Rock Engineering
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Lecture – 27
Strength Criteria for Isotropic and Anisotropic Rock – 02,
Mohr's Failure Theory

Hello everyone. In the previous class, we learnt about the strength criteria for isotropic and anisotropic rock. We started with this topic with the comparison between soils and rocks. We learnt about the stress-strain relationship which is applicable for the linear elastic constitutive relation and then we also learnt about the plane strain loading.

So, today we will learn about the plane stress loading, axisymmetric loading and then we will see; what is the effect of confining pressure on the strength characteristics of rock and rock masses. And then we will also learn about the Mohr's failure theory.

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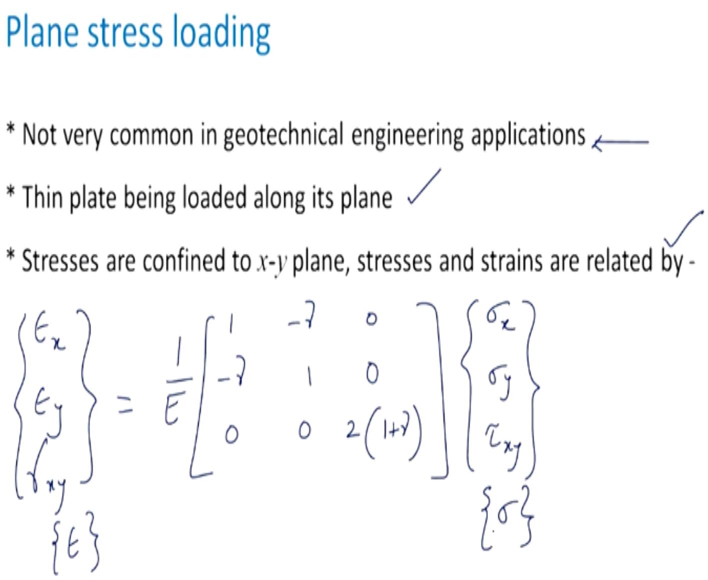

$$\underbrace{\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}}_{\{\epsilon\}} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix} \underbrace{\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}}_{\{\sigma\}}$$

So, we have to start with plane stress loading. This is not very common in the geotechnical applications. In the previous class we saw that the plane strain loading which was quite applicable in areas related to geotechnical engineering. For example retaining walls, strip loading, etc. An example of the plane stress loading includes thin plate which is being loaded along its plane.

So, like we discussed the stress-strain relationship in case of the plane strain loading condition. Let us see how in case of plane stress loading these stresses and strains are being related. Take a note here that in this case of plane stress loading stresses are confined to x-y plane while in case of the plane strain loading, we have seen that strains were confined to x-y plane. The strain in z plane that was the perpendicular to x-y plane was 0 in case of plane strain loading, however σ z was non-zero.

$$\begin{Bmatrix} \in_x \\ \in_x \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E} \begin{Bmatrix} 1 & -\upsilon & 0 \\ -\upsilon & 1 & 0 \\ 0 & 0 & 2(1+\upsilon) \end{Bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

So here in this case this is related to stresses which are confined to x-y plane. So let us see how the stresses and strains are related in this plane stress loading case. So, I will first write the strain vector which is ∈ x, ∈ y and γ xy so that is the strain vector this is equal to 1 upon E times 1 –ν, 0, –ν, 1, 0, 0, 0, 2 times (1+ν) and this will be multiplied by the stress vector which is σ x, σ y and tau xy.

So, this is how your strain vector is related to the stress vector. Now as we did in the previous case, we can also do here like how to represent the stresses in terms of strains.

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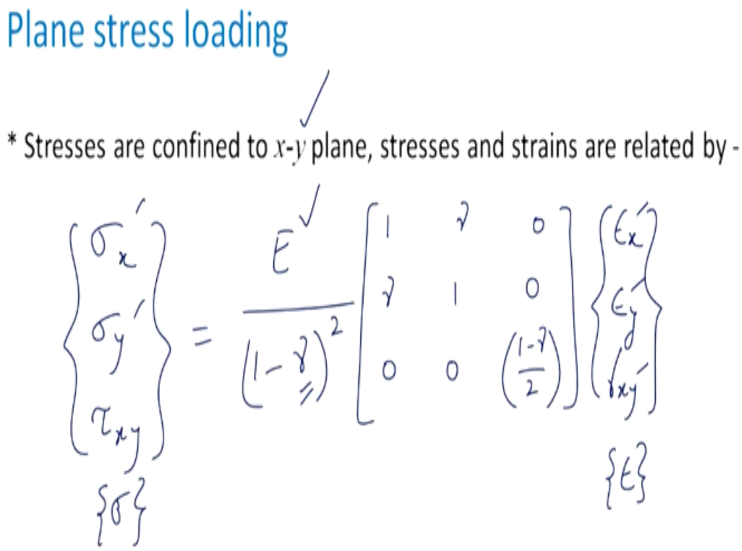

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1-\upsilon)^2} \begin{Bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{Bmatrix} \begin{Bmatrix} \in_x \\ \in_x \\ \gamma_{xy} \end{Bmatrix}$$

So, again stresses are confined to the x-y plane and I am going to write the same expression in terms of the stresses and strains. So, you see that the stress vector σ x, σ y and tau xy which is the stress vector that is equal to E upon 1-ν whole square and 1, ν, 0, ν, 1, 0, 0, 0, and 1–ν upon 2. This multiplied by ∈ x into ∈ y into γ xy. So, this is your strain vector.

Keep in mind that the stresses σ x and σ y these are the normal stresses, tau xy is the shear stress while ∈ x and ∈ y these are normal strains and γ xy is the shear strain. E and ν these are the elastic modulus and Poisson’s ratio for the material. Again, here the assumption is involved that the material is following the Hook’s law.

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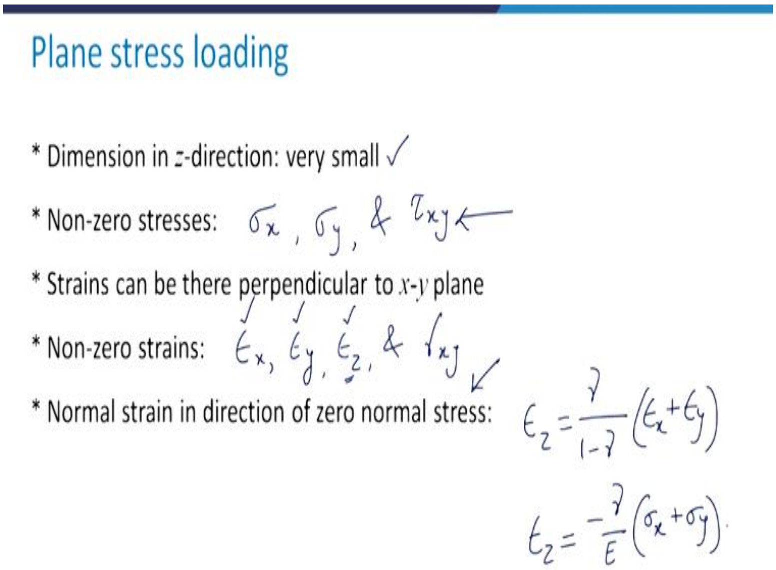

So, dimension in the z direction is very small in case of the plane stress loading. What was the situation in plane strain loading? The dimension in the z direction was pretty long and therefore we could take the strain in that direction to be equal to 0. Now what are going to be the non-zero stresses in case of the plane stress loading? They are going to be σ x, σ y and tau xy. Strains can be there perpendicular to x-y plane.

So, what all are the non-zero strains in this case? That is going to be ∈ x, ∈ y, ∈ z and γ xy. What does this mean is that all the three component of normal strains that is ∈ x, ∈ y and ∈ z are going to be non-zero in case of the plane stress loading. Now what will happen? When you have the normal strain in the direction of 0 normal stress because you can see from here that σ z is 0 in this case, but in z direction ∈ z is non-zero.

So, what is the expression for this ∈ z?

$$\in_z = \frac{\upsilon}{1-\upsilon}\left(\in_x + \in_y\right)$$

$$\in_z = \frac{-\upsilon}{E}\left(\sigma_x + \sigma_y\right)$$

So, we will get ∈ z as ν upon 1–ν into ∈ x + ∈ y or this can also be represented as –ν upon E σ x + σ y. So, this is how all the components of stresses and strains can be determined in case of the plane stress loading. Once again what is the difference between plane strain loading and plane stress loading?

Say if x-y is the plane of the consideration in case of the plane strain loading the strain in the z direction is going to be equal to 0 but it will have non-zero stress in z direction. However, in case of the plane stress loading you will have non-zero strain in the z direction but you will have 0 stress in the z direction. So, you should be able to understand the difference between plane strain and plane stress loading in a very clear manner because this is very much important from geotechnical point of view.

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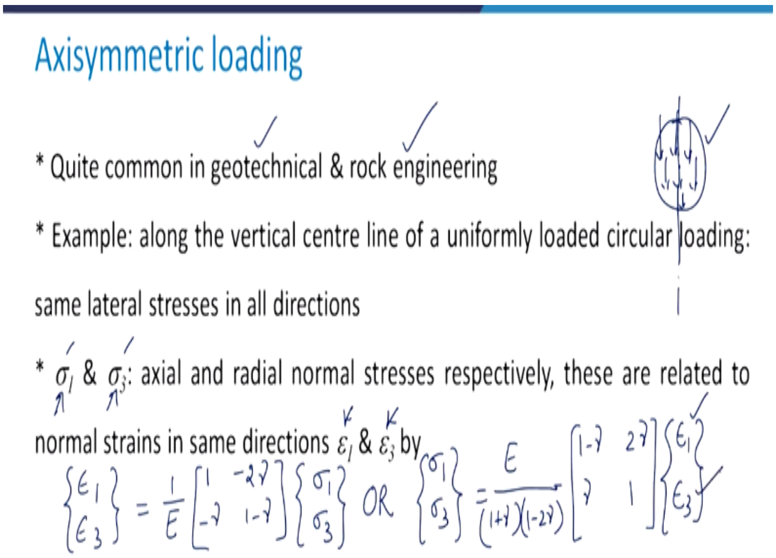


So, the next type of loading is the axisymmetric loading. Again, this is very common in case of the geotechnical and the rock engineering. An example includes that along the vertical center line of a uniformly distributed load on the circular loading because it has the same lateral stress in all the direction, this is what is called as the axisymmetric loading. So, you see it looks like this.

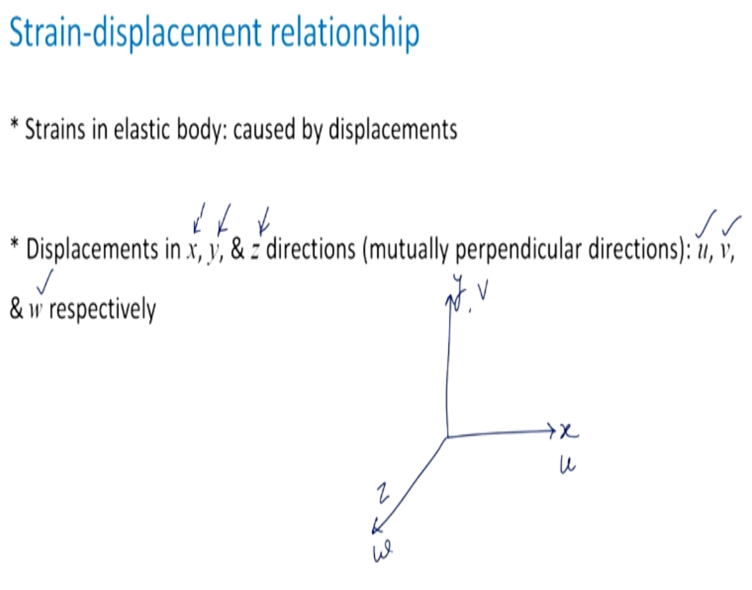
So, you have a circular footing let us say and it is subjected to the uniformly distributed load and if this is being the axis so it is the lateral stress is same in all the directions, so this problem is solved by this axisymmetric loading. Now this σ 1 and σ 3 these are the axial and radial normal stresses ah respectively. These are related to the corresponding normal strains ∈ 1 and ∈ 3.

$$\begin{Bmatrix} \in_1 \\ \in_3 \end{Bmatrix} = \frac{1}{E} \begin{Bmatrix} 1 & -2\upsilon \\ -\upsilon & 1-\upsilon \end{Bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix}$$

So, this ∈ 1 is the strain in which this axial stress is acting and ∈ 3 is the strain in the direction of σ 3. So, let us see that how these are related to each other? So in this case you have the two components ∈ 1 and ∈ 3, one is axial another one is radial that is given as 1 upon E 1 –2 ν –ν 1–ν and multiplied by the stress vector this is σ 1 and σ 3 or this expression can also be written in the other way round that is σ 1 σ 3.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_3 \end{Bmatrix} = \frac{E}{(1+\upsilon)(1-2\upsilon)} \begin{Bmatrix} 1-\upsilon & 2\upsilon \\ \upsilon & 1 \end{Bmatrix} \begin{Bmatrix} \in_1 \\ \in_3 \end{Bmatrix}$$

This is equal to E upon 1+ν, 1–2, ν multiplied by 1–ν, 2ν, ν and 1 and this should be multiplied by the strain vector which σσcomprises of the two components which is the axial and the radial normal strain. ν**(Refer Slide Time: 12:16)**



So, the strain in the elastic body they are caused by the displacement. So, there should be a relationship between strain and displacement and we should be aware of that. So, let us say that displacements in x, y and z directions they are respectively u, v and w. So these x, y, z these three directions they are mutually perpendicular to each other. So, let us say this is your x, this is y and this is going to be z.

So, these three are the mutually perpendicular directions. So, in x direction the deformation is u, y direction it is v, and z direction it is small w.

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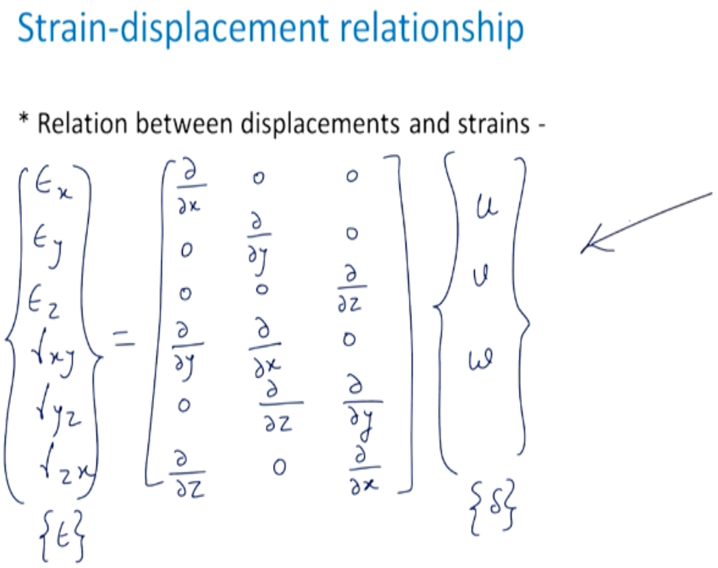

$$\begin{Bmatrix} \in_x \\ \in_y \\ \in_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial}{\partial_x} & 0 & 0 \\ 0 & \frac{\partial}{\partial_y} & 0 \\ 0 & 0 & \frac{\partial}{\partial_z} \\ \frac{\partial}{\partial_y} & \frac{\partial}{\partial_x} & 0 \\ 0 & \frac{\partial}{\partial_z} & \frac{\partial}{\partial_y} \\ \frac{\partial}{\partial_z} & 0 & \frac{\partial}{\partial_x} \end{Bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$$

What is the relationship between displacements and strains? Let us see. Again, we will write it in the form of the matrix. So, we have the strain vector as ∈x, ∈y, ∈z. Then you have the shear strain γ_{xy}, γ_{yz}, and γ_{zx}. So, this is our strain vector that is equal to a matrix which is ∂/∂_x,0, 0, 0, ∂/∂_y,0, 0, 0, 0, ∂/∂_z, then ∂/∂y, ∂/∂x and 0, then 0, ∂/∂_z,∂/∂y, then ∂/∂ z, 0, ∂/∂_x.

So, this matrix into multiplied by you will have the displacement vector which is u, v and w. So, this is your strain vector and this is your displacement vector. So this is what defines the relationship between displacement and strains. So, in case if the material is following the linear elasticity then this is how one can obtain the stress-strain relationship for different types of loading condition. It can be plane strain, plane stress and axisymmetric loading.

Now we will learn about the effect of confining pressure on the strength characteristic of rock and after this we will learn about the Mohr's failure theory. So, most of the rocks they are significantly strengthened by confinement. In some of our earlier lectures when we were discussing the laboratory testing on rocks, we touched upon this particular aspect that when we were increasing the confinement, we were seeing the betterment in the strength characteristic of the rocks.

What is the reason behind that? There are various theories. So, the one theory we are going to discuss now. So, this significant strengthening by the confinement is really very striking in a highly fissured rock.

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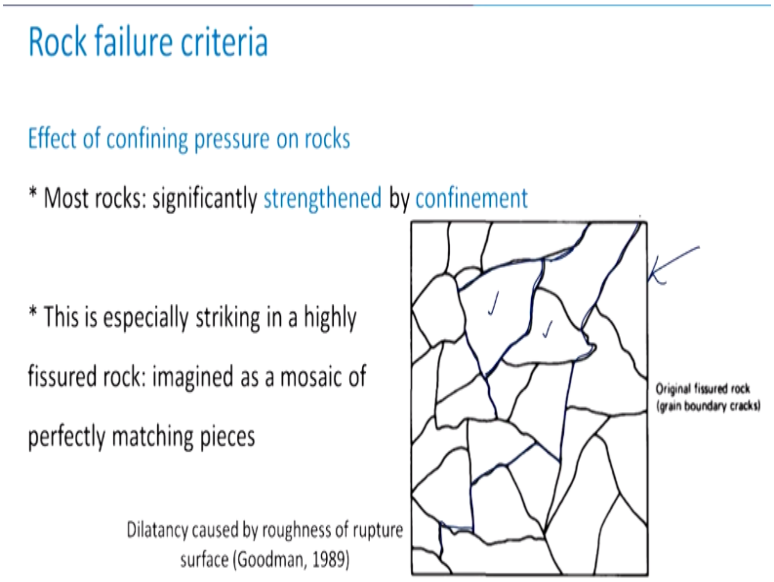

So, a highly fissured rock can be imagined as a mosaic of perfectly matching pieces like it has been shown in this figure. You can see that these are the planes of the discontivity in case of your fissured rock and these discontivity planes they are matching perfectly with the next piece. So, let us say this is one piece, this is another piece, so it is matching with this other piece in a very nice manner.

Now for this fissured rock to deform what should happen? There should be the application of the energy or exertion of the energy which should be there in order to have the movement along any fracture plane. Now that fracture plane can be anything. So, let us say for example that you have an average fracture plane say along this okay like this.

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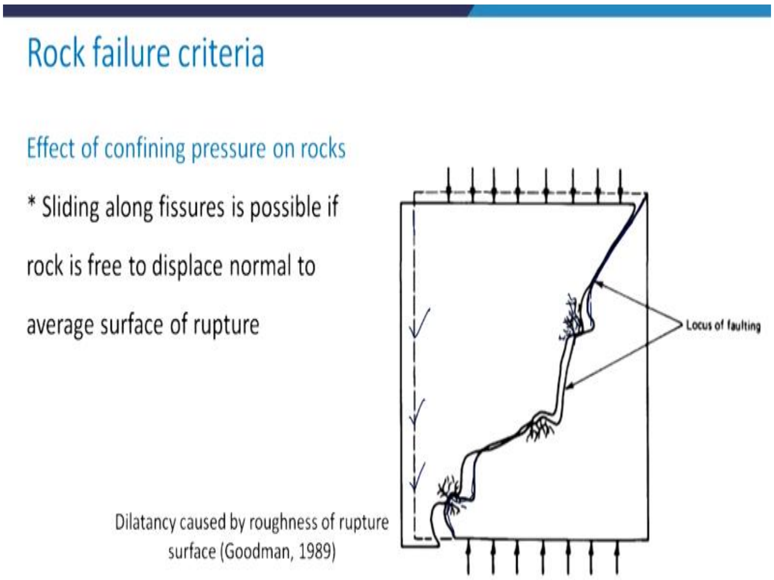

Say for example you can have any fracture plane along any of the combination which is shown here in this figure. So, you see that a typical fracture plane has been shown here. So this dotted line portion that is this portion is the original version and now you see when the load is applied, so sliding along the fissure is possible only if the rock is free to displace normal to the average surface of rupture.

So, say this is the average surface of rupture, this one. This is the average surface of rupture, so sliding will be possible only when this rock is free to displace normal to this plane. So, you can see here that some branches have been shown like this. So, these are typically showing the cracks which are taking place in the direction normal to the average surface of rupture. Now under confinement what will happen?

That the normal displacement which is required to move along such a jagged rupture path will require additional energy input. So, you see if this specimen has no confinement what will happen? It will be easy for the rock to displace along that average rupture surface. But when you have the presence of the confining pressure what will happen? Because of that confining pressure it has to exert more energy to displace along that jagged rupture spot or the surface.

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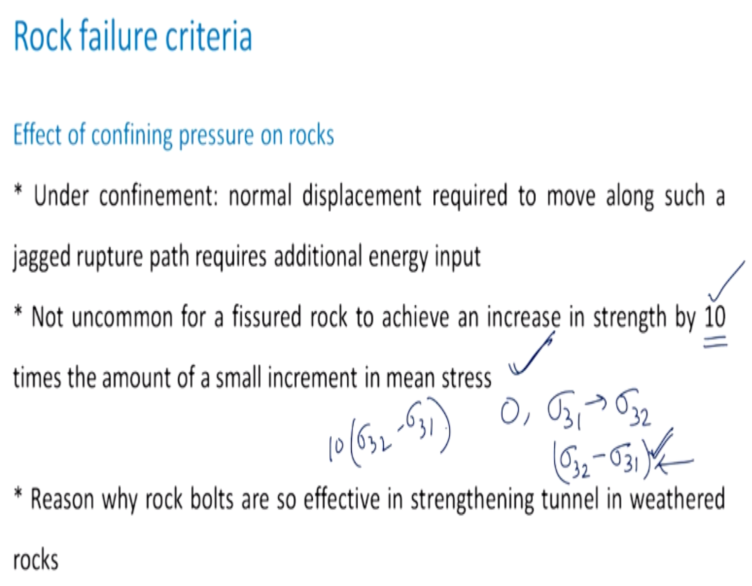

Now this is not uncommon for a fissured rock to achieve an increase in strength by about 10 times the amount of a small increment in mean stress, so what do we mean by this? Let us say that you have initially 0 mean stress or the confinement, then we increase it to let us say σ_{31}, then it has to exert some additional energy in order to have the displacement along the jagged rupture path. Now what I do is I further increase it to σ_{32} let us say.

So earlier it was 0, then it was σ_{31}, and now it says σ_{32}. So, the difference between this confining pressure which I have increased is this much. Now just by increasing the confining pressure by this amount one can observe the increase in the strength as 10 times this difference that is 10 times $\sigma_{32} - \sigma_{31}$, this is so prominent. That is the reason why rock bolts are quite effective in strengthening the tunnels especially in case of the weathered rock.

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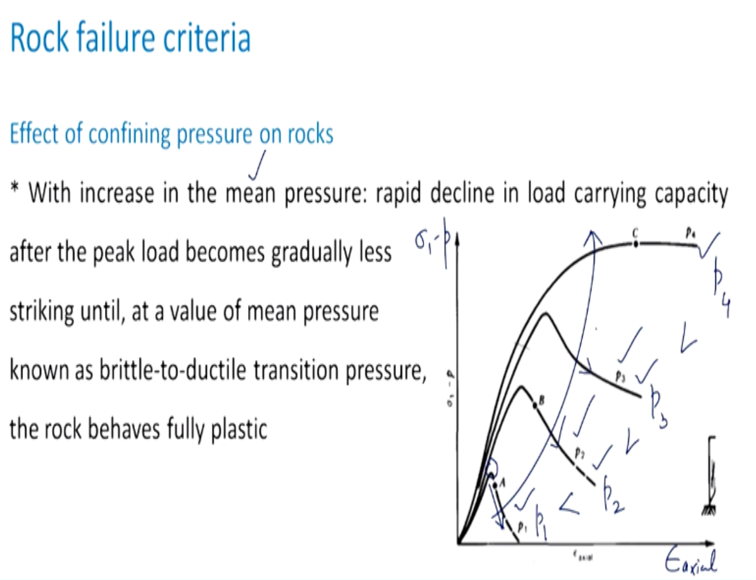

Now another aspect that what happens when we increase the confining pressure or we are calling here as mean pressure. Have a look here, there is a plot between ∈ axial and σ 1 – p, p is being represented as the mean pressure which is nothing but the confining pressure. And you can see that the 4 plots are there, they are corresponding to the mean pressure of p 1, p 2, p 3 and p 4.

So, the condition here is that p 1 is less than p 2, p 2 is less than p 3, and p 3 is less than p 4 in this case. That means in this direction we are increasing the mean pressure. Now what is the observation? Take a look at these four plots. When this mean pressure is small you can see that after the peak that means this location there is a sudden reduction in the stress-strain plot. Then as you increase the mean pressure that is when you go to this p 2.

You can see that the slope of this post peak part it reduces and when you increase it further that is to p 3 this slope gets even milder and a stage will come where you will have this type of situation where after the peak the branch does not come down but it goes like this here. So, with increase in the mean pressure there is a rapid decline in load carrying capacity after the peak load and this rapid decline becomes gradually less striking as we increase the mean pressure.

At the value of the mean pressure which is known as brittle-to-ductile transition pressure, the rock behaves fully elastic. This is a very important phenomenon in case of the rock mechanics. That means that the same rock can behave as brittle as well as ductile depending upon the value of the confining pressure at which the test on that specimen has been conducted. Now the question comes what can be the value of this transition pressure or at what value of the confining pressure the material will start behaving from brittle-to-ductile? Let us see that.

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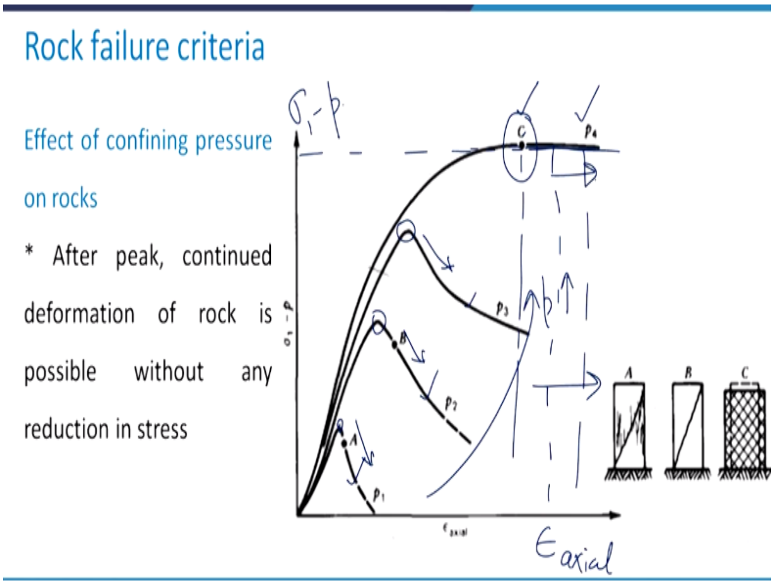

After the peak the contived deformation here in this case, see here after the peak in all these three cases there is a sudden reduction and this is less striking when you are increasing the value of p. But when the ductile behavior is there that is in this case, so the contived deformation is going to take place, so you see that this is after c. That means c is the point kind of a peak and beyond this the contived deformation of the rock is possible without any reduction in the stress.

So, if you just take more or less here the stress level is more or less constant and contivously you can see that the strain is increasing, here it is ∈ axial and this is σ 1 – p. So, contivously strain is increasing without any reduction in stress. Now how should the failure mode will look like when we increase the confining pressure on the rock? Is it going to be the same throughout or with increase in the confining pressure there is going to be any change in that as well? Let us see.

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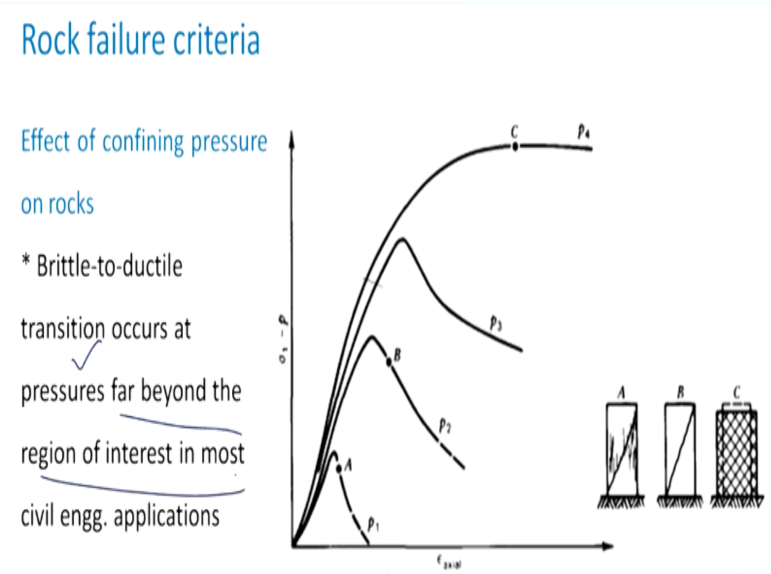

This brittle-to-ductile transition that occurs at a pressure which are far beyond the region of interest in most of the civil engineering application.

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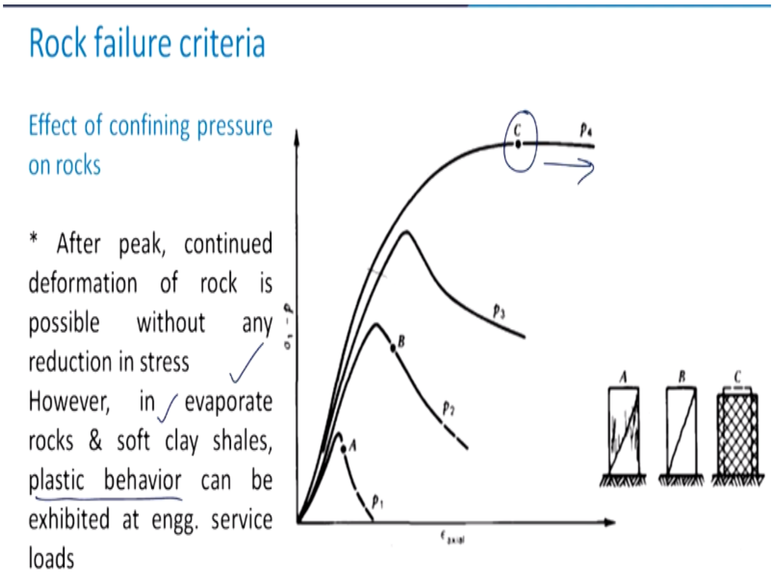

However, in some rocks like evaporate rocks or soft clay shales, this plastic behavior can be seen at engineering service loads itself. So, I mentioned to you once again when you have the ductile kind of behavior after the peak you are going to get the contivous deformation without any reduction in stresses. This brittle-to-ductile transition pressure in most of the rocks is beyond the engineering service loads, but in case of the soft clay shales or evaporate rocks it can be seen at engineering service loads.

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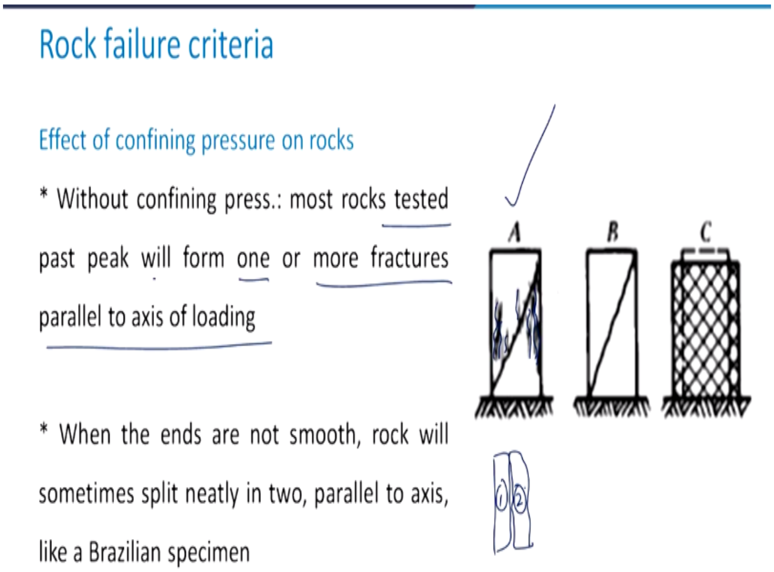

Now without the confining pressure when we test the rocks, most of the rocks they show one or more fractures parallel to the axis of loading. Take a look here. You have more or less parallel to the plane of the loading you have these fracture planes. When the ends are not smooth, the rock will sometimes split in nearly two which is parallel to the axis like a Brazilian specimen.

So, in this case you see it will be something like this. So, it will be like the one piece and this is going to be the other piece just in case if the ends are not smooth then you can get this type of situation.

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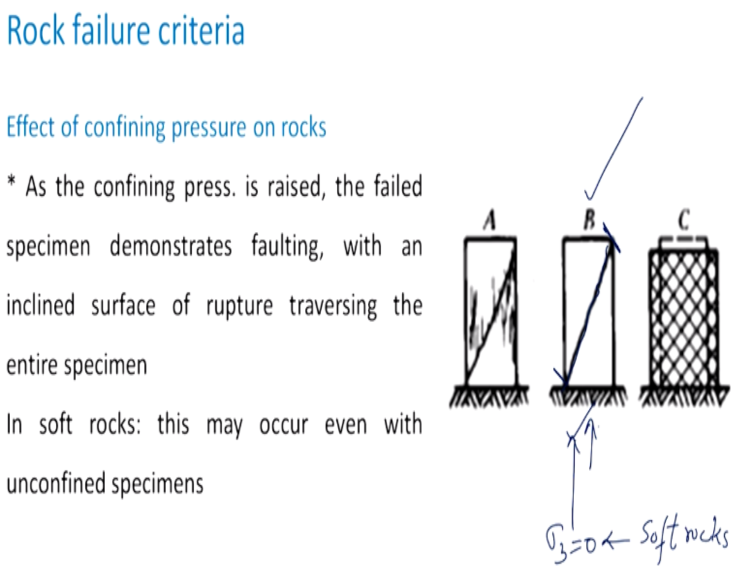

Now when you increase the confining pressure, the failed specimen it demonstrates faulting with an inclined surface of rupture which is traversing the entire specimen. Take a look at the figure vmber B in this case. So, this means here there is the more confining pressure and you can see this kind of inclined surface of rupture which is traversing the entire specimen that is from this end of the specimen to this end of the specimen.

Now in case of these soft rocks, this phenomenon may occur even with unconfined specimen that means when you say σ 3 = 0 in case of the soft rocks then also you can get this type of situation.

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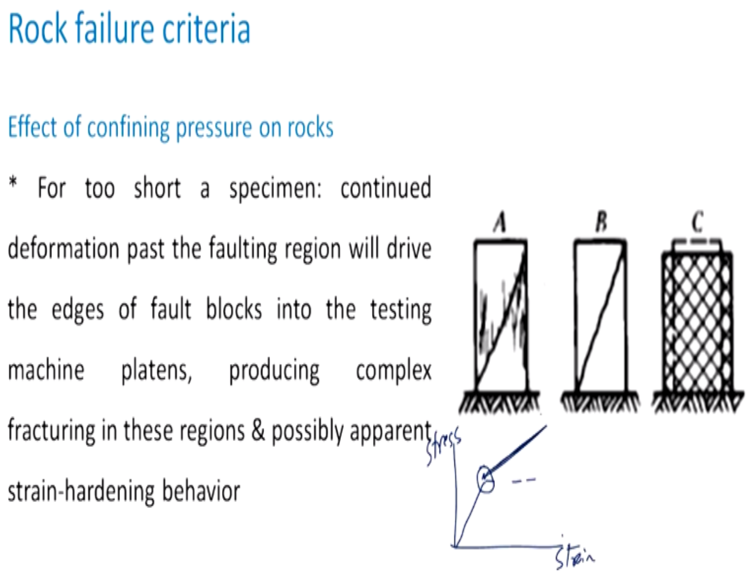

For too short a specimen: Contived deformation past the faulting region will drive the edges of the fault blocks into the testing machine platens thereby producing complex fracturing in these regions and possibly apparently we will get strain hardening kind of behavior. What do we understand by the strain hardening kind of behavior? That means if we have the stress here and strain here, then what we will get is this kind of behavior. That means beyond this yield limit, the resistance will be there in the specimen like this.

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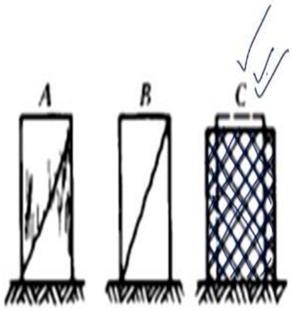
Rock failure criteria

Effect of confining pressure on rocks

* At pressures above brittle-to-ductile transition, there is no failure, but the deformed specimen is found to contain parallel inclined lines that are the loci of intersection of inclined rupture surfaces & the surfaces of specimen



So, at the pressures above brittle-to-ductile transition, there is not going to be any failure because the material is behaving in a ductile fashion. But the deformed specimen is found to contain parallel inclined lime which are the loci of intersection of the inclined rupture surfaces and the surfaces of the specimen that is the condition vmber C. So, you can see here that you are getting these two sets of inclined lines which are intersecting the surface of these specimens.

So, one is in this direction and another one is in this direction which is kind of perpendicular to the earlier one. So, this is the situation that you will get at the pressures above brittle-to-ductile transition.

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Rock failure criteria

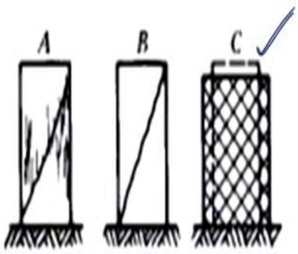
Effect of confining pressure on rocks

* Examination of deformed rock will show intra-crystalline twin gliding, inter-crystal slip, & rupture

* Mogi (1965): at brittle-to-ductile transition:

$$\sigma_1 - \sigma_3 = 3.4\sigma_3$$

* Later: $\sigma_1 = (3-5)\sigma_3$



Now the examination of the deformed rock will show intracrystalline twin gliding, inter-crystal slip and finally the rupture in such situation. Now Mogi in 1965 he talked about this brittle-to-ductile transition pressure and suggested that when your σ 1 – σ 3 is approximately equal to 3.4 times the confining pressure or σ 3. Then this condition, this stress state will correspond to the brittle 2 ductile transition.

Later on, when other researchers carried out the research in this direction, they said that when you have σ 1 in the range of 3 to 5 times σ 3, then you can have the brittle 2 ductile transition. Now after getting the idea about the effect of confining pressure on rocks, let us start our discussion on the first failure theory which is relevant to the material rocks. Let us see what is this failure theory.

Because this was one of the earlier works that is being referred even today for the representation of the strength characteristic of the geomaterials.

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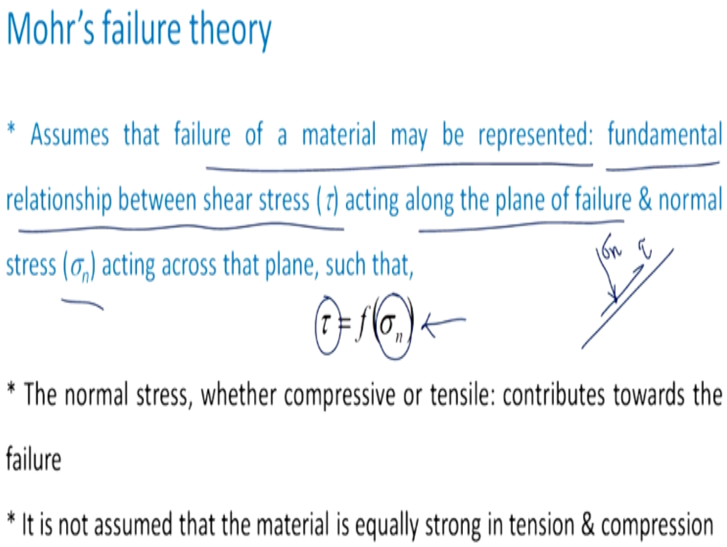

So, in this case the assumption which is made is that the failure of a material is represented by a fundamental relationship between shear stress which is acting along the plane of failure and the normal stress acting across such plane and that relationship is given by tau = f of σ n. That means what? Let us say that you have a plane of failure, then the shear stress along this plane this tau is a function of the normal stress along normal stress on this plane that is σ n.

Now this normal stress whether it is compressive or whether it is tensile, it contributes towards the failure. It is not assumed in this theory that the material is equally strong in tension as well as in compression. Mohr only suggested that there is going to be a relationship between tau and σ n. However, he never said what kind of this relationship it is going to be.

Whether it is going to be linear, whether it is going to be non-linear, it was not mentioned. So, the only thing which was there is that the shear stress is a function of the normal stress on the plane of failure.

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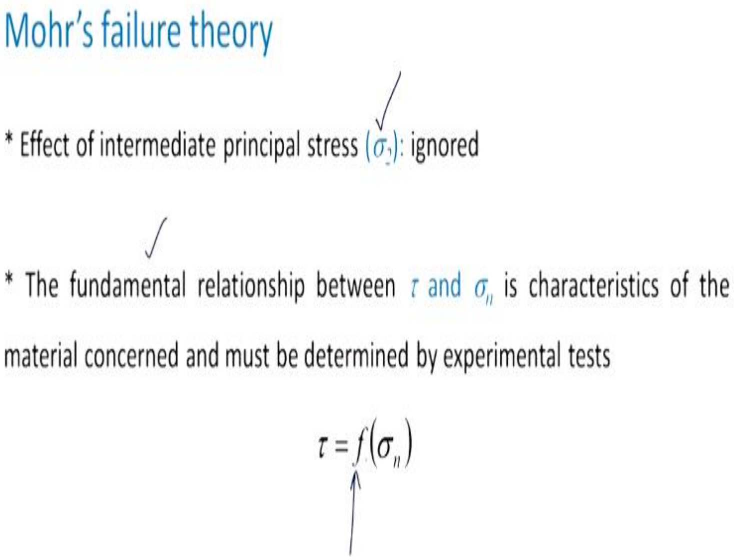

In this theory, the effect of intermediate principal stress which is σ 2 is ignored. As per this theory, it was suggested that the fundamental relationship between tau and σ n is the characteristic of the material concerned and it must be determined by experimental tests. So, say we conduct the tests in the lab, let us say that we conduct the triaxial test in the lab and then we try to plot the relationship between tau and σ n.

And then we try to see what kind of relationship that experimental data is honoring. So that is going to define this function f. Depending upon the characteristic of the material, this function f can be linear or it can be parabolic, it can be hyperbolic, it can be anything. It will be the characteristic of the material. So, Mohr's failure theory in the most simple manner states that the shear stress at a failure plane is a function of the normal stress on that plane.

Now, this theory was further modified by Mohr Coulomb and you know the very well aware Mohr-Coulomb failure criteria and you all know that it is a straight line. So, this we will discuss in the next class. So, to summarize what we discussed today? We discussed about the stress-strain relationship for the plane stress condition, axisymmetric loading. Then we saw what is the effect of the confining pressure on the rocks.

How it is going to influence the failure pattern when you increase the value of σ 3 or the confining pressure? How the failure pattern in the rock specimen got changed? And then we had the discussion on Mohr's failure theory. So, in the next class we will start our discussion with Mohr-Coulomb failure theory. Thank you very much.