

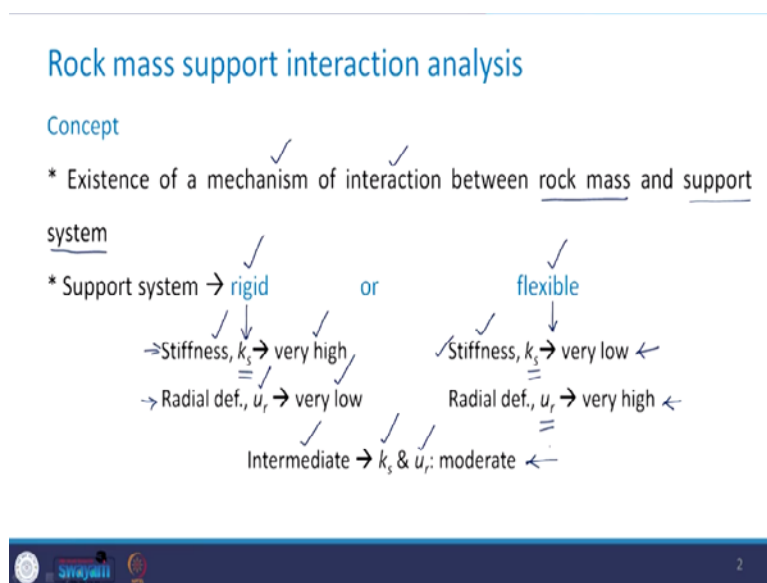
Rock Engineering
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Lecture - 46
Rock Mass Support Interaction Analysis - 01

Hello everyone, in the previous class, we discussed about the structurally controlled failure with reference to the sidewall of the tunnel, where we saw that if a wedge is formed in the sidewall of the tunnel it will slide. And, how these stereographic projections can be used in order to identify the size of these wedges? So, today we will start with a new topic with respect to the tunneling which is the rock mass support interaction analysis.

So, today we will learn about the concept of this interaction. And then I will be introducing you the ground response curve and the support reaction curve with the help of some simple cases. Let us start with the concept of this rock mass tunnel support interaction analysis.

(Refer Slide Time: 01:37)



It includes that there is an existence of a mechanism of interaction between rock mass and the support system. Now, when I say rock mass what does that mean that in that medium the excavation has been made. When I talk about the support system, the moment we excavate, we need to install the support system depending upon the quality of the rock mass or the requirement of the support system over there.

It can be a nominal support system or you may have to go for a heavy support system depending upon that what is the quality of the rock mass surrounding the excavation. So, accordingly this support system can be either rigid or it can be flexible. Now, in case of the rigid support system, its stiffness k_s is going to be extremely high and the radial deformation which will be allowed it will be very low because the support is very stiff.

What happens in case of the flexible support system? Its stiffness k_s is going to be extremely low and the radial deformation will be very high because the stiffness is very low. Somewhere in between these 2 extreme cases, we will have an intermediate condition for which the stiffness of the support system k_s and the radial deformation is neither going to be too low nor going to be too high, but it will be somewhat moderate.

(Refer Slide Time: 03:54)

Rock mass support interaction analysis

Concept

- * **Question:** magnitude of permissible deformation???
- * For the system to be in equilibrium \rightarrow combination of pressure & deformation
- Load imposed on support system = f^n (deformability characteristics of rock mass)
- &
- Deformability of rock mass = f^n (stiffness of support system)
- Interaction = f^n (stand-up time)



The question comes, what should be the magnitude of this permissible deformation? That is u_r which will be the radial deformation. Now, for the system to be in equilibrium there is going to be a combination of pressure and the deformation. Accordingly all these parameters can be decided. And therefore, the load which is imposed on the support system becomes the function of deformability characteristic of the rock mass.

And this deformability characteristic of the rock mass becomes a function of stiffness of the support system. And therefore, the interaction phenomena becomes the function of the stand-up time. You know that what do we mean by this stand-up time?

(Refer Slide Time: 05:02)

Rock mass support interaction analysis

Stress-strain response / constitutive response of rock mass ✓

- * Linear elastic: solid competent rock ←
- * Non-linear elastic: competent rock but with lesser value of modulus ←
- * Non-linear in-elastic: incompetent rock ←
- * Highly non-linear in-elastic: jointed rock mass ←
- * Time dependent response: jointed rock mass ←

Accordingly behavior → elastic / elasto-plastic / visco-elastic / visco-plastic ←



Coming to first the stress strain response or the constitutive response of rock mass, so, we had a dedicated chapter in this course on this topic. We saw that the constitutive response can be linear elastic, and that would be in case of the solid competent rock. It can be nonlinear elastic, in case of the competent rock, but with lesser value of the modulus. It can be nonlinear in-elastic in case of incompetent rock.

Highly nonlinear in-elastic in case of the jointed rock mass, and then you can have the time dependent response also in case of the jointed rock mass. And accordingly, the behavior of the rock mass would be either elastic or elasto-plastic or visco-elastic or it can be visco-plastic.

(Refer Slide Time: 06:21)

Rock mass support interaction analysis

Type of support system

- * Rock bolts → tensile behavior: elastic response ←
- * Shotcrete lining → elastic-plastic response ←
- * Concrete lining → elastic-brittle response ←
- * Steel sets → elastic-plastic ductile response ←
- * Combined support system → complex behavior ←



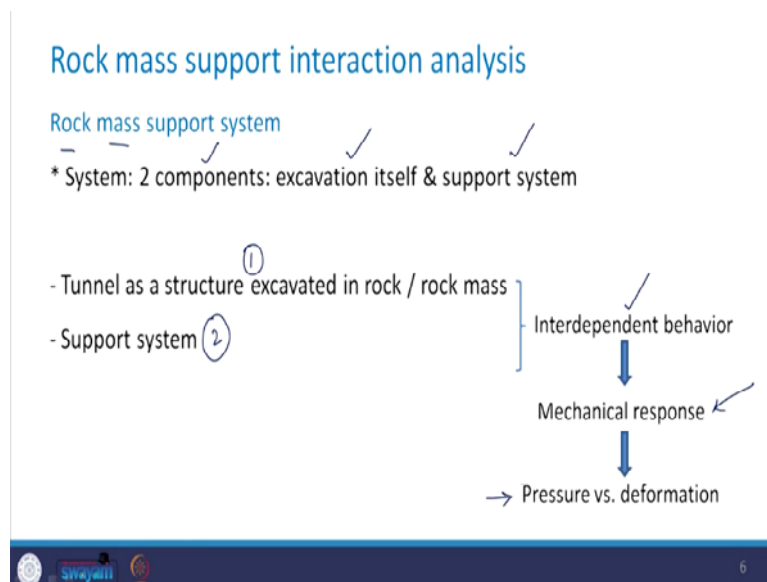
Coming to the types of the support system, various types of support systems can be installed depending upon the requirement. Some of these have been listed here. The first one is rock

bolts, it has the tensile behavior and its response is elastic. In case if you have the shotcrete lining as a support system, its behavior is going to show the elastic-plastic response. In case of the concrete lining, this will be elastic-brittle response.

While in case of steel sets, it is going to be elastic-plastic ductile response. If the requirement is such that that we have to install more than one type of the support system in the same tunnel that is called as the combined support system. And the behavior is quite complex in this case. We really cannot say whether it is going to be elastic response or elasto-plastic or elasto-plastic ductile or any of these which have been listed here.

It is going to be the complex behavior. In case of the rock mass support system, we have 2 components.

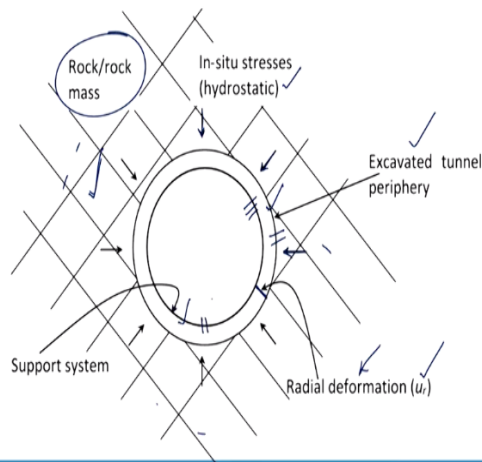
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One is the excavation itself and the other one is the support system. Tunnel as a structure is excavated in rock or rock mass. And then whatever is the requirement accordingly the support system is provided. So, these are the 2 components of this rock mass support system. Their behavior is interdependent, especially in terms of their mechanical response with respect to the pressure versus deformation behavior.

(Refer Slide Time: 08:50)

Rock mass support interaction analysis



Take a look at this figure. We have rock or rock mass here. Then, this was the excavated tunnel periphery. Then, the support system which has been installed is this one. And there is the margin of this radial deformation which is denoted by u_r . And here a typical situation of hydrostatic in-situ stress has been shown. That is from the all the direction you have the same stress. So, that is hydrostatic in-situ stresses. So, please understand these terminology.

This is the excavated tunnel periphery. You have the support system. Then, here is the radial deformation and the rock or rock mass. So, basically, this rock support system includes 2 components, one is this rock or rock mass and the second component is this support system.

(Refer Slide Time: 10:20)

Rock mass support interaction analysis

Radial deformation, u_r

- * Elastic: only in a good quality rock/rock mass
- * Occurs: almost instantaneously for all practical purposes
- * Can also be elasto-plastic
- * Can be time dependent (elastic), if magnitude of in-situ stress < yield stress of rock/rock mass (incompetent rocks, rock mass with relatively higher value of RMR or Q)



This radial deformation will be elastic only in a good quality rock or rock mass. It occurs almost instantaneously for all practical purposes. Depending upon the behavior of the rock mass and

the support system, it can also be elasto-plastic. It can be elastic, but time dependent if the magnitude of in-situ stress is less than the yield stress of rock or rock mass. And this type of situation can be seen in incompetent rocks and rock masses with relatively higher value of RMR or Q.

Do you remember the classification systems? There we learned how to determine this rock mass rating and Q value. Higher of or higher the value of these 2 you will have the better quality of the rock mass. Now, this radial deformation can also be time dependent, but instead of elastic now, it is plastic.

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Rock mass support interaction analysis

Radial deformation, u_r

* Can also be time dependent (plastic), if magnitude of in-situ stress > yield stress of incompetent rock or poor quality rock mass

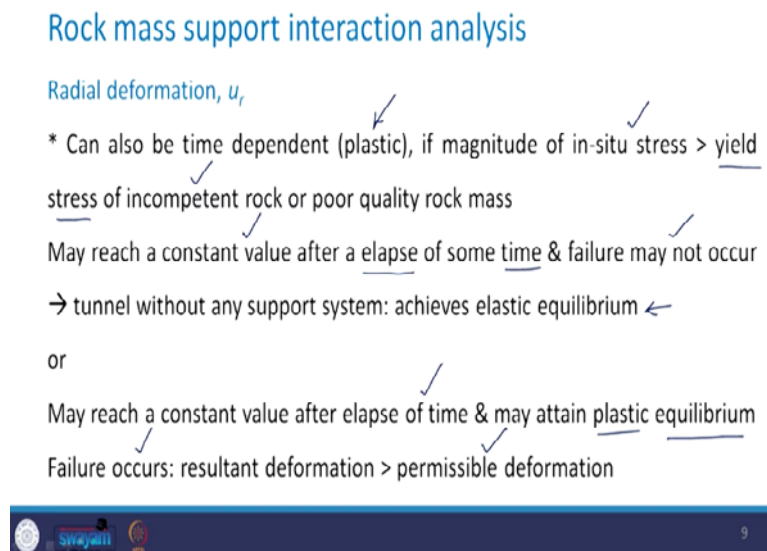
May reach a constant value after an elapse of some time & failure may not occur

→ tunnel without any support system: achieves elastic equilibrium ←

or

May reach a constant value after elapse of time & may attain plastic equilibrium

Failure occurs: resultant deformation > permissible deformation



If the magnitude of in-situ stress becomes more than the yield stress of the incompetent rock or poor quality rock mass, so, as compared to the previous case, here, the value of RMR or Q is going to be low. This value of the radial deformation may reach a constant value after an elapse of sometime and failure may not occur. This is again it is a plastic value, but still failure may not occur.

That means, what that tunnel without any support system achieves the equilibrium. Or, it may reach a constant value after the lapse of time and may attain the plastic equilibrium. In this case, the failure will occur and the resultant deformation is going to be more than the permissible deformation.

(Refer Slide Time: 13:10)

Rock mass support interaction analysis

- * To prevent failure of tunnel: installation of support system after certain amount of time: this time correlated with stand-up time of rock/rock mass
 To be known beforehand: $f^i(\text{RMR or } Q)$
- * In case of low / very low stand-up time: support system be installed within the specified stand-up time



So, to prevent the failure of the tunnel, installation of the support system after certain amount of time should be done. This time is correlated with stand-up time of rock or rock masses, which is to be known beforehand, because it is a function of RMR and Q. And if you recall our discussion, during this, when we were discussing about this systems RMR or Q, we saw that based upon the value of RMR or Q, we can determine the range of these stand-up time.

There were various tables which were available. Now, in case of low or very low values of the stand-up time, the support system need to be installed within the specified stand-up time.

(Refer Slide Time: 14:20)

Rock mass support interaction analysis

- * Purpose of support system: to arrest continuing deformation of rock/rock mass \rightarrow directly a function of stiffness of support system
- u_r : unrestrained before installation of support system
- u_r : restrained due to stiffness of support system (k_s)
- * If u_r to be restrained completely, i.e., no further deformation after installation of support system $\rightarrow k_s \text{ required} = f^i(\text{magnitude of in-situ stress, size of excavation, quality of rock/rock mass, time of installation \& total time elapsed})$



Now, what is the purpose of the support system is to arrest the continuing deformation of rock or rock mass. And therefore, this is directly a function of the stiffness of the support system. So, radial deformation is unrestrained before the installation of the support system. Let us say

that you have made this excavation. Before you install the support system, this free boundary is free to experience the radial deformation like this.

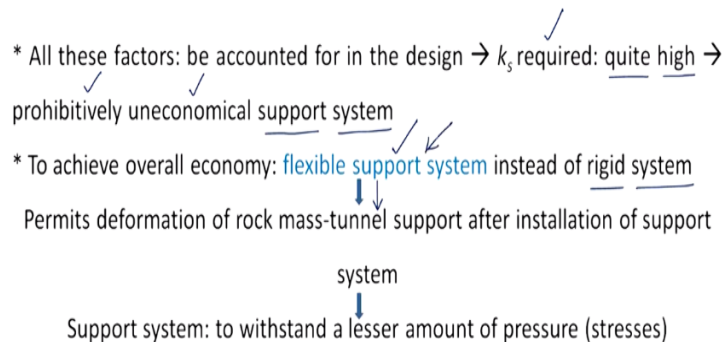
But the moment you provide a support system, what will happen? Now, this free boundary which was earlier deforming freely now will be restricted with the presence of this support system. To what extent that it will be restrained, it will depend upon the stiffness of the support system. Let us say that if you provide very stiff support system, what will happen? The deformation will be completely arrested.

So, this u_r which is this radial deformation gets restrained due to the stiffness of the support system which is being represented by k_s . So, if this radial deformation is to be restrained completely, what does that mean? That is, there is no further deformation after the installation of the support system. Obviously, the stiffness of the support system k_s which is required will be quite high.

And therefore, the required k_s becomes the function of magnitude of in-situ stresses, size of the excavation, quality of the rock or rock mass, time of the support installation and the total time that has been elapsed. So, one need to be careful about it.

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Rock mass support interaction analysis

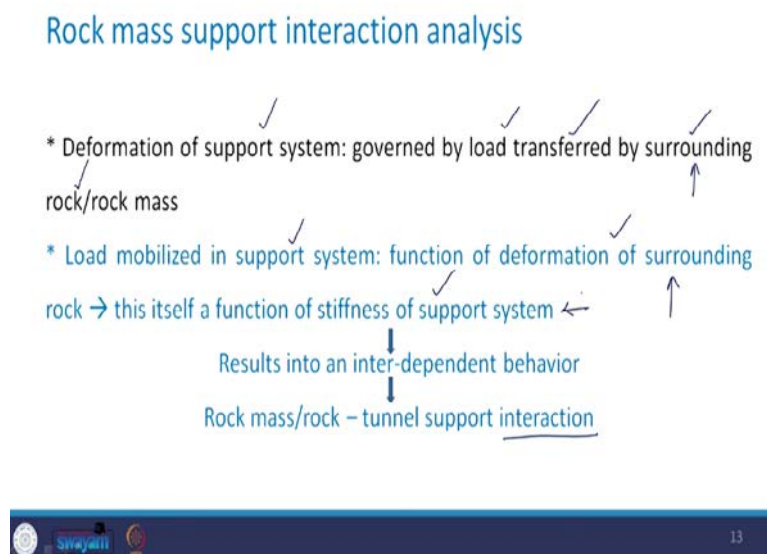


All these factors should be accounted for in the design. Now, as I mentioned that if this radial deformation is to be completely arrested, then the required stiffness of the support system is going to be extremely high. And this would result in to prohibitively uneconomical support

system. Now, in order to achieve the overall economy, one needs to go for the flexible support system instead of the rigid system.

What happens when you go for the flexible support system? This allows the deformation of rock mass and tunnel support even after the installation of the support system. So, what will happen? If the deformation is allowed, the support system has to withstand a lesser amount of pressure or the stresses. So, therefore, in order to have the more economical situation one can go for this flexible support system.

(Refer Slide Time: 18:24)



Therefore, the deformation of the support system is governed by the load which is transferred by the surrounding rock or rock mass. See, we are going to have a very interesting statement just now. One aspect is deformation of the support system, which is governed by the load transferred by the surrounding rock or rock mass. The second aspect is the load which is mobilized in the support system.

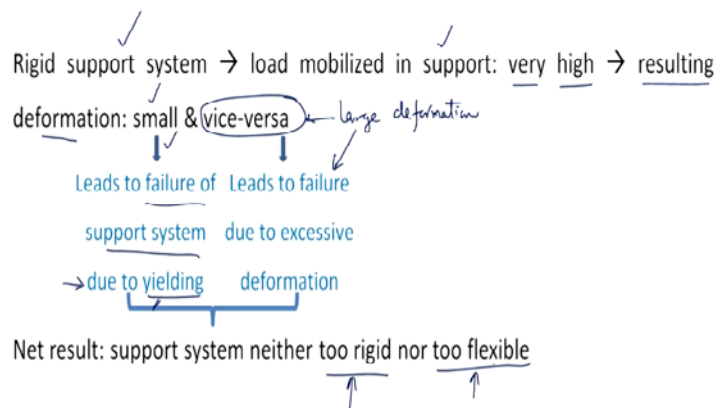
This is the function of deformation of the surrounding rock. And this deformation of the surrounding rock itself is a function of the stiffness of the support system. So, you should be able to visualize that this whole phenomena is resulting into an interdependent behavior. And therefore, this is called as rock mass or rock tunnel support interaction. Why we are calling this as interaction is because of these 2 statements.

That is deformation of the support system is a function of the load transferred by the surrounding rock. This load which is being mobilized in the support system is a function of

deformation of the surrounding rock, which itself is a function of stiffness of the support system. So, this is all interdependent behavior and is called as rock mass or rock tunnel support interaction.

(Refer Slide Time: 20:25)

Rock mass support interaction analysis



In case if you have a rigid support system, load mobilized in the support is going to be very high. And this will result into the deformations which are very small. In case if you have completely flexible support system, the load which will be mobilized in the support is going to be very low. And the resulting deformations are going to be very high. So, that part will fall under this vice versa.

So, in case if the resulting deformation is quite small, this will lead to the failure of the support system due to yielding. Why? Because, in this case, in order to arrest the deformation, we have provided very rigid support system and therefore, the load which is mobilized in the support is very high. So, the failure is going to be there because of the yielding of the support system.

In case if you have very large deformation, which is the other case there the failure is going to be because of this excessive deformation. So, the net result what we get from here is that the support system should neither be too rigid nor it should be too flexible. If it is too rigid, first thing is it will be highly uneconomical. And the failure will be because of the yielding of the support system.

If it is too flexible, then the deformations may become more than the permissible deformation, which is also not desirable. So, we have to compromise on to a situation somewhere in between

these 2 extreme conditions, one is too rigid, another one is too flexible. So, we need to choose an optimum condition. How to do that? For this purpose, 2 curves are defined.

(Refer Slide Time: 22:59)

Rock mass support interaction analysis

- * Ground response curve → defines characteristics of rock mass surrounding excavation
- * Support reaction curve → defines characteristics of support system ←

Take few cases of different rock types to have better understanding of these



One is the ground response curve, which defines the characteristics of the rock mass surrounding the excavation. And the second one is the support reaction curve, which defines the characteristic of the support system. Now, in order to understand these 2 curves in a better manner, we will take few simple cases of different rock types. So, first I will discuss with you the two dimensional situation and then we will go to three dimensional situation.

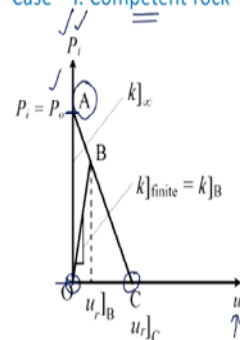
Although in the field most of the time you are going to have the three dimensional situation. But, to understand that situation first you must understand the philosophy of the ground response curve and the support reaction curve. And it will be easy for you to understand with the help of two dimensional situation. So, we take the Case 1 which is related to the competent rock.

(Refer Slide Time: 24:15)

Rock mass support interaction analysis

Two dimensional condition

Case - I: Competent rock



P_i = radial support pressure, u_r = radial deformation

(curve ABC \rightarrow straight line in this case)

\downarrow
Ground Response curve (GRC)

Linear elastic deformation $\rightarrow u_r \rightarrow$ very small

\uparrow
1-10 mm (say)

i) To arrest the def of tunnel periphery to a value, $(u_r = 0)$, stiffness of support system (k) to be provided will be,

$$k_s]_A = \frac{P_0}{0} \rightarrow \infty \leftarrow$$



When I say competent rock, what does that mean? That its behavior will be elastic. Now, this P_i is going to be the radial support pressure. And in this figure you can see that this P_i has been plotted against the radial deformation, which is u_r . So, u_r we are saying that it is the radial deformation. This curve A, B, C in this case of the competent rock, this is the straight line. This is called as the ground response curve. And in short, we call as GRC.

You can see that in the beginning you have the pressure which is equal to P_0 . And then as the deformation is increased, this is coming to this value of zero pressure here. So, here you have the linear elastic deformation. Why linear elastic deformation? Because, we are talking about the competent rock, in this case, u_r is going to be very small, maybe to the tune of say 1 to 10 millimeter. Say this is just to give you the idea.

Now, if we want to arrest the deformation of the tunnel periphery completely. This means that if we want to have u_r equal to 0. So, that is going to be our first case. That is to arrest the deformation of tunnel periphery to a value which is u_r equal to 0. So, the stiffness of the support system which I am writing as k to be provided will be what. See, if I have to arrest this complete radial deformation, u_r has to be equal to 0 here.

So, the stiffness is going to be the pressure which is required to cause the unit deformation. So, that is P_i upon u_r . So, in that case, it is this point which is going to be there because corresponding to this point A, u_r is equal to 0. So, the stiffness k_s at A is going to be P_0 upon 0. Because this is what is u_r at A and this will tend to infinity. So, you see in order to completely

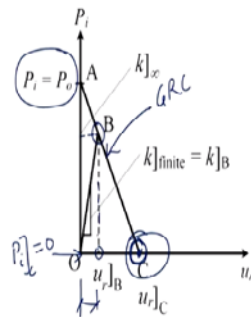
arrest the deformation, u_r equal to 0. I have to go for the stiffness of the support system as infinity means very large.

(Refer Slide Time: 29:03)

Rock mass support interaction analysis

Two dimensional condition

Case - I: Competent rock



(i) If $u_r = u_r]_B$ is to be allowed, then required support pressure: $P_i = P_i]_B$

\therefore Stiffness of support system required:

$$k_s]_B = \frac{P_i]_B}{u_r]_B} \Rightarrow \text{finite value}$$

(ii) If $u_r = u_r]_C$ is to be allowed, the required support pressure $\Rightarrow P_i = P_i]_C = 0$

\therefore Stiffness of support system reqd, $k_s]_C = \frac{P_i]_C}{u_r]_C} = 0$



Now, come to the second case that if this u_r I can allow that. The tunnel can experience some radial deformation. That means, this much from here to here. This much has been permitted is to be allowed. So, in that case, what will be the required support pressure? This will be what. P_i will be equal to. See that the required support pressure will be I take from this point. I take a vertical line like this dotted line.

And I reach here, which is the GRC ground response curve for this competent rock. So, corresponding to this whatever is the value of P_i that I will get it will be equal to say P_i at B. So, therefore, in this case the stiffness of the support system which is required will be k_s at B is equal to P_i at B divided by u_r at B. And this is going to give me the finite value.

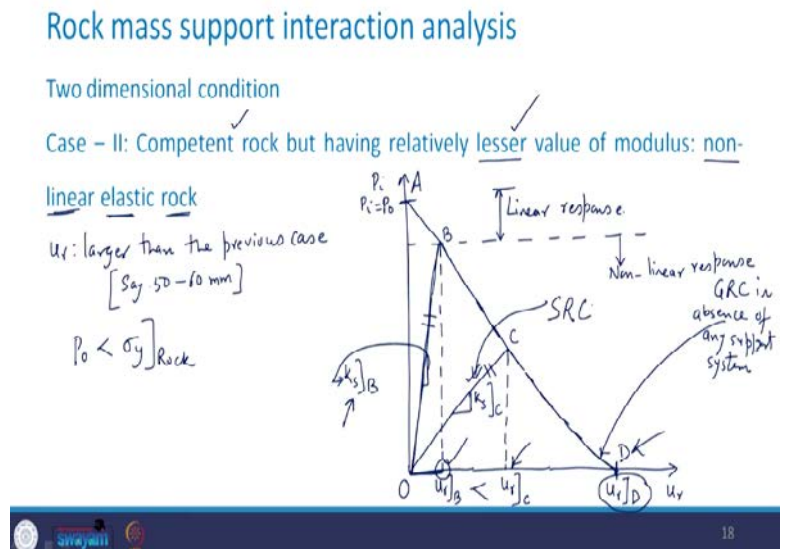
Now, there comes another case that if I allow even more deformation to take place. That means, if $u_r = u_r$ which is happening at C, this point. So, I am even allowing this much of the deformation to take place. Then, what will happen is to be allowed. Then, the required support pressure will be what in this case. Look at this figure and tell that this is going to be here u_r at C. And if I just drop a line here corresponding to this, your P_i value is 0.

See, so, what we are going to get is P_i equal to P_i at C. This is equal to 0. So, therefore, the stiffness of the support system required is going to be equal to say k_s at C which is equal to P_i at C divided by u_r at C. And this will be equal to 0. This means what? That we do not even

need any support system the tunnel or the excavation will be stable even without the support system. So, this is one extreme case.

And this is another extreme case where you do not need the support system. Now, let us take the other case of the competent rock but having relatively lesser value of the modulus.

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That is it is the non-linear elastic rock. Now, since we know that it is non-linear elastic rock, can we draw the ground response curve? As we did in the previous case, let us try to draw that. So, here on the vertical axis, again we will have the radial support pressure, which is P_i . And on the x axis, we will have the deformation radial deformation u_r . Now, in the beginning here, that is the point A as we had in the previous case.

So, I am keeping the same notations. At this point, this $P_i = P_0$. Now, since this is non-linear elastic rock, so, in the beginning some portion you will have the linear response of the ground. That is the linear one and then it will become non-linear. And somewhere here, this becomes equal to 0. So, this is what is going to be the GRC. So, this is ground response curve in the absence of any support system.

Now, from the point A up to this point B here in this zone, we have the linear response. And then here beyond this we have the non-linear response. Now, let us say that we can allow the deformation corresponding to this point B which is u_r at B. Then the stiffness which is required as you learnt just now is going to be something like this. That means you just draw a straight line from this point to B.

In case if you allow let us say some more say this is the another case where you have u_r at point C which is this is this point C. And you join this point here like this. And this point is say D where you have the deformation as u_r] D. This is the point O. Now, the stiffness of the support system in this case is going to be k_s at B. And here it is going to be k_s at C. In this case, u_r is larger than the previous case because in previous case it was the competent rock, but having good or the better value of the modulus.

So, it was it is the behavior was linear elastic. So, in this case, it can be, maybe let us say, it is of the order of say 50 to 60 mm. In this case, this P_0 is less than σ_y of the rock. So, you see that if we are able to allow this much deformation to take place, then we can provide the stiffness like this. But if we want to arrest again the complete deformation, the stiffness of the support system will again tend to infinity.

As you keep releasing or as you keep increasing the permissible value of the deformation like here in this case, you see that u_r]C is more than u_r]B. And you can see that the slope of this line is much less as compared to the slope of this line. That means, that the stiffness of the support required to arrest this much of the deformation will be less than that of the system, which is required to arrest this much of the radial deformation.

And if you want to allow this high value of radial deformation, then you do not need a support system even though it is the rock having relatively lesser value of the modulus. So, this is how one can determine the stiffness of the support system. This is what is called as support reaction curve.

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Rock mass support interaction analysis

Two dimensional condition ✓

Case - III: Incompetent rock $\rightarrow P_0 > \sigma_{y,rock}$

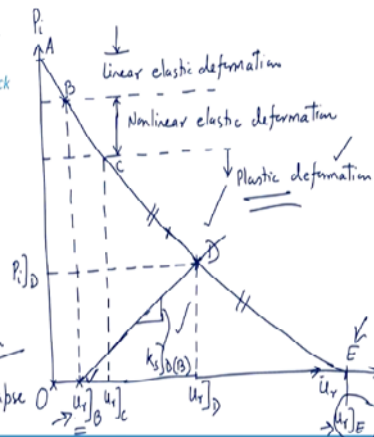
GRC \Rightarrow ABCDE

$u_r]_B \ll \ll u_r]_E$ & even $u_r]_C \ll \ll u_r]_E$

$u_r]_E \rightarrow$ Large deformation

$$k_s]_{D(B)} = \frac{P_i]_D}{u_r]_D - u_r]_B}$$

If $u_r]_E$ exceeds the permissible deformation
loosened rock around tunnel periphery may collapse
into cavity.



Let us go to the next case in which you have the incompetent rock with P_0 is more than the yield stress for the rock. Let us try to draw in this case that how the ground is going to behave without the support system that is on immediate excavation. And if we leave it like that, how it is going to behave? Again here we have this P_i which is the internal radial support pressure and on this axis, we have u_r .

And in this case, we will have say here this point is A. And we will have some different zones here. See the response is like that. So, here in this case from this to this zone, it is the linear elastic deformation. However, from this point B to this point C, this much is non-linear, but it is still in the elastic range. So, it is non-linear elastic deformation. But, what happens beyond C is the plastic deformation.

So, this is the difference between Case 2 and Case 3. But, here in this case we are having some plastic deformation as well. Now, I take a few points. So, this point I name as E. Let us say, I need to arrest the deformation corresponding to this point B. So, what is that? That is $u_r]_B$. So, only this much is allowed. So, what is going to be the stiffness? You join this point with this point and that is going to be pretty high because its slope is going to be very high.

If we want to have this kind of situation that is with respect to the point $u_r]_C$ here, then take some point somewhere here in between which is say the point D and the deformation is say $u_r]_D$. Now, the deal is that before the installation of the support system, $u_r]_B$ has been allowed. So, the support reaction curve will start from this point and not from the origin because before the installation of the support system, this much has already been allowed.

So, you just join these 2 points. And the stiffness of this line, I am denoting as k_s at D when B was allowed. B was allowed means that radial deformation corresponding to this B was already there before the installation of the support system. So, basically in this case now, GRC is going to be the curve A, B, C, D and E. That means this curve. Now, here at this point E, this u_r]E, this u_r at B is much less as compared to u_r at point E.

And even if you take a look at this figure, even u_r]C is quite less than u_r]E because this u_r E is quite large deformation because it is the plastic deformation. So, u_r]E is quite large. So, this k_s at D, when it has started from the point B is going to be what? I take this line here and this is going to be P_i at D. So, find out this stiffness. So, this P_i at D, this will be divided by whatever is the deformation corresponding to this D minus u_r at B.

So, this is what is going to define the stiffness for this support system. If your u_r]E exceeds the permissible deformation then what will happen? What this will do is that there is going to be the loosened rock around the tunnel periphery may collapse into the cavity which is not desirable. So, what we need to do is before the deformation reaches to this high value of u_r at E, we need to arrest it somewhere in between maybe at point D or any other point on this GRC.

And then provide the support system which has some stiffness. And then you will be able to avoid this particular situation. So, this is how we can understand the ground response curve and the support reaction curve in simple 2D conditions. So, for different types of rock, like in case of the competent rock, you had the completely linear response. In case of the incompetent rock, but when this P_0 was less than the yield stress of the rock.

There the whole system was in the elastic domain although it went into the nonlinear regime, but still it was in the elastic domain. However, in case of the incompetent rock, when this P_0 becomes more than this σ_y of the rock or the yield stress of the rock, the deformations enter into the plastic domain, which results into very large deformation, if you do not provide any support system.

You cannot provide very rigid support system because it will become highly uneconomical and you cannot even leave the tunnel unsupported because that may lead to the collapse of the material in the cavity. So, we have to have a deal in between these 2 extreme situation. So, I

tried to explain you the concept of rock mass tunnel support interaction with the help of this simple two dimensional condition.

In the next class, we will take up the three dimensional condition which is relevant in the field. So, we will take an example of a tunnel. And, how it advances slowly? How various types of the ground responses are absorbed along with the required support reaction curve? So, all those things we will discuss in the next class, thank you very much.