Rock Engineering
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Lecture - 51
Rock Slope Stability: Circular Failure

Hello everyone. In the previous class, we discussed about the wedge failure mode of rock slope stability analysis. We saw that how the factor of safety can be obtained for a simple case for the probable mode of failure as wedge failure. So, today, we will take up the next failure mode of the rock slope, which is the circular failure. Let us try to see that how with the help of limited equilibrium analysis, we can find out the factor of safety for this rock slope, which is undergoing the circular failure.

So, this circular failure occurs mainly in rock hills, weathered rocks or rocks with closely spaced randomly oriented discontinuities. This is very much similar to those kinds of failure which occur in case of soil slopes.

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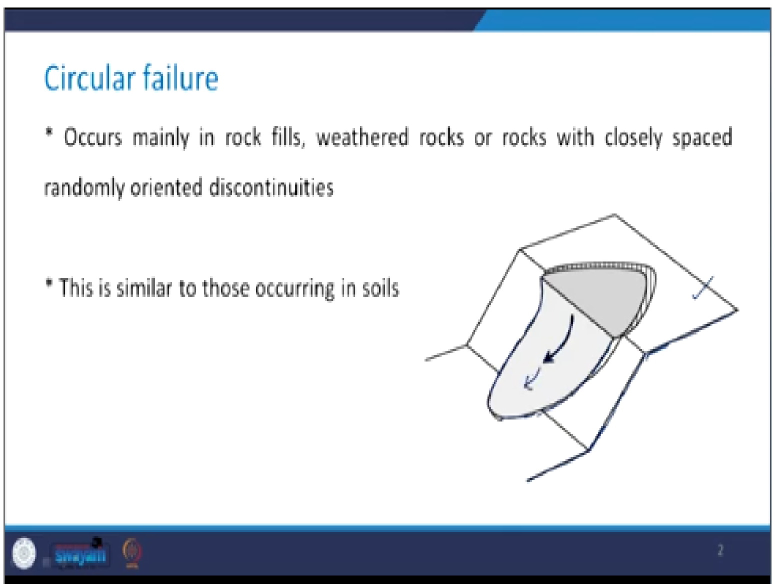

You can take a look in this picture that this is your slope and here the sliding of this mass takes place along this circular plane. So, this whole mass it just slides in this particular fashion.

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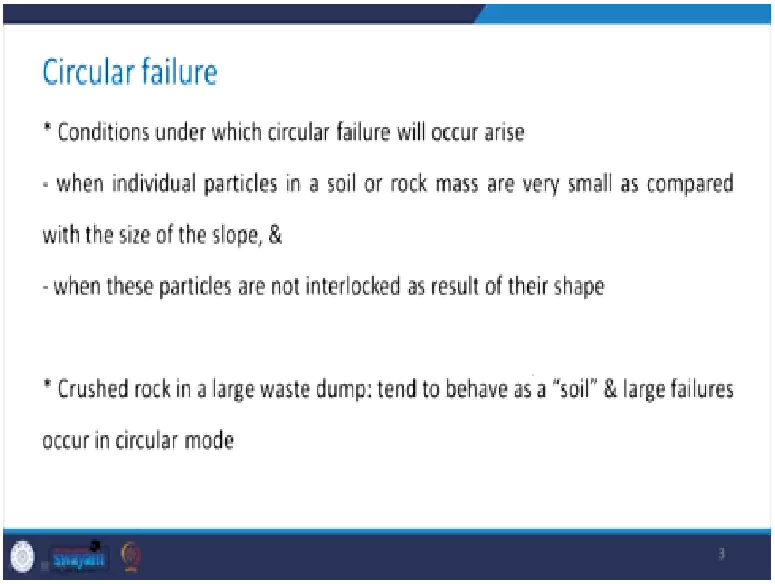

Now, conditions under which the circular failure can occur, which arise when individual particles in a soil or rock mass are very small as compared to with the size of the slope. And the second condition which contributes towards the occurrence of the circular failure is that when such particles are not interlocked as a result of their shape. For example, crushed rock in a large waste dump, tends to behave as the soil and large failures occur in the circular mode.

In case of the rock slope, it is the plane failure and wedge failure which occurs more common. But in case if you have highly weathered rock, then you may observe the circular failure in such type of rock slopes. Alternatively, the finely ground waste material which has to be disposed of after the completion of a milling and metal recovery or process.

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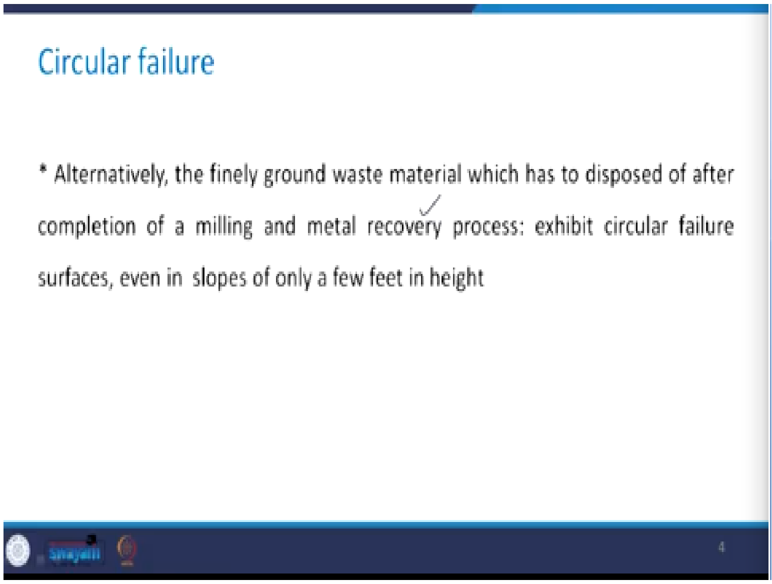

This also exhibit circular failure surface even when the slope of only a few feet in height. So, that means that even if the height of the slope is small, but if you have finely ground waste material that has to be disposed of after these processes of milling and the metal recovery that also can exhibit the circular failure surface. Now, there are various methods for the analysis of the circular failure. We are going to take up one of these that is, Bishop's method of slices.

You must have learned some other methods with respect to the soils in your course on the soil mechanics. But in case of the rocks, circular failure is not very common, except for the case where you have highly weathered rock as a part of the rock slope. Let us try to see that how we go ahead with the analysis using Bishop's method of slices.

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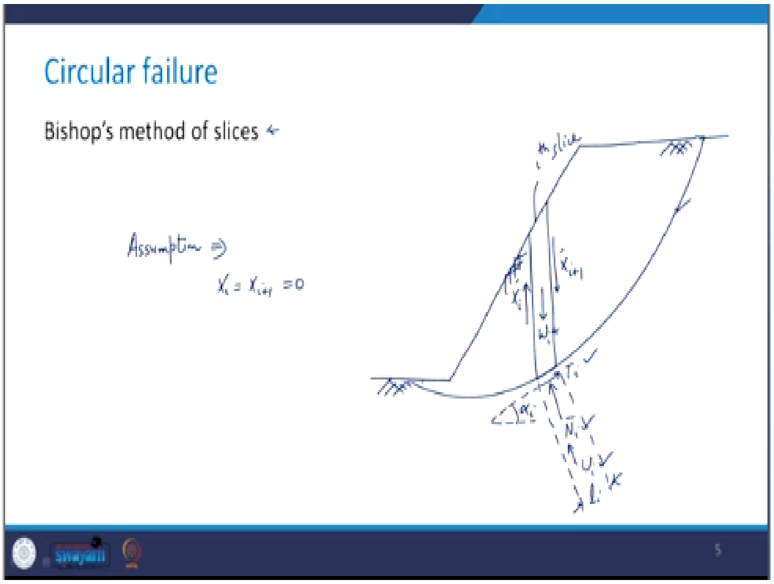

So, I am going to draw a slope here. So, this is the slope face and circular failure can occur along this plane. So, what we do in the Bishop's method of slices that we take a slice like this and we try to get the force equilibrium equation with respect to this slice. So, let us say, this is some ith slice whose weight is vertically downward obviously and I am representing that by W_i. Now, here, along the failure surface, I have a force T_i and normal to that I have N_i bar.

And in case of the pore water pressure and to take care of that, I will have this as U_i which is the force due to pore water pressure. Now, the dimension of this slice along this failure surface is l_i. The inclination here is α_i and you will have the inter slice forces. So, that is going to be like this, which is X_i here and here, it is going to be say X_{i+1}. So, the assumption which is involved here is that X_i will be equal to X_{i+1} and that I am taking it to be equal to 0.

Now, what I will do is you have all these forces T_i, $\overline{N_i}$, U_i, W_i and obviously, because these I have assumed to be equal to 0, so, they will not contribute towards force equilibrium equation. So, what we will do is, we will resolve these forces in the horizontal as well as in the vertical direction.

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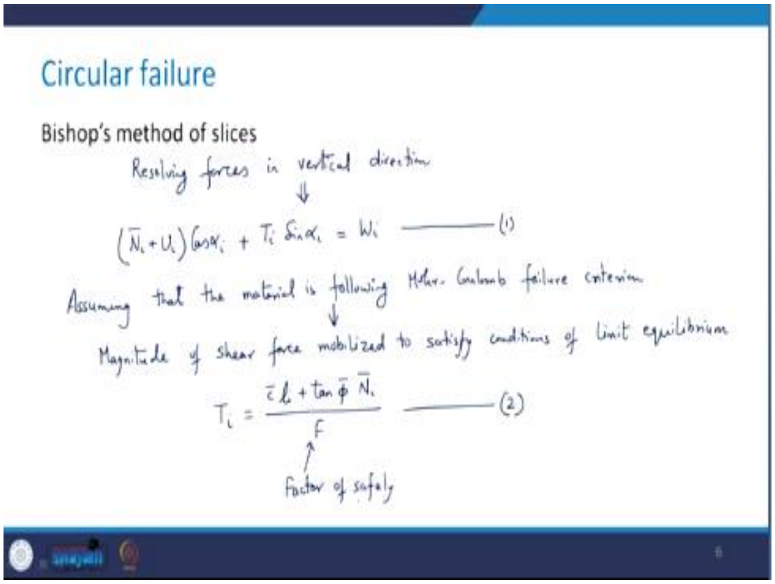

So, if I resolve these forces in the vertical direction. So, resolving the forces in the vertical direction and see what we are going to get is

$$(\bar{N}_i + U_i)\cos\alpha_i + T_i \sin\alpha_i = W_i$$

This equation will become equation number 1. Now, we assume that the material is following Mohr-Coulomb failure criterion. So, from this assumption, the magnitude of shear force which is mobilized to satisfy conditions of limit equilibrium.

That is going to be

$$T_i = \frac{\bar{c} l_i + \tan\phi\, N_i}{F}$$

, this is your factor of safety. So, you see that I am taking the definition of this factor of safety from the first principle that this is equal to the resisting force divided by the force which is inducing the failure. So, the same thing that we are doing here, this is what is going to give us the strength of the material in case the material is following Mohr-Coulomb failure criteria.

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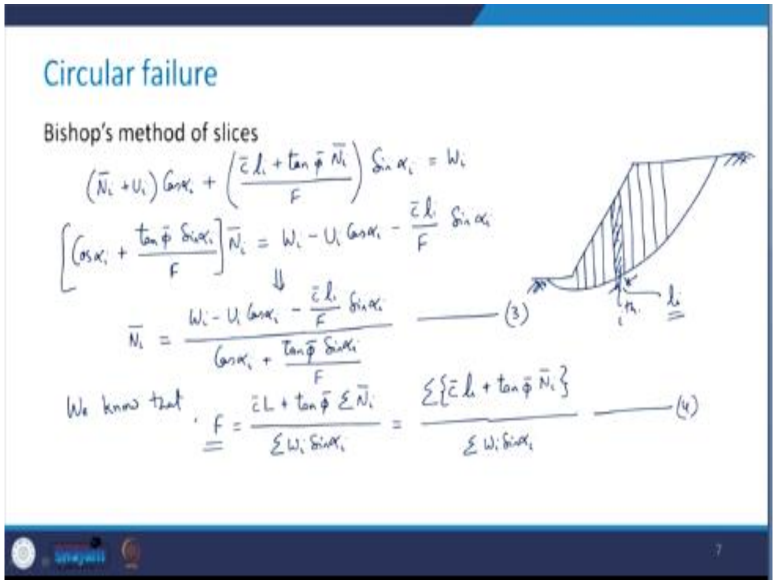

Now, you substitute this in the equation number 1, what we are going to get is

$$(\overline{N}_\iota + U_i)\cos\alpha_i + \frac{\bar{c}l_i + \tan\phi\, N_i}{F}\sin\alpha_i = W_i$$

Now, we try to collect the terms for N_i and see this is what that we are going to get that is

$$\left[\cos\alpha_i + \frac{\tan\bar{\phi}\sin\alpha_i}{F}\right]\overline{N}_i = W_i - U_i\cos\alpha_i - \frac{\bar{c}l_i}{F}\sin\alpha_i$$

Or from here, I can get

$$\overline{N}_i = \frac{W_i - U_i\cos\alpha_i - \dfrac{\bar{c}l_i\sin\alpha_i}{F}}{\cos\alpha_i + \dfrac{\tan\bar{\phi}\sin\alpha_i}{F}}$$

This will be equation number 3. Now, we know that the expression for F can be written as.

$$F = \frac{\bar{c}L + \tan\bar{\phi}\sum\overline{N}_\iota}{\sum W_i\sin\alpha_i} = \frac{\sum\{\bar{c}L + \tan\bar{\phi}\overline{N}_I\}}{\sum W_i\sin\alpha_i}$$

See here, if this is what is the slope which is undergoing this kind of failure? And I took the ith slice. So, you have such type of slices.

So, I workout the force equilibrium equation for a particular slice which is ith slice, let us say and if I integrate it all over this length, then I would be able to get the total factor of safety and that is what we have done in this expression. This is going to be written as summation C bar l i plus tan phi bar N i bar. So, this whole term will come inside this summation. l i is this dimension of the slice which is ith slice divided by summation W i sin alpha i.

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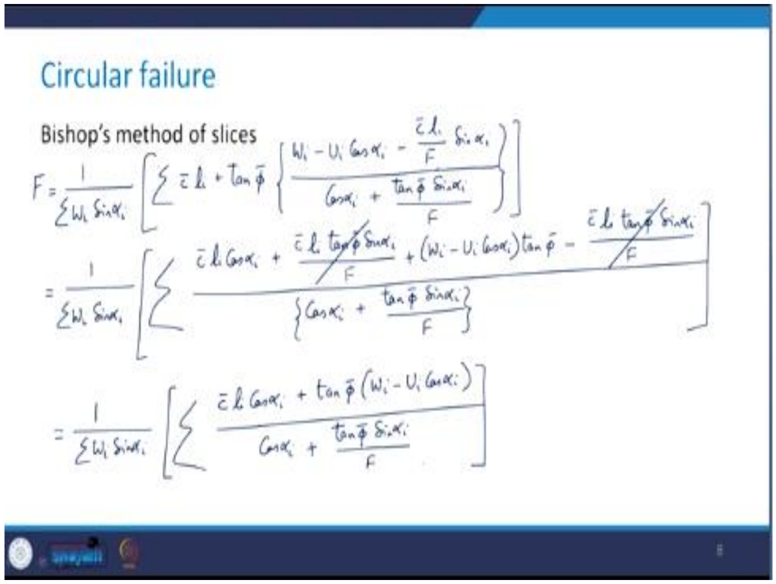

Now, if we just try to substitute this expression of N i bar from equation 3 to this equation 4, what we are going to get is,

$$F = \frac{1}{\sum W_i \sin\alpha_i}\left[\sum \bar{c}L + \tan\bar{\phi}\left\{\frac{W_i - U_i\cos\alpha_i - \dfrac{\bar{c}l_i \sin\alpha_i}{F}}{\cos\alpha_i + \dfrac{\tan\bar{\phi}\sin\alpha_i}{F}}\right\}\right]$$

Now, try to simplify this and see what we get

$$F = \frac{1}{\sum W_i \sin\alpha_i}\left[\frac{\bar{c}l_i\cos\alpha_i + \tan\bar{\phi}(W_i - U_i\cos\alpha_i)}{\cos\alpha_i + \dfrac{\tan\bar{\phi}\sin\alpha_i}{F}}\right]$$

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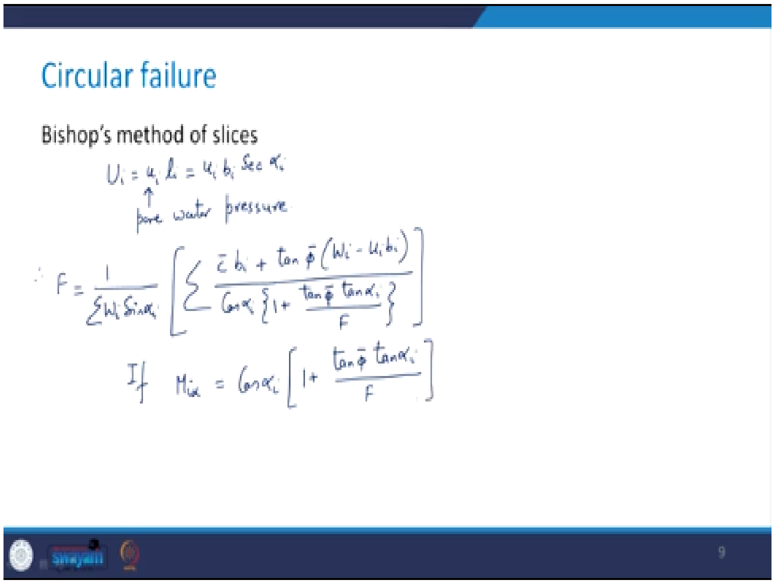

Now, this capital U_i which is the force because of the pore water pressure, this can also be written in terms of the pore water pressure as

$$U_i = u_i l_i = u_i b_i \sec \alpha_i$$

b_i is the width of the ith slice. So, this u_i is the pore water pressure. So, F will become.

This is what we are going to get

$$F = \frac{1}{\sum W_i \sin \alpha_i}\left[\frac{\bar{c} b_i + \tan\bar{\phi}(W_i - U_i b_i)}{\cos\alpha_i\left\{1 + \frac{\tan\bar{\phi}\tan\alpha_i}{F}\right\}}\right]$$

Now, I make an assumption that if M_i alpha is represented by

$$M_i = \cos\alpha_i\left\{1 + \frac{\tan\bar{\phi}\tan\alpha_i}{F}\right\}$$

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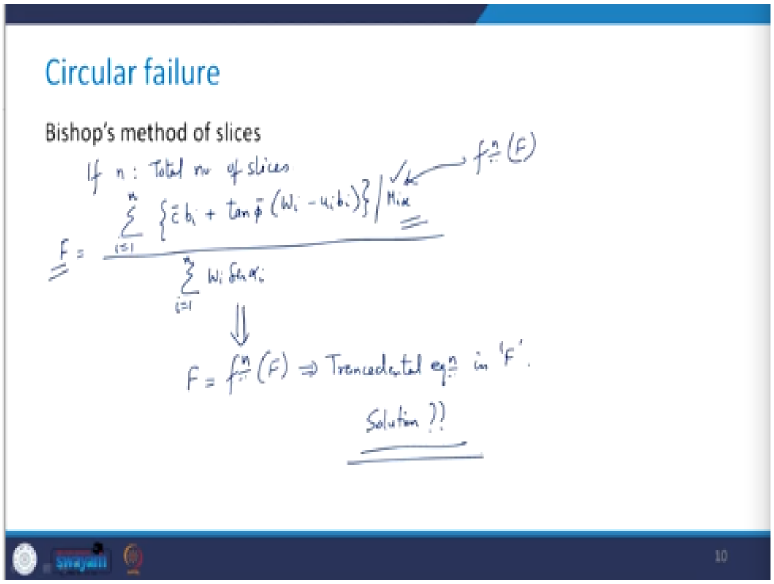

Then if n be the total number of slices, then this F will become equal to summation.

$$F = \frac{\sum_{i=1}^{n} \{\bar{c}b_i + \tan\bar{\phi}(W_i - U_i b_i)\} \Big/ M_{i\alpha}}{\sum_{i=1}^{n} W_i \sin\alpha_i}$$

Now, you see that this is a function of factor of safety F. So, this equation is kind of F is a function of F itself, because on the left hand side, you have F and on the right hand side, also you have F in this term which is $M_{i\alpha}$. So, this is your transcendental equation in F.

The question is its solution. How to obtain the solution in this case? So, what is done here? Since, this is a transcendental equation in F, what is done that you assume some value of F and carry out this analysis and obtain the value of F if the assumed value of F becomes equal to the derived value or the obtained value from this expression, that is what is going to be the final value of the factor of safety.

If the assumed value and what you get from the expression if these 2 values they do not match with each other, then in that case, you have to change the value of the assumed factor of safety and carry out the whole analysis all over again and get the new value of the factor of safety. So, in this particular manner, by trial and error, you will be able to get the factor of safety for the slope which is having the circular failure mode.

So, this was a simple expression that we obtained using the Bishop's method of slices. So, this was all about the circular failure mode in case of the rock slopes. Now, we are left with a one more failure mode, which is the toppling failure that we will take up in the next class. Thank you very much.