# Rock Engineering

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## Lecture - 49
## Rock Slope Stability - Plane Failure

Hello everyone, in the previous class we discussed about the rock slope stability. And I derived a simple expression for the analysis of slope, if it is undergoing deep plane failure. We did not consider any tension crack in that analysis. So, what happens in plane failure? You have a discontinuity plane and a block is considered to slide along that single plane. Now, when we remove the material or during the process of the excavation, when the material is removed, there is a reduction in the lateral constraint.

And because of that, some tension crack may occur in the rock slope. It may occur on the slope face or above the crest of the slope. How to analyze such type of situation in case of the plane failure? This is what we are going to learn today. Excavated slopes, because of the removal of the material, displacement takes place in the horizontal direction. And this results into the development of tension on a vertical plane.

In case if there is insufficient tensile strength, this causes the occurrence of tension cracks. The moment this formation of tension crack takes place, slope may not be stable anymore.

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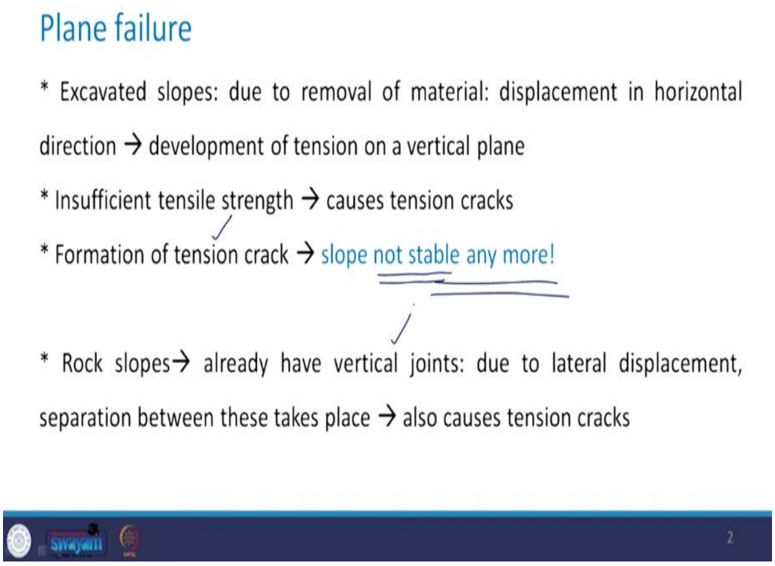

So, that is a very big concern for us that the moment tension crack is formed slope is not stable. Another cause towards the development of tension crack can be that rock slopes already have the vertical joints. Now, due to the lateral displacement, the separation between these vertical joints take place. And this also causes the tension cracks. Tension crack develops up to about 75 percent of the height of the slope. As I mentioned, that these may develop on slope face or upper surface of slope.

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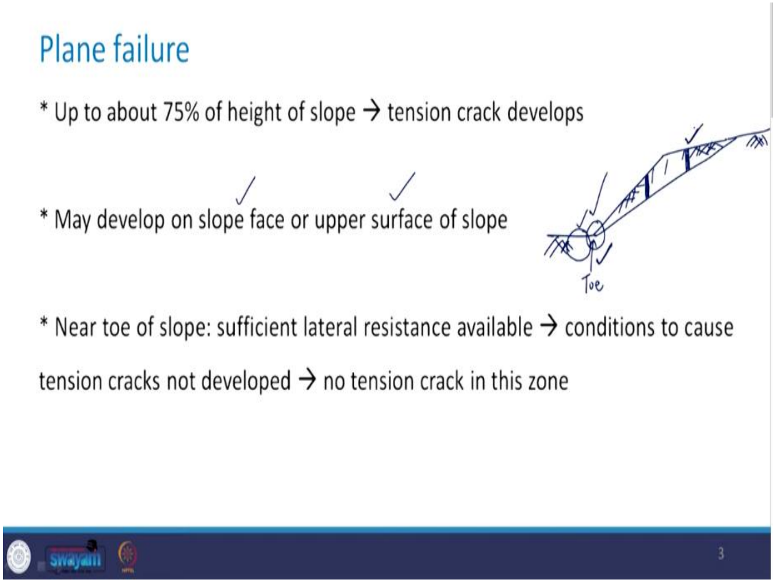


So, let us say that this is what is say the slope and say this is the plane of discontinuity along which the sliding of this block may occur. Now, this tension crack can happen here at the slope face or it can happen in the upper portion of the slope like this. So, depending upon where this tension crack is occurring, we need to carry out the analysis. What happens near the toe of the slope? That means, this point this is what is called as toe.

So, what happens at this point? Here, sufficient lateral strain here the sufficient lateral restraint is available. And therefore, the conditions which are required to cause the tension cracks, they do not develop. And therefore, no tension crack occurs in this zone. So, that tension crack can occur either on the slope face or on the upper surface of the slope.

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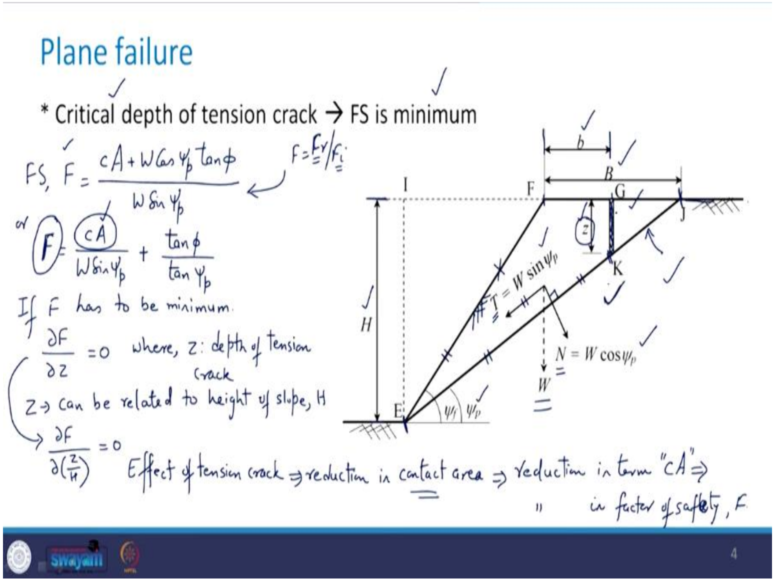


The concern is we need to find out the minimum value of the factor of safety. Because if we know that minimum value of factor of safety and if that minimum value is coming out to be more than 1, then we can say that the slope is stable. Now, for a critical depth of the tension crack, this factor of safety will be minimum. So, our aim in this analysis is going to be to find out the expression for this minimum value of the factor of safety.

Now, take a look at this figure and try to understand the geometry. This is the slope that is EFJ. This portion is the slope. This is the discontinuity plane along which the sliding will take place and the orientation of these 2 planes have been given by angle psi p ($\psi_p$) for the discontinuity plane and angle psi f ($\psi_f$) for the slope face. Height of the slope that is from this level to this level is given by $H$.

You can see here the point G and K and there is the occurrence of the tension crack. So, in this case I am considering that the tension crack is taking place in the upper surface of the slope. That is here, not on the slope face. So, this is the tension crack. And say the depth of the tension crack is z. The width of this zone that is FJ is given by the dimension $B$. However, the distance of the tension crack from this crest of the slope is given by small letter $b$.

The weight of this block which is sliding along this discontinuity plane is represented by $W$ whose component in the perpendicular and the parallel direction can be written as $N = W\cos\psi_p$ and $T = W\sin\psi_p$T equal to $W\sin\psi_i$ respectively. Now, let us try to see, how should we go about the analysis? So, from the previous class, we take the expression for the factor of safety.

So, we saw that this factor of safety which we represented by

$$F = \frac{cA + W\cos\psi_p \tan\phi}{W\sin\psi_p}$$

This is nothing but the simple expression that we had in the earlier class that is

$$F = \frac{F_r}{F_i}$$

So, *Fr* was the resisting force and *Fi* was the force which was inducing the sliding. So, this expression can be written.

Or, we can write this F as

$$F = \frac{cA}{W\sin\psi_p} + \frac{\tan\phi}{\tan\psi_p}$$

Now, if this *F* has to be minimum. Why it has to be minimum? Because, in that case only we are going to get the critical depth of the tension crack, so, in that case, this quantity $\partial F/\partial z = 0$ , where this *z* is what it is the depth of the tension crack. And you can see what *z* here in this figure as well.

Now, this *z* can be related to the height of the slope. So, it can be related to height of the slope. What is the height of slope? Which is *H*. So, I can write it this expression as $\partial F/\partial \frac{z}{H} = 0$. Now, what happens because of the presence of this tension crack? So, the effect of tension crack will be the reduction in the contact area. Recall our discussion of the previous class, when this is not there, our contact area was from here to here.

But since this tension crack occurs here, so, now, the contact area will get reduced to only this that is EK. So, because of the reduction in this contact area, which we are representing here as A, what will happen to the term c? There is going to be the reduction in the term c into A. And this would result in the reduction in the factor of safety which we are denoting by F. You see here if this factor reduces, it will have direct effect in reducing this F.

So, it is important for us to take care of this reduced area and use this in the analysis when the tension crack is present.

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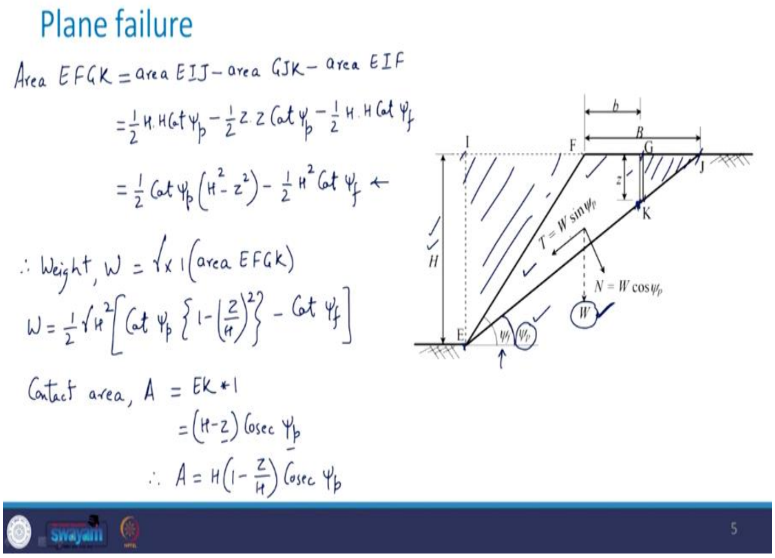


Now, before we go ahead, we would need to find out the weight *W*. So, what is this weight *W* is going to be is going to be the unit weight multiplied by the volume of the wedge that is sliding. And there is an assumption which is involved that we are taking a unit direction in the plane perpendicular to the plane of the screen. So, in order to find out the volume, I need to first find out this area which is EFGH. So, we will have it $\frac{1}{2}H.H\cot\psi_f$. Similarly, GJK, this height of this triangle is H.

And volume is going to be area EFGH into 1 because I am considering the unit dimension in the direction perpendicular to the plane of the screen. So, basically in order to obtain this weight W, I need to first find out the area EFGH K. So, let us try to find this out. That is area EFGK. This is going to be, see, I will write first as area EIJ. That means this EIJ. So, this complete area and then I will subtract area GJK here, so, area GJK.

So, from this complete area, I am removing this area. So, but I am still left with this total area. So, I have to do away with this area as well in order to get the area EFGK. So, I will have another term minus area EIF. Now, use the simple trigonometry and then try to obtain this area EIJ. And see, what we get? See this expression is *H* and this angle is psi ($\psi_i$).

So, I will have $\frac{1}{2}H.H\cot\psi_p$ then minus area EIF. So, this is EIF. And here for this triangle the angle psi f ($\psi_f$) will come into picture. So, we will get $\frac{1}{2}H.H\cot\psi_f$.. So, what we are going to get is

$$\frac{1}{2}\cot\psi_p(H^2 - z^2) - \frac{1}{2}H^2\cot\psi_f$$

Now, weight W will be the unit weight of the rock mass which is there in this sliding block into 1.

Because it is a unit in the direction perpendicular to the plane of screen and this should be multiplied by area EFGK. So, what we will get here as $W = \frac{1}{2}\gamma H^2$. So, I will take $H^2$ common from this expression. And this is what that we are going to get is that cot ($\psi_p$). And I will have here

$$W = \frac{1}{2}\gamma H^2\left[\cot\psi_p\left\{1 - \left(\frac{z}{H}\right)^2\right\} - \cot\psi_f\right]$$

And the contact area will be what, which is represented as *A*.

This is going to be the length EK into 1 because it is again the unit in the direction perpendicular to the plane of the screen. So, this is what you are going to get as $(H - z)\csc\psi_p$, simple trigonometry, EK. So, what we are doing is we are, EK, I can write as EK equal to EJ minus JK. And you know this angle psi p ($\psi_p$). You know *H*. You know *z*. And in terms of *H*, *z* and psi p ($\psi_p$), this can be written.

So, I can write it again in this form which has *z/H* as one of the term $H\left(1 - \frac{z}{H}\right)\text{cosec}(\psi_p)$. So, this is how we can find out the weight of the block here and the contact area.

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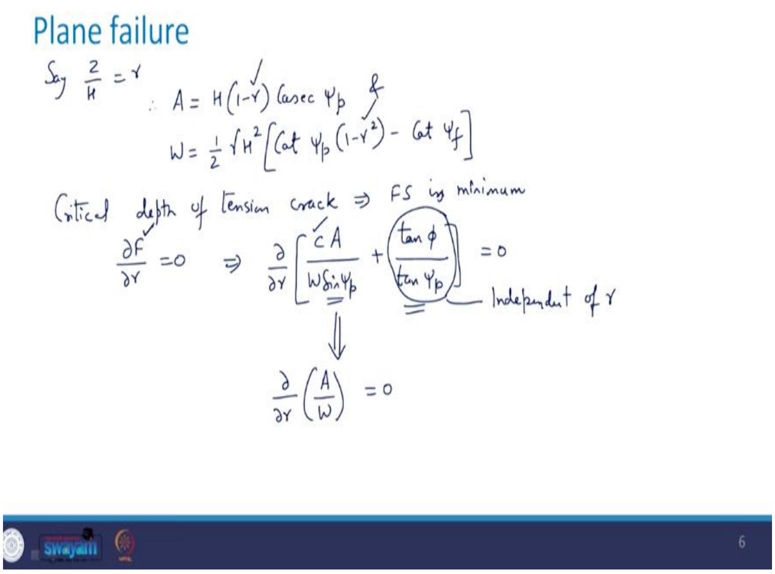

Now, I assume that $\frac{z}{H} = r$. So, therefore, the contact area A is written as$H(1-r)\operatorname{cosec}\psi_p$. And weight W can be written as $W = {}^1/_2\, \gamma H^2[\cot\psi_p(1-r^2) - \cot\psi_f]$. Now, critical depth of the tension crack, we will get when this factor of safety is minimum. So, in order to make the factor of safety as minimum, we can write that $\frac{\partial F}{\partial r} = 0$.

Now, refer back to the expression for F and

$$F = \frac{cA}{W\sin\psi_p} + \frac{\tan\phi}{\tan\psi_p}$$

This is equal to 0. Now, you see this term is independent of r. So, it is differentiation is going to be 0. And c and psi p, they are also constant. So, ultimately this equation will be reduced to $\frac{\partial}{\partial r}\left(\frac{A}{W}\right) = 0$. Now, this A and W, both are the function of (r) as you can see here.

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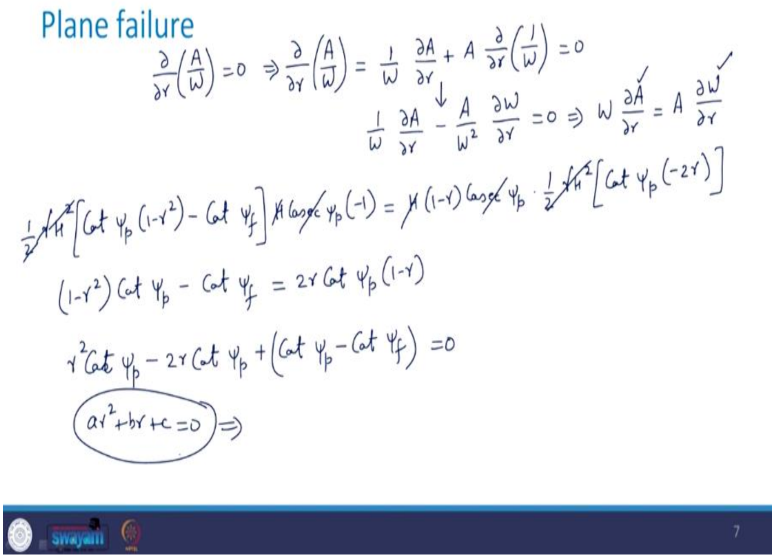


So, we need to do the differentiation accordingly. So, this is how that we need to do is, so, what we have is $\frac{\partial}{\partial r}\left(\frac{A}{W}\right) = 0$. And what we can get from here is that is $\frac{\partial}{\partial r}\left(\frac{A}{W}\right)$ can be written

$$\frac{\partial}{\partial r}\left(\frac{A}{W}\right) = \frac{1}{W}\frac{\partial A}{\partial r} + A\frac{\partial}{\partial r}\left(\frac{1}{W}\right) = 0$$

$$\frac{1}{W}\frac{\partial A}{\partial r} - \frac{A}{W^2}\frac{\partial W}{\partial r} = 0 \Rightarrow W\frac{\partial A}{\partial r} = A\frac{\partial W}{\partial r}$$

Therefore

$$\frac{1}{2}\gamma H^2[\cot\psi_p\{1-r^2\} - \cot\psi_f]H\operatorname{cosec}(\psi_p)(-1)$$

$$= H(1-r)\operatorname{cosec}(\psi_p)\frac{1}{2}\gamma H^2[\cot\psi_p\{-2r\}]$$

This is what the expression that we will get. So, on cancelling similar terms, we will be left with the expression after further simplification like

$$r^2 \cot\psi_p - 2r\cot\psi_p + \left(\cot\psi_p - \cot\psi_f\right) = 0$$

Now, you see this is a quadratic equation in $r$. See, it is like this. Like $ar^2 + br + c = 0$ of this form, this equation is. So, how can we get the roots of this equation?

You know simply, $r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

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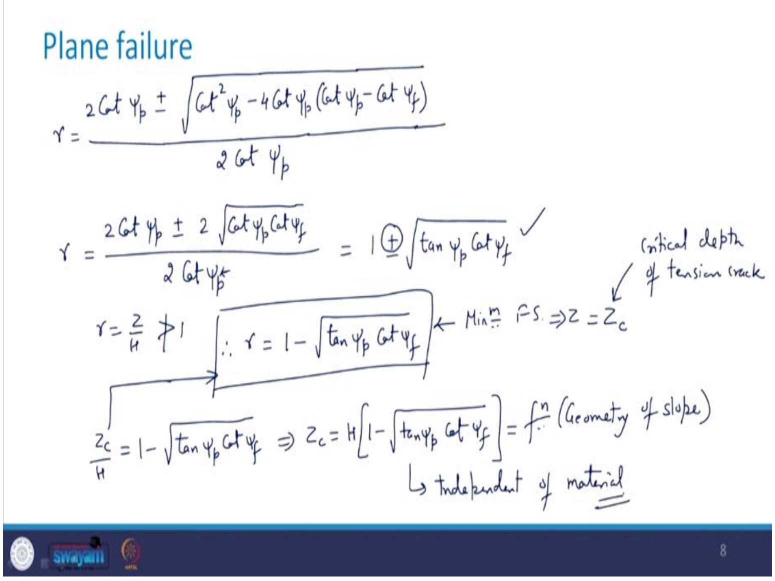


So, we apply that. And this is how that we are going to get the expression for $r$. This is what that we are going to get. Now, if we try to simplify this, what we will get is this expression

$$r = 1 \pm \sqrt{\tan\psi_p \cot\psi_f}$$

See, what will happen? This 2 cot $\psi_p$ 2 cot $\psi_p$ will get cancel out and plus minus this 2 and 2 will go. And when this goes inside the square root sign it will become square. So, 1 cot $\psi_p$ will get cancel out. So, this is what that you will be left with. Now, this r is *z/H* and it cannot be more than 1. So, therefore, this positive sign here is not physically possible.

So, r is going to be equal to $r = 1 - \sqrt{\tan\psi_p \cot\psi_f}$ . Now, since this value is going to give me minimum factor of safety. So, z would be equal to z critical ($z_c$). So, I am going to write here the same expression as $z_c/H$ will be $1 - \sqrt{\tan\psi_p \cot\psi_f}$

Where, what is this $z_c$? This is the critical depth of the tension crack. Or, from here, this z c can be written as $H\left(1-\sqrt{\tan\psi_p \cot\psi_f}\right)$.

So, take a note of it that this is a function of geometry of the slope. And it is independent of the material properties. Keep this in mind. Now, the next step is going to be to know that where this critical tension crack will occur.

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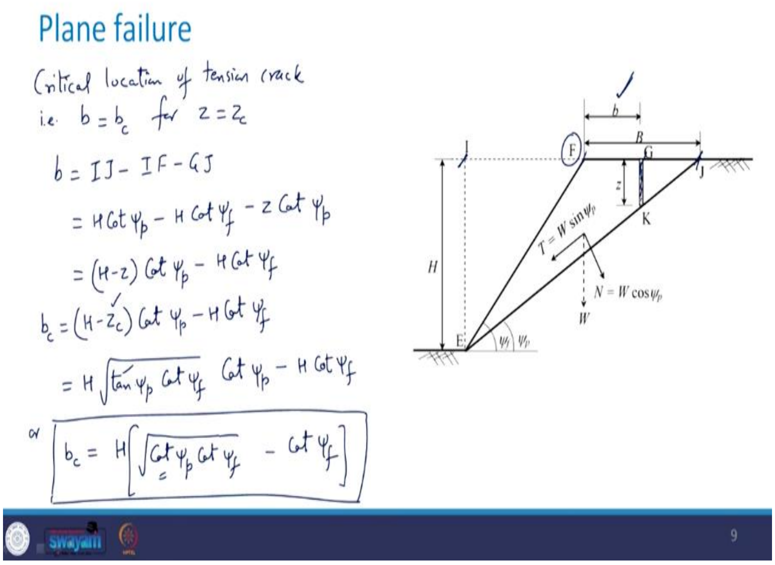


That means critical location of the tension crack. So, you see here the tension crack is occurring at a distance of $b$ from this point F. So, if we want to find out the critical location of the tension crack that is our $b$ will become equal to $b_c$ for $z$ equal to $z_c$. So, now, can we define this $B$? So, $B$ is going to be equal to IJ minus this distance IF minus GJ. You see, this B is going to be this distance minus this distance and minus IF this distance.

$$b = IJ - IF - GJ$$

$$b = H\cot\psi_p - H\cot\psi_p - z\cot\psi_p$$

$$= (H - z)\cot\psi_p - H\cot\psi_f$$

So, $b_c$ is going to be

$$b_c = (H - z_c)\cot\psi_p - H\cot\psi_f$$

Now, we know, what is the expression for $z_c$? Just substituted there, so, what you will get $b_c$ as

$$b_c = H\left[\sqrt{\cot\psi_p \cot\psi_f} - \cot\psi_f\right]$$

See, if this cot goes inside this bracket, so, it will get cancel out with this $\tan\psi_p$. And you will left with only 1 cot term here. So, this is how you can find out the critical location of the tension crack. Now, this was all about when we have the tension crack here in this portion. That is upper portion of the slope.

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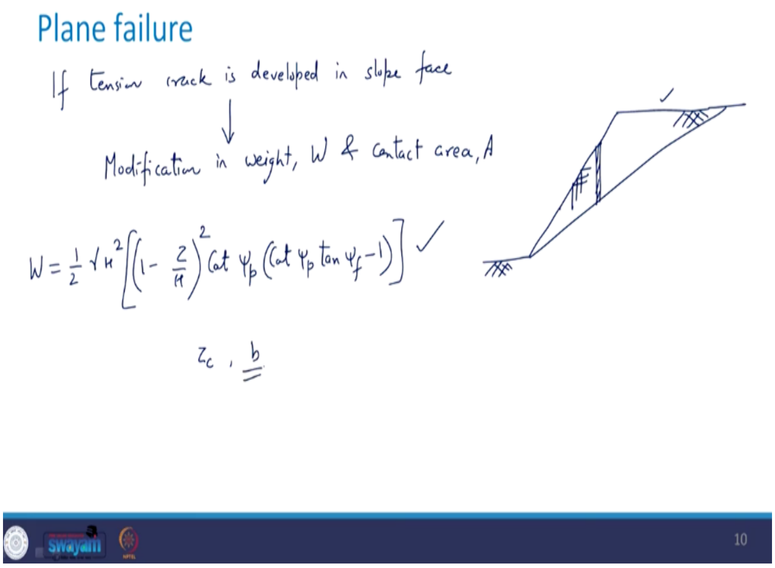


In case if the tension crack is developed in the slope face that means, let us say if this is what is the situation this is your slope face and say some discontinuity is there. And instead of having it here, now, the tension crack is appearing let us say here like this. That is on the slope face. Then, what will happen? Because of this, the procedure of derivation of these expression is going to be same except for the fact that there is going to be the modification in this weight W and the expression for the contact area.

The procedure is going to be same. Now, this weight is going to be you can derive this I am just giving you the expression $W = \frac{1}{2}\gamma H^2\left[\left(1-\left(\frac{z}{H}\right)^2\right)\cot\psi_p\left(\cot\psi_p\tan\psi_f - 1\right)\right]$. So, this is how the weight W can be derived. And accordingly you can find out that what the contact area A and then use those expression as we did in the earlier case.

And obtain the critical depth of the tension crack and its location as well. Now, let us say that that tension crack which is there in the slope.

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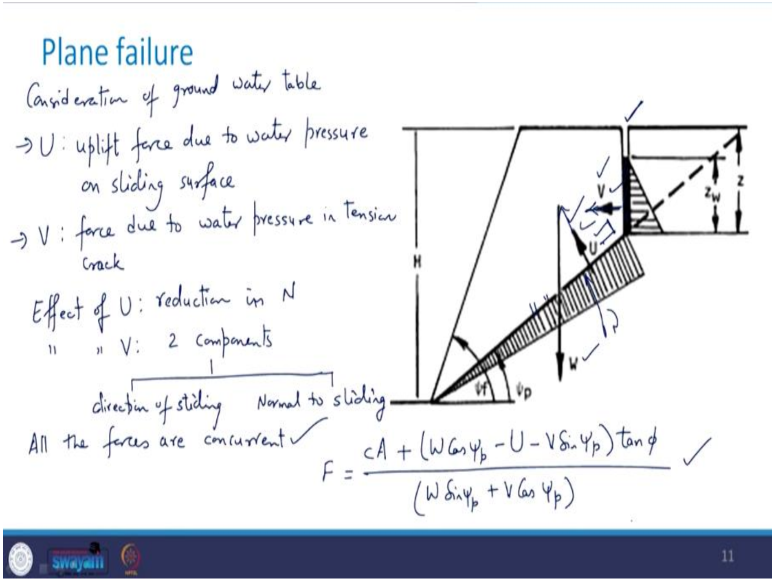


Now, let us say the tension crack that is there in the upper portion of the slope like this as shown in this figure. And there is a presence of the water in the tension crack. Then, how are we going to consider this in the analysis? So, we have it here as the consideration of the ground water table. So, in this case you are going to have the pressure because of the presence of the water and the ground water table in the form of this U and V.

So, what we have is the capital U which is the uplift force due to water pressure on the sliding surface. And then another one we have is V which is the force due to water pressure in the tension crack. And you can see the direction how that these 2 will be acting. So, what is going to be the effect of U? See on this plane, a normal force N was acting, which was some component of W earlier when the water table is not present.

Now, see this U is acting in the other direction, but still it is perpendicular to this plane of discontinuity. So, what will be the effect of this U is that it will result in the reduction in N. And the effect of V is going to be in 2 direction because if you see that it is acting in the horizontal direction. So, it will have its component parallel and perpendicular to this discontinuity plane. So, this V will have 2 components.

One component is going to be in the direction of sliding and another one will be normal to sliding. Now, we need to keep in mind that all these forces are concurrent. So, this expression for factor of safety can be written as

$$F = \frac{cA + \left(W \cos\psi_p - U - V \sin\psi_p\right)\tan\phi}{W \sin\psi_p + V \cos\psi_p}$$

Now, you can substitute the expression for $W$, $A$ etc. And you know how to determine these. And therefore, you should be in a position to obtain the factor of safety under the condition when there is a presence of the ground water table or the water is present in the tension crack. So, this is what that I wanted to discuss with you as far as the plane failure mode of the slope failure is concerned. So, we discussed about few cases of the analysis.

The first case we started with the simplest one without any tension crack simple case where there is one discontinuity plane and the rock block is sliding along that plane. The second case we took with the help of the tension crack, which can be there either on the upper portion of the slope or at the slope face. Then, we saw that how due to the presence of ground water table the factor of safety will be altered. And, how it can be analyzed?

So, this was all about the analysis as far as plane failure is concerned. In the next class, we will take up the next failure mode, which is the wedge failure. Thank you very much.