Water and Waste Water Treatment
Prof. Bhanu Prakash
Department of Civil Engineering
Indian Institute Technology – Roorkee

Lecture – 08
Mass Balance: Nonideal System and Water Quality Parameters

Hello everyone, welcome back to the latest lecture session. A quick recap of what we have been up to in the last session or rather than recap, we will just look at what are the take home messages. We looked at different ideal case reactors, batch reactor, CSTR and plug flow reactor. We looked at how or why they are needed for us in water and wastewater treatment or when we are trying to understand water and wastewater treatment.

Then we looked at applying mass balance equation for the batch reactor, the plug flow and the CSTR. We also looked at the fundamental mass balance equation, which I just threw into the mix to help you understand that there is more complex system at hand, but we typically simplify it to use it or applied it to the ideal case reactors now. Then we looked at comparing CSTR and plug flow reactor. We saw that for the same volume and for the same flow rate, we saw that plug flow reactor typically is more efficient than a CSTR.

Why is that? We saw that the rate throughout the CSTR has to be relatively low, because the outlet concentration is typically low. That is what we want. That is not the case in the plug flow reactor. Why is that because in a plug flow reactor, the concentration changes with distance and is not the same as the concentration in the outlet.

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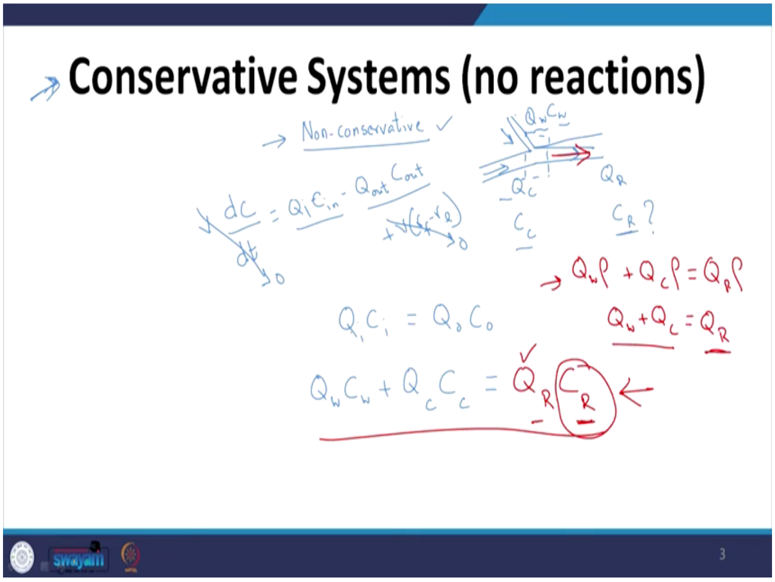

Conservative Systems (no reactions)

→ Non-conservative ✓

$V\frac{dC}{dt} = Q_i C_{in} - Q_{out} C_{out} + V(r_f - r_d)$

$Q_i C_i = Q_o C_o$

$Q_w C_w + Q_c C_c = Q_R C_R$

$Q_w \rho + Q_c \rho = Q_R \rho$

$Q_w + Q_c = Q_R$

Let us move on and see what else it is that I have out here. We have conservative systems no reactions, until now, we looked at non-conservative systems. I should have probably discussed this easier aspect first and come to non-conservative systems. But I mean first discuss the conservative and then come to non-conservative. But I always like to get the difficult part out first, because people typically over think things. Non conservative, we are done with that. But let us look at the case when we have no reactions.

For example, this is my river coming in or canal. This is my wastewater coming in from an industry. Q from the wastewater and Q of the canal and this I am going to call as the Q river let us see and the concentration or the BOD here is C waste and here at C of the canal and C of the river. If I know these 4 variables, how do I get this particular concentration of the or the BOD levels in the mixer stream, which is the river and keep in mind that the flow is going in this direction?

How do I get that mass balance? Here we are assuming steady state. What is the fundamental mass balance equation we have the accumulation term is equal to Q_{in} C_{in} - Q_{out} C_{out} plus volume into rate of formation minus rate of loss. But here we are assuming, or we are saying that it is a conservative system. This term is 0. If it is at steady state, this term is also 0. What is it that? I have Q and C and mass coming in is equal to the mass going out.

That is just it, $Q_{in} C_{in} = Q_{out} C_{out}$. If the mass balance is over, this particular control volume, so mass coming in is due to 2 streams Q of the waste into concentration of the waste plus Q of the canal and concentration of the waste the compound already existing in the canal is equal to the mass going out of the system and how is the mass leaving the system here? This is what we have out here.

That the in the form of the river flow rate of the river into concentration or the BOD in the river, so that is what we have, but how do I get this? It seems like I have only one equation and 2 unknowns here but how do I get it? Here I can also apply it over the mass itself and assuming not assuming. Q waste into density of water plus Q canal into density of water is equal to Q river into density of water.

Assuming that the density of water is the same here, what is it that I end up with $Q_W + Q_C = Q_R$. We can write it or understand that without looking at this equation 2. The waste stream is coming in and can always coming in and both are joining to form the river let us see. That is what we have $Q_{waste} + Q_C = Q_{river}$. From that we have Q river and so, we just need to find C of the river concentration of the compound in the river.

With one particular equation, we can solve for this unknown conservative system. In this way different aspects can be looked at here, we are just looking at 2 streams joining there are other examples, which might be slightly trickier and which we will try to apply out here. Let us move on to the other aspects I wanted to look at.

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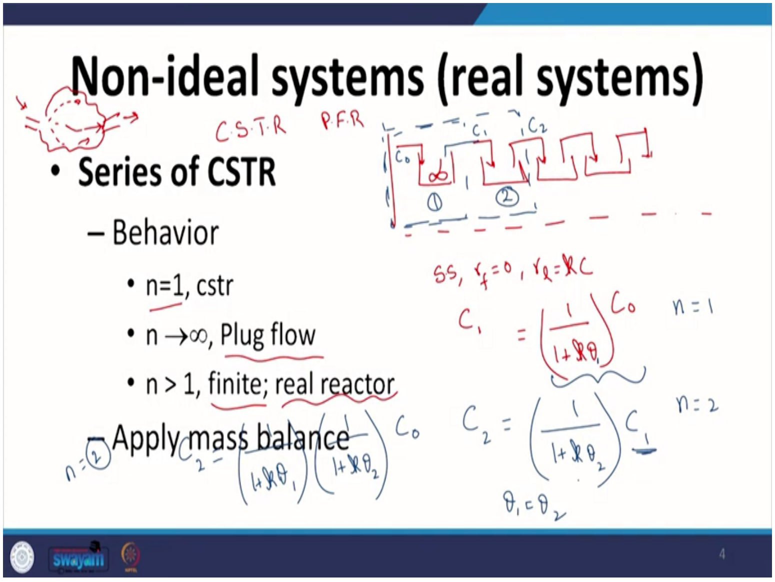

Non-ideal systems (real systems)

- Series of CSTR
  - Behavior
    - n=1, cstr
    - n →∞, Plug flow
    - n > 1, finite; real reactor
  - Apply mass balance

$SS,\ r_f = 0,\ r_R = kC$

$$C_1 = \left(\frac{1}{1+k\theta_1}\right) C_0 \quad n=1$$

$$C_2 = \left(\frac{1}{1+k\theta_2}\right) C_1 \quad n=2$$

$\theta_1 = \theta_2$

$$n = ② \quad C_2 = \left(\frac{1}{1+k\theta_1}\right)\left(\frac{1}{1+k\theta_2}\right) C_0$$

When we are talking about CSTR and plug flow reactor, we always talked about them are referred to them as ideal systems we are assuming complete mixing within our particular CSTR. If I want to understand how a lake behaves, we typically apply the CSTR model to the lake. But in a lake, the inlet is at one end and outlet is at the other end, do we really presume? Or can we presume that the lake is completely mixed? No? It does not really. That is not how it works out there.

$$C_1 = \left(\frac{1}{1 + k\theta_1}\right) C_0$$

But that these are the assumptions we are making. But you can look at different aspects short circuiting when we look at the non-ideal systems, but we are not going to go into that in that detail especially because this is an undergraduate course. But we will just look at understanding some of the real systems. One case is having a series of CSTR and what do we have out here we have flow coming in flow going out and complete mixing.

But if it is a series of reactors, what is it that we have this outlet will be the inlet for the next one ? We have that in series then what is it? Or how do you think that will behave? Let us see. In general, we should not have let the cat out of the bank. , if n = 1 number of reactors is 1, it is a CSTR, it behaves as a CSTR. But if it is n = infinity, let us se, it will end up behaving as a plug flow, we will briefly look at that particular data or set let us see or information.

If n is greater than 1 it is finite, it will behave like a real reactor. In the context of our particular wastewater treatment, I am not talking about it in terms of the lake. If I am talking about the lake and this is the flow coming in and this is my lake, it is not as if , everything will be mixed, some water will go out here, some will be short circuited, indirectly come out here and you have the depth of the reactor also playing depth of your lake also playing a role.

But we are not going to look at the natural systems; we are looking at the engineered systems here. If n greater than 1, it is a finite or real reactor. When we applied the particular mass balance equation to a CSTR at steady state and rate of formation is equal to 0 and rate of floss is based on a first order last reaction, we see that it was kc what is it that we ended up with? I think we had C_{out} or C_{theta} / C_{in} = 1 / 1 + k theta. Let me slightly rewrite it so that I can use it here for my case.

I will write it as C_1 is the out. C_{in} is the C_0 let us see and taking this term out here. What is it going to be? It is equal to this into C_0 C_{out} which is what I am calling as C_1 is going to be 1 / 1 + k theta into C_0 . That is fine when I have just 1 reactor out here. That we just changed the ink color so that we can so if that is fine have just the single reactor but if it is 2 reactors now. How do I go about it? This is the case when n = 1.

But if n = 2, for the first one, fine, this is the case but what is it for C_2 here, for the number reactor number 2, this is for 1. C_0, C_1 I think I missed up something out you. We have this flow going in and this is C_1 and this will be C_2. What is C_2 going to be equal to C_2 is equal to so if this is theta 1 1 / 1 + k theta_2, assuming that the volume is different for the second one, C_1, this is the case.

$$C_2 = (\frac{1}{1 + k\theta_2})C_1$$

But what is C_1, we know that C_1 is equal to this. C_2, which is nothing but the outlet from my whole 2 CSTR in series system is equal to 1 / 1 + k theta_1, you do not need to mug up anything up here. Typically, our exams are going to give you the relevant formulas and you are going to be able to solve it, you just need to display that you have an understanding of the relevant system into C_0. That is what we have C_2 is equal to this when n = 2. If theta 1 = theta 2 let us see.

$$C_2 = (\frac{1}{1 + k\theta_1})^2 C_0$$

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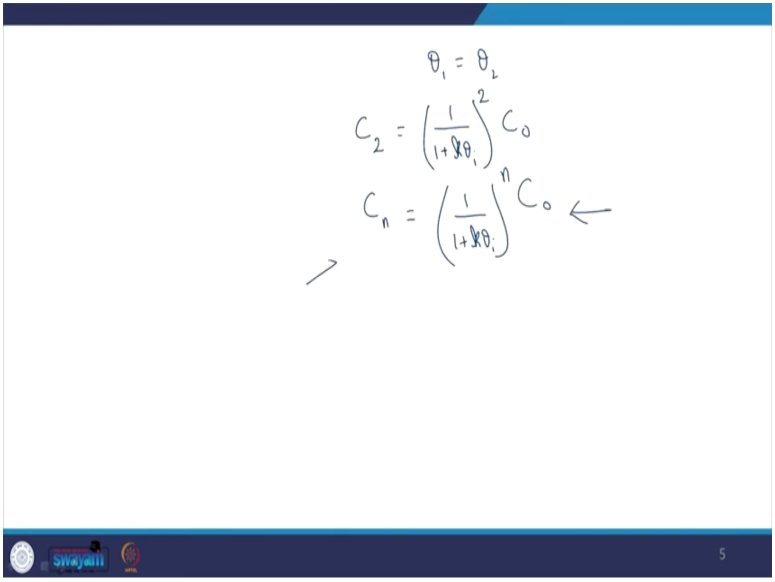

$$\theta_1 = \theta_2$$

$$C_2 = \left(\frac{1}{1+k\theta_1}\right)^2 C_0$$

$$C_n = \left(\frac{1}{1+k\theta_i}\right)^n C_0 \leftarrow$$

I should move on to looking at this particular case. , if theta_1 = theta_2 and if I try to generalize it, what will it become? For C_2, it is going to be equal to 1 / 1 + k into theta_i . theta i is the 1 for each reaction equal to square times C_0. Similarly, C_n, if there are n reactors equal to 1 / 1 + k times theta_i, to the power of n, C_0 or what is coming in? Let us see. This is for the case when CSTR are in series. No need to mug up anything up but just trying to understand the system.

$$C_n = (\frac{1}{1 + k\theta_i})^n C_0$$

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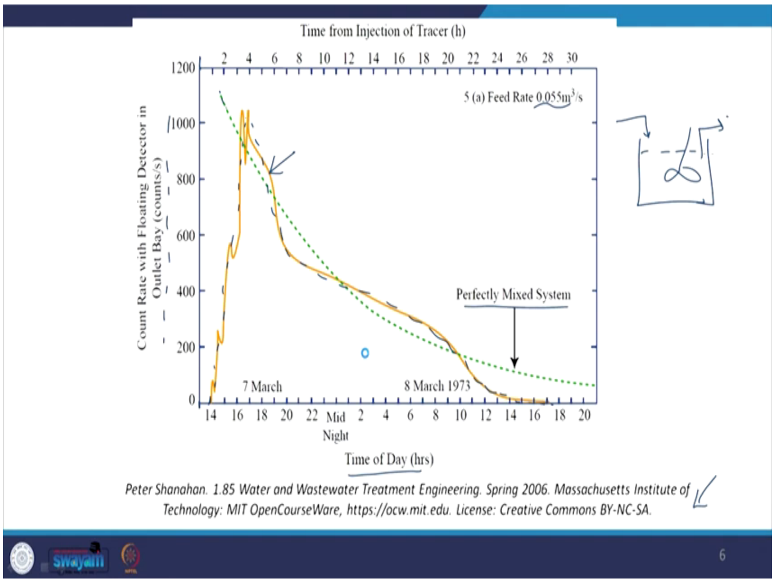

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Let us try to look at some data that we have out here. You have the reference given here. Here, what is it that people do they had a lake out here and feed rate is given out here and they injected a tracer, keep in mind that it is not as if there is a continuous flow of the tracer. Water is flowing into the lake water is flowing out of the lake of your system. While the water is flowing in at one time, at one point, some tracer ejected; it is not a continuous injection, just what one event injection of the tracer.

If it was an ideal system perfectly makes what should happen immediately, everything should be diluted. Let me understand the graph here so that we can go through the relevant data. On the x axis, we have the time and days. They started on 7th March at around, what is it 12 noon and then we have the concentration of the outlet here, a concentration of the tracer in the outlet and that is on the y axis.

If it is completely mixing system, ideal system, what should happen, as soon as I put the tracer in, it should be immediately diluted? That should be shown up immediately in my outlet. That is why we have this theoretical profile of perfectly mixed or ideal CSTR. But what happens it takes time and that is what you see, even though the tracer is put in at this point, it takes some time for the concentration to be become relatively uniform.

Then you see the decrease in concentration out here, was the water is removing the tracer or the tracer in the water is flowing out along with the water. That is what we see, or we see that it is never really ideal reactor, or ideal system. Let us see. That is one case this is for one system now.

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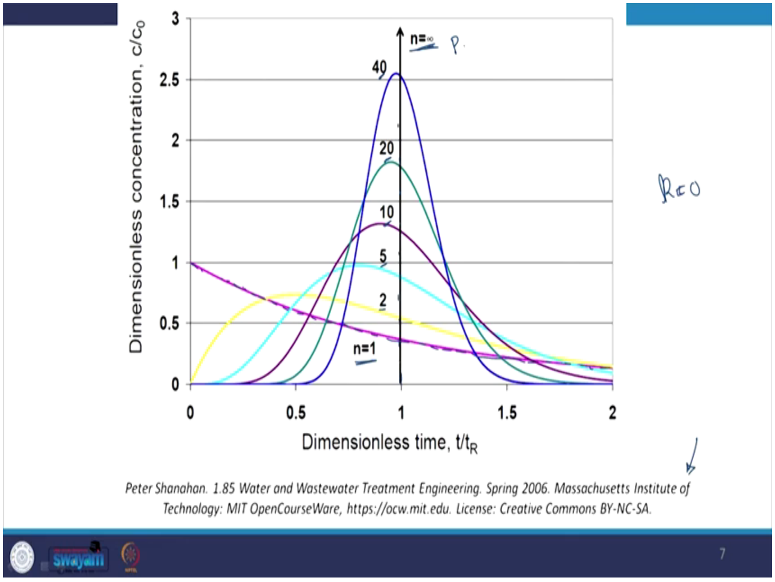

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For a tracer injection and for this particular model, they looked at k = 0, you look and look at the reference in detail if you want to, we had the earlier case, something like this, but I think it is without the steady state assumption. Anyhow, we have at that if n = 1, dimensionless time here and dimensionless concentration C / C$_0$. What is it going to behave like, with great constant equal to 0, this is what it is going to behave like the CSTR?

When n = 1 this is the CSTR. We already had the graph or data for this in the earlier session when we had some rate constants at that time. But never mind, this is the case when we are injecting a tracer. If you are injecting a tracer for a single reactor, this is what it has to look like? That is what we see. But if n = 2, you see that the profile changes and n = 5, slightly. N = 20, I mean 10, 20 and 40 and n = infinity. What does it look like? Whatever I put in out here, it just flows out like that.

This is what a tracer input. That is what you see when n equals infinity, it looks like a plug flow reactor and that is what we discussed out here. When n = infinity more or less this system is like a plug flow reactor when n = 1 CSTR anything in between it looks like a finite or real reactor. That is something to understand.

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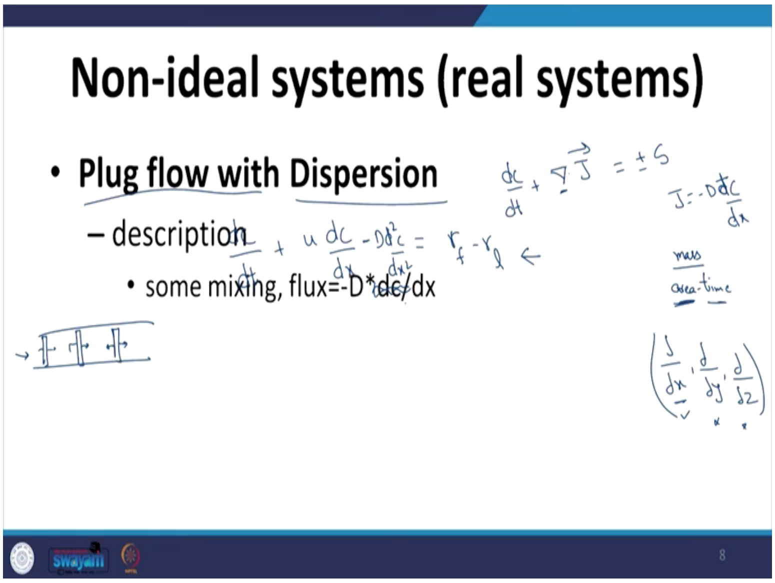

Non-ideal systems (real systems)

- Plug flow with Dispersion
  - description
    - some mixing, flux=-D*dc/dx

$$\frac{dc}{dt} + \nabla\cdot\vec{J} = \pm S$$

$$J = -D\frac{dc}{dx}$$

$$\frac{mass}{area\cdot time}$$

$$\frac{dc}{dt} + u\frac{dc}{dx} - D\frac{d^2c}{dx^2} = r_f - r_l \leftarrow$$

$$\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right)$$

Let us move on and non-ideal systems plug flow with dispersion, let us see, what is the fundamental mass balance equation, we know that it was dc / dt + del product of flux, what is flux Mass per area per time? How much mass is coming in through a particular area per time? Is equal to plus or minus sources or sinks. Del product means I want to understand what or how is my variable changing with x, y and z?

$$\frac{dC}{dt} + \Delta\bar{J} = \pm S$$

When we applied it to our ideal case systems and assuming advection direction only one direction and assuming that there was no diffusion, we ended up with an equation like dc / dt plus assuming that velocity is the same incompressible liquid u and concentration is changing only one direction or the river or the canal is flowing in only one direction. Concentration does not change with y and z, but only changes with x.

Assuming that there is no flux due to dispersion or such, it is equal to plus or minus sources or sinks, meaning rate of formation minus rate of loss. But if there was diffusion, what is it that it would have been, it would have been, flux due to diffusion equal to minus d, should not have written the derivative. Let us go with this derivative dc / dx. It would have been d is the diffusion coefficient or the dispersion coefficient or the turbulent diffusion coefficient.

$$\frac{d}{dx}, \frac{d}{dy} \text{ and } \frac{d}{dz}$$

Second derivative, so, when we assumed ideal case, we assumed that this does not occur, but think about it in a river. Whatever I put in here will not just stay like this. It is going to make slightly this way and that way with the preceding and succeeding parcels, there will be dispersion, because of the turbulence in that particular fluid. If we take this into account and you can solve this knot relatively easily, but people have solid. If you solve this, you will get the case for plug flow with dispersion. Let us see what we have out here some mixing meaning with the flux is equal to - D dc / dx.

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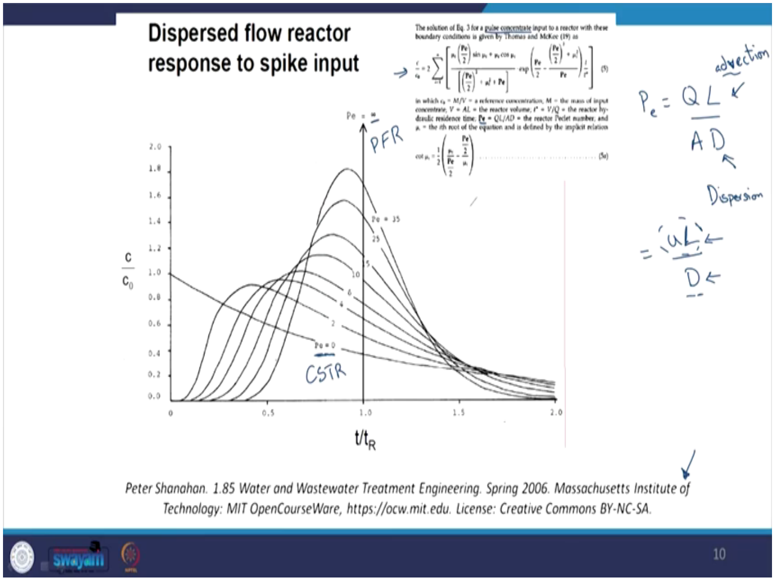

Dispersed flow reactor response to spike input

The solution of Eq. 3 for a pulse concentrate input to a reactor with these boundary conditions is given by Thomas and McKee (19) as

$$\rightarrow \frac{c}{c_0} = 2\sum_{i=1}^{\infty}\left[\frac{\mu_i\left(\frac{Pe}{2}\right)\sin\mu_i + \mu_i\cos\mu_i}{\left[\left(\frac{Pe}{2}\right)^2 + \mu_i^2 + Pe\right]}\cdot\exp\left(\frac{Pe}{2} - \frac{\left(\frac{Pe}{2}\right)^2 + \mu_i^2}{Pe}\right)\frac{t}{t^*}\right] \quad (5)$$

in which $c_0 = M/V$ = a reference concentration; M = the mass of input concentrate; $V = AL$ = the reactor volume; $t^* = V/Q$ = the reactor hydraulic residence time; $Pe = QL/AD$ = the reactor Peclet number; and μ_i = the ith root of the equation and is defined by the implicit relation

$$\cot\mu_i = \frac{1}{2}\left(\frac{\mu_i}{\frac{Pe}{2}} - \frac{\frac{Pe}{2}}{\mu_i}\right) \quad (5a)$$

$$P_e = \frac{QL}{AD}$$

advection

Dispersion

$$= \frac{uL}{D}$$

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Apply mass balance to differential element. Here it is, the reference out here. Here, we let us see what we have we have input to your reactor pulse input, meaning one time injection of your particular tracer or pulse let us see. Tracer apart me and this is the solution. Let us not go into that. But let us just look at what we have out here there is a particular variable called the Peclet number. What is it given us QL / A into D. D is not diameter such it is the dispersion coefficient.

If you look at the numerator and denominator, the numerator will give you an idea about advection flow mass flux or mass transport due to advection. The one or the bottom will be the one due to dispersion. That is equal to velocity ul / D. Advection by dispersion, why is that we will just look at it. If p is equal to infinity, what does that mean? Meaning advection or Peclet number if it is equal to infinity, what does that mean? This numerator is much, much greater than the denominator out there.

Advection predominates and not really dispersion. But if we keep decreasing it and then Pe goes to 0, when P e goes to 0, what does that mean? There is no advection, but only dispersion. , when Pe equals infinity, it is a plug flow reactor. As we decrease it and Pe and sub being 0, it is a CSTR. That is what we C out here. Where it is that your system lies will depend upon this particular Peclet number or such?

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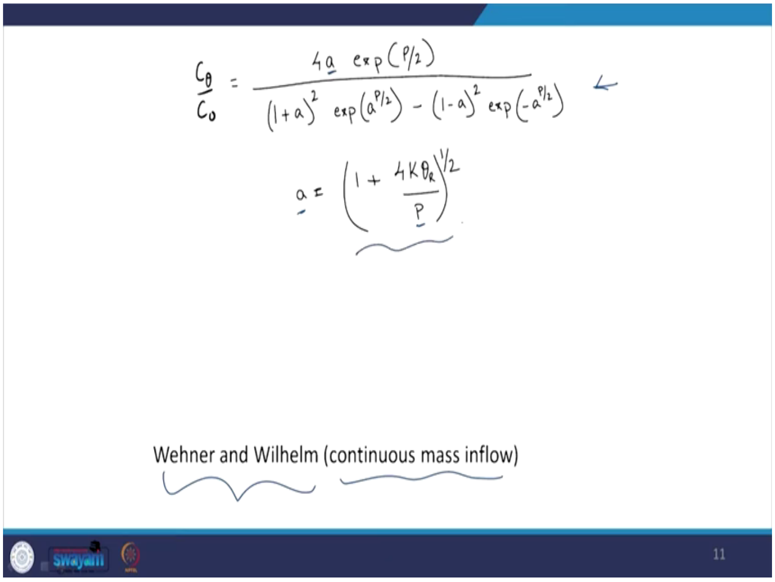

$$\frac{C_\theta}{C_0} = \frac{4a \exp(P/2)}{(1+a)^2 \exp(aP/2) - (1-a)^2 \exp(-aP/2)}$$

$$a = \left(1 + \frac{4k\theta_R}{P}\right)^{1/2}$$

Wehner and Wilhelm (continuous mass inflow)

If it is earlier case was when I injected the tracer just once let us write tracer injection, but if there is continuous flow coming in like my wastewater treatment plant, I have continuous waste coming in. How do I solve that previous equation? People have solved it for the continuous mass flow, and this is what we have , Pe is the Peclet number and a here which is presented here. This is what we have out.

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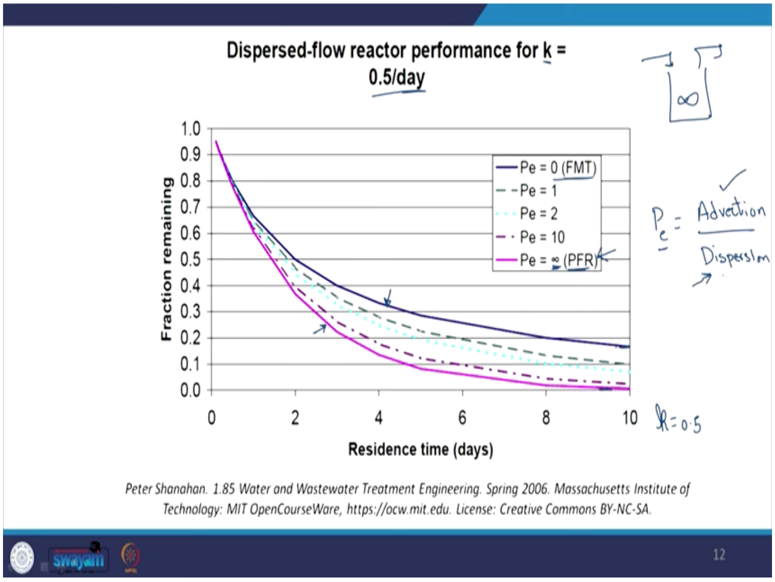

Dispersed-flow reactor performance for k = 0.5/day

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For this , let us see what the plot looks like. , dispersive flow reactors for performance where k = 0.5 minus day inverse. If Peclet number is high, what will it look like plug flow reactor so that is what we have. As we keep decreasing the Peclet number. What did we say it was more or less advection? What is advection now? By dispersion, we are not talking about diffusion itself.

If canal is transporting, some contaminants, so you have a net fluid flow in one direction and thus you have advection, their mass transport due to advection. Diffusion, I think the example that I looked at was I just open the scent bottle. Even though now there is the scent compound, the compound in the scent bottle is only present in this vicinity and not at the end of the rooms or the corner of this room.

Over some time, it will end up there, why is that concentration gradient, the system wants to achieve uniform chemical potential. You are going to have transport molecular diffusion from here to the different coordinates of the room. But that is a very slow process. But if I turn on the fans, I have turbulence and dispersion will take place. It is relatively faster, think of it for fans being turned on or mixing being allowed, there is no advection there is no net fluid flow in any single direction, but you have mixing conditions.

You will have dispersion out here. Advection by dispersion, so advection, when Peclet number is infinity that means advection is very high. That is what we mean when we have plug flow reactor and as we decrease it, that means that dispersion also has a role and when it is only dispersion. There is no advection this is what we have, is there any advection here, no, it is completely mixed.

That is why we end up with this. If you remember some of our systems that we looked at earlier, I drew this for k = 0.52. We also see that plug flow reactor performance is better compared to the CSTR. That is what we see out here.

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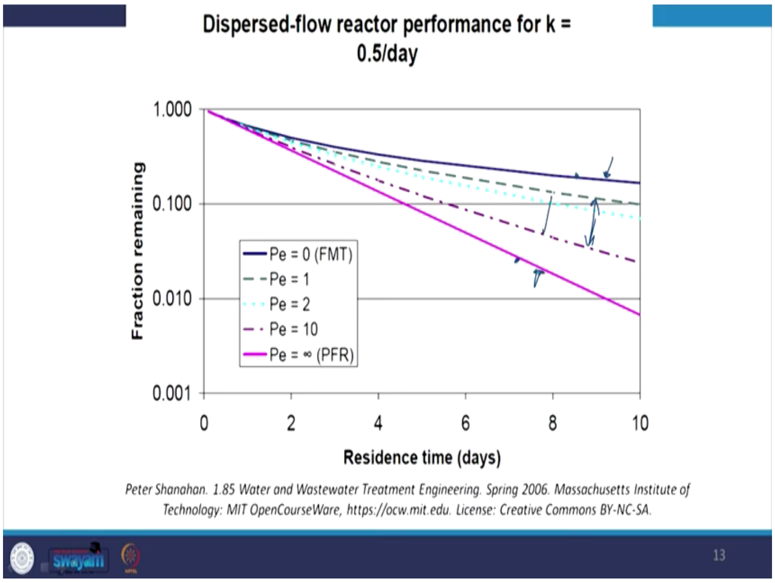

Dispersed-flow reactor performance for k = 0.5/day

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If I take the logarithmic scale on the y axis, this is what it looks like more or less the same information presented in a different manner out here. Real systems, they will be somewhere out here. They would not be typically plug flow reactors, they would not be CSTR. But they will be somewhere in between that something for you to understand. But typically, in wastewater treatment plants depending on how you design it, you people might assume CSTR or plug flow reactor. But you need to understand that the actual performance, the deviation will be due to this case, let us see.

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Primary settling tanks in foreground followed by aeration tanks and circular secondary settling tanks.

Here is one case where they tried to achieve plug flow reactor system. Let us just try to understand this, what we have here we have primary settling tanks in the foreground primary settling tanks as an am just trying to remove the suspended solids which are heavier, by gravity, they will settle to the bottom, they will settle to the bottom and then there are organic compounds, which I will try to remove by the microbes using the microbes.

Microbes though, similar to us or me, they need oxygen, which is an electron acceptor to be able to let the redox process go through. That is why I am going to pump in air. That is why you see this particular turbulence. But did they design it as CSTR? Not really. How did they design it, they designed it as if the flow was going in this direction. This direction, like semi plug flow, they were trying to achieve plug flow reactor conditions.

That is what you see it is neither a CSTR nor certainly a completely a plug flow reactor but they were trying to achieve plug flow reactor like conditions. Then we have the secondary settling tank let us not go there. But I just wanted to show case where it is certainly not CSTR but trying to achieve plug flow reactor conditions.

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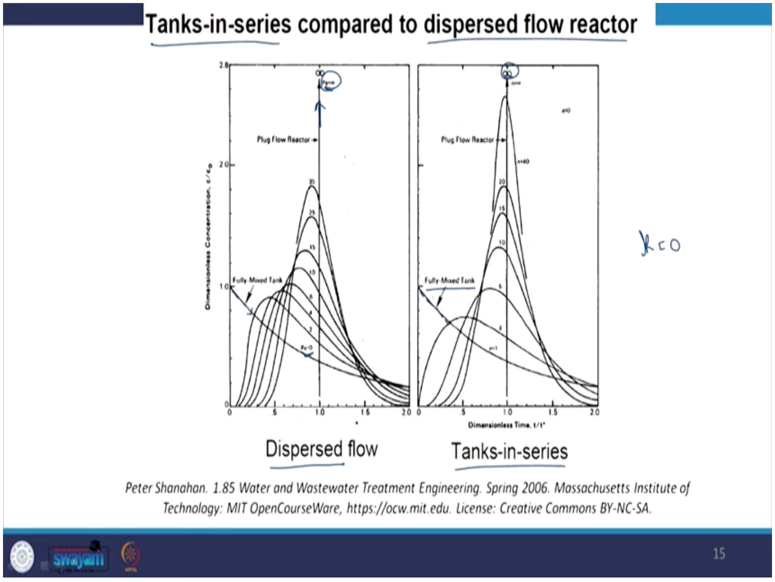

Tanks-in-series compared to dispersed flow reactor

Dispersed flow

Tanks-in-series

$k = 0$

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Let us move on. Lastly, we will compare the tanks-in-series compared to disperse through flow reactor, let us see. This is for the case when rate constant is 0 and for only one-time injection of the tracer. Dispersed flow and tanks-in-series. When n is equal to infinity, what do we see it looks like a plug flow reactor and n = 1 tanks-in-series it is only a CSTR but with dispersed flow too we looked at it Peclet number is infinity.

Meaning advection outweighs dispersion then we see that it behaves as a plug flow reactor when Peclet number comes down and down such that there is no advection and only dispersion.

What is it that you see it acts as a fully mixed tank? From both ways or viewpoints, you can understand the particular system enough of that.

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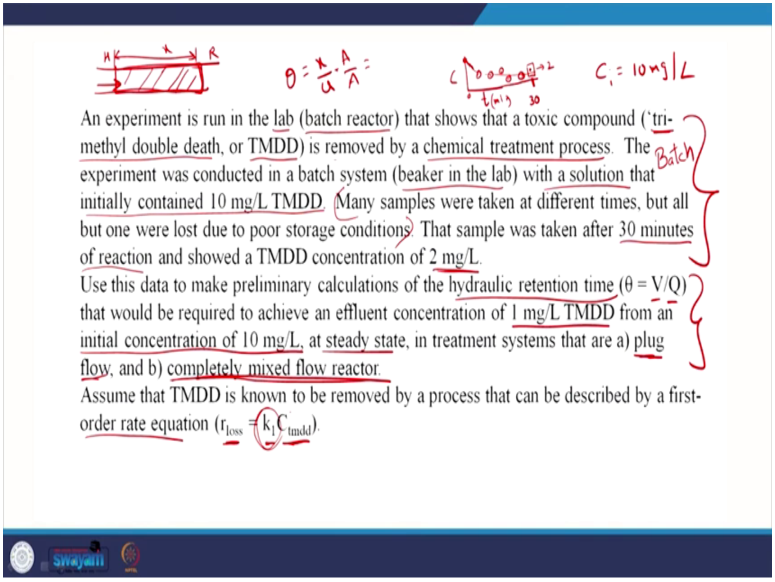

An experiment is run in the lab (batch reactor) that shows that a toxic compound ('tri-methyl double death, or TMDD) is removed by a chemical treatment process. The experiment was conducted in a batch system (beaker in the lab) with a solution that initially contained 10 mg/L TMDD. Many samples were taken at different times, but all but one were lost due to poor storage conditions. That sample was taken after 30 minutes of reaction and showed a TMDD concentration of 2 mg/L.

Use this data to make preliminary calculations of the hydraulic retention time ($\theta = V/Q$) that would be required to achieve an effluent concentration of 1 mg/L TMDD from an initial concentration of 10 mg/L, at steady state, in treatment systems that are a) plug flow, and b) completely mixed flow reactor.

Assume that TMDD is known to be removed by a process that can be described by a first-order rate equation ($r_{loss} = k_1 C_{tmdd}$).

Let us move on. Here we have an example that we will bring in various aspects together. An experiment is run in the lab. Here we in the lab we typically conduct batch experiments and that shows that a toxic compound which we are referring to as tri- methyl double death, or which we are referring to as TMDD. As TMDD is removed or degraded by a chemical treatment process.

The experiment was conducted in a batch system, which is a beaker in the lab with a solution that initially contained 10 milligrams per liter trimethyl double death. Initially is equal to 10 milligram per liter. Many samples were taken at different times, but all but one we are lost due to poor storage conditions. That sample was taken after 30 minutes of reaction and showed a tri-methyl double death concentration of 2 milligram per liter, how much is it? 2 milligram per liter.

Typically, what we try to do is we have concentration here and time here. We have the initial concentration at time 0. We try to, measure the concentration at different times and we fit a model so that we can understand what is the, order of the reaction and try to predict the behavior of the system in other systems.

Here though, looks like we being, the blunderbuss that we are, we lost all these samples. We only have the initial sample and this final sample, so even then you can get some idea about the system. But it is going to be relatively off the mark, more the data more your estimation is going to be nearer to your true rate constant here. Here, we have time taken after 30 minutes.

This is time in minutes. The concentration was 2 milligram per liter, what else do we have we are now asked to calculate the hydraulic retention time. Hydraulic retention time to refresh your memory, we said was theta = x / u. This is typically used in plug flow reactors, a river. River is flowing in this direction.

From Haridwar to Roorkee, this is x. u is the velocity of flow of your fluid, x / u will give you the time that your compound will spend within this system or within the river with between Haridwar to Roorkee. That is what it will give you an idea about hydraulic retention time. If you multiply that by cross sectional area, that is going to be equal to in a way V / Q.

What do we have here? We have or we are asked to calculate the theta that would be required to achieve an effluent concentration of 1 TMDD from an initial concentration of 10 at steady state in treatment systems that are plug flow and also completely mixed flow. Here we have a case where we have a batch reactor, all this data is from batch and then we are asked to approximate or understand or estimate the behavior in a plug flow system and also in a completely mixed flow reactor now.

Assume that the tri-methyl double death is to be removed by a process that can be described as a first order reaction or first order rate equation and rate of losses, k 1 times the concentration of tri-methyl double death. That is what we have out here. Now, what is the, what are the aspects that link, what information you can glean from your batch reactor worth what you are going to use in your plug flow and your CSTR systems?

For example, I want to see how the chemical that is being released by the industrial zone in Bhadravathi is going to affect the population in Roorkee now. If that particular pollutant is being released into the Ganga canal. Now how do I do that? Let us, first I need to understand the reactions, that are going to occur. Within that particular, volume, not volume control volume.

I need to first conduct that experiment in the lab to find out what order of the reaction is it and how is it being affected? I am going to get the rate constant here from the lab, I will be able to get the rate constant. Now I will be able to use that variable to predict what will happen to the compound in the canal. That is what we are trying to do out here. Now let us have you our batch system.

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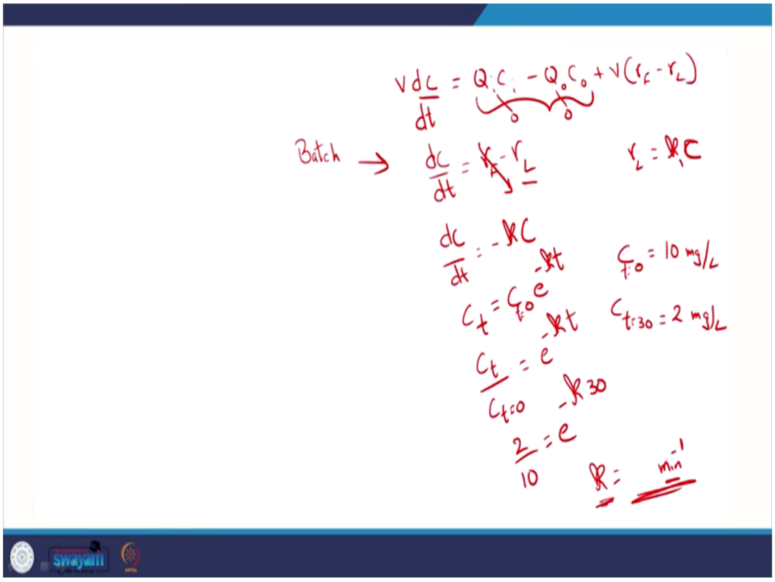

$$V\frac{dc}{dt} = \underbrace{\overset{0}{Q_iC_i} - \overset{0}{Q_oC_o}} + V(r_f - r_L)$$

Batch → $\frac{dc}{dt} = \cancel{r_f} - \underline{r_L}$

$r_L = k_1 C$

$\frac{dc}{dt} = -kC$

$C_t = C_{t=0}\, e^{-kt}$

$C_{t=0} = 10\ mg/L$

$C_{t=30} = 2\ mg/L$

$\frac{C_t}{C_{t=0}} = e^{-kt}$

$\frac{2}{10} = e^{-k30}$

$k = \underline{\quad min^{-1}}$

$$d(VC)/_{dt} = \sum Q_{in}C_{in} - \sum Q_{out}C_{out} + V(r_f - r_l)$$

If I have a blank slate here, so blank slate is here, but let us try to calculate what we have. From the lab scale, what is the not lab scale for the batch reactor V dc / dt = Q_i C_i (mass coming in) - Q_{out} C_{out} (mass going out) plus volume into rate of formation minus rate of loss, as no flow coming in no flow going out. Both of these are equal to 0 let us see, for a batch reactor for a batch reactor.

Now what do we have we been going to end up with dc / dt is equal to rate of formation minus rate of loss. But is tri-methyl double death being formed? No, it is not, but it is being lost. What is the rate of loss, it is given us k 1 times concentration of that particular compound tri-methyl double death. , dc / dt = - kc. You can integrate and we will end up with C_t = C_0 or $C_{initial}$ into e power - k t. That is, it.

$$C_t = C_0e^{-kt}$$

Concentration at time t and what do we have we know that $C_{initial}$ or C_0 here, it is not C_{out}, let me say C at time is equal to 0, we know is 10 milligram per liter. C at time is equal to 30 minutes is equal to what now, I think it was 2 milligram per liter. But typically, we need to bother about the units. But here as you can see, C t / C t is equal to at time equal to 0 = e power – kt, we see that, the terms are the units are cancelling out, so we do not need to really bother about that.

10 or 2 / 10 = e to the power of - k, which is what we are trying to calculate into 30. k value, we can get the value of k in minute inverse but typically, depending on how you are going to use it in parts A and B, you are going to have to change the units here. This is the process. Keep in mind that we are just calculating the rate constant.

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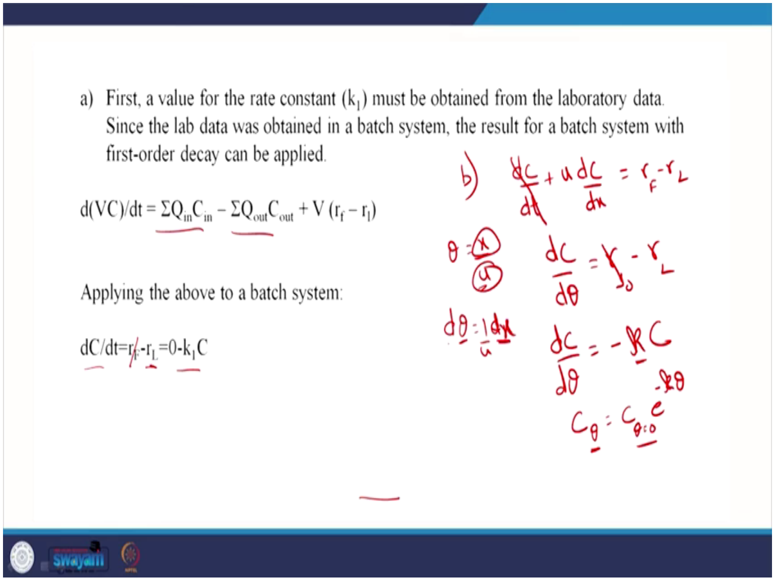

a) First, a value for the rate constant (k_1) must be obtained from the laboratory data. Since the lab data was obtained in a batch system, the result for a batch system with first-order decay can be applied.

$$d(VC)/dt = \Sigma Q_{in}C_{in} - \Sigma Q_{out}C_{out} + V(r_f - r_l)$$

Applying the above to a batch system:

$$dC/dt = \cancel{r_f} - r_L = 0 - k_1 C$$

$$b)\ \cancel{\frac{dc}{dt}} + u\frac{dc}{dx} = r_f - r_L$$

$$\theta = \frac{x}{u}$$

$$\frac{dc}{d\theta} = \cancel{r_f}_0 - r_L$$

$$d\theta = \frac{1}{u}dx$$

$$\frac{dc}{d\theta} = -kC$$

$$C_\theta = C_{\theta=0}e^{-k\theta}$$

We will just regurgitate, what we have been looking at; We have the mass balance equation for a batch system that is going to be relatively simple, because we have no flow coming in no flow going out. No rate of formation of the compound, we know that it is only being lost. , dc / dt = – kc and integrating . Looks like we have to be 0.05 minute inverse, which you can convert into seconds.

Now, the second part of the system was to use this for a batch not batch plug flow system and at steady state, if it was not mentioned, we are assuming steady state. We know from the fundamental mass balance equation u dc / dx is equal to rate of formation - rate of loss. At steady state, this is 0 x / u is theta. Theta = x / u the hydraulic retention time. , dc / d theta is equal to rate of formation, which is 0 - rate of loss.

dc / d theta = - k, which you already calculated in part, not part in the initial cases , when we say rate constants a constant for that particular set of temperature for those conditions, pardon me. Whatever you do in your batch reactor, the rate constant is still going to be the same even in your plug flow system. That is right. , k is going to be constant and kc let us see.

What do we have C at theta = C when theta = 0 into e power - k theta. , keep in mind it is not changing with time, but the C is changing with distance because theta = x / u x is the variable u is the constant here. When we say dc / d theta, it means that we are changing d theta = 1 / u dx we are concerned with the change with the distance rates, or the average time that your molecule is spending in that system.

What did the question ask us for I think we have what do we needed for the plug flow reactor. We want a final concentration of 1 milligram per liter if the initial one is 10 fine. Where are we here so we know that initial one is 10 final one is 1. k is something that , so you can calculate theta in terms of what is it now? Because this is minute inverse, so you can calculate theta in terms of minutes.

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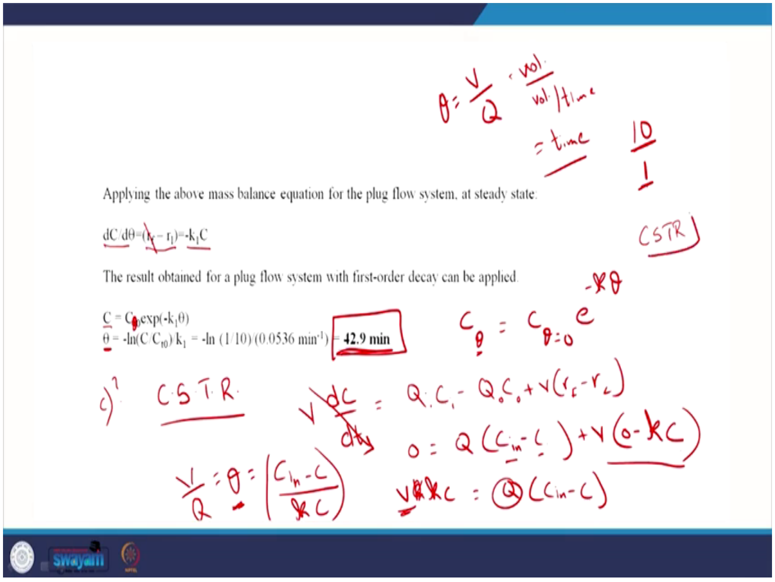

Applying the above mass balance equation for the plug flow system, at steady state:

$dC/d\theta = (\cancel{r_g} - r_1) = -k_1C$

The result obtained for a plug flow system with first-order decay can be applied.

$C = C_{\theta 0}\exp(-k_1\theta)$

$\theta = -\ln(C/C_{t0})/k_1 = -\ln(1/10)/(0.0536\ \text{min}^{-1}) = \boxed{42.9\ \text{min}}$

Even for your other case what is it let us see it calculate for the plug flow reactor? dc / d theta will be rate of formation minus rate of loss rate of formation is 0 - k 1 C, C at theta is equal to this is not time , at theta not, that is a typo on my behalf C at theta = C_1 theta = 0 into e to the power of - k theta. This is for your plug flow, this is not time, but hydraulic retention time and so looks like when , go through the relevant equation, you get 42.9 minutes.

$$\frac{\boldsymbol{V}}{\boldsymbol{Q}} = \theta = \frac{C_{in} - C}{kC}$$

We are in part c, we are going to calculate it for CSTR or the continuously stirred tank reactor. Here we are going to use the macroscopic equation. dc / dt is equal to mass coming in minus mass going out, plus volume into rate of formation minus rate of loss. Here, if it is at steady state, this is 0. What else do we have? , here is 0 = Q into C in - C out plus volume into rate of formation is 0 minus rate of loss is k times C. But C and C out are the same?

This is the term that we have and now we have one equation and one unknown. But what is that unknown though, because we know that C in is 10 and C out or the final one is 1 milligram per liter, but my unknown is theta. But if you look at it, we know that theta = V / Q volume, volume / Q is volume per time. That units of time, theta = V / Q.

V into kc, will be equal to Q into C in - C out or C let us see and theta = V / q. I am going to spread this term out here. V / Q, which is equal to theta = C in - C / kc . That is your hydraulic retention time that we are going to have to calculate. For removal of the compound from 10 milligram per liter to 1 milligram per liter, in a CSTR, the compound or the water has to spend this much of time for a plug flow reactor. It has to be 43 minutes; it has to spend 43 minutes in a CSTR. We are going to have to calculate in this way.

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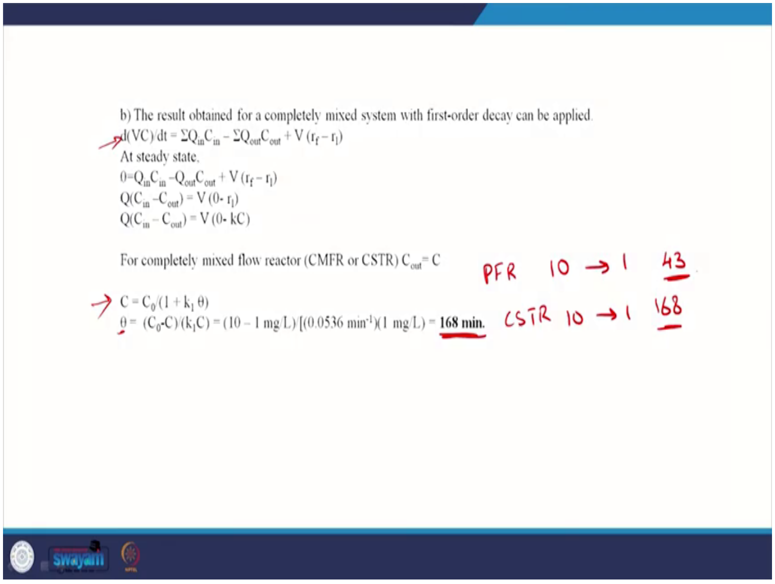

b) The result obtained for a completely mixed system with first-order decay can be applied.

$\rightarrow d(VC)/dt = \Sigma Q_{in}C_{in} - \Sigma Q_{out}C_{out} + V(r_f - r_l)$

At steady state,

$0 = Q_{in}C_{in} - Q_{out}C_{out} + V(r_f - r_l)$

$Q(C_{in} - C_{out}) = V(0 - r_l)$

$Q(C_{in} - C_{out}) = V(0 - kC)$

For completely mixed flow reactor (CMFR or CSTR) $C_{out} = C$

$\rightarrow C = C_0/(1 + k_1\theta)$

$\underline{\theta} = (C_0 - C)/(k_1C) = (10 - 1\ mg/L)/[(0.0536\ min^{-1})(1\ mg/L) = \underline{\mathbf{168\ min.}}$

PFR 10 → 1 $\underline{43}$

CSTR 10 → 1 $\underline{168}$

All this is the same stuff, as we discussed earlier, or now, like the steady state. Where is the so, this is what we have out here, if I made any minor errors in the calculation, so you can correct that. theta is 168 minutes, in a plug flow reactor for the same removal efficiency 10 to 1 we needed, I think 43 minutes. But for plug for a CSTR for the same removal from 10 to 1 you need 168 minutes. We can look at the logic way, but I will leave that to you maybe for an exam question.

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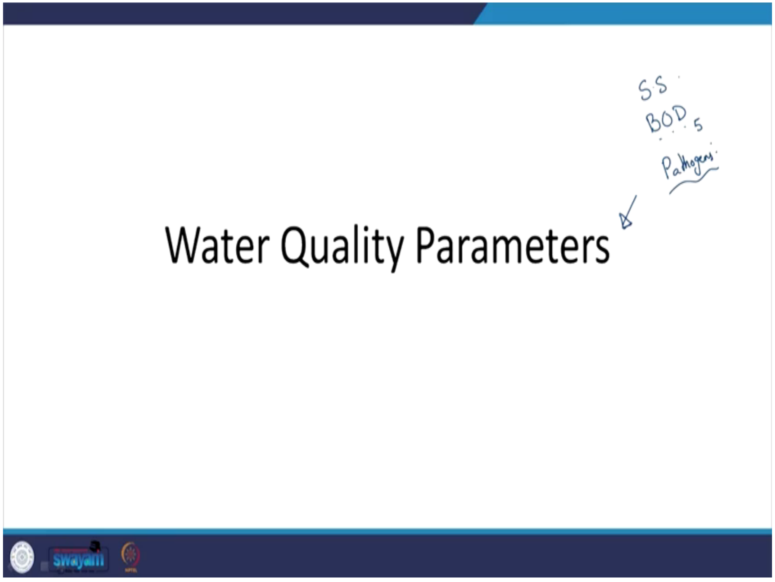

Water Quality Parameters

SS

BOD_5

Pathogens

Let us move on to the other aspects. We have different water quality parameters until now, you heard me talking about suspended solids sometimes about BOD biochemical oxygen demand. Earlier we talked about pathogens or such. Let us just try to understand what it is or what are the general water quality parameters that I am concerned in wastewater treatment and maybe in water treatment?

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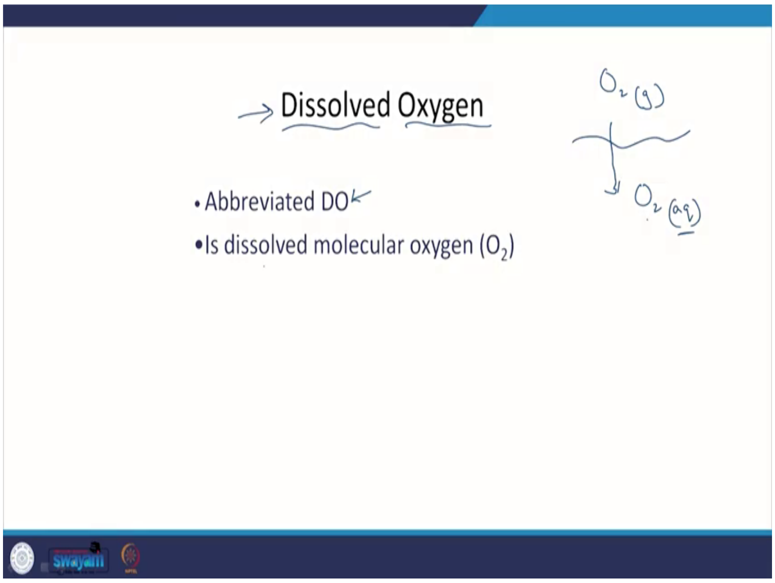

Dissolved Oxygen

- Abbreviated DO
- Is dissolved molecular oxygen (O_2)

$O_2(g)$

$O_2(aq)$

Let us move on without wasting for the time, 1 aspect is dissolved oxygen. We know that oxygen is present in the gaseous phase. If we bubble oxygen into the gaseous aqueous phase or water, it will be dissolved in that and that we are going to say is oxygen in the aqueous phase or the dissolved oxygen. Why is that required? It is called DO. Whenever somebody says DO. This is what they are talking about, it is dissolved molecular oxygen.

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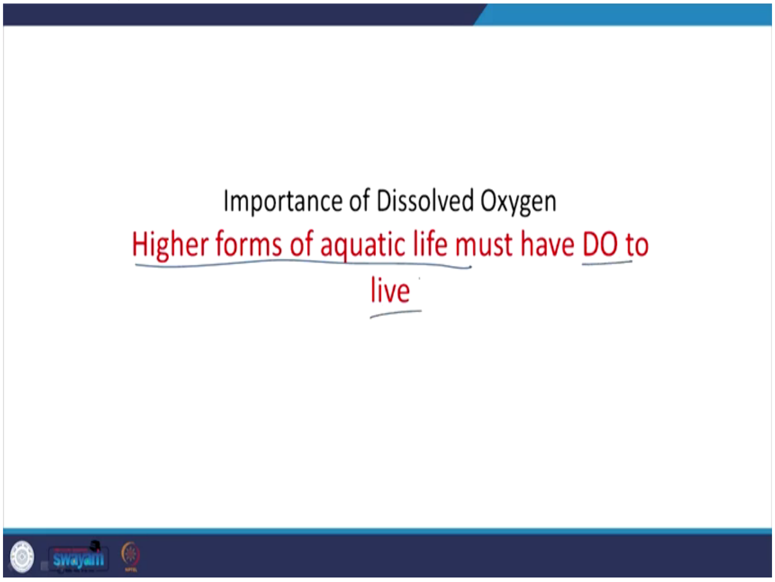

Importance of Dissolved Oxygen

Higher forms of aquatic life must have DO to live

Why is it that we are mean we need to maintain DO in the water, because higher forms of aquatic life fish, different kinds of fish to give just one example need DO to live we need oxygen. Different what we say types of life require oxygen even in the aquatic environment. If the dissolved oxygen is too low, typically, you do not have a thriving ecosystem, you have typically ecological dead systems, you have pathogens teeming out there and remarkably healthy conditions thriving out there.

How is it that dissolved oxygen is going to be affected or decreased? One is if DO less stream comes into play or if I have a lot of organic C x naught H. This is , a simple representation of human waste? I ate something and let it out. Then if I let it out into the nature, what is going to happen it is going to take up oxygen and it is going to go to CO_2 and H_2O or different by-products.

We are using up the dissolved oxygen this is in acquiesce phase dissolved oxygen. Thus, we are creating anaerobic conditions or no oxygen conditions in our environment, which is unhealthy, that is something to keep in mind, what are the different water pollutants.

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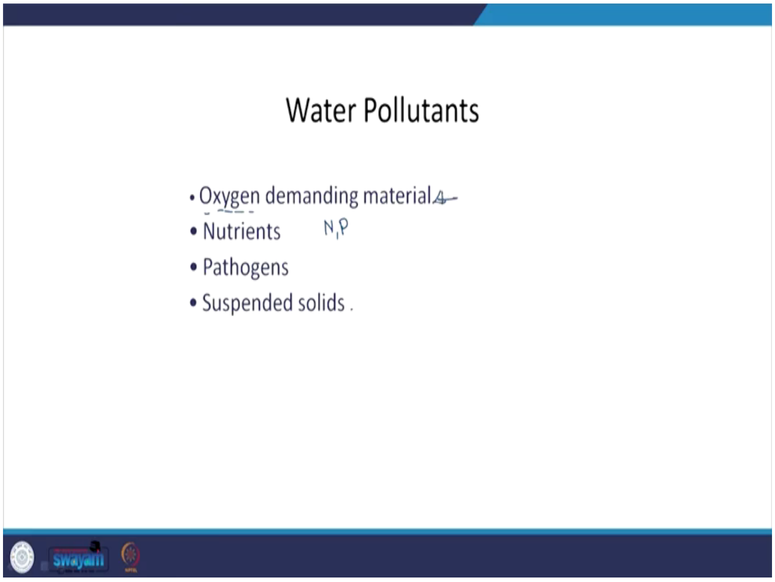

Water Pollutants

- Oxygen demanding material
- Nutrients N,P
- Pathogens
- Suspended solids .

Thus, if I am concerned about DO am concerned about oxygen demanding material and then nutrients, we looked at this nitrogen and phosphorus typically and then pathogens disease are organisms capable of causing disease to humans or such suspended solids we will look at this in detail, but for today, I will try to touch upon this particular aspect of you.

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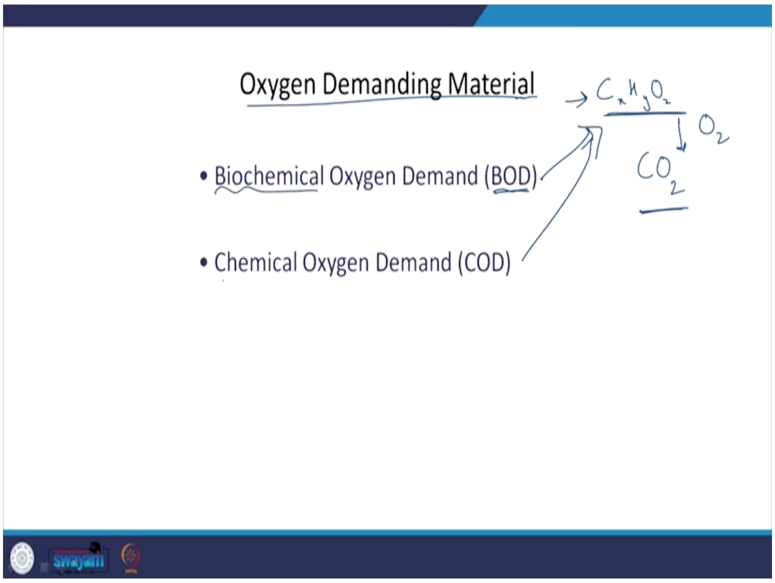

Oxygen Demanding Material

- Biochemical Oxygen Demand (BOD)
- Chemical Oxygen Demand (COD)

$C_xH_yO_z$

O_2

CO_2

Oxygen demanding material, what do we typically have I have biochemical oxygen demand, a way of approximating the amount of oxygen demanding material, but this is relatively slow. Another way which is relatively faster is chemical oxygen demand, a way of approximating the oxygen demanding material and what is the oxygen demanding material, typically our waste or the industrial waste, which are reduced to compounds?

You see, it is relatively reduced out here. We have the H out here. If it is completely oxidized, it would go to carbon dioxide here, it is relatively oxidized here failed to be reduced. How will they go from here to here, they will need oxygen to go from here to here. That is why these compounds we are calling them as oxygen demanding material.

We are using variables such as BOD or COD to try to approximate this oxygen demanding material. The difference between these 2 as you see biochemical oxygen demand and only chemical oxygen demand.

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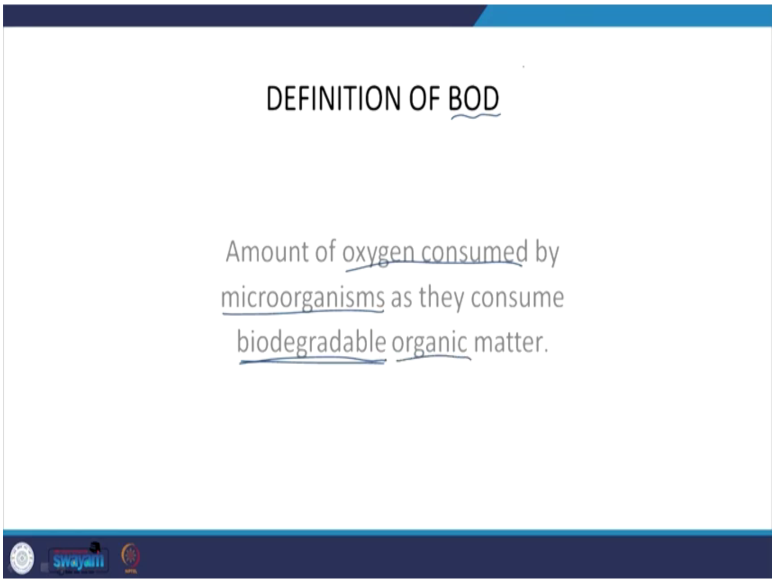

DEFINITION OF BOD

Amount of oxygen consumed by microorganisms as they consume biodegradable organic matter.

Let us go forth and look at what it is that we mean by when we say BOD? Please pay some attention. It is the amount of oxygen consumed by microorganisms as they consume the biodegradable matters. 3 aspects oxygen consumption, so thus oxygen demand and that which is consumed by microorganisms as they tried to consume or degrade biodegradable, organic matter, typically biodegradable organic matter. That is the definition of BOD.

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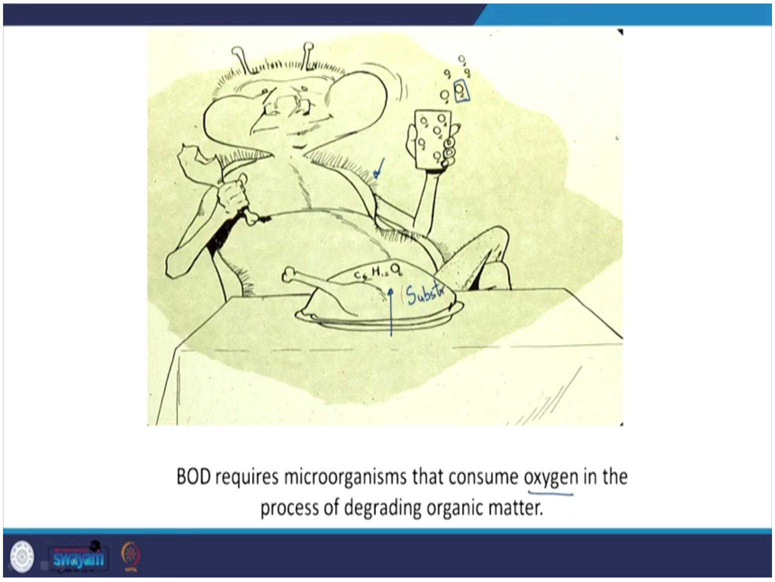

BOD requires microorganisms that consume oxygen in the process of degrading organic matter.

We have the microorganism out here. We have the glucose or such out here, which is the food for the relevant microorganisms, our waste. That is a different form. Here we have $C_6H_{12}O_6$ though is the food for the microorganisms. But what do they need to be able to make use of this particular food for them and our waste? They need oxygen.

They need the dissolved oxygen, so that is what they have. BOD requires microorganisms that consume oxygen in the process of degrading organic matter. Our waste is their food, which is typically called substrate. In general, it is their food meaning microorganisms food.

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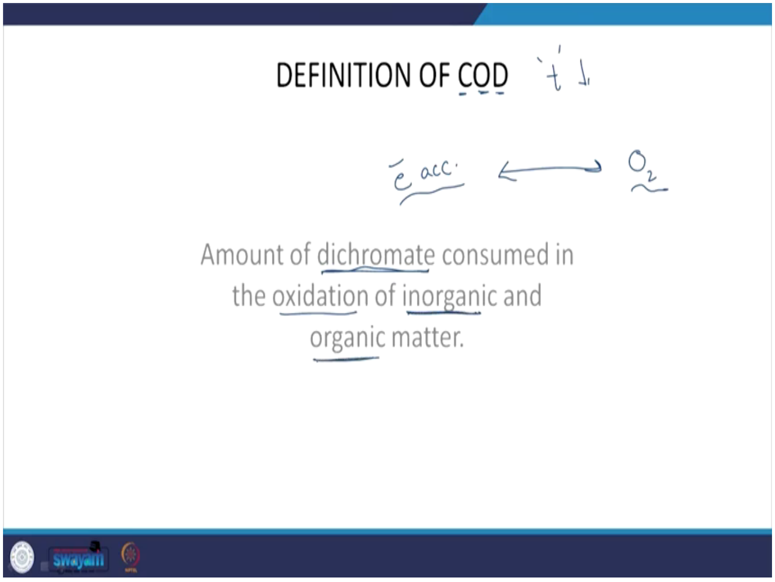

DEFINITION OF COD

Amount of dichromate consumed in the oxidation of inorganic and organic matter.

Definition of COD, it is the amount of dichromate we will see why dichromate, which is consumed in the oxidation of inorganic and usually certainly organic matter. Here, we are trying to get out understanding the oxygen demand, but chemical oxygen demand and I want the time taken for the test to be less. , when we are looking at microbes, the time that it takes is relatively higher.

Test that takes 5 days, I do not have the time and I need the results or need to be able to understand the system relatively earlier. I cannot wait for 5 days to understand the performance of the plant or such. I conduct COD test. That is why instead of the microbes, using oxygen, I tried to indirectly measure the amount of oxygen how oxygen is an electron acceptor.

Rather than electron acceptor, I am going to use dichromate, which is a relatively strong oxidizing agent, meaning a relatively strong electron acceptor. Then I will look at the electronic equivalents to get it in terms of oxygen? It is the amount of dichromate needed to oxidize. The inorganic and organic matter we are not seeing biodegradable organic matter, we are seeing inorganic and organic matter. That is something to keep in mind.

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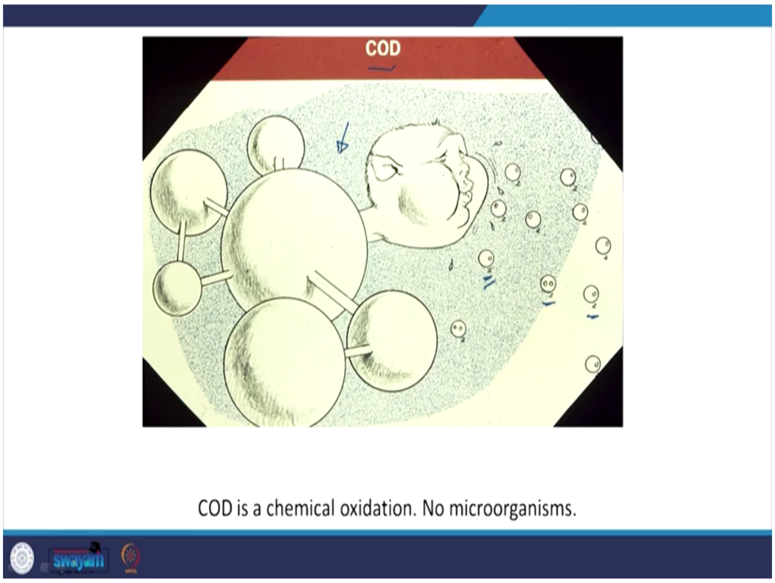

COD

COD is a chemical oxidation. No microorganisms.

COD, it is just the dichromate out here and we are going to give it in terms of oxygen equivalents. But it is not actually a reaction occurring between oxygen and the relevant compound of interest. But we are just trying to understand if all these compounds need or heat up here, oxygen to be oxidized, how much will the oxygen demand be? That is chemical oxygen demand. It is a chemical oxidation. There are no microorganisms here? This is not a microorganism, just the representation of the compound of concern.

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Simple BOD Measurement

Please review:
https://www.youtube.com/watch?v=YJ2UiuSHpuU

How do we look at the BOD or how do we measure it? There are simple ways, but I am out of time. Thank you for your patience. I will end today's session.