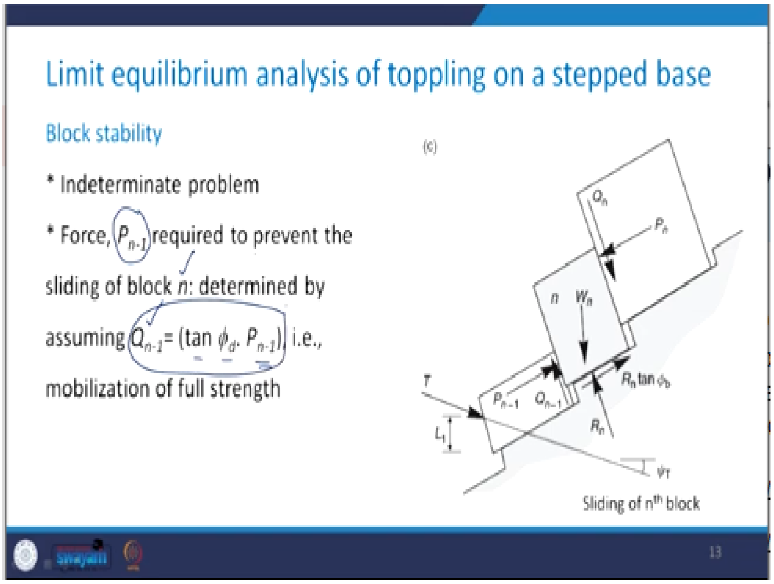

In order to make it determinate, we need to have some kind of an assumption. So, the assumption which is involved here is that force P_{n-1} which is required to prevent the sliding of n[th] block is determined by assuming $Q_{n-1} = \tan\phi_d . P_{n-1}$ that means it is assumed that there is the full mobilization of the strength that is going to take place, because if that full mobilization is not there, this condition will not be satisfied.

So, considering this, the magnitude of Q_{n-1} would be known to us and therefore, it will reduce some number of unknowns and the problem will become determinate.

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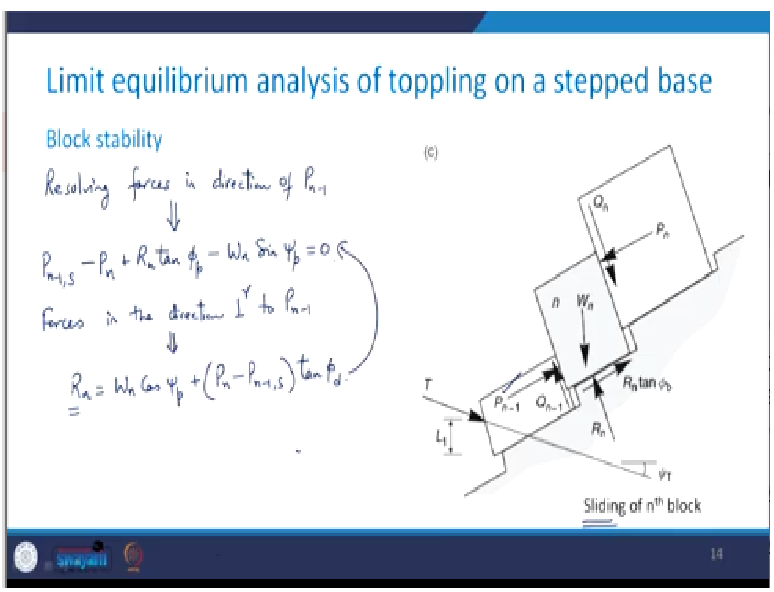

So, how to carry out the analysis? So, what we do is, we resolve the forces in the direction of this , P_{n-1}. So, let us do that resolving the forces in the direction of , P_{n-1}. What we will get is , $P_{n-1,s}$. Why I am using s here because now I am taking the sliding of the n[th] block. So, this is

$$P_{n-1} - P_n + R_n \tan\phi_d - W_n \sin\psi_p = 0$$

Then what are the forces in the direction perpendicular to P n minus 1?

These are going to be R n which will be equal to

$$R_n = W_n \cos\psi_p + \left(P_n - P_{n-1,s}\right) \tan\phi_d$$

So, this is what that we are going to get.

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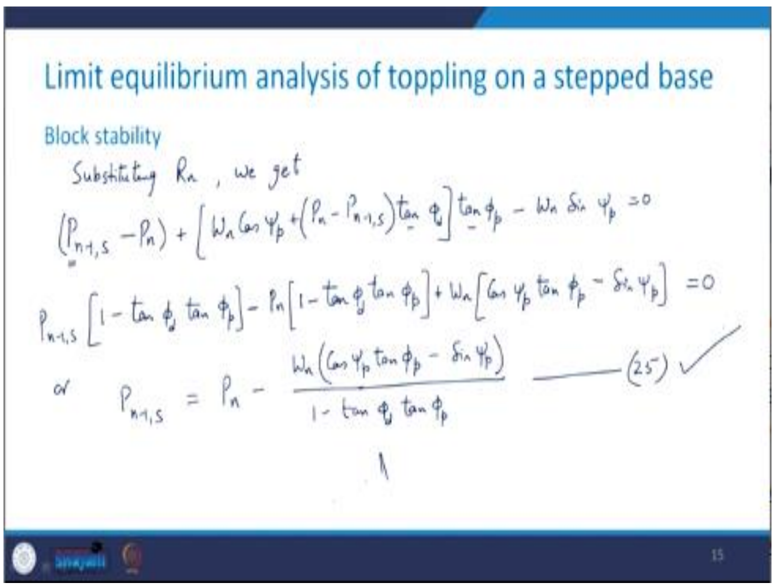

Now if we substitute this expression of R n in this equation, what we are going to get is,

$$P_{n-1} - P_n + \left(W_n \cos\psi_p + \left(P_n - P_{n-1,s}\right) \tan\phi_d\right) \tan\phi_d - W_n \sin\psi_p = 0$$

Now, you take the terms of this $P_{n-1,s}$ together. So, this is what that we are going to get; this will be

$$P_{n-1,s} = P_n - \frac{W_n\left(\cos\psi_p \tan\phi_p - \sin\psi_p\right)}{1 - \tan\phi_d \tan\phi_p}$$

So, this is how we will get the P_{n-1} for the nth block which is undergoing the sliding, make this equation as equation number 25.

So, now, after knowing that how can we calculate this P n minus 1 for the nth slide which is undergoing either the toppling or the sliding. Let us try to see how to go ahead with the complete analysis as far as the toppling failure of the slopes are concerned.

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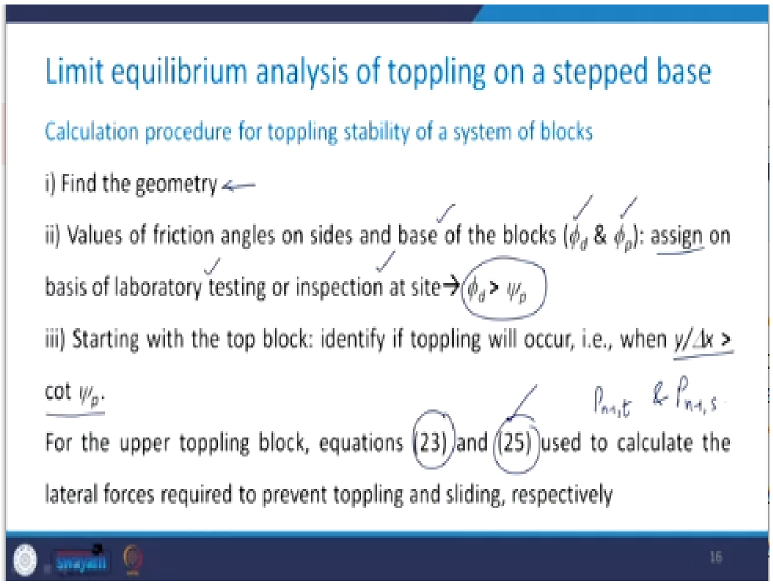

So, it is the calculation procedure for toppling stability has been given here. So, the first step is going to be; you need to find the geometry of every block, then values of the friction angles on the sides and base of the blocks, they are ϕ_d and ϕ_p. We need to assign these on the basis of lab testing or inspection at site and you need to keep in mind that this ϕ_d will be greater than ψ_p. Then we will start with the top block and identify if the toppling will occur or not.

And when the toppling will occur, this condition that is y/Δx has to be greater than cot ψ_p will be true. This, we have seen when we were discussing about the kinematic analysis of the toppling failure. Now, for the upper toppling failure, just now, we obtained equations 23 and 25. These would be used to calculate the lateral forces which are required to prevent the toppling and sliding respectively. So, we will use these equations 23 and 25 in order to get that $P_{n-1,t}$ and $P_{n-1,s}$.

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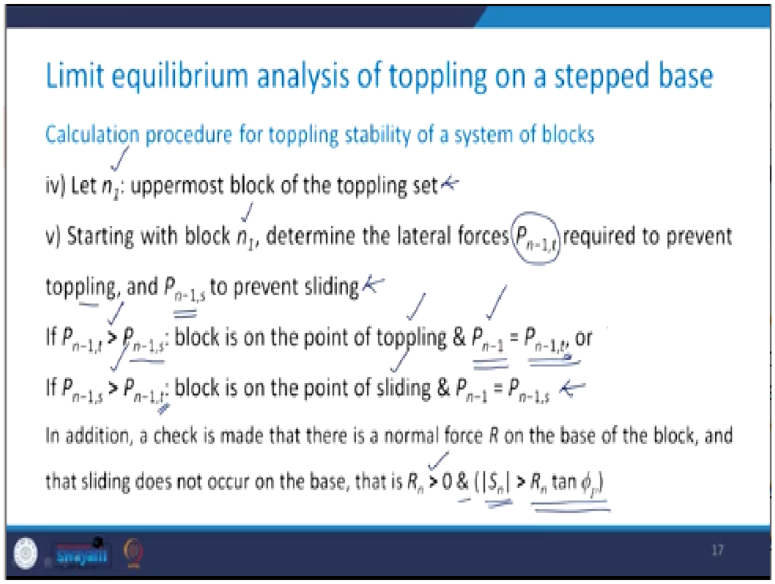

Now, let us say that n_1 be the uppermost block of the toppling set. Now, starting with the block n_1, we will determine the lateral forces$P_{n-1,t}$, which is required to prevent toppling and $P_{n-1,s}$ to prevent the sliding. If the P_{n-1} value for toppling works out to be more than that for the sliding. This means that the block is on the point of toppling and P_{n-1}would be assigned the value which is corresponding to $P_{n-1,t}$.

So, see here what we are trying to check is for every block, I am trying to find out what the force that is required to stop toppling is and sliding both. Now, whichever is more that mechanism is going to govern that particular block. So, in case if you have $P_{n-1,t}$greater than $P_{n-1,s}$. Block is on the point of toppling and accordingly this P_{n-1} will take the value of P_{n-1} corresponding to toppling that is $P_{n-1,t}$.

And in another case, if $P_{n-1,s}$ is more than the toppling value of P_{n-1} that means that the block is on the point of sliding and your P_{n-1} will become equal to $P_{n-1,s}$. So, this is how that for every block is starting from the top, we are going to decide whether it is going to topple or slide and then what is going to be the value of this P_{n-1} depending upon whether the block is toppling or whether the block is sliding.

In addition to this, a check needs to be made that there is a normal force are on the base of the block and that the sliding does not occur on the base that means that R_n has to be equal, more than

0 and your S_n, mod of S n should be more than R_n tan ϕ_p. So, in addition to these conditions, this condition should also be satisfied.

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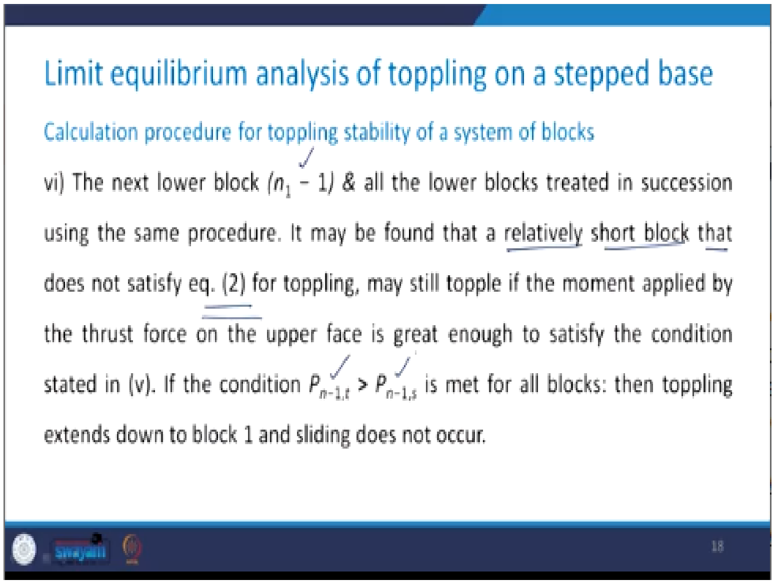

Now, the next lower block which is n 1 minus 1 and all the lower blocks, they will be treated in succession using the same procedure. So, first you take the first block which was n 1, then you go to n 1 minus 1 and then you try to get the P_{n-1} value for the block and n_{1-2} and so on. It may be found that the relatively short block that does not satisfy the equation number 2 for toppling.

It may still topple if the moment applied by the thrust force on the upper face is great enough to satisfy the condition that we have stated in the earlier step which is step number 5. Now, if the condition that is P_{n-1} toppling is more than P_{n-1} sliding is met for all the blocks. What does that mean? That the toppling extends down to block number 1 and sliding will not occur at all for all the blocks which are there on that slope.

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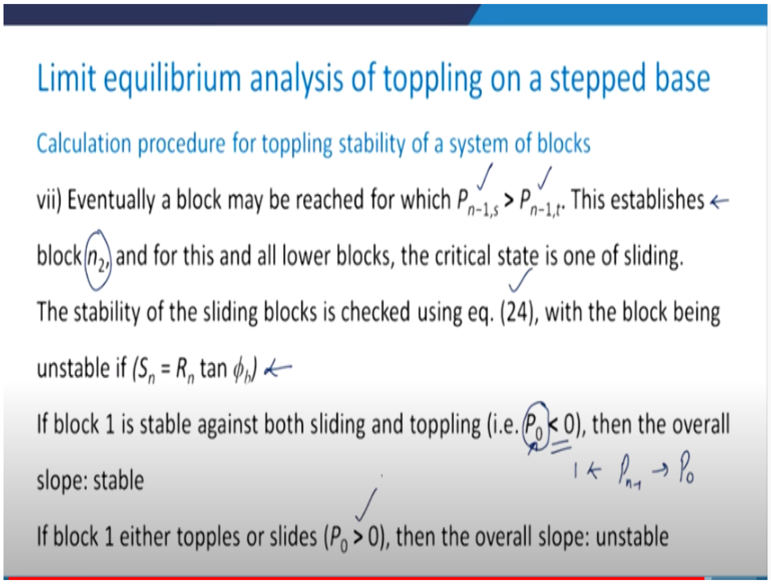

Now, eventually a block may be reached for which it can have P_{n-1}sliding more than P_{n-1}toppling. So, this will establish the block n 2 and for this n, all the lower blocks. Now, the critical state is going to be that of the sliding and not toppling. So, the stability of the sliding block is now going to be checked by using equation number 24. With the block being unstable if S_n is going to be equal to $R_n \tan \phi_p$.

Now, if this block 1 which is the lower most block near the toe of the slope is stable against both sliding and toggling that is P_0. Why we are getting this P_0? Because, it is the block number 1 and for block number 1 P_{n-1}will be P 0. So, if P_0 is less than 0, then the overall slope is going to be stable. Now, if this block 1 either topples or slides that means, if this P_0 is greater than 0, then the overall flow is going to be unstable.

So, this is how we can find out whether the slope is going to be stable or unstable. So, you need to go in a systematic manner, starting from the top most block and then go one by one towards the toe of the slope and then keep finding out that what is going to be the value of $P_{n-1}$for each of these blocks, apply your check which one is the dominating mechanism whether it is toppling or whether it is sliding. Accordingly, you assign the value to this force P_{n-1}.

And ultimately, when you carry out this analysis for the block number 1, which is the lowest block at the toe of the slope that is going to decide whether the slope is going to be stable or not. If the bottom most one is neither sliding nor toppling, that means that the slope is going to be stable, otherwise, it is unstable. If the slope works out to be unstable, you need to provide some kind of a slope stabilization measure and in case of the toppling failure, we provide the anchor there. And with the tension that is mobilized in that anchor, that toe block gets stabilized.

So, there are various ways that we can stabilize these slopes under different types of conditions. So, all these things, we will take up in the next class. Thank you very much.