Rock Engineering
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Lecture – 26
Strength Criteria for Isotropic and Anisotropic Rock - 01

Hello everyone. In the previous class, we finished our discussion on the engineering classification of rocks and rock masses. So, today we will start a new chapter on strength criteria for isotropic and anisotropic rocks. So, before I go to the strength criteria, first we will discuss about the two materials that is soil versus rocks. Then we will see that what do we understand by the stress strain relationship with respect to the rocks.

Then we will learn about the plane strain loading. So, to start with let us first understand about the two materials both are the naturally occurring material that is soils and rocks. So, here we are going to compare these from the strength characteristic point of view. So, the first head under which that I am going to compare these two is that how the material is being formed, like what is the characteristic of the material.

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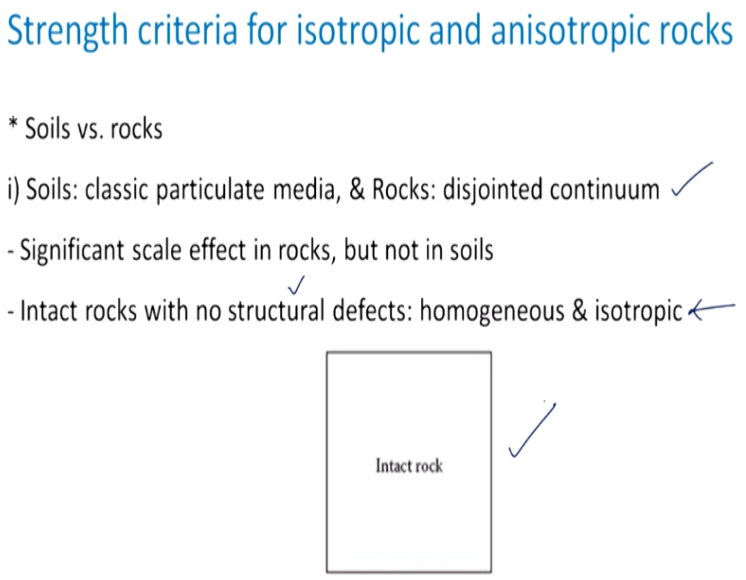

Like soils they are the classic particulate media. You know that these are three phase system and in case of the rocks it is disjointed continuum. You have seen that we have intact rocks as well as the joints in a rock mass. So, basically these are modelled as disjointed continuum. There is significant scale effect in rocks, but it is not there in soils. I have explained it to you that when you increase or reduce the size of the specimen of the rock how the strength is going to be influenced by that.

In case of the soils, we do not have any of such phenomena. In case of the intact rocks which have no structural defects, it can be considered as homogeneous and isotropic as has been shown in this figure. There are no discontinuities and therefore this can be considered as homogeneous and isotropic. But what happens in case of the rock mass?

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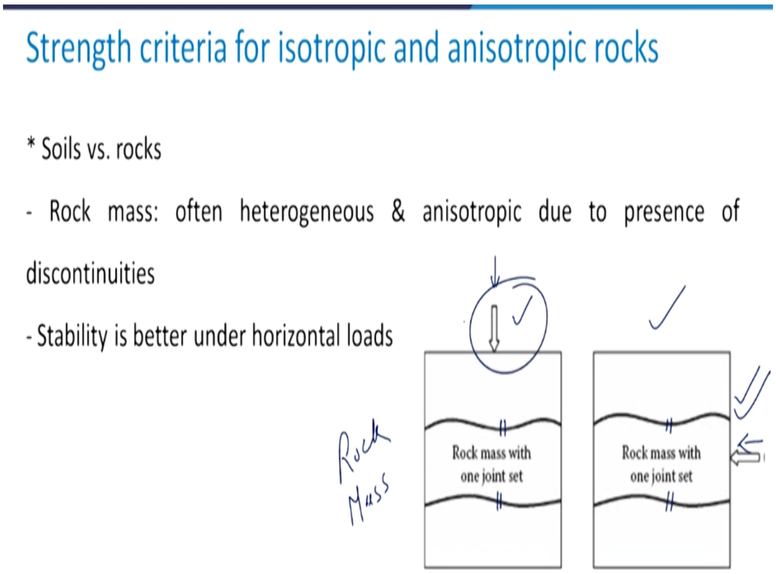

It is often heterogeneous and anisotropic why because of the presence of discontinuities. Take a look at these two pictures. There is one joint set and this I am calling as rock mass. Now in one case the loading is applied in the vertical direction and in another case the loading is applied in the horizontal condition. And if we have such type of joint set like here it is this joint set, then in this case the stability is going to be better under this horizontal load as compared to the vertical load.

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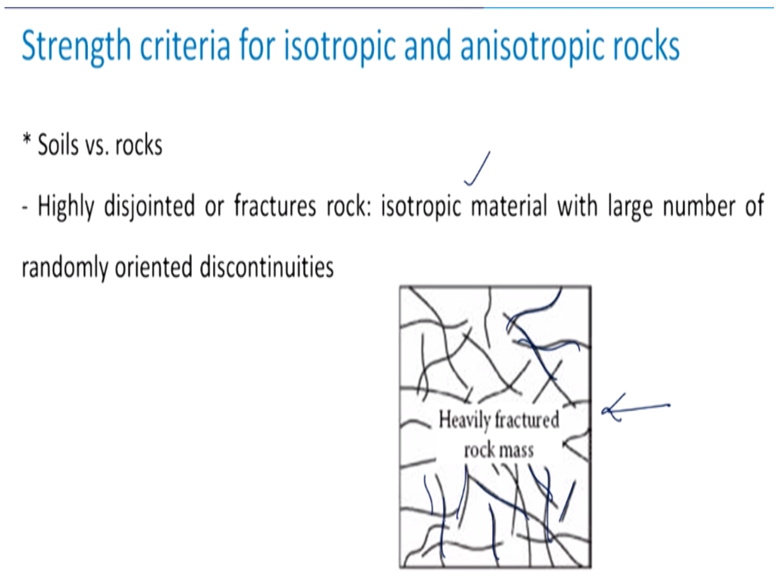

Further if there are more number of joint sets or if the rock mass is highly disjointed or it is the fractured rock, then it can be considered as isotropic material with large number of randomly oriented discontinuities as has been shown in this figure. So, you can see in this figure is that the discontinuities are oriented in such a random manner. There is no pattern that means I cannot say that it has 2 joint sets or 3 joint sets or 10 joint sets. So, we are calling this as heavily fractured rock mass.

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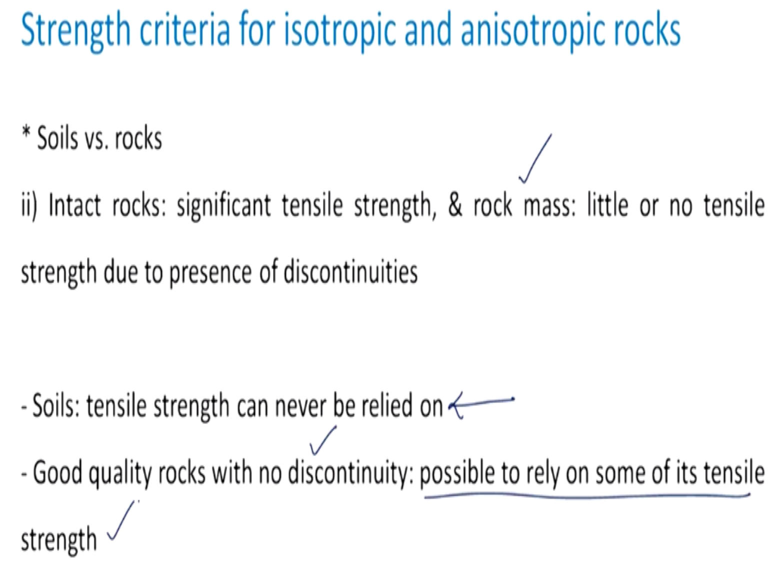

Soils are generally treated as homogeneous and isotropic, although they are not, but most of the theories when they were developed in the beginning that is the first assumption which is involved homogeneous and isotropic are it shear-strength, theories or consolidation, etc. There are no scale effects in case of the soils irrespective of the extent which is considered the behaviour of the soil specimen is going to be same. So, this is no scale effect.

The second aspect where I can do it as soil versus rock is that in case of the intact rock, we get significant tensile strength. And in case of rock mass little or no tensile strength is there because of the presence of the discontinuities. But what happens in case of these soils? Have you heard of tensile strength of the soil; your answer should be no because inside the strength of the soil cannot be relied on and therefore this does not make any sense that is the tensile strength of soil.

Good quality rocks with no discontinuities, it is possible to rely on some of its tensile strength, but not in case of the rock mass but with good quality rocks with no discontinuity we can rely on its tensile strength.

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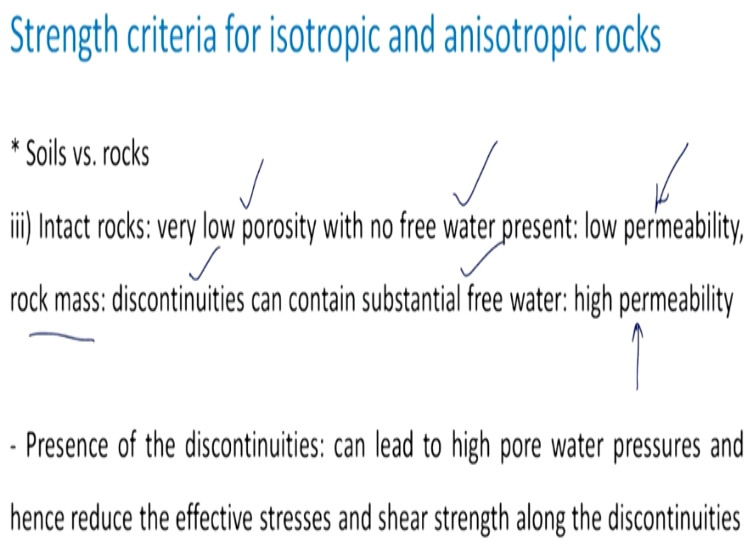

Then, the third aspect which makes the rocks different from the soils is that in case of the intact rock you have very low porosity with no free water that is present and this results into the low permeability. What happens in case of the rock mass? You have the discontinuities and these can contain substantial free water and as a result the permeability of the rock mass becomes very high.

Presence of the discontinuities lead to the high pore water pressure and therefore it reduces the effective stresses and shear strength along the discontinuities. In the presence of the water, the shear strength of the soil also gets altered but the way it gets altered it is altogether different than that of the rock.

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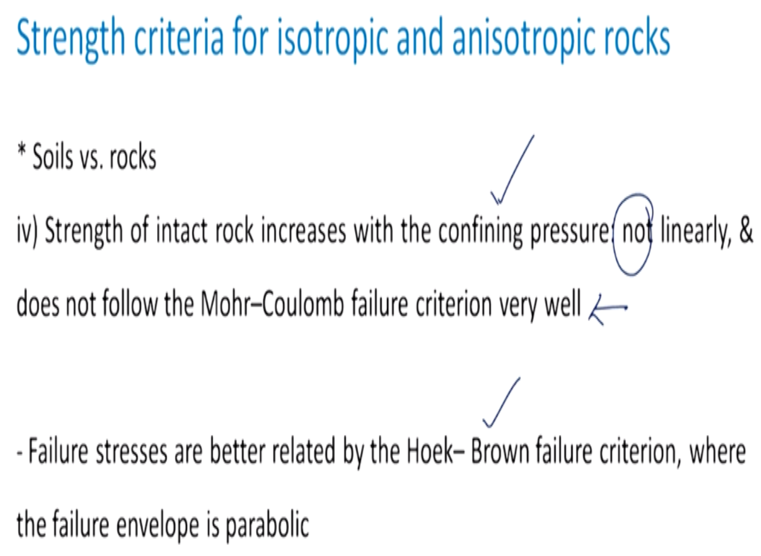

Then, the fourth criteria is the strength of the intact rock it increases with the confining pressure and this increase is not linear and it does not follow the Mohr-Coulomb failure criterion very well. See in this chapter we will learn about the details of the Mohr-Coulomb criterion and other criterion which are more suitable for the rocks and rock masses. So, in the case of the rocks, failure stresses are better related by the Hoek and Brown failure criterion, where the failure envelope is parabolic and not linear.

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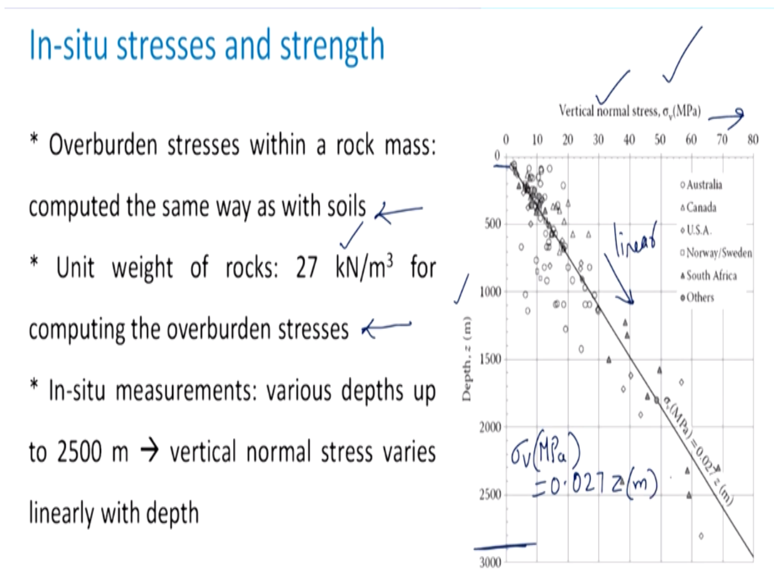

But what happens in case of these soils? Most of the time we say that okay Mohr-Coulomb failure criterion is fairly applicable in case of the soils. So, having these differences in between these two materials in mind let us have a look on the aspects related to in-situ stresses and strength. Overburden stresses within a rock mass they are computed in the same manner as it is done in case of the soils.

Generally, the unit rate of rock is taken as 27 kiloNewton per meter cube (KN/m^3) for computing the overburden stresses. Here in this picture, in-situ measurements are shown corresponding to various steps from let us say very near to the ground surface up to about more than 2500 depth. And it can be seen that the normal stress which is the vertical normal stress here on this axis, it varies linearly with the depth.

That means the variation of vertical normally stress with that depth is the linear variation which is given by this expression, which is written here.

$$\sigma_v = 0.027Z$$

So, sigma v in Mega Pascal (σ_v is in MPa) is given as 0.027 times Z which is in meters.

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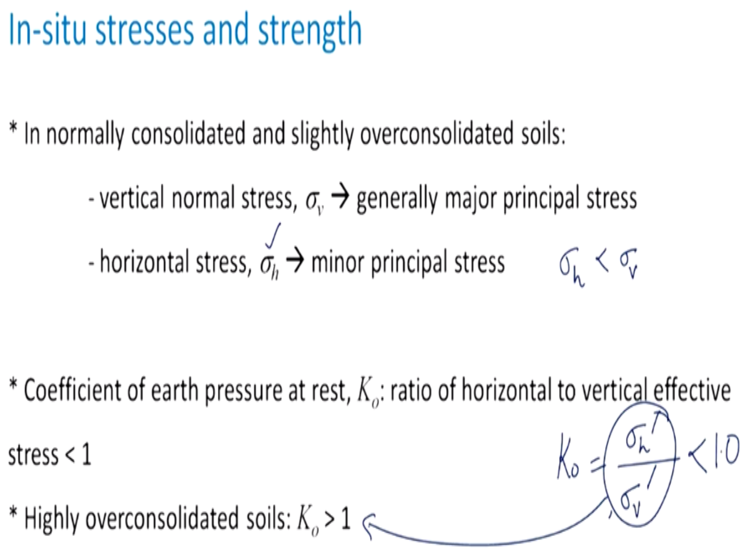

In normally consolidated and over consolidated soils, you all know that vertical normally stress sigma v (σ_v) is generally the major principal is stress and the horizontal stress sigma h (σ_h) is minor principal stress and therefore sigma h is less than sigma v ($\sigma_h < \sigma_v$). In case of the normally consolidated and slightly over consolidated soils, the coefficient of pressure at rest which is defined as the ratio of horizontal effective stress to the vertical effective stress.

This is usually less than 1. But in case of the highly over consolidated soils, this ratio that is this quantity becomes greater than 1. What happens in case of the rocks?

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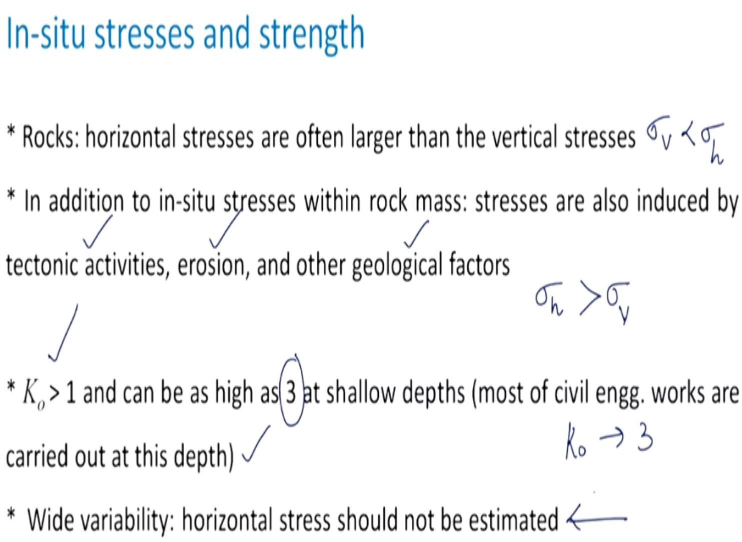

In case of the rocks horizontal stresses are often larger than the vertical stresses that is generally your sigma v will be less than sigma h. In addition to in-situ stresses within the rock mass, stresses are also induced by various tectonic activities, erosion and other geological factors and these contribute towards sigma h being more than sigma v in case of rocks. In this case as against the soil K_0 is greater than 1 and it can be as high as 3 at shallow depths.

Which are usually applicable in most of the civil engineering works that is most of the works are carried out at this depth only. So, within all the practical possible range of the depths, this K_0 can take the value as high as 3. Because of this wide variability, horizontal stress should not be estimated in case of the rocks.

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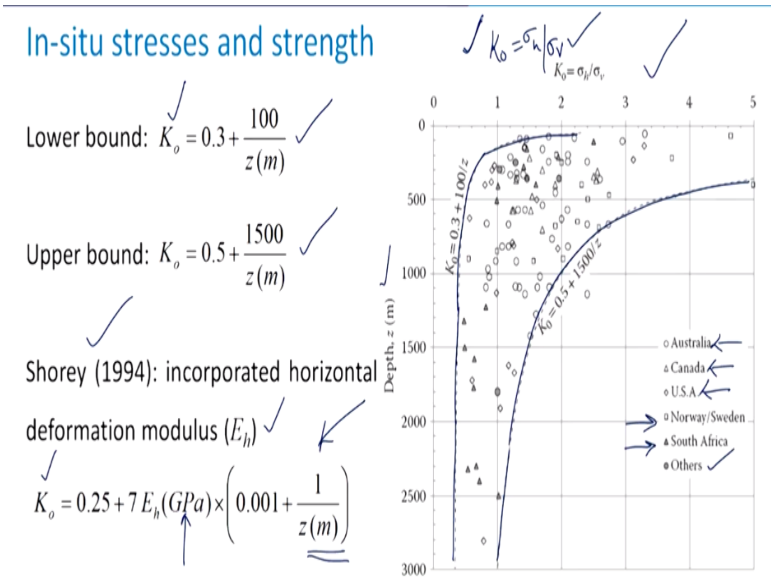

So, in order to get the idea about the in-situ stresses, let us take a look at this figure where the data from various countries with respect to K_0 which is sigma h upon sigma v (σ_h / σ_v) has been presented along with the depth. So, these countries from where the data has been taken is Australia, Canada, US, Norway or Sweden and South Africa and there are other countries also. And then the two bounds have been drawn on this data.

As you can see that this is the upper bound with respect to the larger values of K and this is the lower bound on the values of K and the equation of this lower bound is given by K_0 = 0.3 + 100/z in meter and the upper bound is given by 0.5 + 1500 /Z which is there in meter. Then in 1994, Shorey Incorporated horizontal deformation modulus in order to find this coefficient K_0 in terms of E_h and the depth z and this is given by this expression.

$$K_0 = 0.25 + 7E_h\left(0.001 + \frac{1}{z}\right)$$

So, here this E_h which is the horizontal deformation modulus this is substitute gigapascal and the depth, z is written in meter. So, this is how the coefficient which is defined as the ratio of horizontal to vertical stress can be determined. So, I mentioned that because of the large variability (σ_h) cannot be estimated. So, what we do is we estimate (σ_v) and then using this expression of K_0 which is (σ_h / σ_v) we evaluate the value of (σ_h) and then use it for the analysis purpose.

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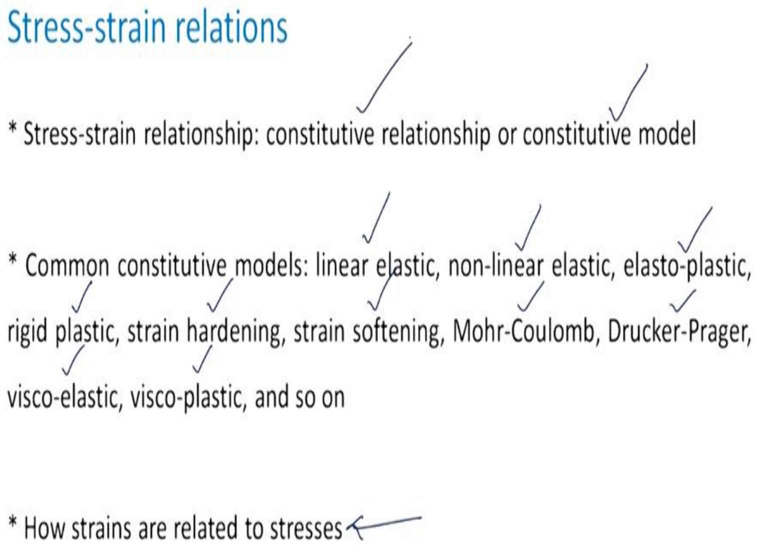

Coming to now the stress-strain relationship; you know that the stress strain relationship is also known as constitutive relationship or the constitutive model. Common constitutive models that you must have heard of they include linear elastic, nonlinear elastic, elastoplastic, rigid plastic, strain hardening, strain softening, then in the elastic plastic mode you have the Mohr-Coulomb failure criterion, you have Drucker-Prager.

Then you can have viscoelastic stress-strain relationship, visco-plastic and the list is long. The question is how the strains are related to stressors? And the answer to this is given by this stress-strain relationship or the constitutive relationship.

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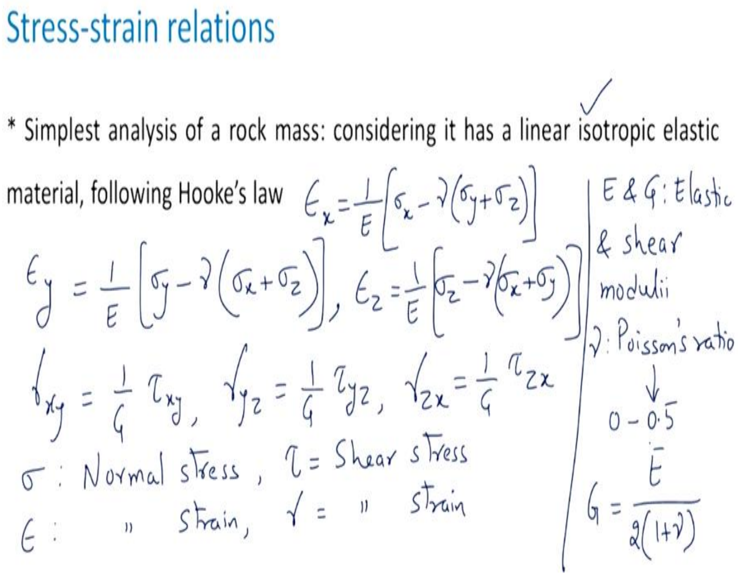

So, the simplest analysis of a rock mass can be carried out considering it as a linear isotropic elastic material, which is following the Hooke's law. So, how can we do that? So, you see that I write the strain components.

$$\in_x = \frac{1}{E}\left(\sigma_x - \upsilon\left(\sigma_y + \sigma_z\right)\right) \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\in_y = \frac{1}{E}\left(\sigma_y - \upsilon\left(\sigma_x + \sigma_z\right)\right) \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz} \qquad G = \frac{E}{2\left(1+\upsilon\right)}$$

$$\in_z = \frac{1}{E}\left(\sigma_z - \upsilon\left(\sigma_x + \sigma_y\right)\right) \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

E and G these are the elastic and shear moduli, μ is the Poisson's ratio which varies between 0 to 0.5. So, you are aware of all these relationships. It is the basic elastic relationships when you have linear isotropic elastic material.

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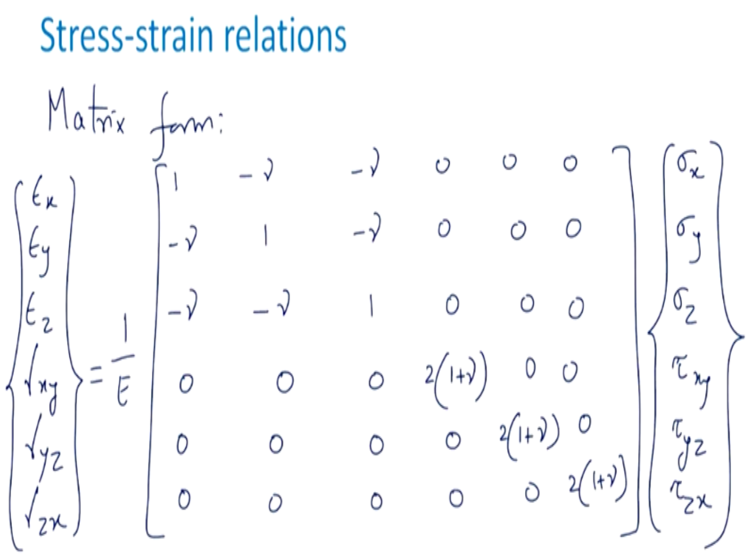

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\upsilon & -\upsilon & 0 & 0 & 0 \\ -\upsilon & 1 & -\upsilon & 0 & 0 & 0 \\ -\upsilon & -\upsilon & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\upsilon) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\upsilon) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\upsilon) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

Now in the matrix form, how these stress-strain relationships can be written? Let us see, so it is in the matrix form. So, this is how the strain can be written in terms of stresses.

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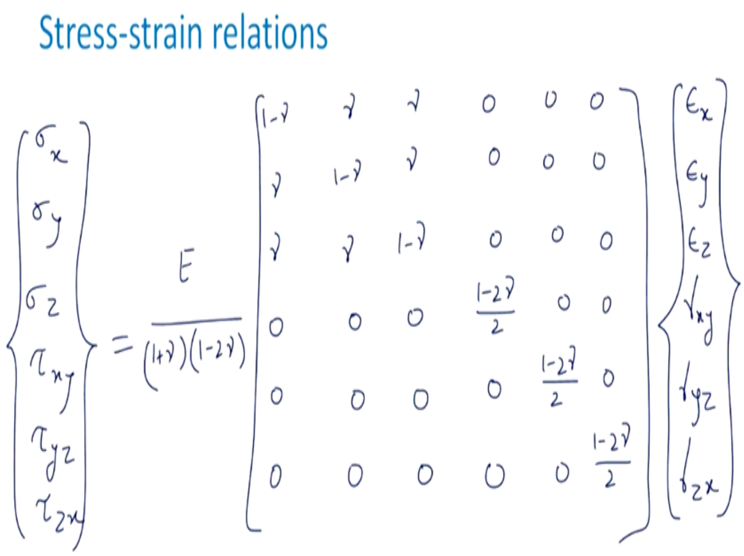

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{(1+\upsilon)(1-2\upsilon)} \begin{bmatrix} 1-\upsilon & \upsilon & \upsilon & 0 & 0 & 0 \\ \upsilon & 1-\upsilon & \upsilon & 0 & 0 & 0 \\ \upsilon & \upsilon & 1-\upsilon & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\upsilon}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\upsilon}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\upsilon}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

Similarly, you can write these stresses in the form of strains, let us see how. So, here now we will have these stress vector. So, it is one and the same thing. The equations they can be written in the matrix form, stress can be represented in the form of strains or the strains can be represented in the form of stresses.

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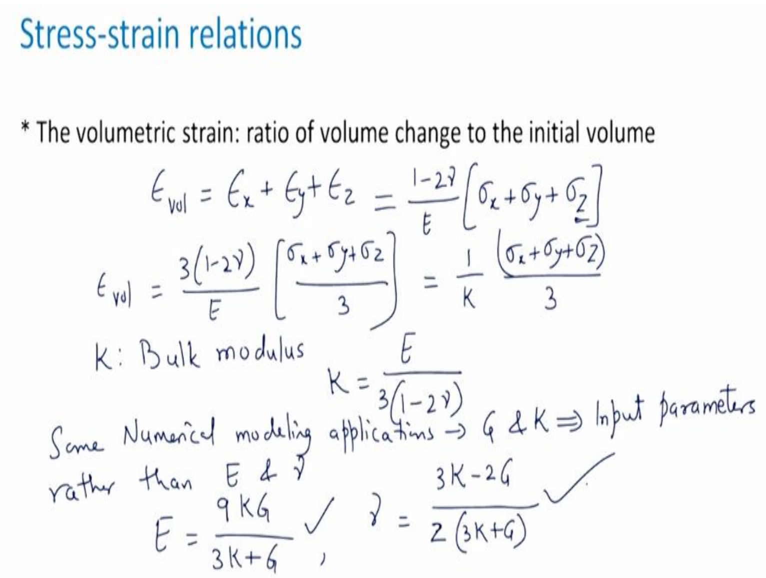

$$\epsilon_{vol} = \epsilon_x + \epsilon_y + \epsilon_z = \frac{1-2\upsilon}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\epsilon_{vol} = \frac{3(1-2\upsilon)}{E}\left[\frac{\sigma_x + \sigma_y + \sigma_z}{3}\right] = \frac{1}{K}\left[\frac{\sigma_x + \sigma_y + \sigma_z}{3}\right]$$

$$E = \frac{9KG}{3K+G} \qquad \upsilon = 3K - 2G \big/ 2(3K+G)$$

So, let us proceed further that we have the volumetric strain, which is the ratio of volume change to the initial volume.

Now, in some numerical modeling approaches or applications these G and K they are used as the input parameters rather than E and nu. So, therefore it is important for us to know about all these. So, this is how that you can have the relationship between E, K, G and Poisson's ratio.

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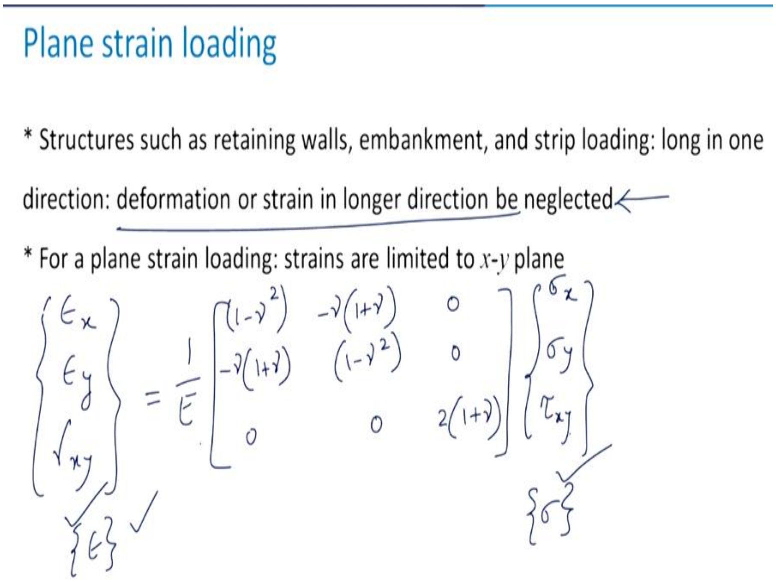

Now, for these structures related to geotechnical engineering include retaining walls or embankments and strip loading, they are long in one direction and therefore the deformation or the strain in the longer direction it can be neglected. So, such type of the loading is called as plane strain loading. So, for a plane strain loading these strains they are limited to x-y plane. So, let us write in the light of this property de stress strain relationship for plane strain loading.

$$\begin{Bmatrix} \in_x \\ \in_x \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E} \begin{Bmatrix} (1-\upsilon)^2 & -\upsilon(1+\upsilon) & 0 \\ -\upsilon(1+\upsilon) & (1-\upsilon)^2 & 0 \\ 0 & 0 & 2(1+\upsilon) \end{Bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

So, this is how that we can write it. So, this can also be written as stress vector and this is the strain vector.

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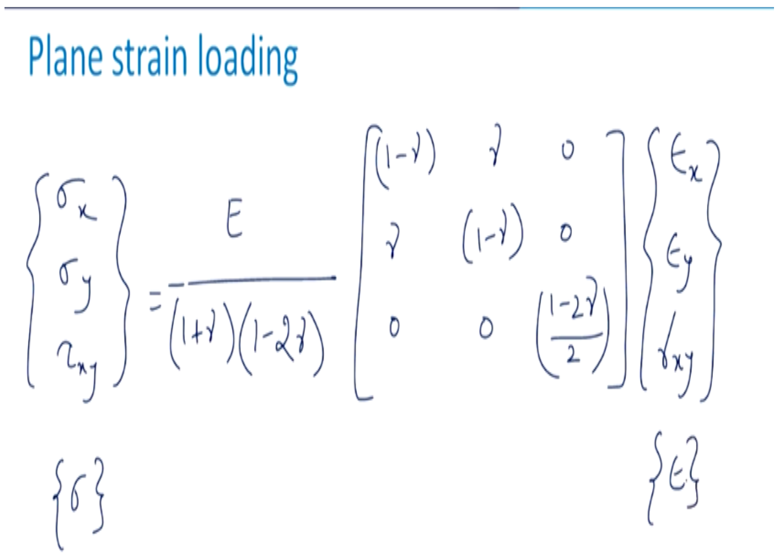

Or like we did in earlier case stresses also can be represented in the form of strains. So, this is how we have stress vector here and this is the strain vector.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\upsilon)} \begin{Bmatrix} (1-\upsilon) & \upsilon & 0 \\ \upsilon & (1-\upsilon) & 0 \\ 0 & 0 & \frac{1-2\upsilon}{2} \end{Bmatrix} \begin{Bmatrix} \in_x \\ \in_x \\ \gamma_{xy} \end{Bmatrix}$$

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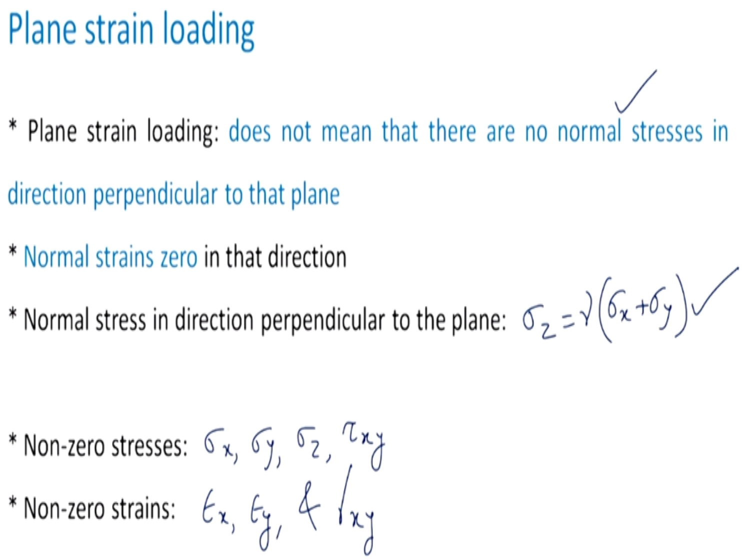

Now, what will happen to the normal stress which is perpendicular to the plane. So, you see that the plane strain loading it does not mean that there are no normal stresses in the direction perpendicular to that plane. So, normal strains zero in that direction which is perpendicular to the plane and in this case, we have taken that x y plane and so perpendicular to that plane is the z axis. So, normal strains are going to be zero in that direction, but not the normal stress.

So, what is going to be the normal stress in the direction which is perpendicular to the plane.

And therefore, in case of the plane strain loading you will have the nonzero stresses .So, this is how the stress-strain relationship or the constitutive relationship for the plane strain loading can be obtained or you can use these in the analysis of the structures which are going to be there in the rock or rock masses.

So, this is what that I wanted to discuss with you. So, just to summarize we had the discussion about the soil versus rock. Then we discussed about these constitutive relationships that there can be different types of the constitutive relationship. And to start with we discussed about the linear elastic theory and I mentioned to you about the stress-strain relationship. And then I explained you the concept of plane strain loading.

In this context, I suggest you to remember that the normal strains they are zero in the direction perpendicular to the plane where we are considering the structure, but not the normal stresses. So, in the next class we will discuss about the plane stress loading, axisymmetric loading and then we will continue with different failure criterion which can be applicable in case of the rocks and rock masses. Thank you very much.