Rock Engineering
Prof. Priti Maheshwari
Department of Civil Engineering
Indian Institute of Technology-Roorkee

Lecture-32
Hoek and Brown Criterion (1980)

Hello everyone. In the previous class we learnt about some of the basics of the regression analysis. Then I introduced you to some of the empirical criteria which were applicable in case of rocks and rock masses. Today we will learn about the most commonly adopted empirical failure criterion. In case of rocks and rock masses which is known as Hoek and Brown criterion, this was given in 1980 by these 2 authors Hoek and Brown.

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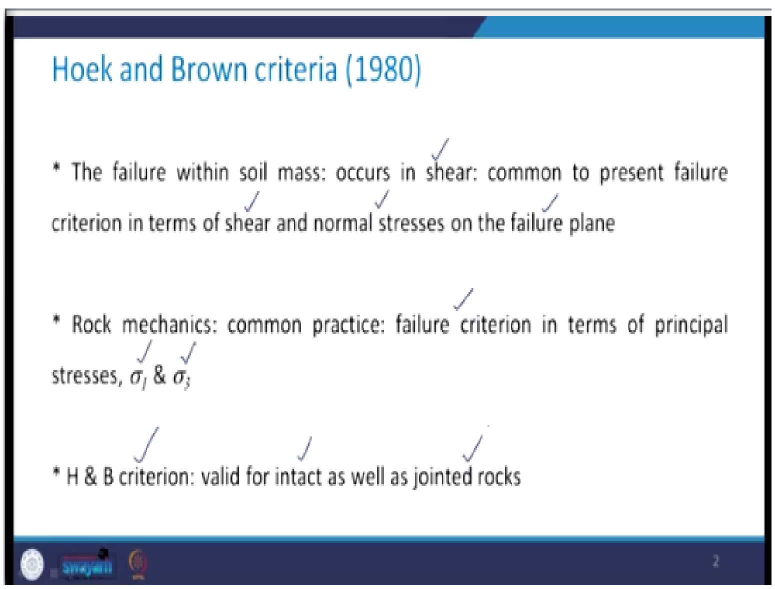

The failure within the soil mass, it occurs in shear, we all know this. And therefore, it is common to present the failure criterion in terms of shear and the normal stresses on the failure plane. But in case of rock mechanics, it is the common practice to represent the failure criterion in terms of principal stresses which are σ_1 and σ_3. It is the convention to represent the major principal stress by σ_1, and minor principal stress by σ_3.

So, therefore for most of the failure criterion in case of rock mechanics, it is a functional relationship between σ_1 and σ_3. This Hoek and Brown criterion is valid for intact rock as well as the jointed rocks. And that is one of the reasons that it is quite widely used in rock mechanics. So, for the intact rock, we have seen that there were deficiencies of Mohr coulomb criterion when this was applied in case of the rocks, what were these deficiencies?

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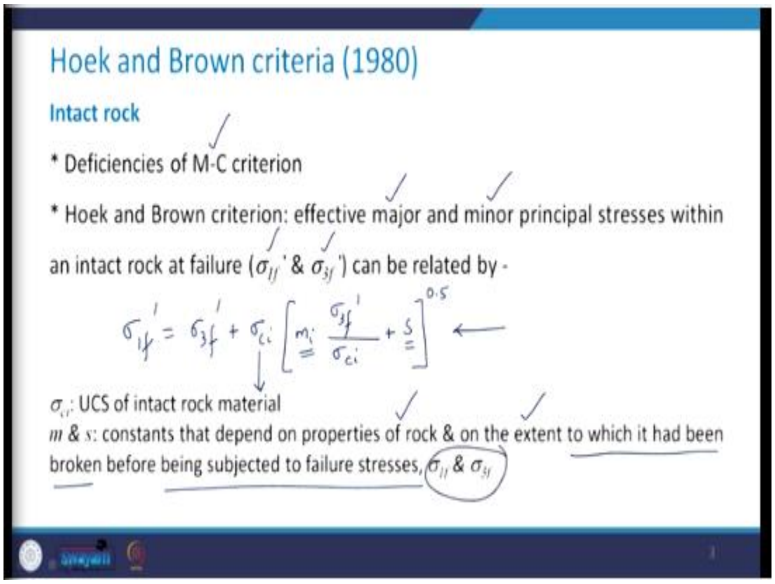

If you recall our discussion with respect to this Mohr coulomb criterion, we discussed that in case of the rock mass where the discontinuities are also present it is not really possible to get the parameters of Mohr coulomb failure criterion in terms of effective stresses. And therefore, we just left there with a big question that how to determine these parameters. So, in view of those deficiencies this criterion which was given by Hoek and Brown, it takes care of some of those deficiencies.

So, let us see first that how the Hoek and Brown criterion is applicable in case of the intact rock? And then we will see that how it has been extended to the rock mass? And then we will see how the parameters of Hoek and Brown criterion can be used to determine the parameters of Mohr Coulomb failure criterion? So, this Hoek and Brown criterion in terms of effective major and minor principal stresses on an intact rock at the failure which is σ_{1f}' and σ_{3f}' respectively.

It can be related by this expression which is

$$\sigma_{1f}' = \sigma_{3f}' + \sigma_{ci}\left[m_i \frac{\sigma_{3f}'}{\sigma_{ci}} + s\right]^{0.5}$$

So, this is the equation for Hoek and Brown criterion. Where the σ_{ci} is the UCS of the intact rock material m and s, these are the constants that depend on the properties of rock. And also, on the extent to which it had been broken before being subjected to failure stresses, which are σ_{1f} and σ_{3f}.

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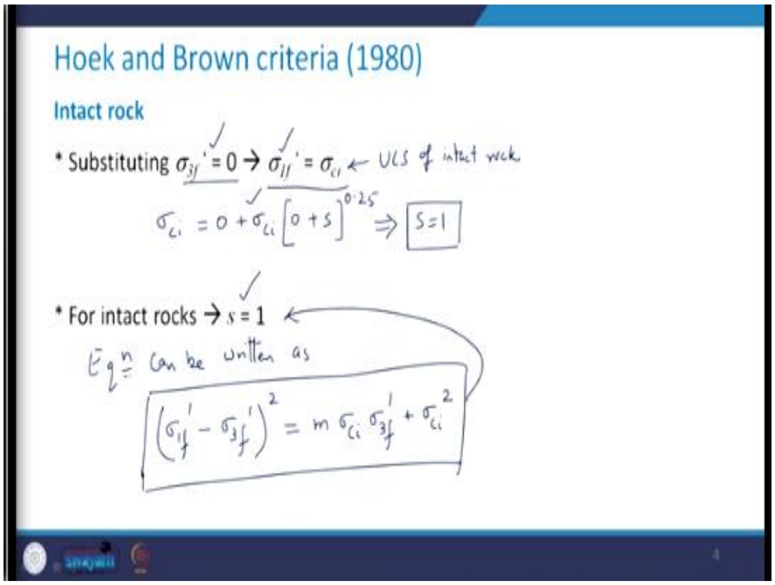

Now in case of this equation, if we substitute this $\sigma_{3f} = 0$, that means confining pressure is equal to 0, so what we are going to get as an axial stress? As it will be the UCS of the intact rock, that is what that we are going to get. So, just substitute these values in the expression and what you are going to get is, see I just do it.

$$\sigma_{ci} = 0 + \sigma_{ci}[0 + s]^{0.25} \Rightarrow s = 1$$

And what this equation gives me is that, for this equation to be satisfied this s should be equal to 1. And therefore, we can conclude that for intact rocks this parameter of Hoek and Brown criterion which is s that will be equal to 1. Now if we just substitute this s is equal to 1, in now if we just substitute this s is equal to 1, in the Hoek and Brown criterion equation. So, that equation can be written as

$$\left(\sigma_{1f}' - \sigma_{3f}'\right)^2 = m\sigma_{ci}\sigma_{3f}' + {\sigma_{ci}}^2$$

This is for the intact rock with s = 1.

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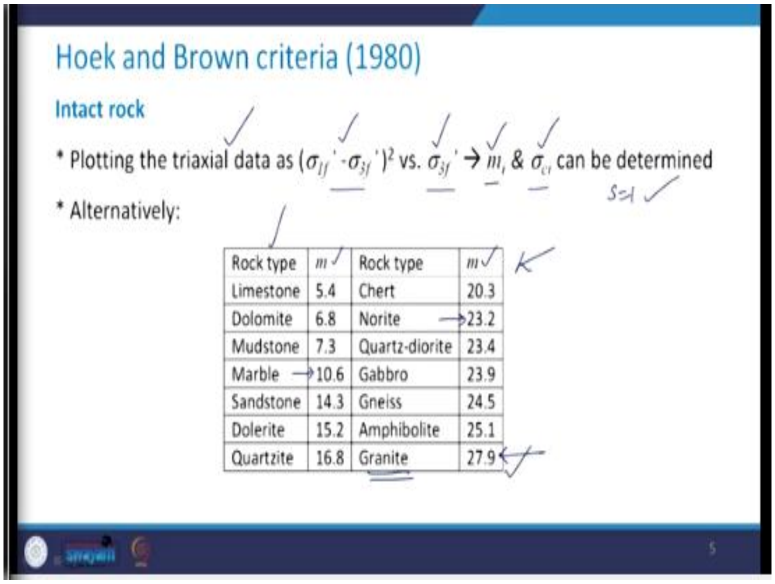

Rock type	m	Rock type	m
Limestone	5.4	Chert	20.3
Dolomite	6.8	Norite	23.2
Mudstone	7.3	Quartz-diorite	23.4
Marble	10.6	Gabbro	23.9
Sandstone	14.3	Gneiss	24.5
Dolerite	15.2	Amphibolite	25.1
Quartzite	16.8	Granite	27.9

Now we have seen that how the equation for the intact rock looks like. So, if we plot the triaxial data as $(\sigma_{1f}' - \sigma_{3f}')^2$ versus σ_{3f}. We can plot a straight line between these two and from there we can obtain m_i and σ_{ci}. So, the parameters were s, m and σ_{ci}. So, this is the way we can find out m_i and σ_{ci}, and you have seen that s is going to be equal to 1 in case of the intact rock.

Now if it is not possible to get this m_i from the triaxial test data, then we can find out from this table that is depending upon the various rock types. Here the value of m has been given. We have discussed this table earlier as well. Say in case of the marble it is 10.6, and in case of norite it is 23.2. As the quality of the rock gets better, it corresponds to the larger value of m, you can see here that granite has large value of m as 27.9.

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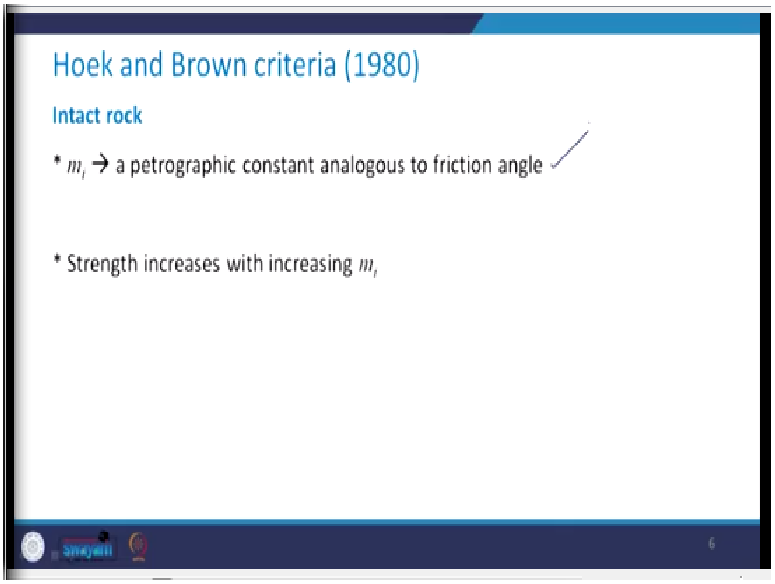

So, basically this m_i is a petrographic constant which is analogous to the friction angle. And the strength increases with increasing values of m_i.

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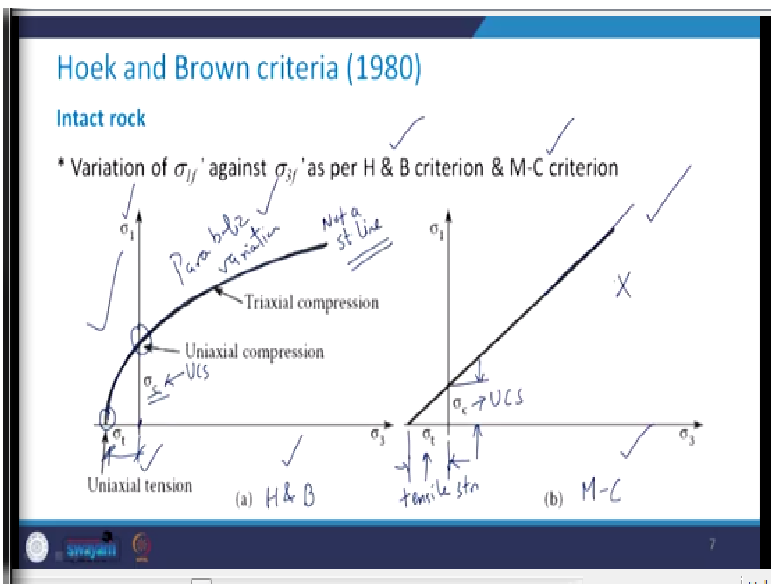

For the intact rock, the variation of σ_{1f} against σ_{3f} as per Hoek and Brown criterion and Mohr Coulomb criterion has been shown here. So, the first plot which is (a), this corresponds to Hoek and Brown criterion, and (b) corresponds to Mohr Coulomb criterion. You have seen in case of the Mohr coulomb criterion, this relationship was a straight line where the intercept on σ_1 axis was giving us the idea about the UCS.

And the intercept on x axis is giving us the idea about the tensile strength. But in case of the Hoek and Brown criterion, this is how the variation is going to be. So, in this case it is not a straight line as you all can see. So, this is kind of a parabolic variation, where this point corresponds to the uniaxial compression. Because here σ_3 is equal to 0, so whatever is the value of σ_1 that is going to give you the UCS which is σ_c?

And this point corresponds to the results for the uniaxial tension, so this σ_t is going to give us the tensile strength under uniaxial tension. So, if you compare these two criteria, this is straight line, it is the parabolic variation. Why it is more commonly adopted in case of rocks? Because most of the times when you conduct the test on the rock when you conduct the triaxial test on the rock this is the kind of variation that you will get and not the straight line.

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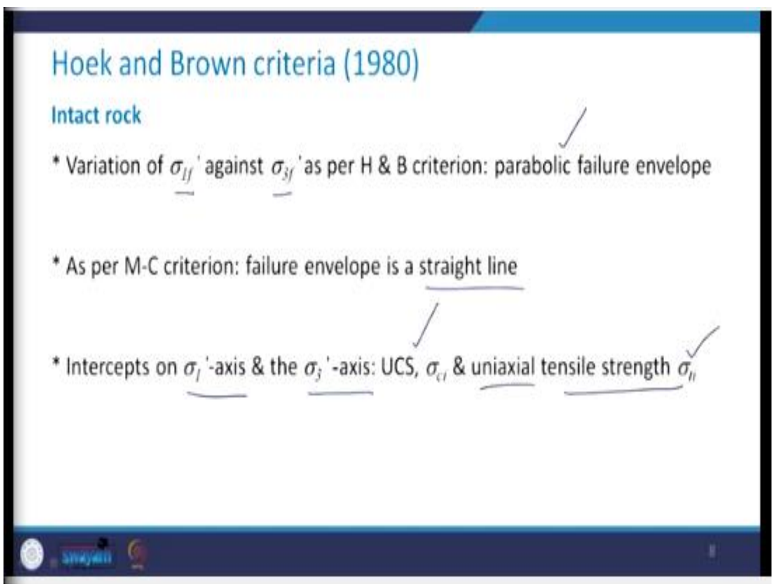

So, as I mentioned it is variation of σ_{1f} again σ_{3f} as per Hoek and Brown criterion is the parabolic failure envelope. In case of the Mohr coulomb criterion, it was a straight line and the intercept on σ_1 axis and the σ_3 axis gave us respectively UCS of the intact rock and uniaxial tensile strength of the intact rock.

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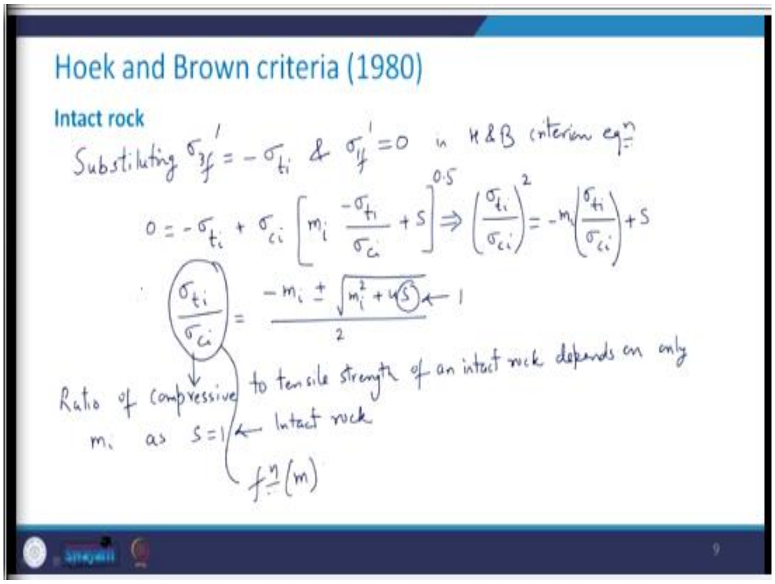

Now we just substitute this σ_{3f} is equal to σ_{ti} with it is appropriate sign. So, we are substituting

$$\sigma_{3f}{}' = -\sigma_{ti} \; and \; \sigma_{1f}{}' = 0$$

in Hoek and Brown criterion equation. So, what we are going to get is, take a look,

$$0 = -\sigma_{ti} + \sigma_{ci}\left[m_i \frac{-\sigma_{ti}}{\sigma_{ci}} + s\right]^{0.5} \Rightarrow \left(\frac{\sigma_{ti}}{\sigma_{ci}}\right)^2 = -m_i\left(\frac{\sigma_{ti}}{\sigma_{ci}}\right) + s$$

$$\left(\frac{\sigma_{ti}}{\sigma_{ci}}\right) = \frac{-m_i \pm \sqrt{m_i{}^2 + 4s}}{2}$$

So, from here we can see that this ratio which is the ratio of compressive to tensile strength of an intact rock. This depends upon only m i as s = 1for intact rock**.** So, you see that for the intact rock this quantity is going to be equal to 1, and therefore this ratio will be only the function of *m*.

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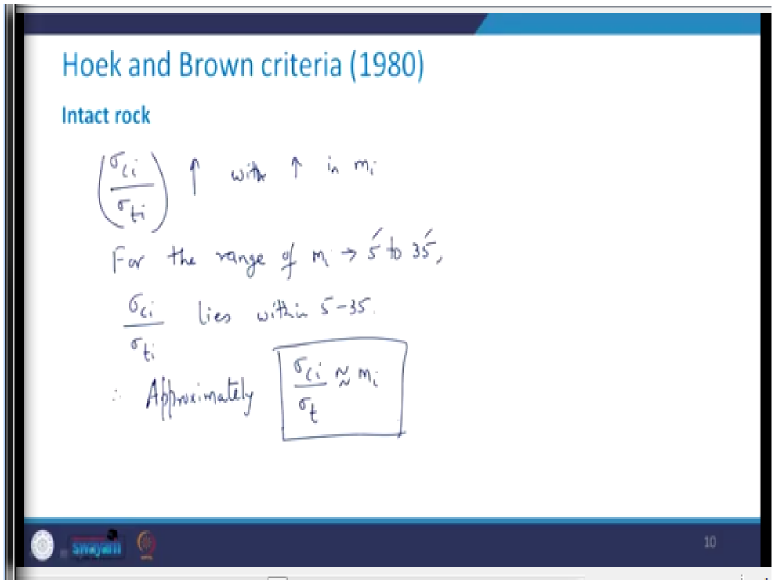

Now this ratio $\left(\frac{\sigma_{ci}}{\sigma_{ti}}\right)$ increases with increase in m_i, and if we take a range of m_i, so, from 5 to 35, so for the range of m_i from 5 to 35, this $\left(\frac{\sigma_{ci}}{\sigma_{ti}}\right)$lies within 5 to 35. You can just check it with the help of the calculator, just substitute the value of m between any value between 5 and 35 and you will get that this will be approximately equal to the value of m_i which you have taken. So, we can say that approximately

$$\left(\frac{\sigma_{ci}}{\sigma_{ti}}\right) \approx m$$

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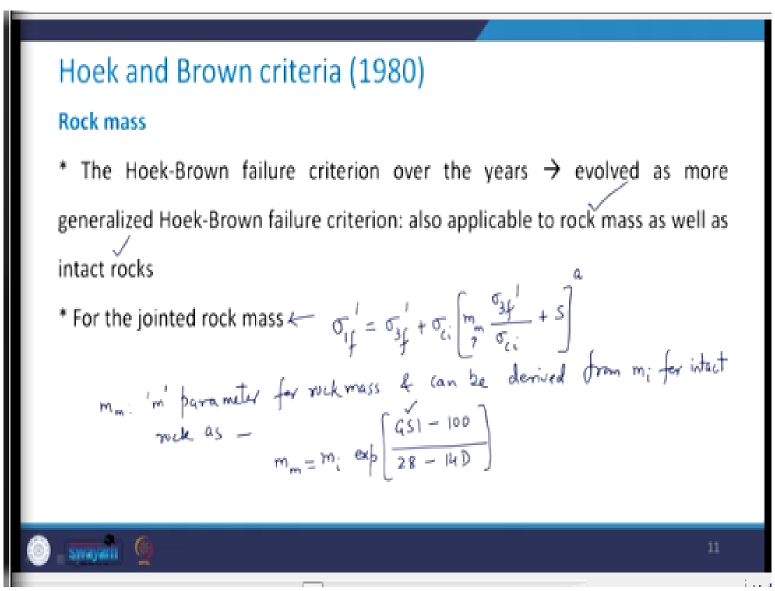

Now this was all about the intact rock, I mentioned to you that this criterion can be extended or applicable in case of the rock mass, let us see how this is done. So, over a period of many years this Hoek and Brown failure criterion has been evolved as more generalized Hoek and Brown failure criterion and is also applicable in case of the rock mass along with the intact rocks. So, for the jointed rock mass this is how the expression is written for the Hoek and Brown failure criterion,

$$\sigma'_{1f} = \sigma'_{3f} + \sigma_{ci}\left[m_m \frac{\sigma'_{3f}}{\sigma_{ci}} + s\right]^a$$

Now here this parameter m_m is nothing but the m parameter which we had in case of the intact rock as well for the rock mass. And because it is for rock mass therefore this subscript has been added here. And this can be derived from m_i which is for intact rock, how? See m_m is written as

$$m_m = m_i\, exp\left[\frac{GSI - 100}{28 - 14D}\right]$$

You remember this *GSI* we discussed when we were discussing the chapter on the classification of the rock mass, this is what is your geological strength index?

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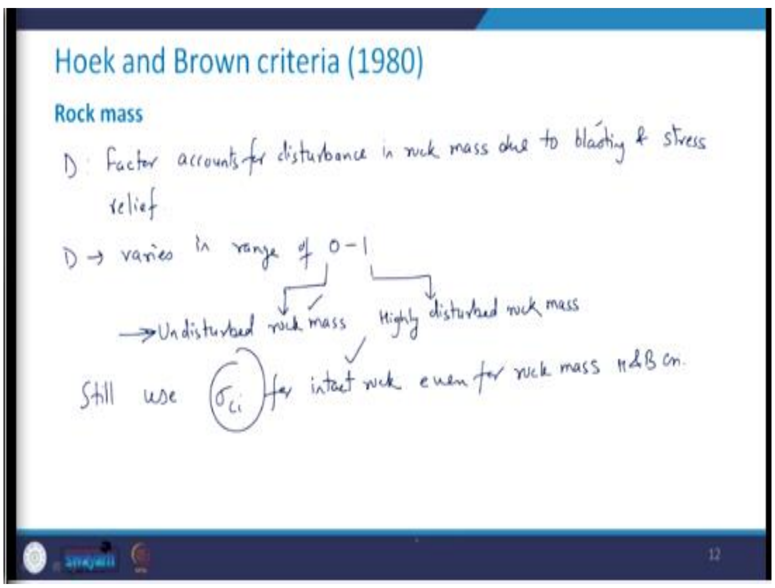

And this *D* factor is a factor which accounts for the disturbance in the rock mass. So, this *D* is the factor accounts for the disturbance in rock mass due to blasting and the stress relief. I will tell you little later that how this parameter is determined. For the time being just keep in mind that this *D* varies in the range of 0 to 1, 0 stands for the undisturbed rock mass while this 1 is for highly disturbed rock mass.

So, basically this undisturbed rock mass means there is no disturbance because of this blasting or there is no stress relief because of the excavation phenomena, so this is like kind of undisturbed situation. But then in this case you have seen that we still use σ_{ci} for intact rock even for the rock mass Hoek and Brown criterion. So, although it is for the rock mass but the term σ_{ci} which is there for the intact rock comes into picture.

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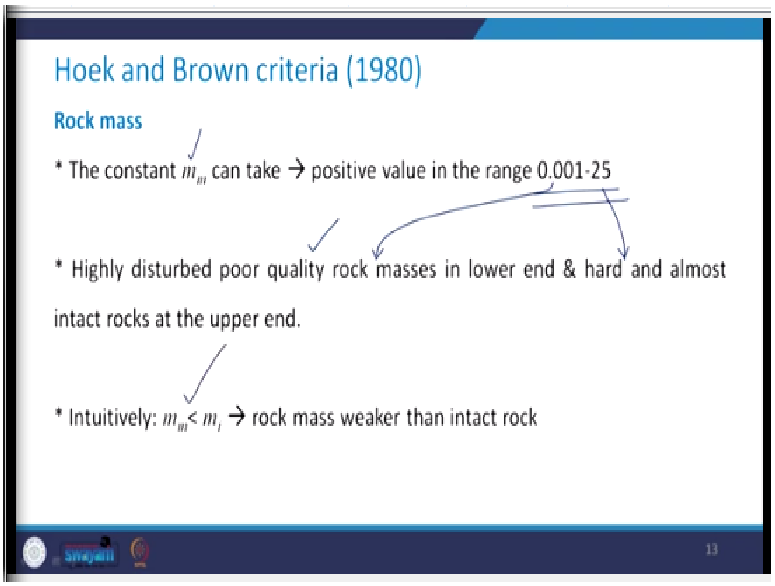

Now this constant m_m which is for the rock mass, it can take the positive value in the range of 0.001 to 25. Highly disturbed poor quality rock masses they will fall towards the lower range and hard and almost intact rock will be at the upper end. Intuitively we can say that because of the fact that rock mass they are weaker than the intact rock. Because of the presence of the discontinuity, intuitively m_m will be less than m_i.

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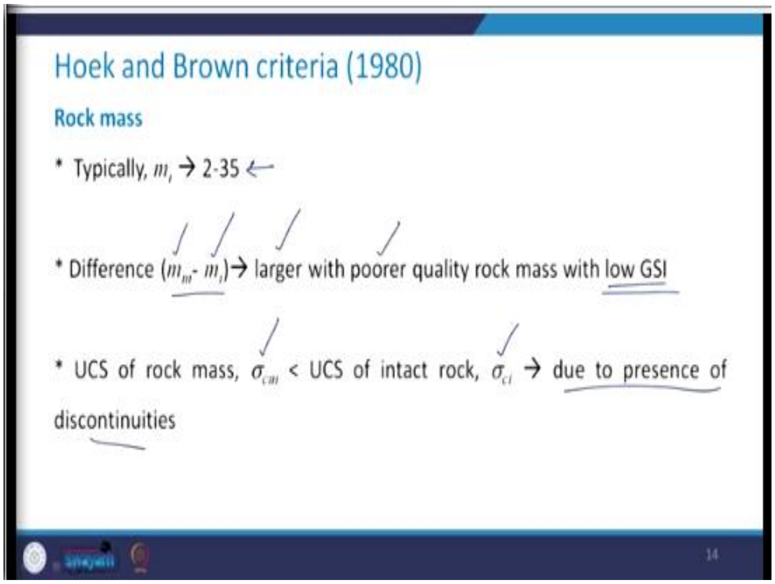

Typically, this m_i is going to be in the range of 2 to 35, this difference between parameter m for the rock mass and for the intact rock which is m_m - m_i is larger with the poorer quality rock mass, which has low value of GSI, that is geological strength index. Due to the presence of the discontinuities UCS of the rock mass which is represented by σ_{cm} will be less than the UCS of the intact rock which is represented by σ_{ci}.

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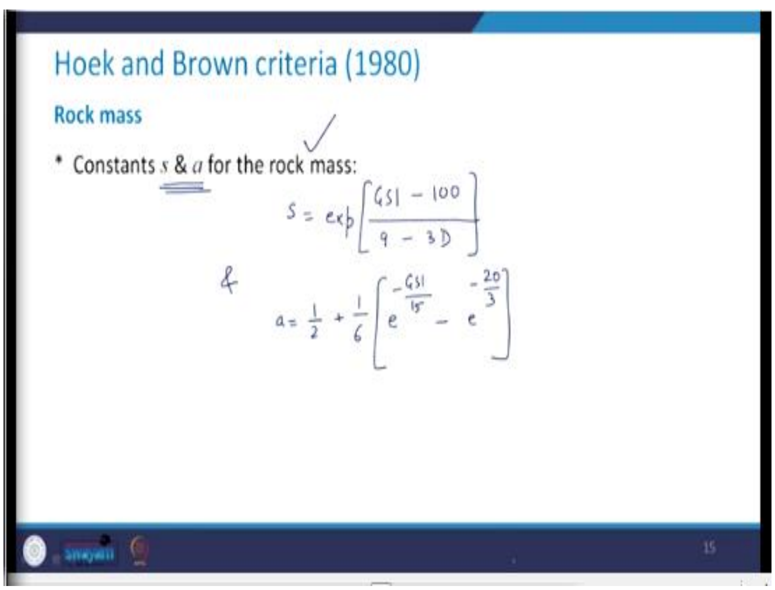

These constant s and a for the rock mass can typically be determined by these expressions which are given as s

$$s = exp\left[\frac{GSI - 100}{9 - 3D}\right]$$

$$a = \frac{1}{2} + \frac{1}{6}\left[e^{\frac{-GSI}{15}} - e^{\frac{-20}{3}}\right]$$

So, this is how the constants for the rock mass which are *s* and *a* these can be determined, and we have already seen how the parameter *m* for the rock mass can be determined.

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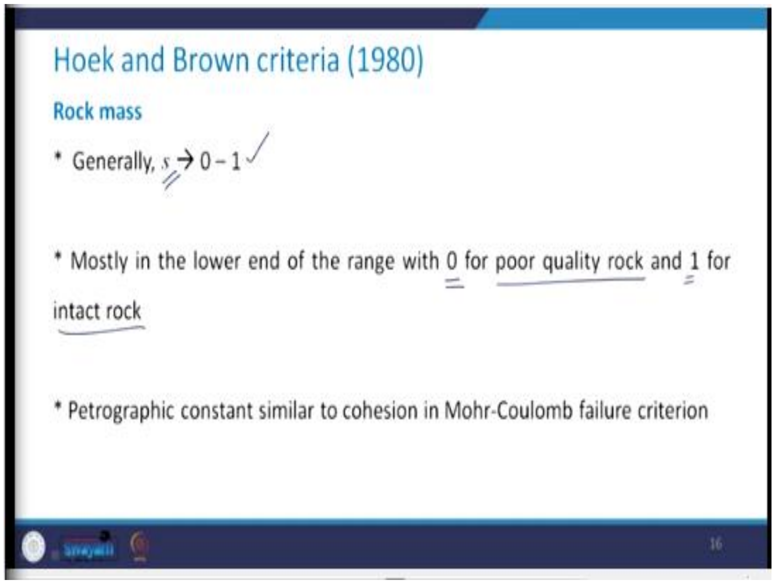

So, this generally this s for the rock mass it varies in the range of 0 to 1. And as explained earlier, mostly in the lower end of the range with 0 corresponds to the poor-quality rock and 1 for the intact rock. The petrographic constant which is *s* only is very much similar to the cohesion in Mohr coulomb failure criterion.

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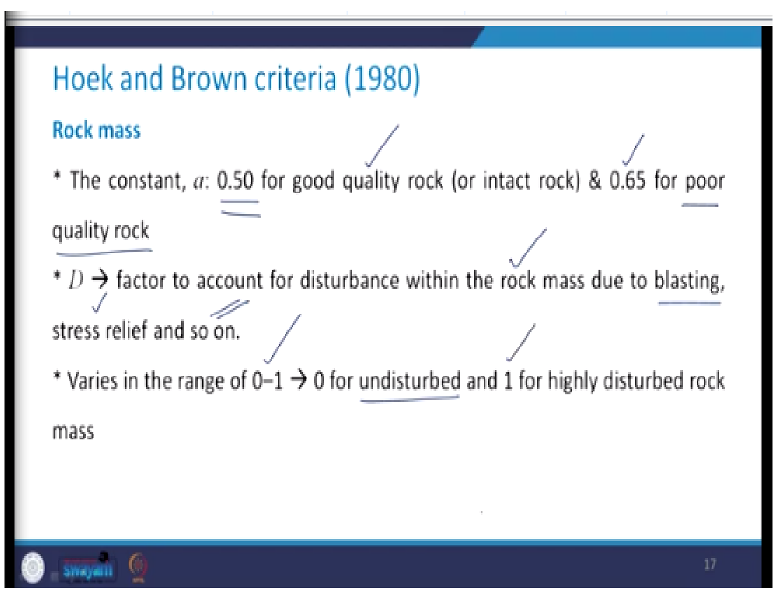

The constant *a* is 0.5 for the good quality rock or the intact rock, and it takes a value of 0.65 for poor quality of rock. As mentioned, *D* is a factor to account for the disturbance within the rock mass due to blasting stress relief and other such phenomena. And it varies in the range of 0 to 1, 0 for the undisturbed rock mass and 1 for the highly disturbed rock mass.

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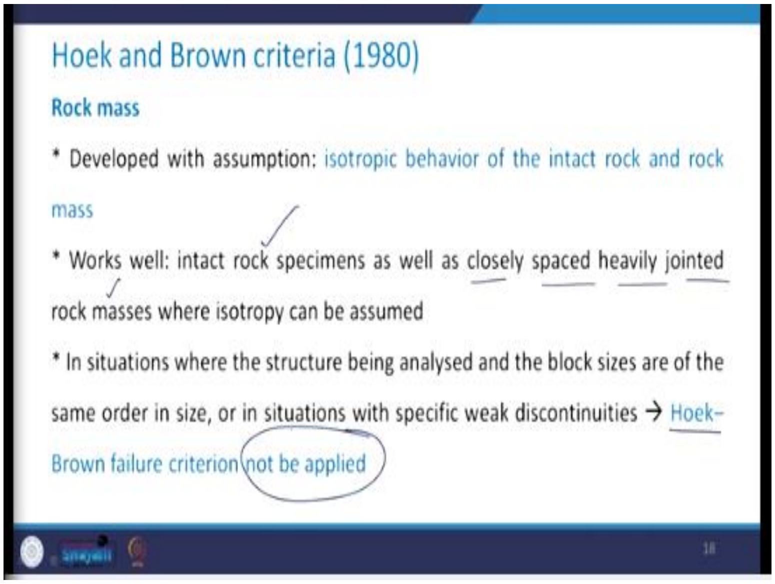

In case of the Hoek and Brown criterion, the assumption with it is development was that the intact rock and the rock mass they are going to behave in an isotropic manner. So, this criterion has been found to work well in case of the intact rock specimen. And the closely spaced heavily jointed rock masses where you can assume the isotropy, but, where you have the situations that the structure that you want to analyze and the block sizes which are there in the rock mass if they are of the same order in size.

Or if you have the situations with weak discontinuities, then in that case this criterion should not be applied. So, once again please keep in mind as long as the material or the rock mass or the intact rock can be assumed as an isotropic material Hoek and Brown criteria works well in such cases but in these cases, it is not applied.

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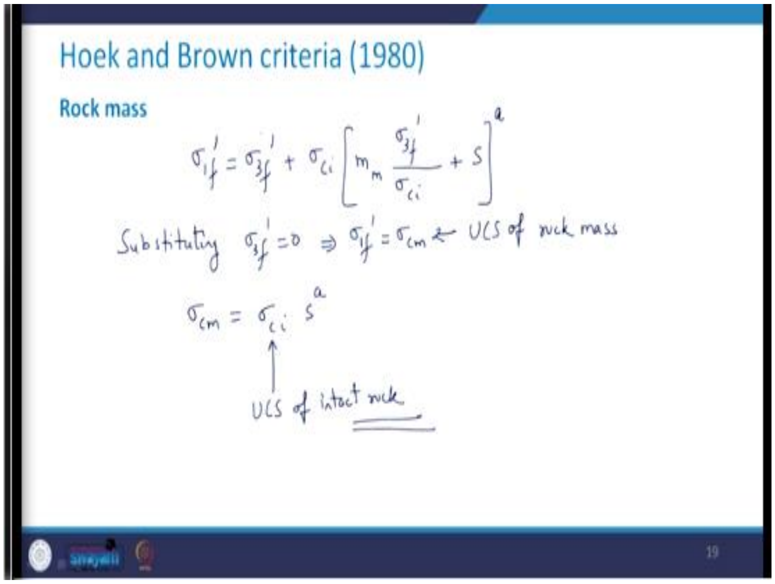

Therefore, for the rock mass the criteria as,

$$\sigma'_{1f} = \sigma'_{3f} + \sigma_{ci}\left[m_m \frac{\sigma'_{3f}}{\sigma_{ci}} + s\right]^a$$

So, what we are going to get is $\sigma_{1f'}$ will be σ_{cm} which is UCS of the rock mass. So, just substitute it here and see what you get, so this is what is going to be σ_{cm}, this term will become equal to 0, and this term will also become equal to 0.

So, ultimately you are going to get

$$\sigma_{cm} = \sigma_{ci} s^a$$

And we know this σ_{ci} is UCS of the intact rock. Now how to get the σ_{cm} from the other way?

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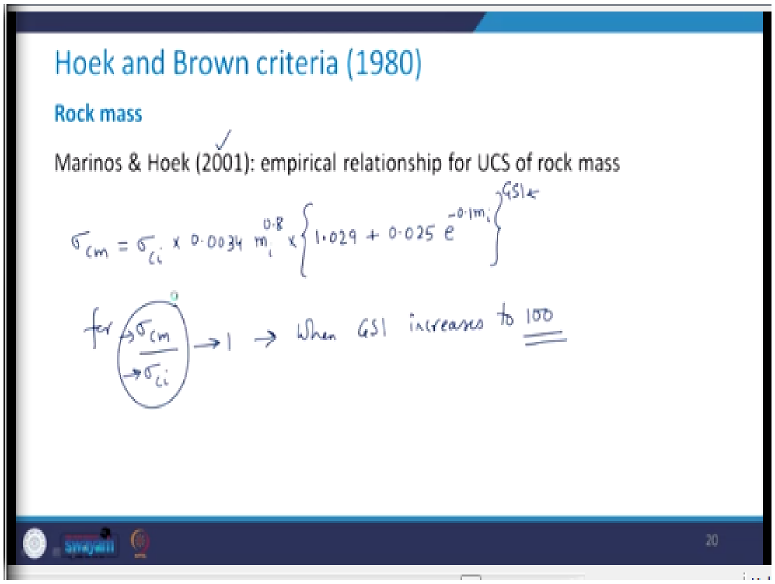

So, Marinos and Hoek in 2001 gave empirical relationship for the UCS of rock mass. And that is given as

$$\sigma_{cm} = \sigma_{ci} \times 0.0034\, m_i^{0.8} \times \{1.029 + 0.025e^{-0.1m_i}\}^{GSI}$$

So, for $\sigma_{cm}/\sigma_{ci} \rightarrow 1$, what does that mean? That the UCS of the intact rock and UCS of the rock mass they are equal to each other, this would correspond to the situation when this GSI increases to 100.

So, you just see that, substitute this here as 100 and then you will be getting that this ratio will be approximately equal to 1 or it will be tending to 1.

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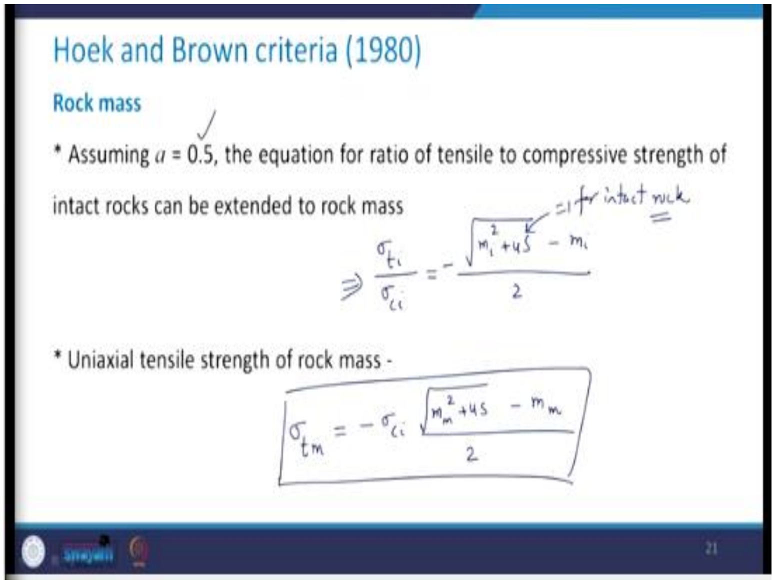

Then assuming a = 0.5, let us try to find out the equation for the ratio of tensile to compressive strength of the rock mass. So, first let us try to see what was that equation for the intact rock?

$$\left(\frac{\sigma_{ti}}{\sigma_{ci}}\right) = \frac{\sqrt{{m_i}^2 + 4s} - m_i}{2}$$

Obviously, this *s* would be equal to 1for the intact rock. But right now, I am not substituting it equal to 1 because I want to extend this expression in case of the rock mass.

So, that is how what we are going to get as a uniaxial tensile strength of the rock mass that I would represent

$$\sigma_{tm} = -\sigma_{ci}\frac{\sqrt{{m_m}^2 + 4s} - m_m}{2}$$

So, just take a note of it, here that this negative sign would be here. So, this is how that we are going to get the uniaxial tensile strength of the rock mass.

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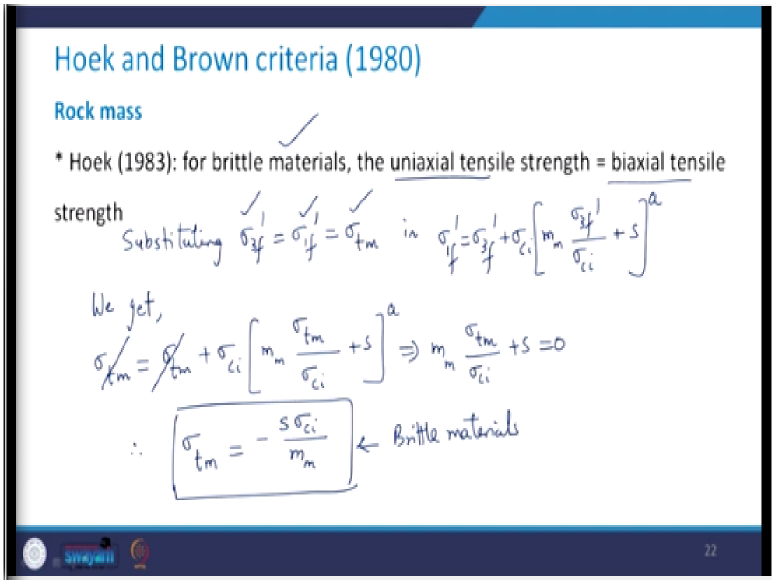

For the brittle materials, the uniaxial tensile strength is going to be equal to the biaxial tensile strength. And therefore, just substitute $\sigma_{3f'} = \sigma_{1f'} = \sigma_{tm}$, in this equation which was

$$\sigma'_{1f} = \sigma'_{3f} + \sigma_{ci}\left[m_m \frac{\sigma'_{3f}}{\sigma_{ci}} + s\right]^a$$

Just substitute it in this what we get is

$$\sigma_{tm} = \sigma_{tm} + \sigma_{ci}\left[m_m \frac{\sigma_{tm}}{\sigma_{ci}} + s\right]^a \Rightarrow m_m \frac{\sigma_{tm}}{\sigma_{ci}} + s = 0$$

$$\sigma_{tm} = -\frac{s\sigma_{ci}}{m_m}$$

So, this is how in such situation for brittle materials only, this is for brittle materials. You can get the tensile strength of the rock mass in such manner. So, today we discussed about the Hoek and Brown criteria which is one of the empirical criterions most commonly adopted in case of rock and rock masses.

We saw the different aspects related to this criterion. So, in the next class we will learn that how the parameters of Hoek and Brown criterion can be useful in order to get the Mohr coulomb failure criterion parameters under effective stress condition. Thank you very much.