Water and Waste Water Treatment
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Lecture - 07
Mass balance: Comparison of CSTR and Plug Flow Reactor

Hello everyone, welcome to the latest lecture session, we been discussing mass balance and before we dig further, let us have a quick recap of what we discussed in the previous session. Mass balance, we are trying to understand what is happening to the mass in a particular system or reactor volume or a control volume. Why are we doing that? Because we are dealing with wastewater and water.

And we have different unit process where we remove different kinds of compounds. For example, in the activated sludge process, we tried to remove the organic content mostly the dissolved organic content, which is our waste. How do we do that? We have the mass coming in or the wastewater coming in and the treatment occurs in this tank and then the waste leaves the system, treated waste leaves the system.

To be able to design, how the volume of this particular tank or how much time the water has to spend in this tank. How much or how many microorganisms, I need to maintain in the tank. I need to be able to understand the aspects in terms of mass balance mass coming in mass going out. And what is happening to the mass within that particular system?

That will give me an idea about the accumulation with time or accumulation with the power compound in that particular system.

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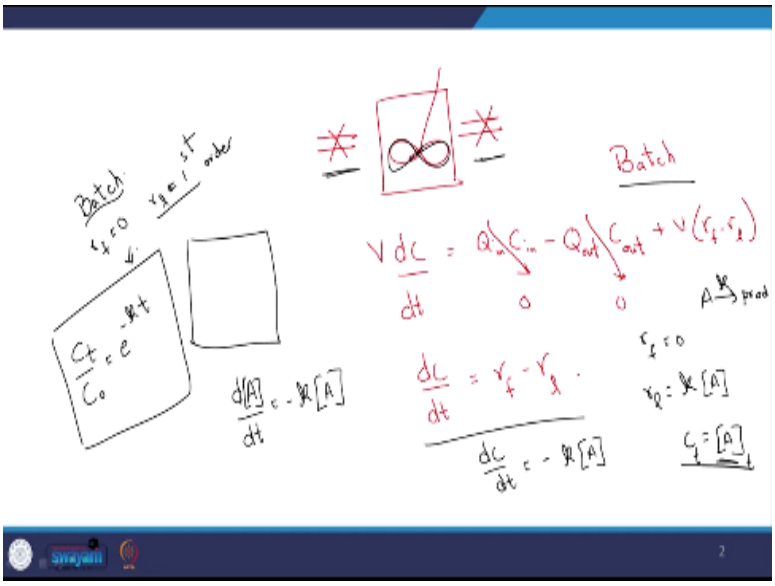

$$d(VC)/_{dt} = \sum Q_{in}C_{in} - \sum Q_{out}C_{out} + V(r_f - r_l)$$

In that context, we looked at 3 ideal case reactors; let us just have a quick recap. The first thing is the batch reactor, where we had no mass coming in, no mass going out but continuous stirring. For that case, we see this is what it answers to be. If my rate of formation of a compound is 0 and rate of loss is for, a first order reaction, A goes to products and the rate constant is k. Rate of loss is k times the concentration of A.

If I plug that in here, what will I get? I get dc by dt is equal to minus rate constant times concentration of A. C here is nothing but the concentration of A in this context. d (concentration) of A by dt is equal to minus the rate constant times concentration of A. Integrate and so on and so forth, what will I get? I will get if saying that this is I am going to represent concentration of A by t C t.

C t by C 0 or initial concentration is equal to u to the power of minus k_t. And again, this is the case, for that reactor batch and then when we have no rate of formation of the compound and we have rate of loss to be first order loss first order. That is what we have and this is what we came up with. If we have rate of formation and such, you will plug it into this equation and you will transform the equation.

Then we move on to looking at the other 2 ideal reactors, continuously stirred tank reactor and a plug flow reactor. Let us just have a very quick recap about them.

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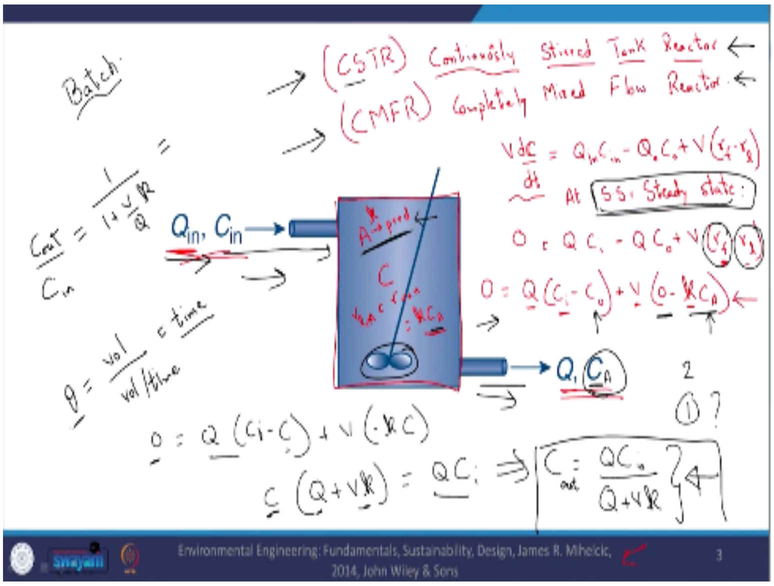

Environmental Engineering: Fundamentals, Sustainability, Design, James R. Mihelcic, 2014, John Wiley & Sons

Continuously stirred tank reactor or completely mixed flow reactor. What is it that they mean? First, we have flow coming in and flow going out and then we have continuous mixing. For the same case at steady state, again at steady state and assuming that there is no rate of formation and rate of loss is first order, we came up with this equation. C is equal to this particular term here, where C_i is equal to C_{in} and C equal to C_{out}.

$$C_{out} = \frac{QC_{in}}{Q + Vk}$$

Let me just play around with this equation, just a bit more. I should have done that last time. C_{out} which is this by C_{in} is equal to and if I divide by Q here, what will I get? I will get 1 by 1 plus V by Q into the rate constant. But V by Q, we know that we represented by theta, hydraulic retention time. For example, what are the units out here. V is volume flow rate is how much volume is coming in per time.

These are more or less in units of time. What is it that this time is giving me now? Practically, what is it that I understand from this hydraulic retention time. It tells me how much time this water or the compound of concern which is dissolved in the water is pending in my reactor. That is an aspect that we need to note and which we take into when designing. It is equal to 1 by 1 plus k theta.

Please note that this is the case when we have CSTR at steady state, no rate of formation and rate of loss is first order. If it is a different case, obviously, you will have to get at different aspect.

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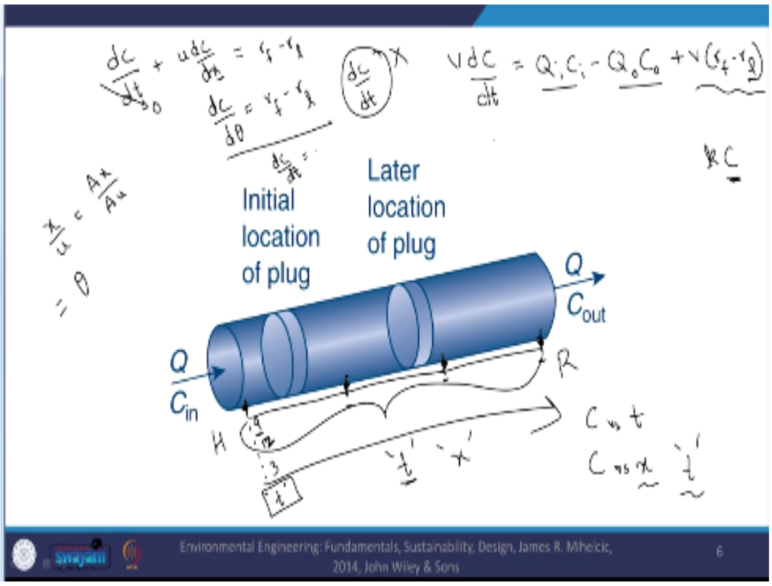

And then we moved on to another ideal case reactor, a plug flow reactor. Plug flow as in, there is no mixing within the reactor. And whatever comes in does not mixed with the packet that preceding and succeeding that particular packet or plugged, flows as a plug. Whatever I put in, just flows through like this and goes through. That is what we have.

For plug flow reactor, obviously, we cannot use this particular equation, the one that we have been looking at until now, the macroscopic mass balance equation. Why is that? Let us just take a look. Why cannot I use this V dc by dt is equal to Q in C in minus Q out C out plus volume into rate of formation minus rate of loss. Why cannot I use this? First, this is the mass coming in and mass going out of the system.

If I want to write the rate of loss as k C. First order loss. What is C here? Because the concentration of my compound within the reactor is not uniform. It is changing. Within the reactor, the concentration is changing. It is not the same within the reactor.

We all V as an alternative, we looked at our fundamental mass balance equation and assuming that advection is only in one direction, right, advection u dc by dx, x direction and assuming that there is no dispersion and or diffusion. Ideal case reactor, what do we have is equal to rate of formation minus rate of loss. At steady state, this term turns out to be 0.

u dc by dx, but what is x by u. That is again, if you multiply it by cross sectional area, A into x by A into U, what will that be more or less, it is hydraulic retention time volume by flow rate, hydraulic retention time. dc by d theta is equal to rate of formation minus rate of loss. As you can see here, here we see that this equation tells me that the concentration is changing with distance. That is what is taken into account here.

Some people erroneously write this equation for plug flow is dc by dt. That is huge blunder. Why is that? For example, this is Haridwar and downstream, this is Roorkee and the Ganga canal is always flowing in this direction. If I want to look at the change in concentration, or understand the system, so, what do I need to do? I either need to just sit at Haridwar and take the sample at 9 am, 12 am, 12 noon and 3 pm and so on and so forth.

Then I will get an idea about the change in concentration with time. But I cannot say I will take a taxi and I will take a sample here. I will take a sample here. Then I will end up at Roorkee at , 2 hours later and take the sample here. And by comparing the samples at these different locations, I cannot say that you know, give any idea about how the concentration is changing with time.

We will only be able to give an idea about how concentration is changing with distance but that too poorly because at the same time, time has also changed. If you want to look at it, you have to take a sample at 9 am at the same time and you will understand how the concentration is changing with distance. The point I am trying to make is you either have to keep the space constant when you are trying to measure the effect of time or the variation of concentration with time.

Or you have to keep the time constant when you are trying to measure the effect of or how concentration changes with distance. That is one aspect to keep in mind. And as we solve the fundamental mass balance equation, we can get this pretty easily dc by dt is equal to rate of formation minus rate of loss. And we see that it is relatively similar to the equation for batch reactor dc by dt equal to rate of formation minus rate of loss.

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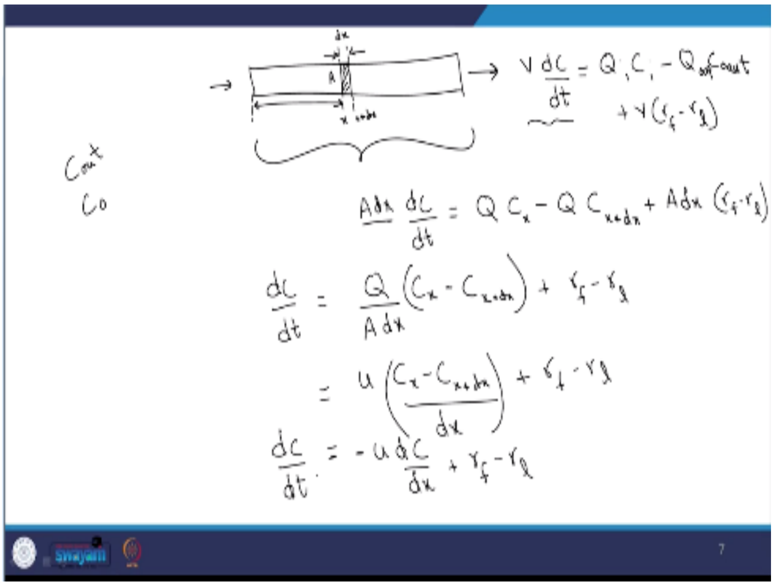

$$V\frac{dC}{dt} = Q_iC_i - Q_{out}C_{out} + V(r_f - r_l)$$

$$A\,dx\,\frac{dC}{dt} = QC_x - QC_{x+dx} + A\,dx\,(r_f - r_l)$$

$$\frac{dC}{dt} = \frac{Q}{A\,dx}\left(C_x - C_{x+dx}\right) + r_f - r_l$$

$$= u\left(\frac{C_x - C_{x+dx}}{dx}\right) + r_f - r_l$$

$$\frac{dC}{dt} = -u\frac{dC}{dx} + r_f - r_l$$

But for better understanding, we are going to look at this equation and try to apply that for the plug flow reactor. Plugs, which do not mix with each other. The distance here from 0, this is 0, this is x and the thickness of this particular plug, within which I am assuming that the concentration is the same as dx, the thickness is dx.

$$\frac{dC}{dt} = -u\frac{dC}{dx} + r_f - r_l$$

Now, instead of applying the mass balance or the centre section, we will be applying it over the small section that we have out here, within which we are assuming that the concentration is uniform because we are only considering a small plug within this plug flow reactor. We can assume that the concentration with that party within that particular plug is constant.

We know that our mass balance equation is V dc by dt accumulation term, how is my compound changing within the reactor or accumulating is equal to mass coming in Q_{in} C_{in} minus Q_{out} C_{out} plus volume into rate of formation minus rate of loss, mass coming in and mass going out and rate of formation and rate of loss. Please note that C_{out}. We should not, what do we say, confuse that with C_0.

Let us wait so, that is confusing depending on the way I write it sometimes Q_{in} and Q_{out} here is Q_0 here. Let me write it for the sake of understanding, Q_{out} and C_{out}. Here, what is my volume now? Assuming that the area, cross sectional area is A, the area will be A into dx not area, pardon me, volume is A dx dc by dt is equal to Q coming in and Q going out, I am assuming to be the same.

I will just use the term Q and concentration coming in at distance x so, C_x minus Q_{out} and Q are the same and concentration going out meaning at x plus dx distance C_x plus dx plus volume which is A dx into rate of formation minus rate of loss. Dividing by this particular volume, which is A dx, I have dc by dt is equal to Q by A dx C_x minus C x plus dx or x dx, this is what I have, plus rate of formation minus rate of loss.

That will be equal to what now? Q by A will be, Q is the flow rate, volume per time and here we have cross sectional area. We will end up with velocity of flow of the fluid in the numerator and we will have here C_x minus C x plus dx by dx plus rate of formation minus rate of loss. What is it that I end up with? I will end up with minus U this term is dc change in concentration with distance minus 2 because it is not C_x plus dx minus C_x.

But the other way around that is why the negative term plus rate of formation minus rate of loss. Let me just write this equation in the next page dc by dt is equal to this particular term.

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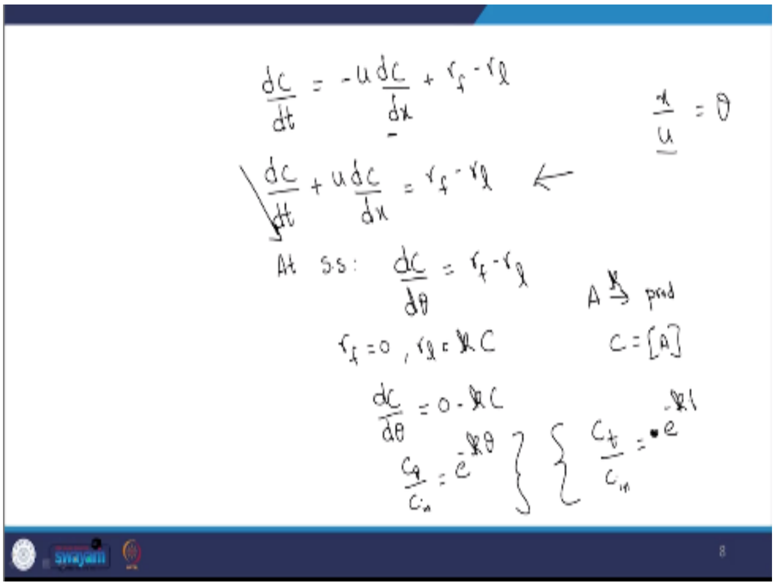

$$\frac{dc}{dt} = -u\frac{dc}{dx} + r_f - r_l \qquad \frac{x}{u} = \theta$$

$$\frac{dc}{dt} + u\frac{dc}{dx} = r_f - r_l \leftarrow$$

$$\text{At S.S: } \frac{dc}{d\theta} = r_f - r_l \qquad A \xrightarrow{k} \text{prod}$$

$$r_f = 0,\ r_l = kC \qquad C = [A]$$

$$\frac{dc}{d\theta} = 0 - kC$$

$$\left.\frac{C_e}{C_{in}} = e^{-k\theta}\right\} \left\{\frac{C_t}{C_{in}} = e^{-kt}\right.$$

What do we have? We have dc by dt is equal to minus u dc change in concentration with distance plus rate of formation minus rate of loss. If I bring this term out to the left hand side dc by dt plus u dc by dx is equal to rate of formation of the compound minus rate of loss of the compound. If you look at it, this is nothing but what we have or what we derived from the fundamental mass balance equation, when we assume that there is no diffusion and dispersion and that there is flow only in one direction this is what we had earlier.

$$\frac{C_{out}}{C_{in}} = e^{-kt}$$

Let me just go back and point that out. This is what we had the fundamental mass balance equation and we came up with it. Or we looked at it or solved it out here. We can use or go from macroscopic equations to the fundamental mass balance equation to or the mass balance equation such that it is suitable to be applied to the plug flow reactor.

As you can see now, we now understand why you cannot directly apply this macroscopic mass balance equation to the plug flow reactor. This equation as we did earlier if it is at steady state, this term concentration does not change with time or no variable will change with time steady state. What is it that we have u dc by dx u by or x by u, is equal to theta hydraulic retention time.

What will I have? dc by d theta is equal to rate of formation minus rate of loss. Assuming that it is rate of formation of a compound is 0, rate of loss is based on a first order degradation reaction where compound A is going to products and rate constant is k. It is equal to k times concentration of A; C is nothing but concentration of A here. What will this equation turn out to be? Dc by d theta is equal to 0, rate of formation is 0 minus k C.

C at any theta will depend upon C in or C okay I will write in C in coming into the system into u to the power of minus k theta. This is the same or not same similar to what we had earlier C t by C 0 or C in equal to e to power of minus k t, this is for the batch. You can compare the similarity out here. That is what we have. And until now, we looked at applying the mass balance equation when we had mass coming in and mass going out and such. Let me look at what else I have out here.

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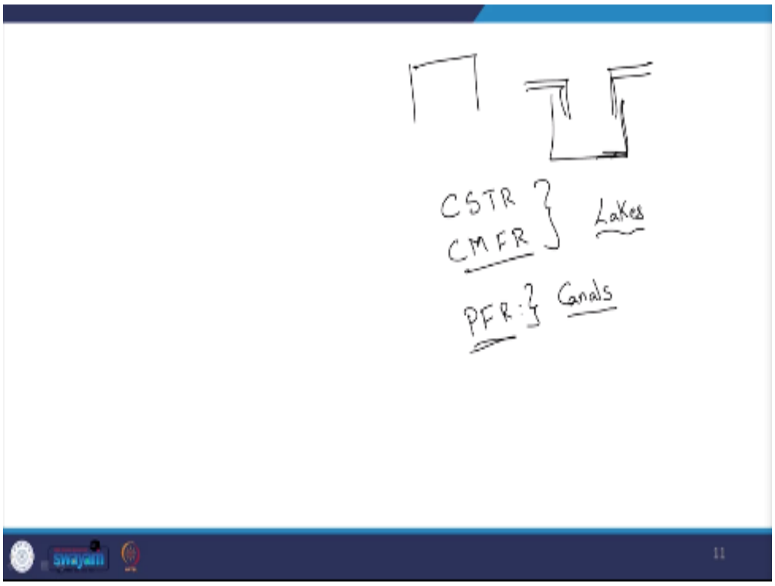

Until now, what did we look at? , this is my relevant system mass coming in, mass going out. I want to design this system as in what is the volume, how much time do I need to let my water stay in the system. One example not one example, some aspects that I want to mention here are that CSTR or completely mixed flow reactor can be used to model what can I say, lakes, which we are assuming to be well mixed.

Not a great assumption, but not a very poor assumption for some back of the page calculations. Plug flow reactor, what can I use it for? I can use it for , canals where the turbulence is relatively less or such that canals are reverse. Not great assumption, because we are talking about ideal reactors here but you understand where it is or what it is that I am trying to hint at.

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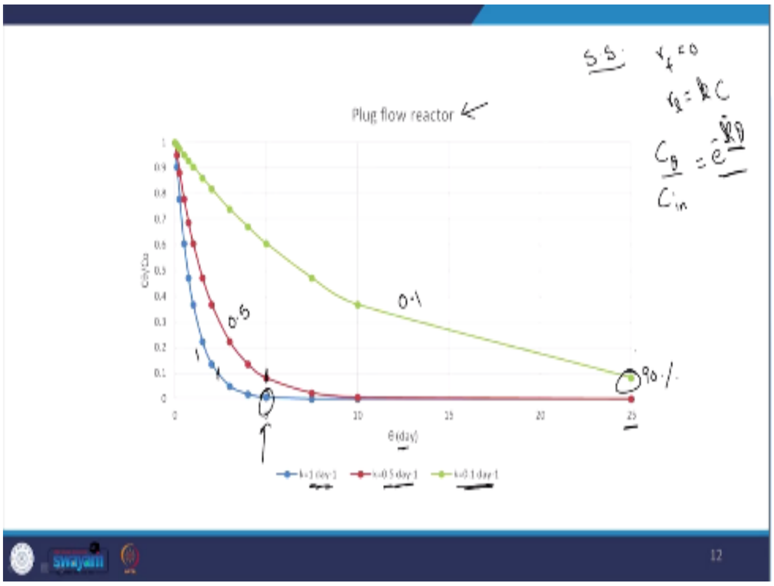

Plug flow reactor

Let me move on and try to compare different aspects out here. Here, I asked the TA to plot different sets of data. Here we have plug flow reactor and this is the case when the system is at steady state and rate of formation is 0. And rate of loss is equal to k C. This is more or less based on C theta by C in is equal to e to the power of minus k theta. For random k and random units day inverse and obviously days, k theta will be dimensionless. Yes, k theta will be dimensionless.

I plotted or the student plotted the relevant data here. First, what do we see? This is the one with rate constant of 0.1. This is the one with rate constant of 0.5. This is the one with rate constant of one day inverse. I mean, it can as well be any units out here, but these are the rate constants, we chose here.

Because the constant the rate constant is less, in this case, the green colored one, we see that it takes longer as much as 25 days for 90% removal to take place 0.1 by one or approximately 90% . That is what you see. But with, 5 times faster, 90% is achieved within , around 5 days. 90% removal is achieved within 5 days.

Compared to 0.1, if it is 10 times faster one day inverse 90% is achieved, within 2 and a half days. It is exponential obviously out here. You see the role of the rate constant, in deciding the rates of the reaction. Why is this important? For your wastewater treatment process now, during some summer, the temperature is high rate constant, which is dependent upon temperature and pressure will obviously be affected by it.

Within reasonable limits, we will look at these empirical values later, the rate constant will increase with increasing temperature. During the December period or during the winter months, the temperature comes down, at least in Roorkee comes down to 6, 7, sometimes 2, 3. The temperature decreases. Obviously, the rate constant will decrease. Why is that an issue?

During summer, we have this profile. During winter, we have this profile. It will not change as drastically though, but there will be considerable change or some change rather, if you do not see to it that the temperature is more or less maintained, but in general sewage has some temperatures. It should not be a great deal of issue in India. In winter, obviously, the performance can be affected depending on how you design the system and how you run it. These are the aspects that come into play.

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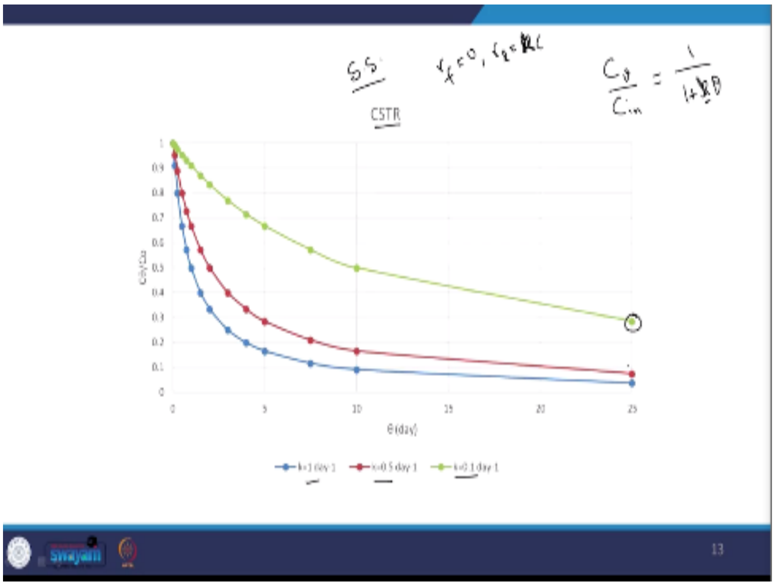

$$\frac{C_\theta}{C_{in}} = \frac{1}{1 + k\theta}$$

Let us move on and look at the CSTR or the completely mixed flow reactor. CSTR or completely mixed flow reactor so, same case, we took k to be these 3 values. For CSTR, what is the relevant, what is it, equation for rate of formation is equal to 0 and rate of loss is equal to k C or first order loss and at steady state, what is it C theta by C in, is equal to 1 by 1 plus k theta.

For this, what do we have, we see this profile, obviously, the lower rate constant. That it is going to take more time that is what we see to achieve a certain rate of removal when compared to the one with the higher rate constant. Again, that is the profile is similar to that of the plug flow reactor.

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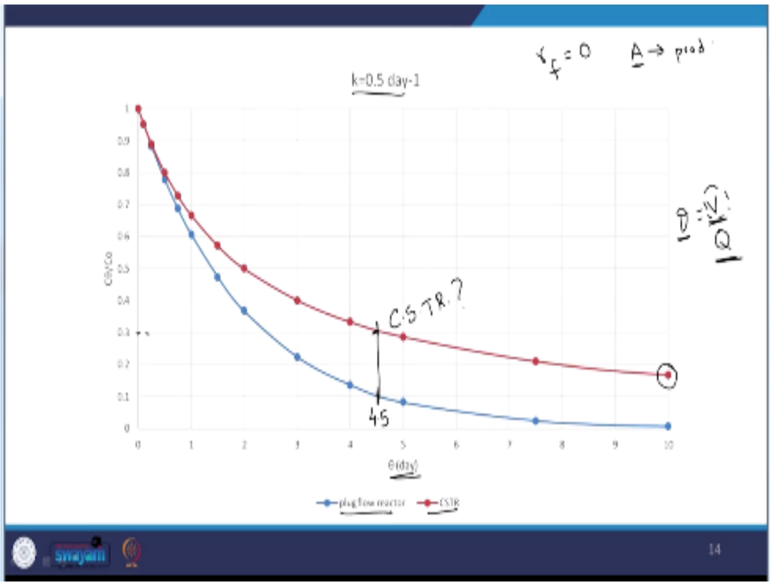

But if I move on and try to compare the plug flow reactor and CSTR for the same rate constant of k and for the same case, when rate of formation is 0 and it is a first order loss A going to products, I am trying to model the concentration of A, C_θ by C_0. Here to achieve again, blue is plug flow reactor to achieve 90%, it takes around, assuming that point 1 by 1, my guess is 90% or approximately that.

We see that it takes around 4 and a half days, 4.5 days. But for a CSTR, as you can see, it takes much longer as an even after 10 days, we see that 90% has not been removed now. That is something for you to see or not see, think about. Why is it that for the same theta? For the same theta, theta is V by Q so, if the flow rate is constant, when we say this theta is the same, what is it that means, from the same volume, flow rate coming in as, the same for the plug flow reactor and your CSTR.

When we say theta of the CSTR is the same as theta of the plug flow reactor, what does that more or less mean? For a given flow rate, the volume is also the same. For the similar volume, why is it that? While almost 90% is removed by 4 and a half days, it is only, what is it now, we still have considerable fraction left, 70% or so, is left not left but with 30% or so, is left after 4 and a half days for CSTR. Why is that?

These are questions we need to understand Let me see if I have some graph here okay. Let me try to draw it out here.

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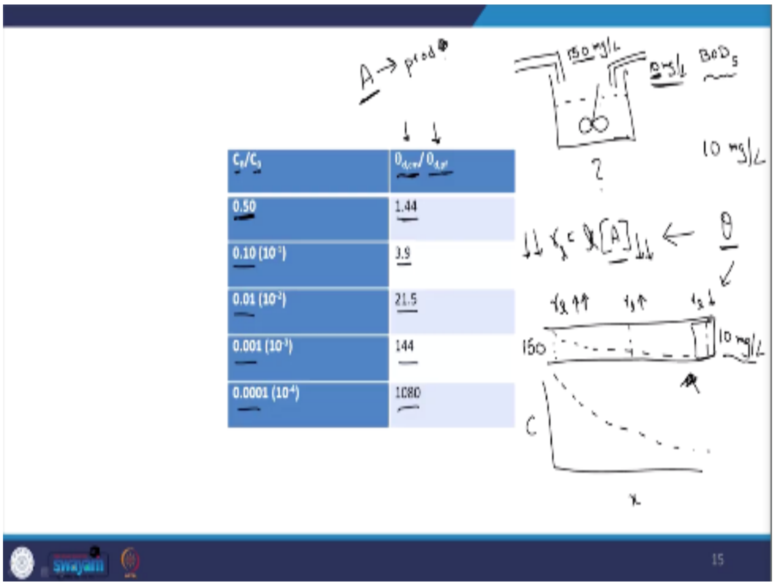

C_θ/C_0	$\theta_{d,cm}/\theta_{d,pf}$
0.50	1.44
0.10 (10^{-1})	3.9
0.01 (10^{-2})	21.5
0.001 (10^{-3})	144
0.0001 (10^{-4})	1080

Taking that line of thought further, if C_θ by C_0 ratio is 0.5, the ratio of theta to achieve the similar level of removal is thus, completely mixed by plug flow, hydraulic retention time with a completely mixed flow reactor divided by the hydraulic retention time with a plug flow reactor. We see that the ratio is higher or we know. It takes more time with the completely mixed flow reactor. Yes. And what do we see if it is by 0.1.

We see, it is more and 0.01 much more and greater and so on and so forth, the concentration or not concentration, the when we compare the CSTR and plug flow reactor, we see that CSTR takes a lot more time. Why is that now? Let us understand the system here. We have our CSTR here and we have the concentration coming in, at 150 milligrams per liter, this is bod at 150 milligrams per liter, BOD_5 days.

We will look at what BOD_5 is in the next session and 150 milligrams per liter is coming in and my discharge standards now into the river or such as 10 milligrams per liter. What is it that can go out? It can only be 10 milligrams per liter. But in CSTR, what is the whole point is that? It is continuously mixed and completely mixed. Whatever is coming in, 150 will have to be immediately diluted to a very low value which is 10 milligrams per liter.

Why is that? Because whatever is in the reactor is what is leaving the reactor. If I want to achieve a low outlet concentration, the concentration of the compound within my reactor also has to be 10 milligrams per liter or the very low value. Why is that an issue? Because think of the rate, what will be the rate of loss of this particular compound A, if it is going from A to products.

What is the rate dependent upon it is? A is dependent upon not A the rate is dependent upon the concentration of A. Here we see that the concentration of A is pretty low, because that is what we want to achieve. If the concentration of A is low, what does that mean? The rate of loss of your particular compound is also going to be pretty low.

Before we finish our thought, let us compare it to the plug flow reactor. This is the plug flow reactor. Here, it is coming in at 150 and here, it is going out at 10 milligrams per liter, we would 150 milligrams and 10 but is the concentration of the reactor throughout this, I mean, compound throughout this reactor the same, it is not. We know that here the concentration changes with distance, maybe the concentration will be something like this if this is the distance x. Concentration versus x.

Why is that of importance? Because here, when the concentration of the relevant compound is high, the rate of loss will also be high. Here also, it is not at 10 milligrams per liter so, rate of loss is still high, maybe not as high. Only at the backend of the water, I mean, reactor will we have rate of loss to be low because the concentration of A is low.

Only within small zone of your plug flow reactor will you have low rates which are comparable to the rates which typically exist throughout your system in the CSTR. Here we see that the rate of removal is higher right or in greater parts of the particular plug flow reactor. We have higher rates of removal compared to the CSTR. That is why we have better removal with plug flow reactors for the same theta, meaning same V.

When we say same theta, what is it that we mean? We mean that for a given flow rate volumes of both the reactors are the same. We see that plug flow reactors are relatively more efficient. Keep in mind that these are ideal case reactors as in we are assuming that there is no dispersion here in plug flow reactor or we are assuming that it is instantaneously diluted and so on and so forth.

We will look at these aspects but I am out of time. We will look at these aspects in the next session. And I thank you for your patience and I will end the session now.