Rock Engineering
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Lecture No -29
Mohr- Coulomb Failure Criterion, Coulomb-Navier Failure Criterion

Hello everyone. In the last class we discussed about Mohr-Coulomb failure criterion and I mentioned to you that how this is modified in the tensile region in order to make this failure criterion applicable to rocks. Today, we will learn few other things related to this failure criteria such as its physical interpretation and then we will learn about another failure criterion, which is a Coulomb-Navier failure criterion. So, in order to have the physical interpretation related to Mohr-Coulomb failure criterion;

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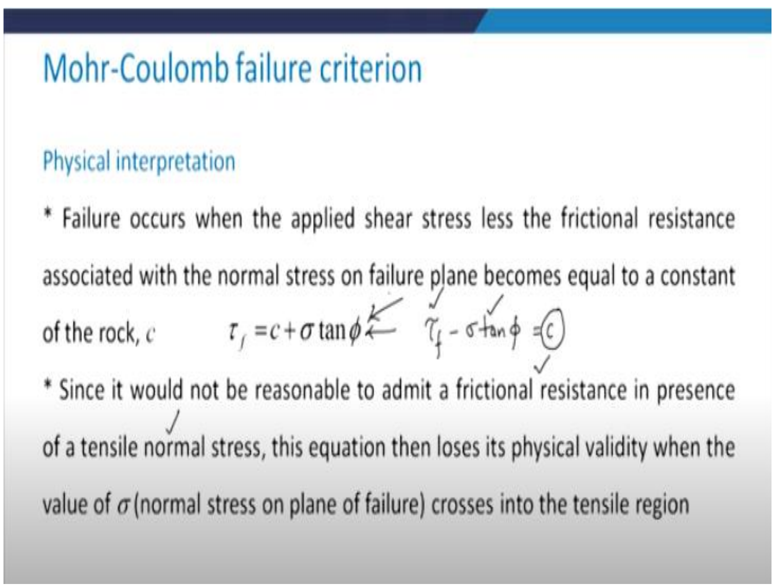

We saw that this is the equation for the Mohr-Coulomb failure criterion and the failure will occur when this quantity

$$\tau_f - \sigma \tan\phi = c$$

This means that the failure occurs when the applied shear stress less the frictional resistance which is associated with the normal stress on the failure plane becomes equal to the constant of the rock which is *c*. Since it would not be reasonable to admit a frictional resistance in the presence of tensile normal stress.

This equation loses its physical validity when the value of σ which is the normal stress on the plane of failure goes into the tensile region. So, we saw in the previous class also that this equation will work fine as long as you have this normal stress σ in the compressive region. The moment it goes to the tensile region or the moment its value becomes negative this equation which is,

$$\tau_f - \sigma \tan \phi = c$$

it loses its physical validity.

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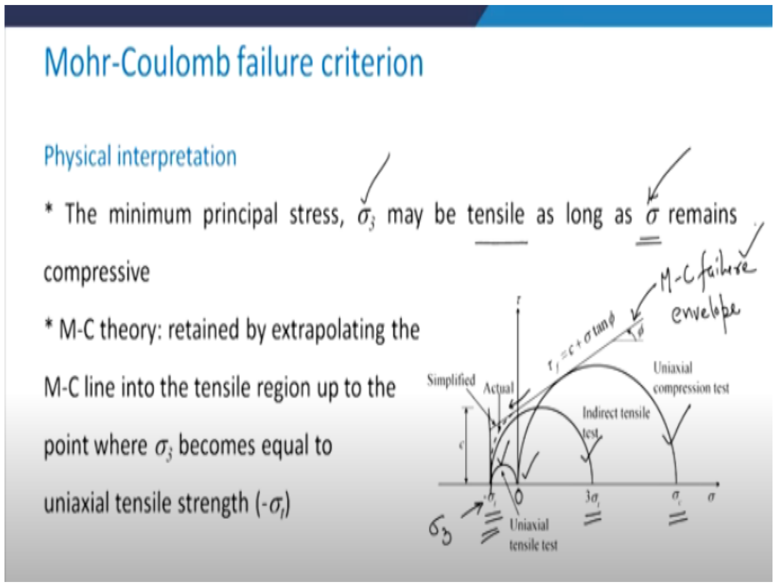

The minimum principal stress which is σ_3 it may be tensile as long as this σ remains compressive. This σ is the normal stress on the failure plane. Take a look here in this figure that this Mohr circle is corresponding to uniaxial compression test. This corresponds to the indirect tensile test which was Brazilian test and this small one is for uniaxial tensile test and here you can see that σ_1 here for this small circle is 0 and σ_3 becomes equal to $-\sigma_t$.

However, for the indirect tensile test the major and minor principal stresses are 3 σ_t and σ_t. In case of the uniaxial compression test the major principal stress is given by σ_c and the minor principal stress is going to be equal to 0. So, this Mohr-Coulomb theory it was retained by extrapolating the Mohr-Coulomb line into the tensile region up to a point where σ_3 becomes equal to uniaxial tensile strength.

What does this mean that we have this as Mohr-Coulomb failure envelope. This was extrapolated in this direction in the tensile region and then we put a cut off here where this is σ_3 value. So, this is how the Mohr-Coulomb failure envelope was extrapolated in the tensile region.

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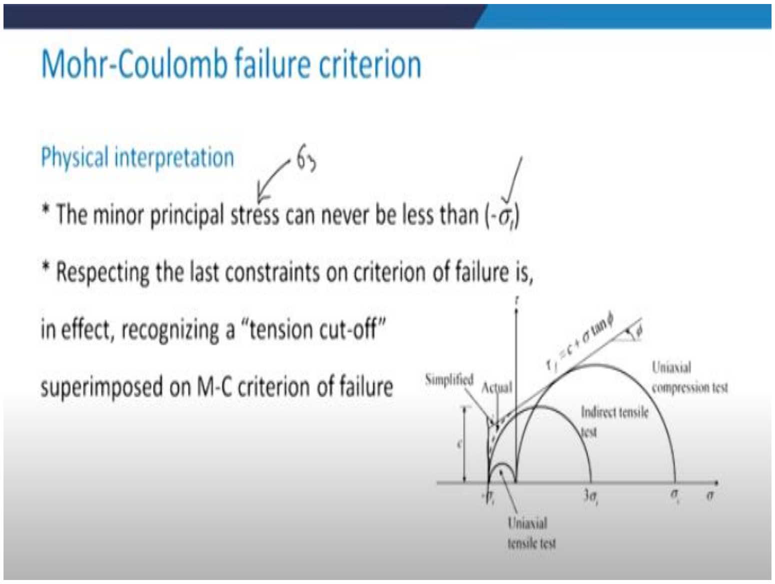

The minor principle stress which is σ_3, it can never be less than $-\sigma_t$ and therefore we need to respect this last constraint on the failure criterion and in order to do this we need to recognize a tension cut off which is given by this line. And this cut off is superimposed on the Mohr-Coulomb criterion and therefore what you get is this vertical line and then this failure envelop.

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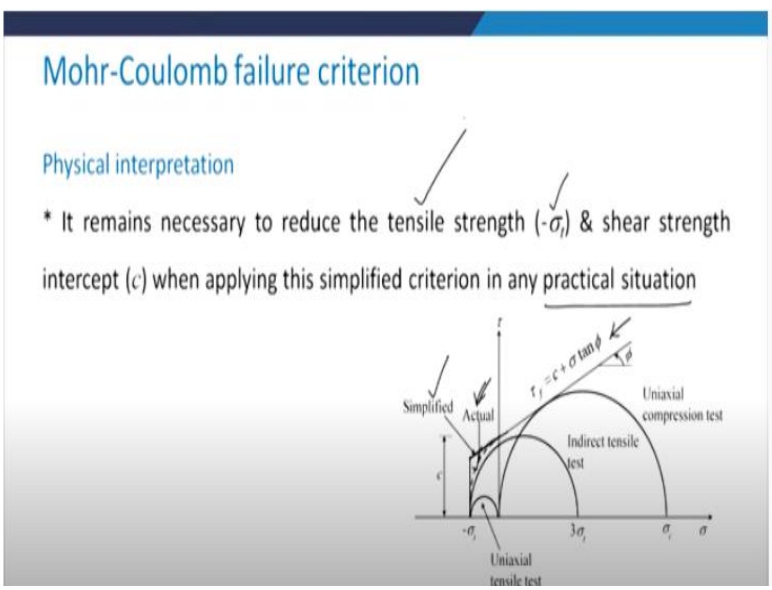

Now it remains necessary to reduce the tensile strength and the shear strength intercept when we apply this simplified criterion in any practical situation. Why is it so? You can see here that this dotted line gives us the actual value of the criterion or the actual value of the strength characteristic. However, by extrapolating the Mohr-Coulomb failure criterion we have got this simplified version.

Now, if you take a look, the shear strength characteristic by this simplified version will always be higher than the actual one and therefore when we use it for any practical situation, we need to reduce the tensile strength and the shear strength intercept which is *c*.

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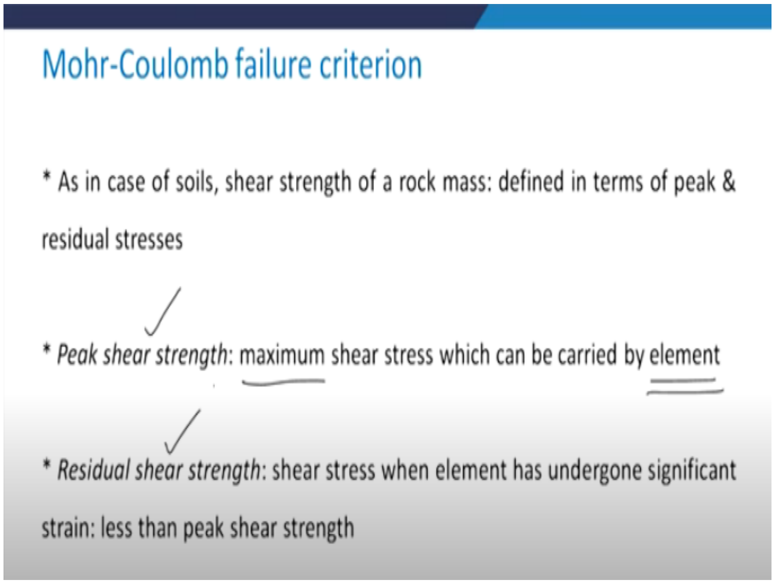

As in the case of soils shear strength of the rock mass is defined in terms of peak and the residual stresses. You all know that the peak shear strength is the maximum shear stress which can be carried by the element, when I say element, this means that the element of the rock or soil. The next one is the residual shear strength which is the shear stress when this rock element has undergone significant strain. This residual shear strength is always less than the peak shear strength. Take a look at this picture.

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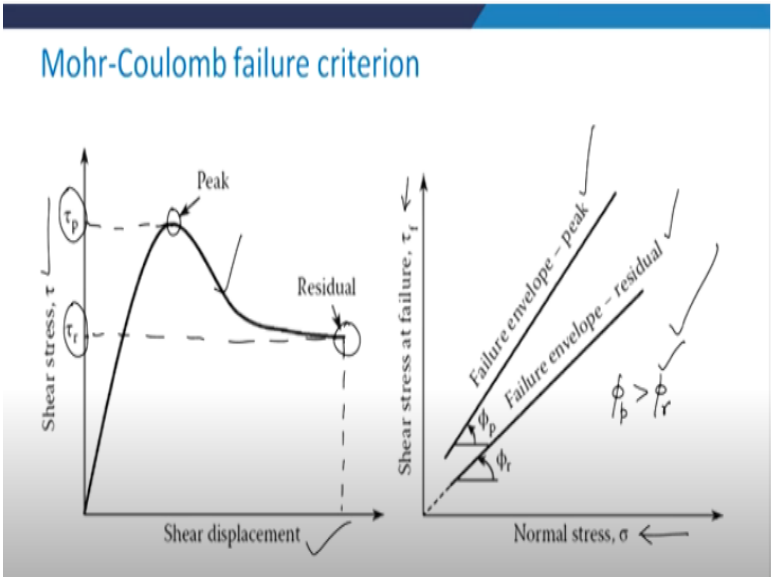

Here on the y axis, you have the shear stress and on x axis you have the shear displacement and you can draw the stress displacement curve like this. So, here wherever you get this maximum value, this is going to give you the peak value of the shear stress. However here at the end when you have a large value of the shear displacement here corresponding to this value you have the residual shear strength, which is represented as τ_r.

Look at this figure, where the shear stress at failure has been plotted with the normal stress. So, you get two plots or two envelopes. One corresponds to the peak values the next one corresponds to the residual value. So, you can see here that $\phi_p > \phi_r$. This means that the frictional characteristic corresponding to the peak condition is more than the friction angle corresponding to the residual condition.

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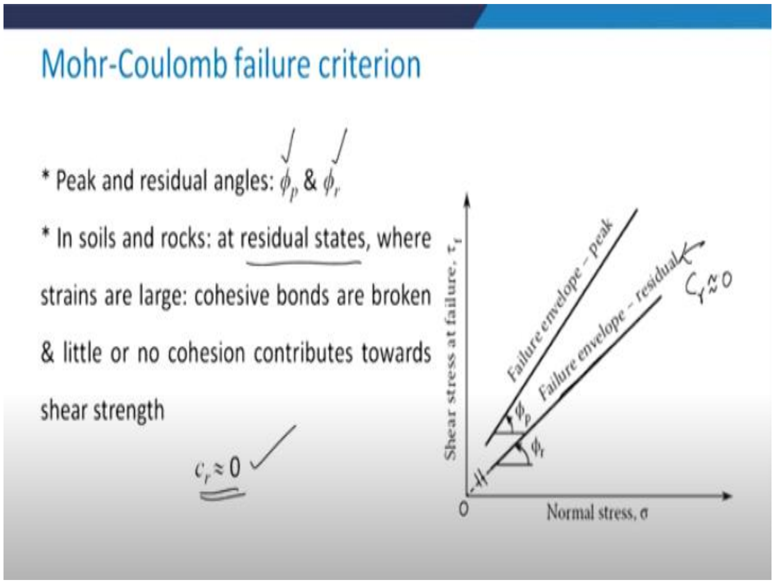

Now peak and residual angles I mentioned, that these are denoted by ϕ_p and ϕ_r respectively. In soils and rocks at the residual states where you have large strains, cohesive bonds are broken and therefore little or no cohesion contributes towards this shear strength. That is the reason that the cohesion at the residual condition is approximately equal to zero. So, you just extrapolate this failure envelope for the residual condition just extrapolated which is shown by this dotted line and you will see that it is intersecting at the origin O.

This means that the cohesion intercept in the residual condition is equal to zero. That is c_r will be approximately equal to zero in the residual state.

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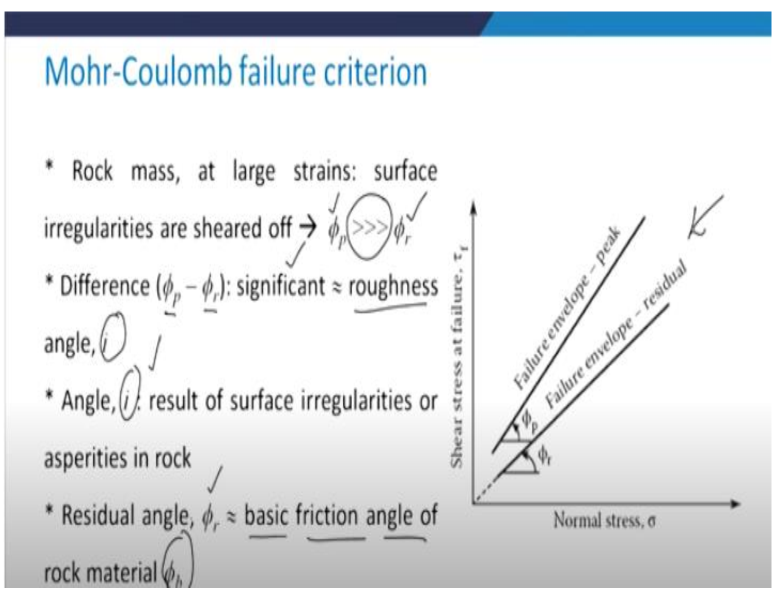

Rock mass, what happens to these at large strains? All the surface irregularities would be sheared off and therefore this peak friction angle will be much larger than the residual friction angle which is evident from this figure as well. So, the difference between peak and the residual friction angle is going to be significant which will be approximately equal to the roughness angle *i*. This angle *i* is the result of surface irregularities or asperities in rock.

When we discussed about the classification of the jointed rocks or the rock mass, we had some discussion about this angle *i*. So, this is the same as what we discussed earlier. The residual angle will be approximately equal to the basic friction angle of the rock material ϕ_b. When I say rock material, means I am referring to the intact rock. As far as intact rocks are concerned these are relatively impervious.

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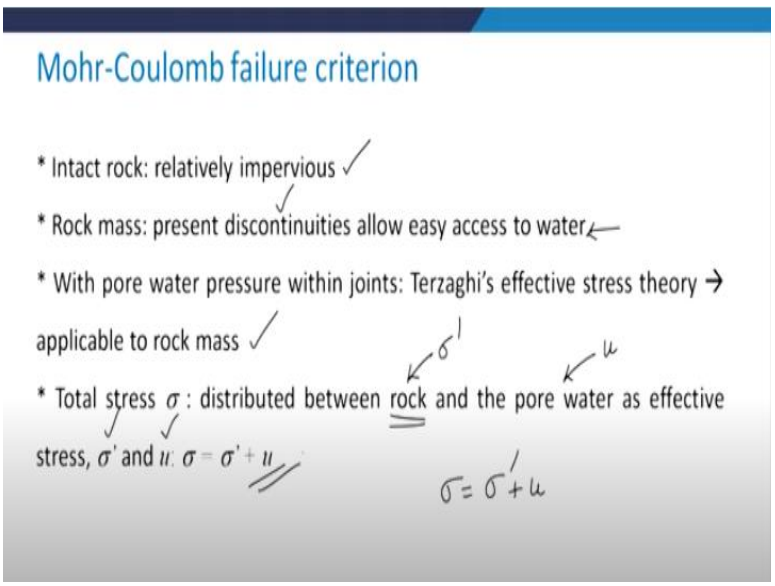

We had the discussion on this aspect as well that because of the absence of any kind of discontinuities in the intact rock these are relatively impervious. However, in case of the rock mass due to the presence of discontinuities there is easy access to the water. Because these discontinuities they allow the water to flow through them. Now what happens because of that? When the water is there within the joints it exerts pore water pressure and therefore Terzaghi's effective stress theory becomes applicable to the rock mass in such situations.

So, what this theory suggests? That is the total stress is distributed between the rock and the pore water as effective stress σ' and u. That means whatever is distributed with rock this is σ' and the pore water pressure is u. And you all know that what is this effective stress theory that is the total stress is equal to effective stress plus the pore water pressure. So, this will be applicable to the rock mass.

Now once this is applicable then we need to represent the Mohr-Coulomb criterion to rock mass in terms of effective stresses.

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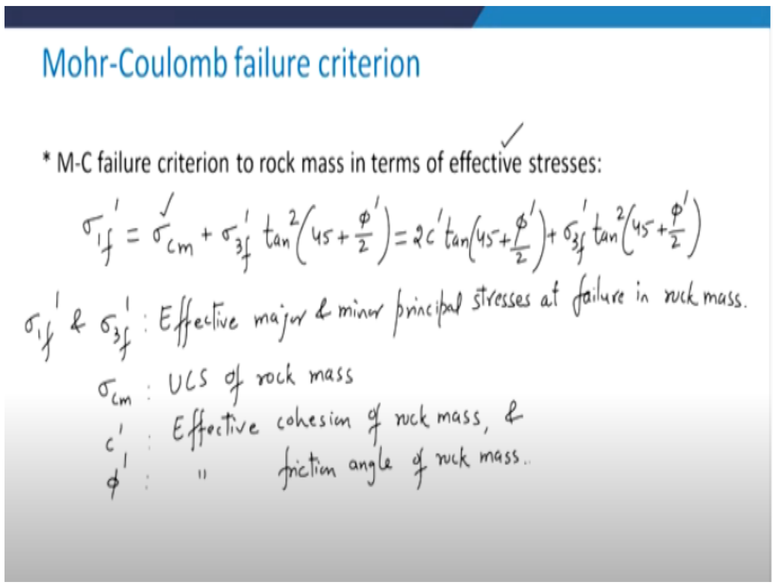

So, let us see that how this can be done. This is written as

$$\sigma'_{1f} = \sigma_{cm} + \sigma'_{3f} tan^2\left(45 + \frac{\phi'}{2}\right) = 2c' \tan\left(45 + \frac{\phi'}{2}\right) + \sigma'_{3f} tan^2\left(45 + \frac{\phi'}{2}\right)$$

σ'_{1f} and σ'_{3f}: Effective major and minor principal stresses at failure in rock mass.

σ_{cm}: UCS of rock mass

c': Effective cohesion of rock mass

ϕ': Effective friction angle of rock mass

So, this is how Mohr-Coulomb failure criterion for the rock mass in terms of effective stresses can be written.

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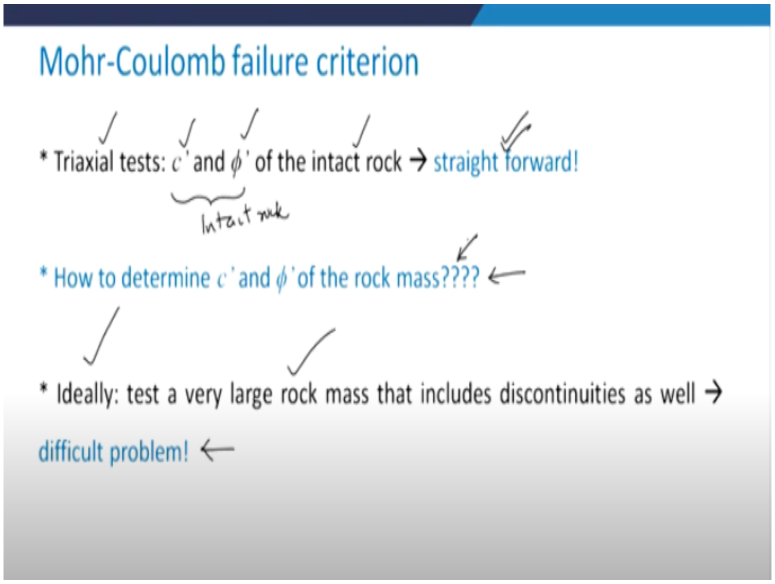

The question is how to determine the parameters c and ϕ in case of the effective stress condition for the rock mass. As far as c' and ϕ' for the intact rock is concerned, we can straight forward conduct the triaxial test and analyze the data of distance in order to get this c' and ϕ' for the intact rock. This is pretty straight forward. The question is how to determine these for the rock mass? So, in order to do that ideally what we should do is we should carry out the test on a very large rock mass which includes discontinuity as well.

But to conduct such a test is very, very difficult or it may not be really possible to conduct the triaxial under such ideal conditions. So, this is a difficult problem. The question is then how to determine this?

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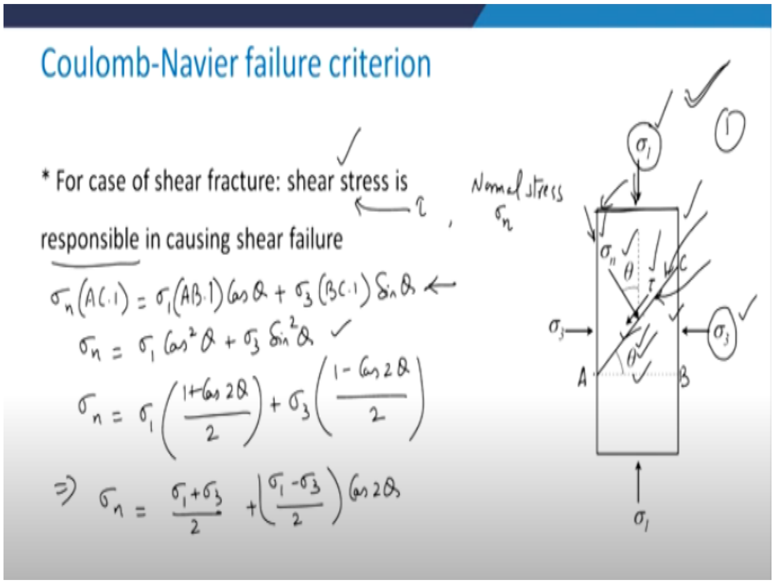

The answer you will get, may be after a few classes when I will discuss with you the Hook and Brown criteria. There we will see that how the parameters of Hook and Brown criterion can be used to determine the c prime and phi prime for the rock mass. So, this was all about that. I wanted to discuss with you as far as Mohr-Coulomb failure criterion was concerned. Now to start with now let us discuss the other failure criterion which is the Coulomb-Navier failure criterion.

This is more or less similar to the Mohr-Coulomb failure criterion however the process of derivation of the model for this failure criterion is little bit different. So, we will see the detailed derivation now. In this case for the shear fracture to occur in the rock mass the shear stress is responsible. So, here this is a specimen where this is being subjected to the stress condition in such a manner that the major principal stress is σ_1 and minor principal stress is σ_3. There is a joint which is this one which is subtending an angle of θ with the horizontal.

And on this plane, there is going to be shear stress and normal stress. This stress we are calling as τ and the normal stress is σ_n. So, we can write the force equilibrium equations in the direction parallel and perpendicular to the plane of failure, which is this plane. How? Let us see. So, here we have that σ_n is acting. So, say that this is a point A, this is point B and this point is C.

So, we will have σ_1 into AC into one that is equal to σ_1 and this σ_1 is acting on this side that is this one which has AB so that is going to be AB into 1 and it is acting in the vertical direction. So, I need to find out the component of this in this σ_n direction. And because this angle is θ this angle will also be equal to theta. So, the component of this σ_1 in this σ_n direction is going to be

$$\sigma_n(AC.1) = \sigma_1(AB.1)\cos\theta + \sigma_3(BC.1)\sin\theta$$

$$\sigma_n = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$$

$$\sigma_n = \sigma_1\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_3\left(\frac{1-\cos 2\theta}{2}\right)$$

$$\sigma_n = \left(\frac{\sigma_1+\sigma_3}{2}\right) + \left(\frac{\sigma_1-\sigma_3}{2}\right)(\cos 2\theta)$$

Why we are multiplying it by 1 is that the dimension in the plane perpendicular to the plane of this screen that is being taken as 1.

So, this is the equation that we wrote that is the force equilibrium equation which was in the direction normal to the failure plane. Now similarly we can write the force equilibrium equation parallel to the plane of failure which is this. So, see here how I will write that.

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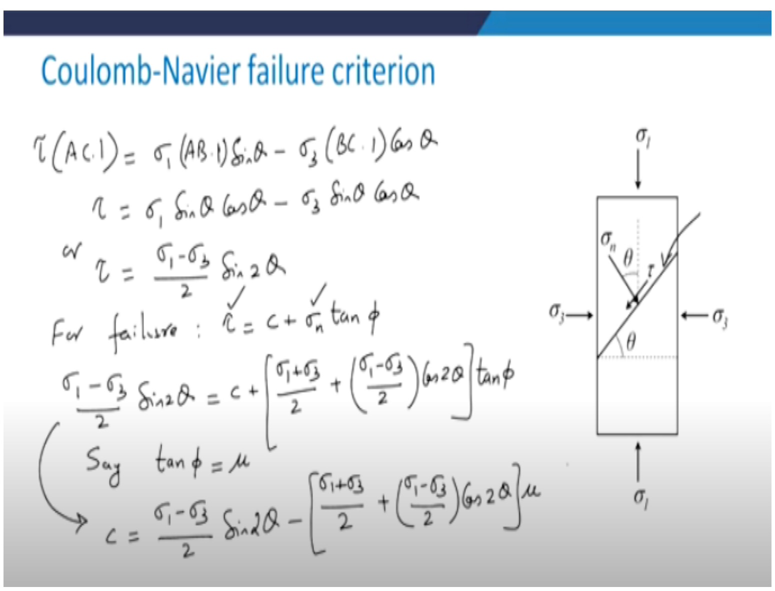

$$\tau(AC.1) = \sigma_1(AB.1)\sin\theta - \sigma_3(BC.1)\cos\theta$$

$$\tau = \sigma_1 \sin\theta \cos\theta - \sigma_3 \sin\theta \cos\theta$$

$$\tau = \left(\frac{\sigma_1 - \sigma_3}{2}\right)(\sin 2\theta)$$

$$For\ failure: \tau = c + \sigma_n \tan\phi$$

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right)(\sin 2\theta) = c + \left[\left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right)(\cos 2\theta)\right]\tan\phi$$

$$\text{Say}, \tan\phi = \mu$$

$$c = \left(\frac{\sigma_1 - \sigma_3}{2}\right)(\sin 2\theta) - \left[\left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right)(\cos 2\theta)\right]\mu$$

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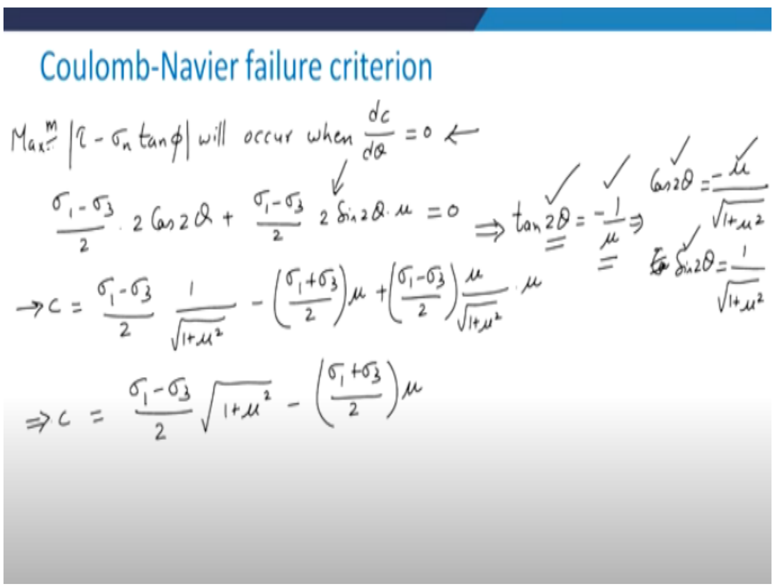

So, when this maximum value will occur? So, I take a mod of this quantity.

$$Maximum\ of\ |\tau - \sigma_n \tan\phi|\ will\ occur\ when\ \frac{dc}{d\theta} = 0$$

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) 2(\cos 2\theta) + \left[\left(\frac{\sigma_1 - \sigma_3}{2}\right) 2(\sin 2\theta)\right]\mu = 0$$

$$\tan 2\theta = -\frac{1}{\mu}$$

$$\cos 2\theta = -\frac{\mu}{\sqrt{1+\mu^2}}$$

$$\sin 2\theta = \frac{1}{\sqrt{1+\mu^2}}$$

$$c = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\frac{1}{\sqrt{1+\mu^2}} - \left(\frac{\sigma_1 + \sigma_3}{2}\right)\mu + \left(\frac{\sigma_1 - \sigma_3}{2}\right)\frac{\mu}{\sqrt{1+\mu^2}}\mu$$

$$c = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sqrt{1+\mu^2} - \left(\frac{\sigma_1 + \sigma_3}{2}\right)\mu$$

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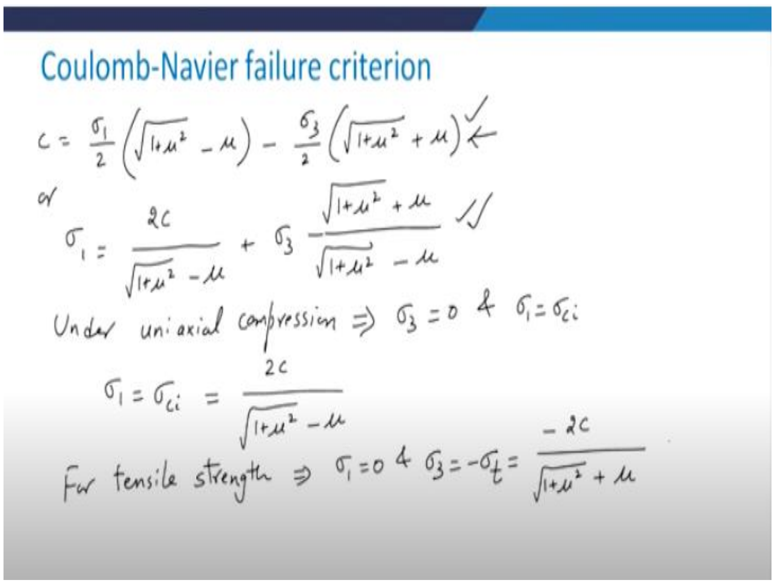

$$c = \frac{\sigma_1}{2}\left(\sqrt{1+\mu^2} - \mu\right) - \frac{\sigma_3}{2}\left(\sqrt{1+\mu^2} + \mu\right)$$

$$\sigma_1 = \frac{2c}{\sqrt{1+\mu^2} - \mu} + \sigma_3 \frac{\sqrt{1+\mu^2} + \mu}{\sqrt{1+\mu^2} - \mu}$$

Under uniaxial Compression→ $\sigma_3 = 0 \; and \; \sigma_1 = \sigma_{ci}$

$$\sigma_1 = \sigma_{ci} = \frac{2c}{\sqrt{1+\mu^2} - \mu}$$

For tensile strength →

$$\sigma_1 = 0 \; and \; \sigma_3 = -\sigma_t = \frac{-2c}{\sqrt{1+\mu^2} + \mu}$$

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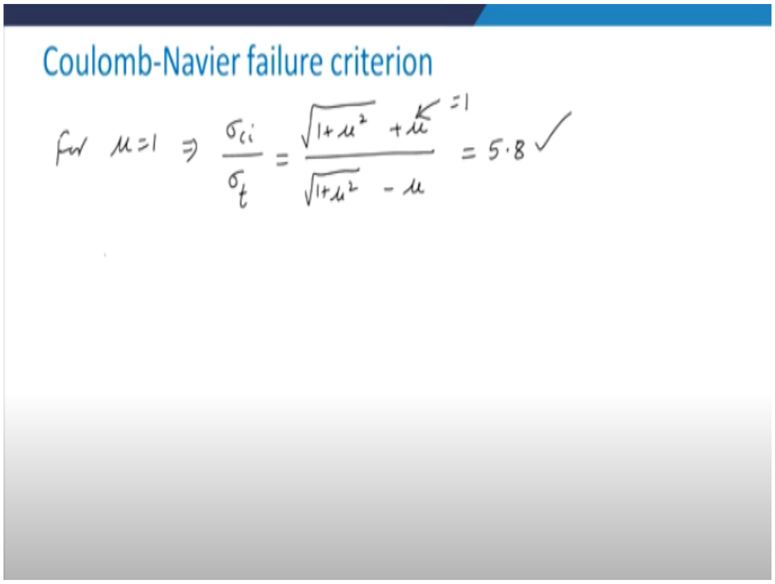

For μ=1,

$$\frac{\sigma_{ci}}{\sigma_t} = \frac{\sqrt{1+\mu^2}+\mu}{\sqrt{1+\mu^2}-\mu} = 5.8$$

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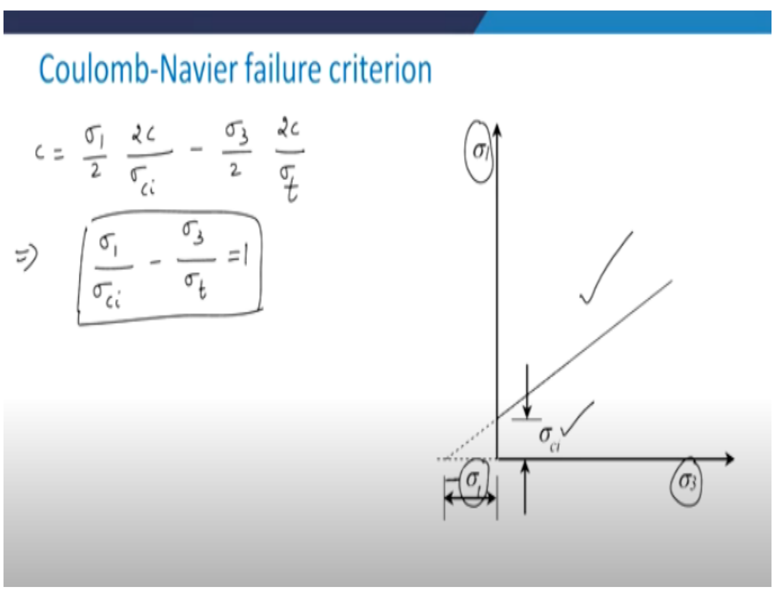

So, whatever are the equations that we derived just now if you just take a look what we are going to get is:

$$c = \frac{\sigma_1}{2}\frac{2c}{\sigma_{ci}} - \frac{\sigma_3}{2}\frac{2c}{\sigma_t}$$

$$\frac{\sigma_1}{\sigma_{ci}} - \frac{\sigma_3}{\sigma_t} = 1$$

That is going to be equal to 1. And this is what which has been plotted here in this figure which is a plot between σ_1 and σ_3 on the σ_1 axis. You have σ_{ci} as the intercept and it is positive and on the σ_3 axis you have σ_t as the intercept which is with the negative sign.

So, this is how the derivation of Coulomb-Navier failure criterion can be done. So, in today's class we discussed about the physical interpretation of Mohr-Coulomb failure criterion and then we saw that how the expression for Coulomb-Navier failure criterion can be obtained. This is very much similar to the Mohr-Coulomb failure criterion. However, during the derivation little bit of difference is there. So, in the next class we will learn about the concept of instantaneous c and ϕ, thank you very much.