Rock Engineering
Prof. Priti Maheshwari
Department of Civil Engineering
Indian Institute of Technology, Roorkee

Lecture No -28
Mohr- Coulomb Failure Criterion

Hello everyone. In the previous class, we discussed about the plain stress loading, axisymmetric loading followed by the discussion on the effect of confining pressure on the strength characteristic of the rocks and then we discussed about Mohr's failure theory. So, now the next step is Mohr-Coulomb failure criterion. Mohr's failure theory only mentioned that there is a relationship between shear stress and the normal stress on the failure plain.

Mohr-coulomb failure criterion kind of established a relationship between this shear stress and the normal stress. So, we will today learn that how the Mohr-Coulomb failure criterion can be applicable in case of the rocks. What all are the deviation or what all are the modification that one needs to make in the Mohr-Coulomb criterion which was applicable in case of the soil, so that the same criterion is applicable in case of rocks. So, we are going to discuss all these things in detail today.

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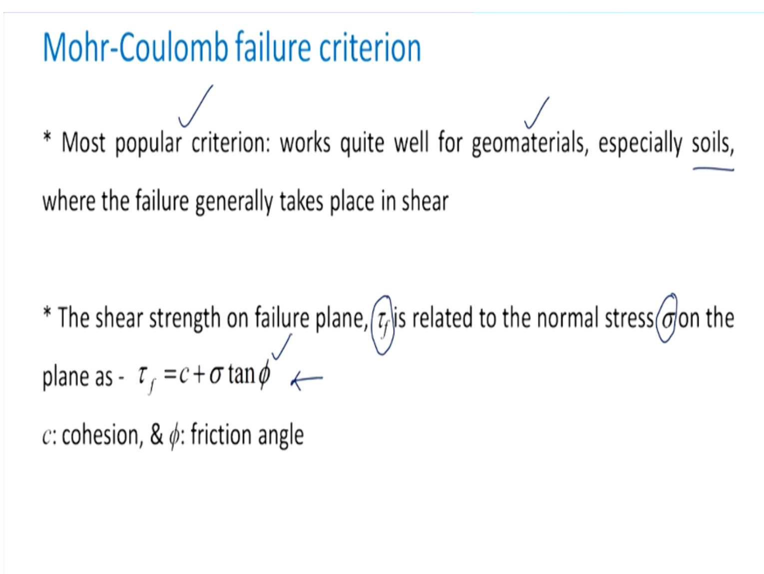

We all know that the Mohr-Coulomb failure criterion is one of the most popular criteria and it works very well for the geomaterials, specially for the soils, where the failure generally takes place in shear. As per Mohr-Coulomb failure criterion, the shear is strength on the failure plane, which is represented by

$$\tau_f = c + \sigma \tan \phi$$

On the plane as this expression, where c is the cohesion and ϕ is the angle of friction or angle of internal friction.

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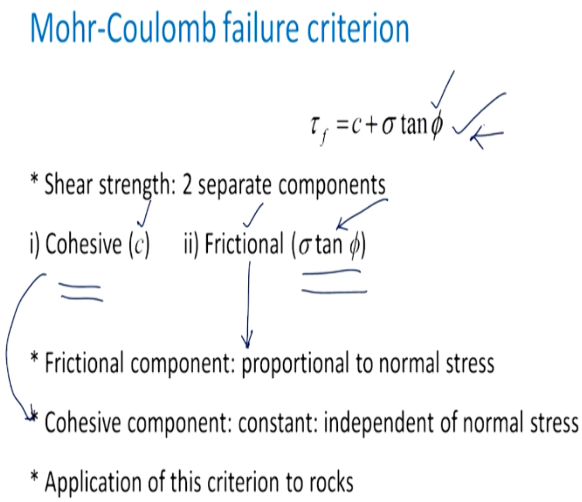

Now this equation, it has two separate components, one first component is the cohesive one which is represented by c and the second one is the frictional component. Which is given by $\sigma \tan \phi$, take a look at this frictional component which is proportional to the normal stress. So, frictional component is equal to $\sigma \tan \phi$, so we can say it is proportional to the normal stress, however this cohesive component is constant and it is independent of the normal stress.

We all know that this criterion is quite well applicable in case of the soils. Let us see whether this can be applied in case of rocks or not in this particular manner itself. If it is not applicable in this form, to the case of rocks, then we will see that what should be the modification be made to this criterion, so that it can be applicable in case of rocks.

(Refer Slide Time: 04:20)

Mohr-Coulomb failure criterion

* Uniaxial compressive strength test: very common on soils, rocks and concrete

* UCS → σ_c

* When the specimen fails: $\sigma_1 = \sigma_c$ & $\sigma_3 = 0$

* Direct tensile test on rocks: difficult to conduct in view of requirement of special shaped specimen, non-uniformity of stresses etc.

* Theoretically, for a uniaxial tensile strength test: tensile strength → σ_t

* At failure: $\sigma_1 = 0$ & $\sigma_3 = \sigma_t$

Uniaxial compressive strength tests, they are very common in case of soils, rocks as well as concrete. Uniaxial compressive strength is represented by σ_c. Now when the specimen fails that is at the failure state, the applied load σ_1 is going to give me the UCS of these specimens and since it is the uniaxial compressive strength test all around pressure or the confining pressure is going to be equal to 0.

And therefore, σ_3 will be equal to 0 so for the UCS test. The state of stress in terms of σ_1 and σ_3 is going to be σ_c and 0 respectively. We have discussed this in some of the earlier classes that in case of the determination of the tensile strength of rocks one needs to go for the indirect methods like Brazilian tests, what was the reason behind that? The reason was when we go for the direct tensile test on rocks.

There were many difficulties which one faces. The first one was that it was very difficult to conduct the tests in view of the requirement of this special shaped specimen, because when I am saying that it is the direct tensile test, so the specimen should be subjected to the direct pull or the direct tensile force. For such case the gripping of this specimen becomes very, very crucial.

And one needs to go for the special shaped specimen, which may not be that easy to make in case of rocks. Further even if you make the special shaped specimen, but then also add the contact point of the loading platen, there is going to be very high stress concentration which will cause the non-uniformity of these stresses in the whole specimen and therefore it was told that direct tensile test on rocks, they are not that feasible to conduct.

But, say theoretically if we are conducting the direct uniaxial tensile strength test in that case, add the failure what we will get is the tensile strength say σ_t. So, remember that we are talking about our theoretical condition. So, at failure what is going to be is that you will have σ_1 equal to 0 and σ_3 is going to be equal to the tensile strength. Now you take a note of this fact here that I am putting a negative sign here, negative sign.

Why? Just to show that it is tensile in nature. So, what we are going to do is that, to follow the conventions that compression is being presented in positive sign and tension with a negative sign. So, in the two cases that is uniaxial tensile strength test, which is a theoretical one and uniaxial compressive strength tests one can draw the Mohr's circle at failure. Take a look here.

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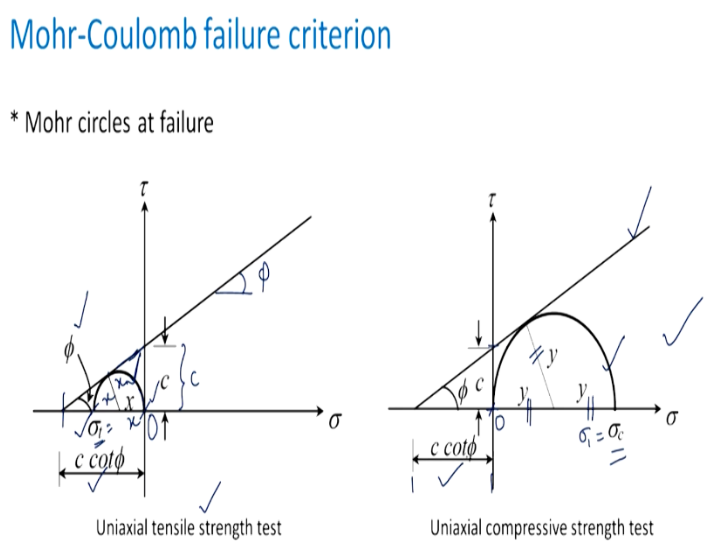

The first figure gives us the idea about the uniaxial tensile strength test. So, whatever is the strength that we have is σ_t. This point is 0, so it is on this side of 0, so it is negative. And that is the reason that we wrote that σ_1 will be equal to 0 and σ_3 is going to be equal to minus σ_t, because you know that by definition σ_1 is the representative of the major principal stress and σ_3 is the representation for the minor principal stress.

Now, I assume that the Mohr-Coulomb failure criterion, which was

$$\tau_f = c + \sigma \tan\phi$$

is applicable in the tensile zone also. Then we can extrapolate it in this direction like this and therefore, this will be the tangent to this circle, which is shown here this circle. Now the radius of this circle is going to be $\sigma_t/2$ because this total is σ_t. This is x, this is also x and since this distance is c.

This angle is ϕ as shown here this distance from this point to this point by simple trigonometry is going to be $c \cot\phi$. Now take a look at the other figure which is for the UCS test. So, in case of the UCS test D value of σ_1 is going to be equal to σ_c which is the uniaxial compressive strength and σ_3 here which is equal to 0. Now, obviously because it is only positive side.

So, Mohr-Coulomb failure envelope is valid in this case, which will be tangent to the more circle at failure representing this state of stress. So, this distance, this distance and this distance, they are going to be same as the radius of the Mohr's circle. Similarly, from the simple trigonometry if this distance is c, this distance from here to here is going to be $c \cot\phi$.

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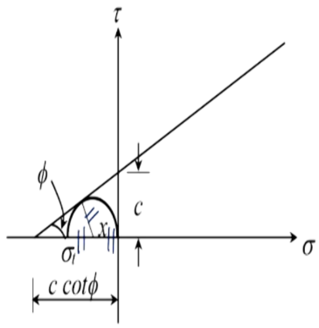
Mohr-Coulomb failure criterion

* Assumption: M-C criterion valid also in tensile region



$$x = \frac{c \cos\phi}{1+\sin\phi}$$

$$\sigma_t = 2x = \frac{2c\cos\phi}{1+\sin\phi}$$

Now let us have a look once again on this figure, so if you just try to apply simple trigonometry relationships what you are going to get is,

$$x = \frac{c\cos\phi}{1+\sin\phi}$$

$$\sigma_t = 2x = \frac{2c\cos\phi}{1+\sin\phi}$$

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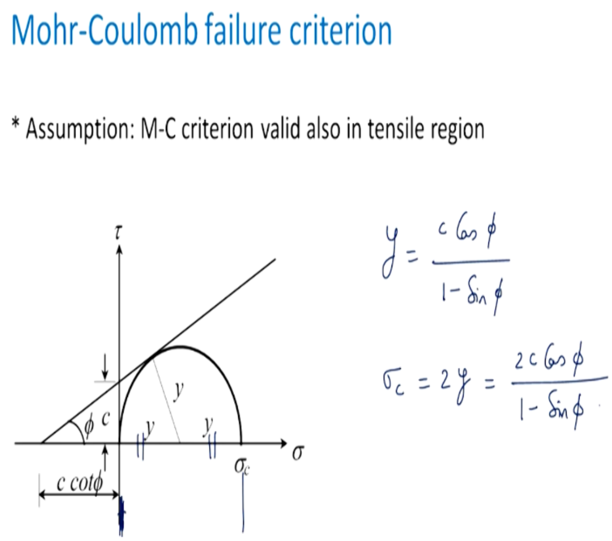

Similarly for the uniaxial compressive strength test, we have

$$y = \frac{c\cos\phi}{1-\sin\phi}$$

$$\sigma_c = 2y = \frac{2c\cos\phi}{1-\sin\phi}$$

So, it is a simple trigonometry that we can find out these relationships.

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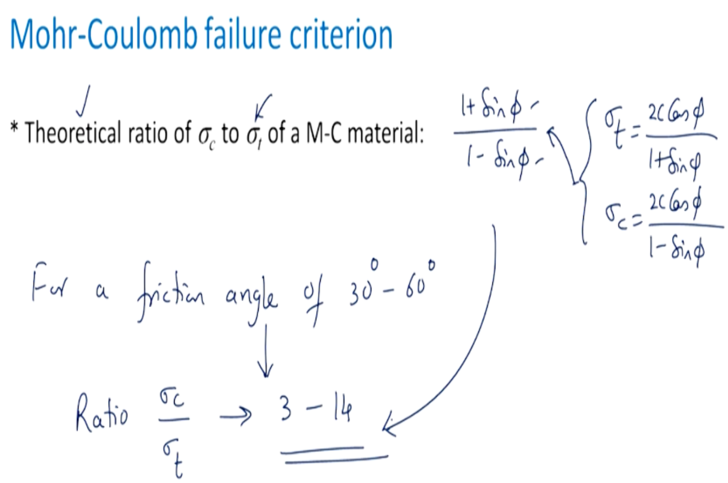

Now what is going to be the theoretical ratio of σ_c / σ_t, why I am saying the theoretical ratio, because this σ_t is what we are getting from a theoretical uniaxial tensile strength test. So, this is going to be

$$\frac{\sigma_c}{\sigma_t} = \frac{1 + \sin\phi}{1 - \sin\phi}$$

So, from these two we can get this ratio. Now for a friction angle of say 30° to 60° this ratio which is σ_c / σ_t, it will be in the range of 3 to 14. So, you just substitute the value of 30° here in this expression and value of 60° and you will be able to get this range.

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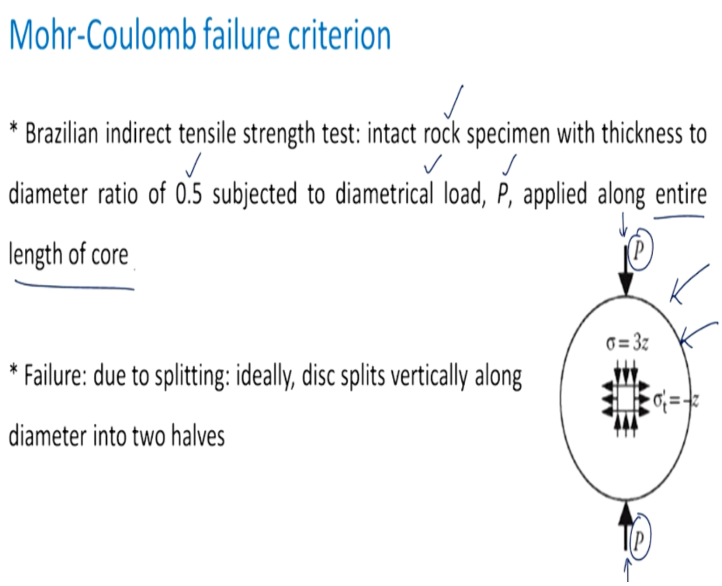

Now I mention to you that it is the Brazilian indirect tensile strength test, which is conducted on the intact rock specimen to obtain the tensile strength of the rock. The condition for the sample preparation was that thickness to diameter ratio should be 0.5 and it was subjected to a diametric load *P*. Look at this figure. This is that specimen, which is it will look like and it is subjected to this diameter load *P*.

And this is applied along the entire length of the code. The failure takes place due to splitting and ideally this should split vertically along the diameter into 2 halves.

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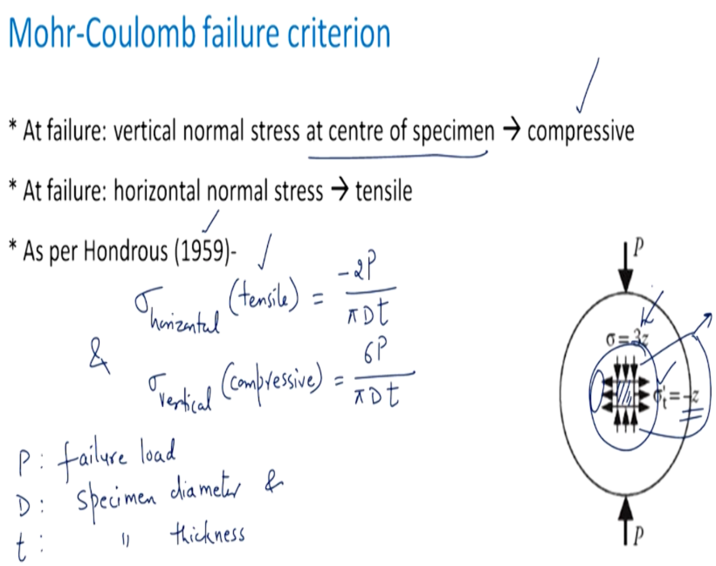

Then the failure will be at the center of the specimen and the vertical normal stress at the center is going to be compressive, please take a look here. If we take an element at the center of this specimen, you see that this vertical. In the vertical direction you have the normal stress which is compressive in nature. And at the failure you will have the horizontal normal stress that is this, which is tensile in nature.

So as per this author, who did this work in 1959, this σ_h or σ horizontal, which is tensile in nature.

$$\sigma_{horizontal}(tensile) = \frac{-2P}{\pi Dt}$$

$$\sigma_{vertical}(compressive) = \frac{6P}{\pi Dt}$$

where this *P* is the failure load, *D* is the specimen diameter and *t* is the specimen thickness. So, basically you see here that three times of the magnitude of the horizontal stress is the compressive stress in the vertical direction.

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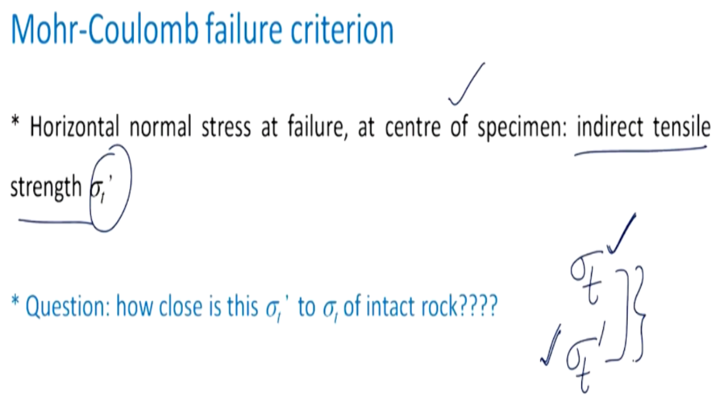

So, taking a clue from here, we need to see how we can modify the Mohr-Coulomb failure criterion in case of the rocks. Because in case of these soils tensile strength did not make any sense. But in case of the rocks one can rely on some tensile strength of the rock. So, the horizontal normal stress at failure at the center of the specimen, it is going to give me the indirect tensile strengths and say I am representing it by σ_t'

So now we are going to have two, tensile strength. One was the one which I got theoretically which was direct tensile strength that was σ_t and another is the indirect tensile strength that is σ_t'. The question is how close are these two values to each other or how close this σ_t, which ideally it should be the tensile strength of the interact rock. But we are not able to find this out practically.

What we can find out is σ_t'. So, the question is how close these two values are to each other?

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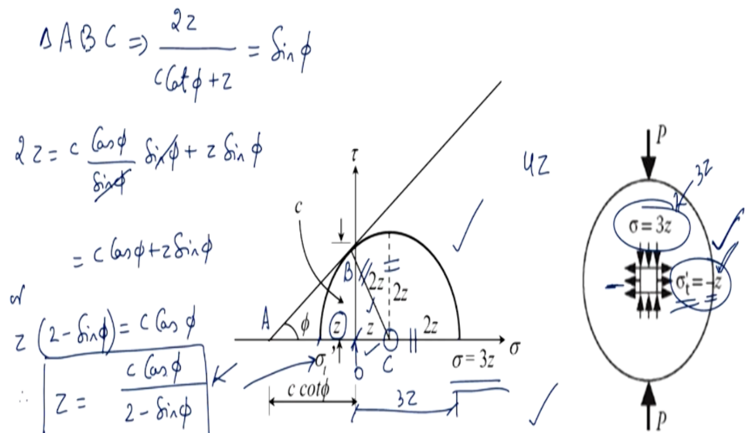
Mohr-Coulomb failure criterion



Mohr circle at centre of specimen: Brazilian test

Take a look here at this figure, so this is the Mohr's stress circle for the Brazilian test, so what is the state of stress? This is 3 times the stress in this direction. This means that if the stress in the horizontal direction that is in this direction is represented by a unit *z* or it is *z* unit then in the vertical direction it is 3z unit. Now look at this Mohr circle σ_1 here is 3z and σ_3 is going to be -z.

So, this is the origin 0 and here you have $\sigma_t{}'$ which is -z. So, from 0, this is -z which has been taken here. Now from this to this it is going to be the total distance is going to be what, from here to here it is 3z + z, so this diameter of this circle is going to be 4z. So, it is radius is going to be 2z and that is why we have written it like this that this is 2z and this is also 2z.

Now since this distance is z, so the remaining from 0 to the center of the circle, which is this point it has to be equal to z. Taking the note of all this, take this triangle, let us say A at this point is B and this point is say C. So, take in this triangle, in triangle ABC what you are going to get is,

$$\sin\phi = \frac{2z}{c\cot\phi + z}$$

$$2z = c\frac{\cos\phi}{\sin\phi}\sin\phi + z\sin\phi$$

$$= c\cos\phi + z\sin\phi$$

$$z(2 - \sin\phi) = c\cos\phi$$

$$z = \frac{c \cos \phi}{(2 - \sin \phi)}$$

So, this is what is the expression that I am going to get for the z. So, this is representing you see here z is representing the tensile strength. So, the tensile strength in case of the Brazilian test is going to be given by this expression.

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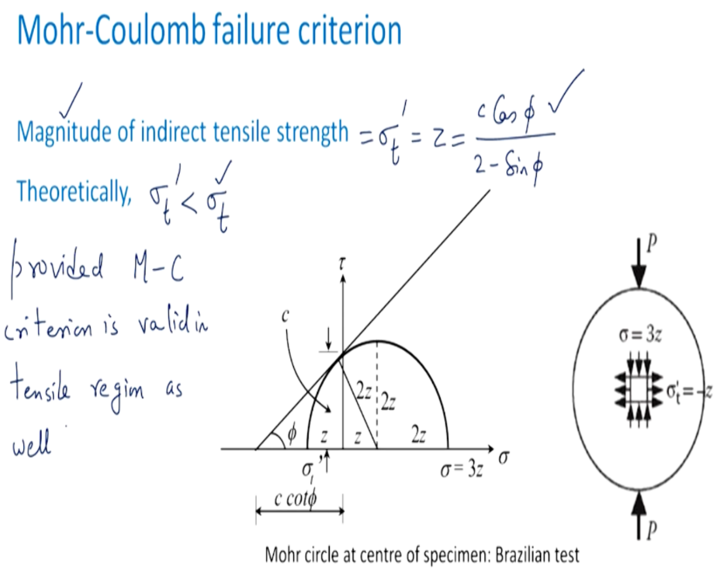

Now what will happen to the magnitude of the indirect tensile strength, this is going to be σ_t' which is equal to z. See I am talking only in terms of the magnitude, therefore. I am not putting a negative sign here. Which we just derived.

$$\sigma_t' = z = \frac{c \cos \phi}{(2 - \sin \phi)}$$

Now theoretically this $\sigma_t' < \sigma_t$. Please compare this σ_t expression, which we obtained earlier with this expression of σ_t' and you will get this.

Now, this is theoretically true provided Mohr-Coulomb criterion is valid in tensile region as well. So, we need to be careful about it that the one which we are obtaining from the Brazilian test is less than the direct tensile strength of the rock.

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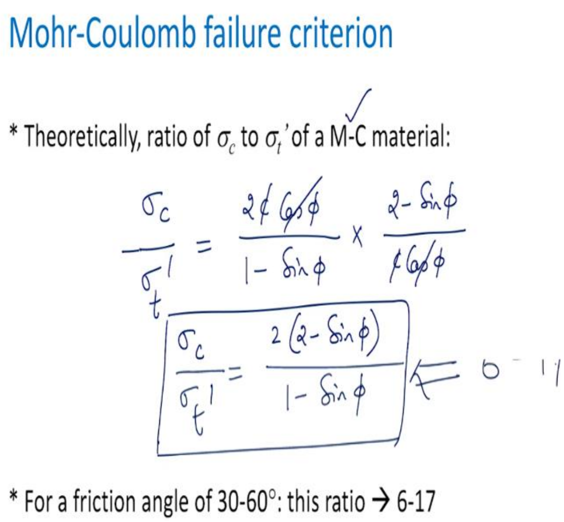

Now theoretically the ratio of σ_c to σ_t' of Mohr-Coulomb material is going to be given by this expression,

$$\frac{\sigma_c}{\sigma_t'} = \frac{2c\cos\phi}{1-\sin\phi} \times \frac{2-\sin\phi}{c\cos\phi}$$

$$\frac{\sigma_c}{\sigma_t'} = \frac{2(2-\sin\phi)}{1-\sin\phi}$$

Now for a friction angle of 30° to 60° this ratio works out to be 6 to 17.

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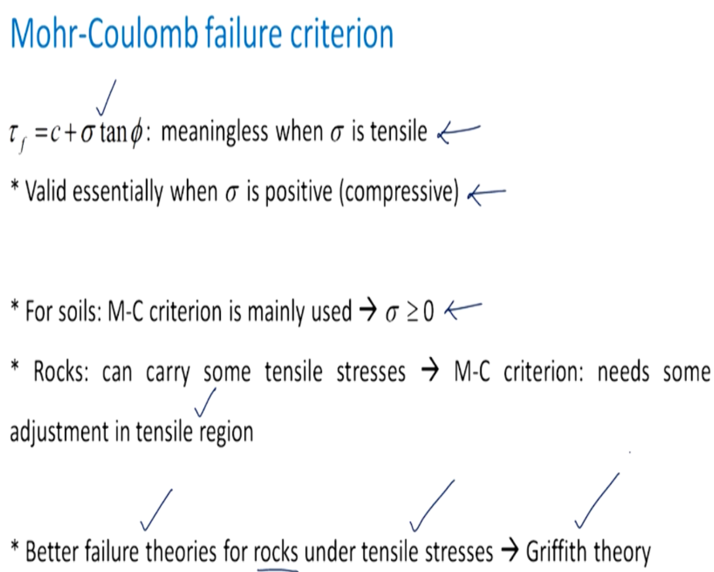

$\tau_f = c + \sigma \tan\phi$ is basically meaningless when σ is tensile. So, this is valid essentially when σ is positive or compressive in nature for these soils, this Mohr-Coulomb criterion is mainly used. As this σ is greater than or equal to 0, but rocks can carry some tensile stresses and in view of this, if we want to apply Mohr-Coulomb failure criterion in case of the rocks.

We need to make some adjustment in the tensile region; as far as the compressive region is concerned, we do not have any problem. There are better failure theories which are available for rocks under the tensile stresses and one of such theory is the Griffith theory. Although this is beyond this scope of this course, but this is just for your information that better theories than Mohr-Coulomb failure criterion.

They are available to represent the strength characteristic of rocks under tensile stresses. Now since we have to do some kind of adjustment that is what we saw what kind of adjustments should it be?

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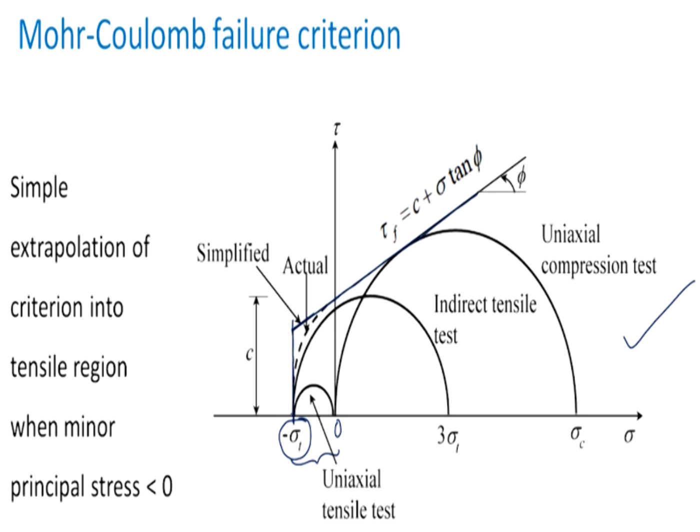

M-C failure criterion with adjustment for tensile normal stresses

So, you see here in this figure that it is the simple extrapolation of the criterion. So, this criterion was like this in the compressive zone. So, I will just simply extrapolate it in the tensile region like this where the minor principal stress is less than 0. That means this is your zero point in this zone and obviously this minor principal stress cannot be less than tensile strength of the rock. And therefore, the limit to this extrapolated line is going to be given by this vertical line where this is the state of stress represented by $-\sigma_t$.

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Mohr-Coulomb failure criterion

* Three Mohr circles simulating

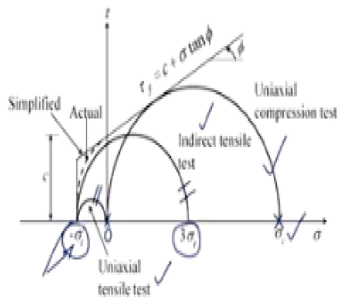
i) UCS test, ii) Brazilian tensile test, &

iii) Uniaxial tensile strength test

* Assumption: tensile strength from

Brazilian tensile test & Uniaxial tensile

strength test: same



In this figure, there are three Mohr circles which are simulating, number one is the uniaxial compression test in which σ_3 equal to 0 which is here and σ_1 equal to it is UCS value which is σ_c. The second one is the Brazilian tensile strength test or the indirect tensile test, which is given by this circle and we have seen that in that case, you have σ_1 as equal to $3\sigma_t$.

And σ_3 is equal to $-\sigma_t$. Then the third circle is the representation of the state of stress. In uniaxial tensile test, which is the direct uniaxial tensile strength test and, in that case, we have the σ_1 is equal to 0 and σ_3 as $-\sigma_t$ assumption which is involved here is that the tensile strength from the Brazilian tensile test and uniaxial tensile direct strength test they are same. Otherwise, we will not be able to draw this figure.

(Refer Slide Time: 31:34)

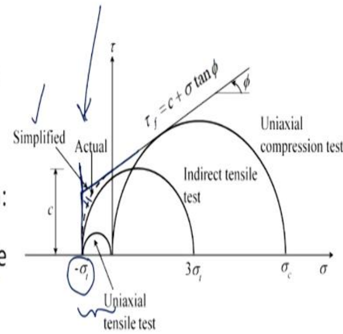
Mohr-Coulomb failure criterion

* Curved dashed line: actual envelope in tensile region

Simplified M-C extrapolation: overestimate the strength in tensile region

Lower values of c and σ_t: used judiciously

Left side of M-C envelope: worrisome!



Now take a look at this curved dashed line which is the actual envelope in the tensile region, but what we are doing is we are extrapolating this Mohr-Coulomb failure envelope in the tensile region and making a tension cut off here corresponding to σ_3 equal to σ_t that is in the negative side. So, the actual one is this dotted one and the simplified one is this one. So, we can see that from the figure itself it is clear that this simplified Mohr-Coulomb extrapolation it overestimates the strength in the tensile region, that is this region.

So, one needs to be careful about this issue, because whatever that you will get from the more coulomb failure criterion, which has been extrapolated in the tensile zone you are going to get better strength characteristic because it is overestimating the strength. So, one need to be careful about the choice of the values of c and σ_t has to be used judiciously. So, basically this tensile part or the left side of this Mohr-Coulomb envelope is worrisome. We need to be careful about it.

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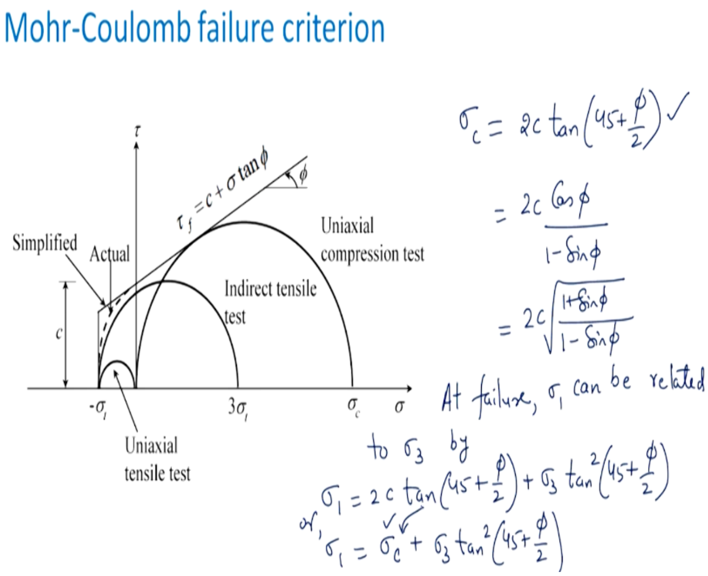

Now take a look at it, here so we will apply this tension cut off and then we will try to see that what can be the situation here.

$$\sigma_c = 2c\tan\left(45+\frac{\phi}{2}\right)$$

$$= \frac{2c\cos\phi}{1-\sin\phi}$$

$$= 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

This at failure, σ_1 can be related to σ_3 by this expression

$$\sigma_1 = 2c\tan\left(45+\frac{\phi}{2}\right) + \sigma_3\tan^2\left(45+\frac{\phi}{2}\right)$$

$$\sigma_1 = \sigma_c + \sigma_3\tan^2\left(45+\frac{\phi}{2}\right)$$

So, we will have the subsequent discussion with respect to this Mohr-Coulomb failure criterion in the next class, so what we discussed in today's class is with respect to the Mohr-Coulomb failure criterion, we saw that it is quite widely applicable in case of soils. And in case of the rocks, we need to make some adjustment. As far as the tensile region is concerned to apply this criterion in case of rocks. So, we will continue with this discussion in the next class.