

Rock Engineering
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Lecture-33
Parameters of Failure Criteria

Hello everyone. In the previous class we discussed about Hoek and Brown failure criterion. And we saw that this failure criterion which is an empirical criterion is applicable in case of intact rocks as well as for the rock masses. The only change which differentiates between the failure criterion applicable to intact rock and the rock mass with reference to Hoek and Brown criterion is their parameters.

In case of the intact rocks, it is the parameters corresponding to the intact rock while in case of the rock mass you will have the respective parameters. And you have also seen that the parameters of the rock mass, they can be correlated with the parameters of the intact rock especially the parameter m . So, today we will learn few aspects related to parameters of the failure criteria.

And these failure criteria will include Mohr Coulomb failure criterion and Hoek and Brown failure criterion. You have seen that there are some difficulties which are associated with the determination of Mohr Coulomb parameters under effective stress conditions. And we left the discussion with a question mark there that how to determine these effective strength parameters. So, today we are going to have the answer to that question, let us see how?

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M-C parameters: c' & ϕ' from H-B parameters

- * Derivation of H-B parameters for rock mass from intact rock: straightforward!
- * For intact rock: parameters are m_i and σ_{ci} ($s = 1$ & $a = 0.5$)
- * For the rock mass: parameters are m_m , σ_{cm} , s , & a
- * These two sets of parameters: related by GSI and D that reflect the quality of the rock mass and the degree of disturbance it has undergone during excavation, blasting and so on

The derivation of Hoek and Brown parameters for rock mass from the intact rock, it is pretty straightforward. For intact rock, the parameters are m_i and σ_{ci} while the other two parameters which were s and a , they can be assigned a value of $s = 1$ and $a = 0.5$ for intact rock. In fact, why this $s = 1$ in case of the intact rocks? That also we have seen, in fact we proved it. For the rock mass, the parameters are m_m , σ_{cm} , s and a .

So, we have in all four parameters here, keep in mind that the subscript m here for these two constants, these two parameters, this corresponds to the rock mass while in this case for intact rock you have seen that we are using the subscript i . Now these two sets of parameters, they are related by GSI that is geological strength index. And a disturbance factor D that reflect the quality of the rock mass.

And the degree of disturbance it has undergone during excavation, blasting and so on. So, today towards the end of this lecture, we will also learn about the values related to this D , that how this can be assigned?

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M-C parameters: c' & ϕ' from H-B parameters

- * M-C failure criterion: quite popular among geotechnical engineers → tendency to apply this to rocks too.
- * Main difficulty: to derive the shear strength parameters c' & ϕ' in terms of effective stresses for the rock mass
- * (Not practical to test a representative rock mass in a triaxial cell: it can only be carried out through a simulation exercise

Now coming to the Mohr Coulomb failure criterion, we have seen that it is quite popular among, we geotechnical engineers and in case of the soils it is quite widely used. And therefore, we have this tendency to apply this to the rocks too. And when we were discussing this failure criterion in detail, we have seen that what is the modification that we need to make in the Mohr coulomb criterion, so that it can be applicable in case of the rocks.

Do you remember the tension cutoff? The main difficulty which was there with respect to Mohr Coulomb failure criterion was to derive it is shear strength parameters under effective stress condition for the rock mass, which we were representing as c' and ϕ' . It is really not practical to test a representative rock mass in a triaxial cell. It can only be carried out through a simulation exercise.

So, when I say simulation exercise means we have to conduct some kind of numerical kind of an experiment in order to have the representation of the same condition as it would have been, if we would have been able to test the rock mass in a triaxial cell.

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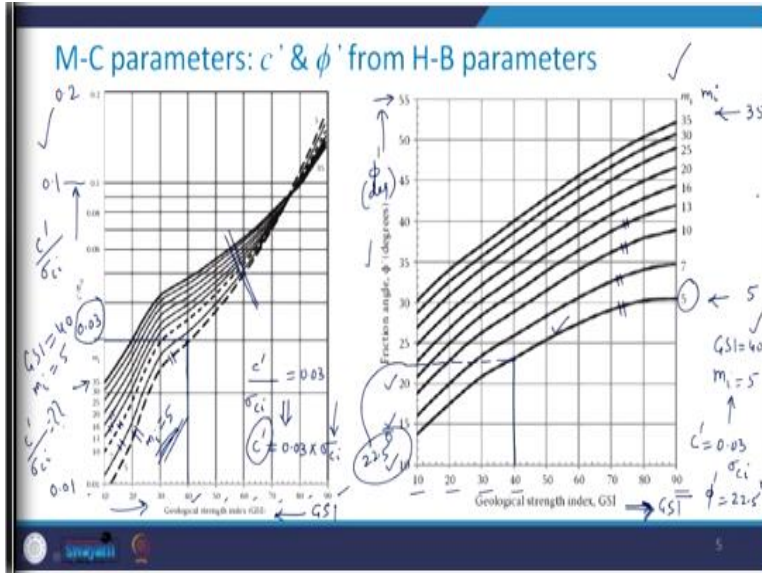
M-C parameters: c' & ϕ' from H-B parameters

- * Hoek and Brown (1997): simulated a series of triaxial test data for the rock masses of different GSI, m_i and σ_{ci} values → confining pressure $\sigma_{3f}: 0 - (\sigma_{ci}/2)$
- * M-C envelopes drawn with these simulated data from which c' & ϕ' for the rock masses were determined

So, Hoek and Brown in 1977, they simulated a series of triaxial test data for the rock masses with different GSI, m_i and σ_{ci} values, what does this mean? This is the intact rock parameter m ; σ_{ci} is the UCS of the intact rock. And when they simulated this, they took the confining pressure range between 0 and half of this UCS value, which is this. So, Mohr Coulomb envelopes, they were drawn with these simulated data from which these two parameters c and ϕ for the rock masses were determined.

So, you see that there is a difference between conducting the triaxial test data and the data which we have got from the simulation. So, in this case they did not conduct the triaxial tests, but they simulated these results.

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Now the result of that simulation can be seen here in these two figures. You can see here on the first figure, this scale is giving us GSI and this is increasing in this direction, that is 10, 20, 30, 40, 50, 60, 70, 80 and 90. And on y axis, you have this term c'/σ_{ci} , which is on the log scale starting from 0.01, this is 0.1 and so on, this is here 0.2 which has been shown. Then you can see some series of these curves, these correspond to the curve related to various values of m_i .

For example, this first dotted line corresponds to $m_i = 5$, the next one is for 7, then for 10, 13, 16 and so on, it goes up to 35. So, how to make use of this? You know what is the GSI for that rock mass? Then from the intact rock property, you know what is this parameter m for that rock? So, for example, let us say that you have GSI as say 40 and say m_i is 5. Now how can I find out c'/σ_{ci} .

Take a look here, this is 40, so I will draw a line from here and it is intersecting this curve which is corresponding to $m_i = 5$ here. Then I join a horizontal line from this particular point in this fashion. And see, this is 0.01; this is 0.02, so this will be corresponding to 0.03. So, what I am going to get from here is c prime divided by σ_{ci} is going to be 0.03. So, from here, I can evaluate $c' = 0.03 * \sigma_{ci}$.

This value of σ_{ci} corresponds to UCS of the intact rock and that can be obtained with quite a precision in the lab. So, this is how the parameters c' for the rock mass under effective stress condition can be determined. Now, take a look at this figure. Again, here on this scale we have

GSI and it is increasing in this direction 10, 20, 30, 40 and so on up to 90. Then on this y axis, we have the friction angle ϕ' in degrees, which is increasing in this direction, starting from 10, 15, 20 and it goes up to 55.

Then, here you can see the series of these curves are there, these correspond to various values of m_i , it is varying from $m_i = 5$ to $m_i = 35$. So, in this case, how can we obtain the friction angle ϕ' ? So, let us say I take the same example, that is we had $GSI = 40$, and $m_i = 5$. So, let us take the vertical line from $GSI = 40$, so let us follow this. And this is the plot corresponding to $m_i = 5$, it is intersecting it here.

So, I will just draw a horizontal line from this point and more or less you can see that it intersects is here. So, here this is corresponding to say approximately 22.5° . So, for a rock mass for which $GSI = 40$ and the corresponding value of m_i parameter is 5, you can have $c' = 0.03 * \sigma_{ci}$ and you can have $\phi' = 22.5^\circ$. So, this is how from the geological strength index value and from the parameter of the Hoek and Brown criterion for intact rock which is m_i .

We can determine the values for c' and ϕ' which are the Mohr Coulomb parameters for the rock mass under effective stress conditions, which are not possible to determine from the conduct of triaxial tests.

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M-C parameters: c' & ϕ' from H-B parameters

- * Synthetic data were generated to follow a parabolic failure envelope in $\sigma_1' - \sigma_3'$ space
- * Linear M-C envelope fitted to these data will vary depending on the stress range covered (σ_{1i} , σ_{3f} , $\sigma_{3,max}$)
- * M-C parameters c' & ϕ' \rightarrow vary depending on the range of values selected for σ_{3f}

So, synthetic data were generated as I mentioned to you to follow the parabolic failure envelope in σ_1' - σ_3' space. And then linear Mohr Coulomb envelope fitted to these data, this will vary depending upon the stress range which was covered. So, in this case the stress range which was covered was given by this that is σ_{3f} was in between the tensile strength of the rock mass and a stress which is σ_{3max} .

This Mohr Coulomb parameters c and ϕ they vary depending upon the range of values selected for σ_{3f} , we will see how?

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M-C parameters: c' & ϕ' from H-B parameters

* Hoek et al. (2002): use of curve fitting exercise gives

$$\rightarrow \sin \phi' = \frac{6a m_m (s + m_m \sigma'_{3n})^{a-1}}{2(1+a)(2+a) + 6a m_m (s + m_m \sigma'_{3n})^{a-1}}$$

$$\rightarrow \frac{c'}{\sigma_{ci}} = \frac{[(1+2a)s + (1-a)m_m \sigma'_{3n}] (s + m_m \sigma'_{3n})^{a-1}}{(1+a)(2+a) \sqrt{1 + (6a m_m (s + m_m \sigma'_{3n})^{a-1})} ((1+a)(2+a))}$$

where, $\sigma'_{3n} = \sigma'_{s, max} / \sigma_{ci}$

So, using these things Hoek gave these 2 expressions and these were obtained using the curve fitting exercise. And likewise, here you can see this is the expression

$$\sin \phi' = \frac{6a m_m (s + m_m \sigma'_{3n})^{a-1}}{2(1+a)(2+a) + 6a m_m (s + m_m \sigma'_{3n})^{a-1}}$$

. Here you can see that you have a term σ_{3n}' , in these expressions everywhere you have the σ_{3n} prime. So, this σ_{3n}' is defined as $\sigma_{3max}' / \sigma_{ci}$. So, the next question with comes is that how to determine this σ_{3max}' ?

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M-C parameters: c' & ϕ' from H-B parameters

- * $\sigma'_{3,max}$: upper limit of σ'_{3f} → selected depending on project and stress levels
- * General guideline for tunnels and underground excavations:

$$\frac{\sigma'_{3,max}}{\sigma'_{cm}} = 0.47 \left(\frac{\sigma'_{cm}}{\gamma H} \right)^{-0.94}$$

where, H : depth below the surface & γ : unit weight of rock mass

This is the upper limit of σ'_{3f} and it is selected depending upon the project and stress levels. So, as a general guideline for tunnels and underground excavation, we use this expression where the $\sigma'_{3,max}/\sigma'_{cm}$, these two empirical numbers are there, this H here is the depth below the surface that means depth of the overburden for this tunnels or underground excavations and γ be the unit weight of the rock mass.

$$\frac{\sigma'_{3,max}}{\sigma'_{cm}} = 0.47 \left(\frac{\sigma'_{cm}}{\gamma H} \right)^{-0.94}$$

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M-C parameters: c' & ϕ' from H-B parameters

- * σ'_{cm} : as per Hoek and Brown (1997): global rock mass strength: determined from M-C failure envelope fitted to simulated data
- * σ'_{cm} : better representation of average UCS of rock mass
- * σ'_{cm} : simply the UCS determined from M-C criterion → generally larger than the rock mass strength, $\sigma_{cm} = \sigma_{ci}^{s^u}$

σ'_{cm} as per the Hoek and brown 1997 is defined as the global rock mass strength which can be determined from Mohr Coulomb failure envelope fitted to the simulated data. So, the σ'_{cm} gives us

the better representation of average UCS of the rock mass. This σ_{cm}' is simply the UCS which is determined from Mohr Coulomb criterion, just by fitting to the simulated data.

And it is generally larger than the rock mass strength that you obtain from this expression. How we get this expression? This we have seen in the last class, rock mass strength is given by the intact rock UCS multiplied by the parameters of the Hoek and Brown criterion. In this manner,

$$\sigma_{cm} = \sigma_{ci} S^a$$

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M-C parameters: c' & ϕ' from H-B parameters

* For slopes, $\sigma'_{3,max} / \sigma'_{cm} = 0.72 \left(\frac{\sigma'_{cm}}{\gamma H} \right)^{0.91}$

where, H : height of slope

* Assumptions involved: 2-D failure surfaces in form of circular arcs and Bishop's method of slices

In case of slopes, this expression is used. So, you can see that the empirical constants here they have changed, H be the height of the slope and again γ is the unit weight of the rock mass. One needs to keep in mind that when you apply this expression in case of the slope, the assumption which is involved is this, that is 2-D failure surfaces in the form of circular arcs and one needs to use Bishop's method of slices.

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M-C parameters: c' & ϕ' from H-B parameters

* From M-C envelope: $\sigma'_{cm} = \frac{2c' \cos \phi'}{1 - \sin \phi'}$ ←

* In the normal stress range of $\sigma'_x, \sigma'_y, 0.25 \sigma'_{ci}$:

$$\sigma'_{cm} = \sigma'_{ci} \frac{[m_b + 4s - a(m_b - 8s)](0.25m_b + s)^{a-1}}{2(1+a)(2+a)} \leftarrow$$

↑
Rock mass

Now from the Mohr Coulomb envelope up you will get this expression which is

$$\sigma'_{cm} = \frac{2c' \cos \phi'}{1 - \sin \phi'}$$

This we have seen earlier. Now in the normal stress range of tensile strength to 0.25 times the UCS, you will have an expression like this, which will give you the value of σ'_{cm} .

$$\sigma'_{cm} = \sigma'_{ci} \frac{[m_b + 4s - a(m_b - 8s)](0.25m_b + s)^{a-1}}{2(1+a)(2+a)}$$

And you can see that this is a function of σ'_{ci} which is the UCS of the intact rock plus the parameters of the rock mass Hoek and Brown parameters of the rock mass. See here, instead of m_m , m_b is written but this is also for the rock mass.

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Deformation modulus

- * Can be estimated from index Q (Grimstad & Barton, 1993):

$$E_d = 25 \log Q \quad \text{[for } Q > 1]$$
- * Bieniawski (1978):

$$E_d (GPa) = 2 RMR - 100 \quad \text{[for } RMR > 55]$$
- * Serafim & Pereira (1983):

$$E_d (GPa) = 10^{\left(\frac{RMR-10}{40}\right)}$$

Now let us have some discussion about the deformation modulus. I have already defined to you that what do we mean by this? Usually, in the case of rock mass, we talk in terms of deformation modulus and not in terms of the elastic modulus. Now this deformation modulus can be estimated from index Q using this expression, which was given by these two authors in 1993. So, this expression gives you that deformation modulus is equal to 25 times $\log Q$.

$$E_d = 25 \log Q \quad \text{for } Q > 1$$

And the condition on the Q was that Q should be greater than 1. The next one was given by Bieniawski in 1978, that is

$$E_d (GPa) = 2RMR - 100 \quad \text{for } RMR > 55$$

And this is applicable when you have RMR greater than 55. Then this expression you are aware of that is

$$E_d (GPa) = 10^{\left(\frac{RMR-10}{40}\right)}$$

This we have discussed earlier as well.

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Deformation modulus

* Hoek et al., 2002: $E_d(GPa) = \left(1 - \frac{D}{2}\right) \sqrt{\frac{\sigma_{ci}}{100}} \times 10^{\left(\frac{GSI-10}{40}\right)}$ for $\sigma_{ci} < 100 MPa$

$E_d(GPa) = \left(1 - \frac{D}{2}\right) \times 10^{\left(\frac{GSI-10}{40}\right)}$ for $\sigma_{ci} > 100 MPa$

* Hoek and Diederichs (2006):

$E_d(GPa) = 100 \left(\frac{1 - D/2}{1 + e^{(75 + 25D - GSI)/11}} \right)$

$E_d = E_i \left(0.02 + \frac{1 - D/2}{1 + e^{(60 + 15D - GSI)/11}} \right)$

Then, there are few other correlations which are available. So, as per Hoek et al in 2002, this deformation modulus for the rock mass having $\sigma_{ci} < 100 MPa$ is given by this expression,

$$E_d(GPa) = \left(1 - \frac{D}{2}\right) \sqrt{\frac{\sigma_{ci}}{100}} \times 10^{\left(\frac{GSI-10}{40}\right)}$$

And in case σ_{ci} is $> 100 MPa$, it is given by the second expression,

$$E_d(GPa) = \left(1 - \frac{D}{2}\right) 10^{\left(\frac{GSI-10}{40}\right)}$$

Then in their further analysis, these two authors modified or proposed a new correlation where they had some different form with respect to D and GSI. Take a look here,



$$E_d(GPa) = 100 \left(\frac{1 - D/2}{1 + e^{(75 + 25D - GSI)/11}} \right)$$

In other way, you can also use this expression in case if you have the elastic modulus in case of the intact rock, which is given as E_i here.

$$E_d = E_i \left(0.02 + \frac{1 - D/2}{1 + e^{(60 + 15D - GSI)/11}} \right)$$

So, you can use either this expression or this expression in order to get the deformation modulus as per these authors. The question here comes how to determine this parameter D which is the disturbance factor. Because how to know this GSI, you have seen it when we were discussing about various classification systems for the rock mass.

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Appearance of rock mass	Description of rock mass	Suggested value of D
	Excellent quality controlled blasting or excavation by tunnel boring machine here results in minimal disturbance to the confined rock mass surrounding a tunnel.	D = 0
	Mechanical or hand excavation in poor quality rock masses (no blasting) results in minimal disturbance to the surrounding rock mass. Where squeezing problems result in significant floor heave, disturbance can be severe unless a temporary invert, as shown in the photograph, is placed.	D = 0 D = 0.5 No invert

Handwritten notes: TBM, D=0, Crown, Invert



So, take a look here, the first column gives you the picture of the rock mass that is related to its appearance, the second one describes the rock mass and the third one gives you the value of D. So, here as the picture looks like this corresponds to excellent quality-controlled blasting or excavation by tunnel boring machine, this in short, we call as TBM as well. This results in the minimum disturbance to the confining rock mass surrounding a tunnel.

So, if this is the tunnel whatever is the surrounding rock mass it is least disturbed because of the process of excavation. And therefore, the suggested value of D is taken to be equal to 0, that is $D = 0$ in this case. Come to the second category where the rock mass will have the appearance like this, and it is described as the mechanical or hand excavation in poor quality rock masses, and there is no blasting.

So, you see that either it is the mechanical or hand excavation and it is in the poor-quality rock masses. This also results in the minimum disturbance to the surrounding rock mass. But in case if you have the squeezing problems, these results in significant floor heave and disturbance can be severe unless a temporary invert which is shown here in the figure is placed. So, you see here, see if this is the tunnel, the bottommost portion point is called as invert and this one is called as crown.

So, here you can see that the temporary invert has been placed here. So, in this case you will have the suggested values of D, maybe $D = 0$ or $D = 0.5$ and no invert case you will have $D = 0.5$.

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Appearance of rock mass	Description of rock mass	Suggested value of D
	Very poor quality blasting in a hard rock tunnel results in severe local damage extending 2 or 3 m, in the surrounding rock mass.	D = 0.8 <u>D = 0.8</u>
	Small-scale blasting in civil engineering slopes results in modest rock mass damage, particularly if controlled blasting is used as shown on the left-hand side of the photograph. However, stress relief results in some disturbance.	D = 0.7 Good blasting → D = 0.7 D = 1.0 Poor blasting → D = 1.0

Then, the third appearance of the rock mass looks like this, where you have very poor-quality blasting in a hard rock. Hard rock and very poor-quality blasting, so what will happen? On one hand it is the hard rock and on second hand you are having very poor quality of blasting. So, the disturbance is going to be severe in this case. And that is what has been given as a part of the description here, that in this case tunnel results in severe local damage extending 2 or 3m in the surrounding rock mass.

And in that case, obviously you will have larger value of this disturbance factor which is given by this $D = 0.8$. Then, the fourth category looks like this, where you have small scale blasting in civil engineering slopes results in modest rock mass damage. Particularly, if the control the blasting is used, as it has been shown here in this figure. However, the stress relief results in some disturbance. So, accordingly in case if you have the good quality blasting, then you will have $D = 0.7$, and in case of the poor blasting D will become even higher which is 1.0.

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Deformation modulus



✓
Very large open pit mine slopes suffer significant disturbance due to heavy production blasting and also due to stress relief from overburden removal.

In some softer rocks, excavation can be carried out by ripping and dozing and the degree of damage to the slopes is less.

$D = 1.0$ ←
Production blasting ←
 $D = 0.7$ ←
Mechanical excavation ←




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The last category looks like this, where you have very large open pit mine slopes, which suffer significant disturbance due to heavy production blasting, and also due to stress relief from the overburden removal. Now in case of some softer rocks, excavation can be carried out by ripping and dozing and the degree of damage to the slopes is less in that case. So, in case if you have the production blasting, this D is as high as 1 and in case you have mechanical excavation, this value of D is 0.7.

So, from this information, you can obtain the value of D substitute it in the appropriate expression in order to get the deformation modulus. And earlier also we have seen the use of this D in various expressions. So, depending upon what is the situation and decide, what is the type of the structure? What is the type of the blasting that is being adopted at the site? Accordingly, the value of this D can be assigned for further use in various expressions has been discussed with you.

So, in today's class, we discussed about some aspects related to the parameters of Mohr Coulomb failure criterion and Hoek and Brown criterion. And we could see that from the parameters of the Hoek and Brown criterion, one can obtain the Mohr Coulomb failure criterion parameters for the rock mass under effective stress conditions. Then we saw some aspects related to deformation modulus along with the discussion on how to assign an appropriate value to this disturbance factor D .

This was all about that I wanted to discuss with respect to the intact rock and rock masses and the empirical criteria that is Hoek and Brown criterion. In the next class we will learn about few other failure criterions which are applicable in case of the rock mass. One is the single plane of weakness theory, and the second one is the shear strength of the joint walls. Thank you very much.