Water and Waste Water Treatment
Prof. Bhanu Prakash
Department of Civil Engineering
Indian Institute of Technology, Roorkee

Lecture - 05
Mass Balance

Hello, everyone, welcome back to the latest lecture session a very quick recap of what we discussed in the last session. We were looking at different units, molar mass per volume, and also we looked at units in terms of equivalents, equivalents in terms of typically H^+ or charge and then we looked at the conversion.

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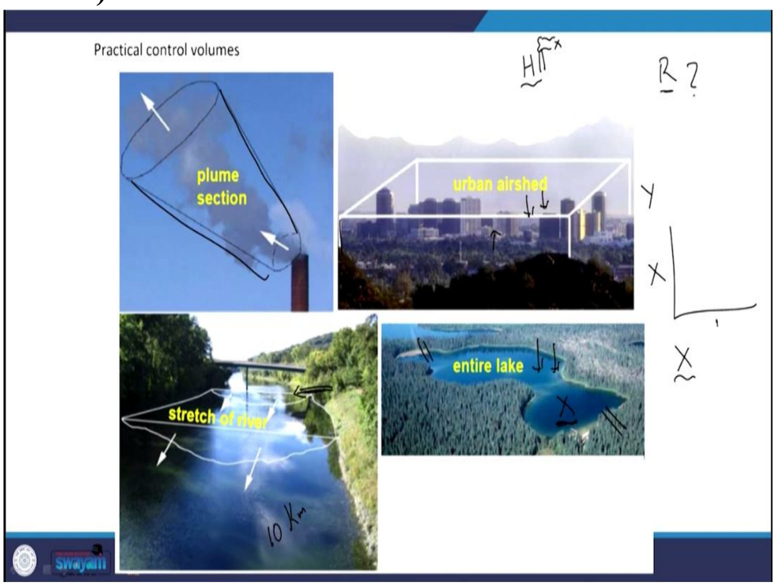

Let us move on to the major aspect that is going to be relevant throughout this course. I have some pictures out here. Let us just look at it. First, before we move on to the relevant session, here I have what we say for lack of a better term a chimney that is releasing some particles into the atmosphere, and this is the plume section. We have Haridwar, assume that this chimney is in Haridwar and Roorkee is out here, and I want to know what are the adverse effects at Roorkee?

Due to the plume or the pollutants being given out at Haridwar. How do I go about that? Or rather than leaving it to be pretty generic, let me be a bit more precise, I want to predict the concentration of pollutant x, which is being given out through this particular chimney at Roorkee. How do I do that? I have an urban airshed here. We are assuming that this is the control volume, the whole, atmosphere, about the city is the urban airshed, I want to know what is the concentration of let compound Y?

Throughout the year, how is it changing? Let us say if there are different reactions, or if the pollutant is settling down? Or such and how is it changing? Or I have a river here, or a canal? Let us say, and I am letting out a pollutant out here and the pollutant is joining the stream out here. 10 kilometers down the line or along the river, what is the concentration going to be assuming that I know the velocity of flow of the river the reactions that might be affecting this compound or such and then here we have the example of lake.

Here, if there is an inlet and outlet, or even without an inlet or outlet, but if we are just seasonal rainfall and some reactions affecting my compound X, what is the concentration of X with time. How is it going to look I want to understand these aspects? Now we are going to use these aspects or one comes across these questions quite often and the field of environmental engineering closer home to this course that is water and wastewater treatment. What are some of similar scenarios that one can come across?

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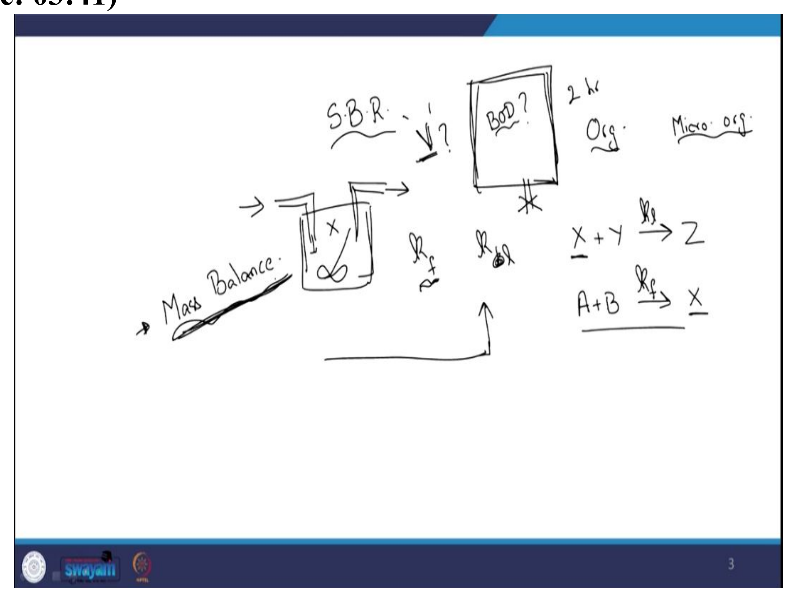

As we looked at earlier, we saw the video of a wastewater treatment plant in that video we had what we looked at was a sequential batch reactor. Let us say are about we look at the details later. We have this particular reactor fine, and we had water coming in, and then it is filled to a particular threshold and then we turn off the inlet flow and then aeration occurs where during which microbes degrade your substrate or the organic compounds.

I want to know what is the volume of this tank that required what is the required volume? How we are having we need to design the volume? The capital cost that are going to be incurred? Yes. You need to design the volume. Or you also need to design, how much the concentration of microorganisms or how many microorganisms need to be present in this particular tank of volume V. Such that after certain time 2 hours. The concentration of The BOD decreases to a particular value how do I do that?

These are the questions that come up when we are designing different unit process and water and wastewater treatment. The key here is understanding that mass is conserved, it cannot be destroyed at the best it transforms into another form. If I have a continuous flow system, water is continuously coming in water is continuously going out, and, let us assume that this is always mixed or stirred.

I am concerned about the concentration of a compound X given that the rate of formation of the compound can be calculated using the rate constant k f and we also have k backward and assuming that X + Y goes to Z and A + B goes to X rather than forward formation and loss, for example, here this is k loss rate constant for this reaction, where which X is being launched is k out and rate constant, why are those reaction where X is produced, we are saying is k f rate constant of the formation reaction.

$$X + Y \xrightarrow{k_l} Z$$

Assuming that I know this information, I can calculate different unknown at steady state, given a flow coming in and initial concentration of the compound X what is the concentration of the compound in the effluent? Let us say, how do I calculate it? We can calculate this by applying mass balance, and I would suggest that people pay attention to this aspect, because you are going to use this considerably or quite often during this class mass balance.

$$A + B \xrightarrow{k_f} Z$$

How does this help me now? Or where does this come from? How does this help me we just looked at it we know we looked at the different applications out here and in the previous slide. Principle is that mass that comes in either has to go out or has to be transformed, mass is conserved, if there are reactions occurring not conserved, but then the transformations also will have to be looked at. We are going to look at the principle of or consider the approach of mass balance.

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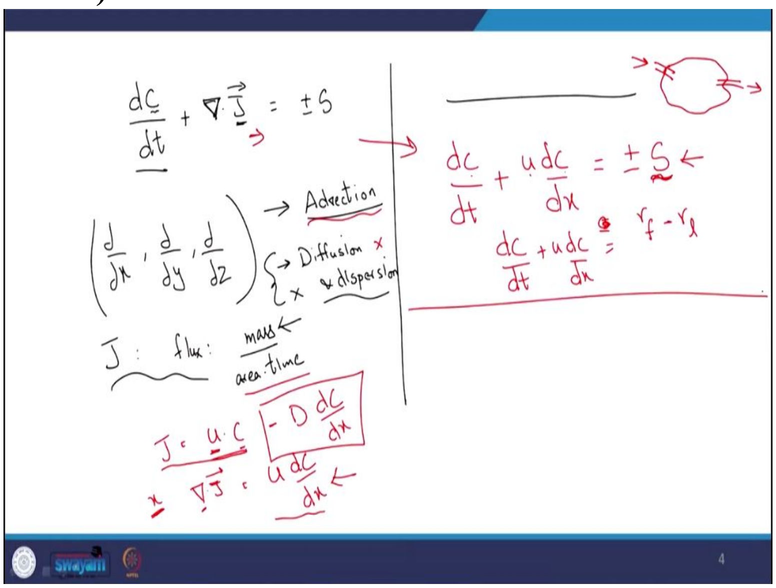

For that, what do we need to do? Before I go further to that slide, note that in this class, we are going to use one approach for fundamental or for mass balance, and we look at that equation. But before I do that, I would like to acquaint you with the fundamental mass balance equation. dc / dt, c is the concentration of the compound, the interest, plus the del product of the flux is going to be equal to plus or minus sources and sinks.

$$\frac{dC}{dt} + \Delta\bar{J} = \pm S$$

Accumulation term concentration change per time, this is the del product, meaning I am trying to get dou / dou X dou / dou Y and dou / dou Z. That is when I am talking about del product. Flux units or mass per area per time. For example, now the door to this room is closed, if I open up this door and open up another door out here and say that a particular compound is coming through that particular door. I am interested in the flux, meaning mass coming in through that unit area of the door per time.

$$\frac{d}{dx}, \frac{d}{dy} and \frac{d}{dz}$$

In the context of flux, there are 2 types of transport. One is due to advection, and the other one is due to diffusion and dispersion. In this room, the doors are closed, no noise. The atmosphere is relatively stable or pretty stable rather, and we have no net fluid flow in any direction.

We do not have the fans turned on. If I open up a scent bottle here. If I open up a scent bottle, it typically means a volatile compound, a volatile compound typically wants to stay in the gaseous phase, rather than in the aqueous phase or the solvent phase. Now, once I open it I can smell it only out here.

If I leave the bottle out here and go ahead and sit at the end of this, 15 meter room, I am not going to be able to sense it now, but after 10 minutes or 1 hour or 10 hours, I can sense it at the end of the room. Why is that? due to the concentration gradient, the concentration of this compound in the gaseous phase is higher, and they are at 0.

Over time, to achieve similar chemical potential, due to diffusion, you are going to have transport the molecules from here to the end of the class. That is described by diffusion Fick's law. Another example is if I turn on the fans, then I will be able to sense the smell at the end of that room or this room sooner. Why is that?

Now I have turbulent conditions in this particular room. The dispersion or turbulent diffusion is going to lead to faster transport of the compound within this particular room. Diffusion and dispersion, we are not going to look at that in this class. Advection what is that about? Again, a better example is out here.

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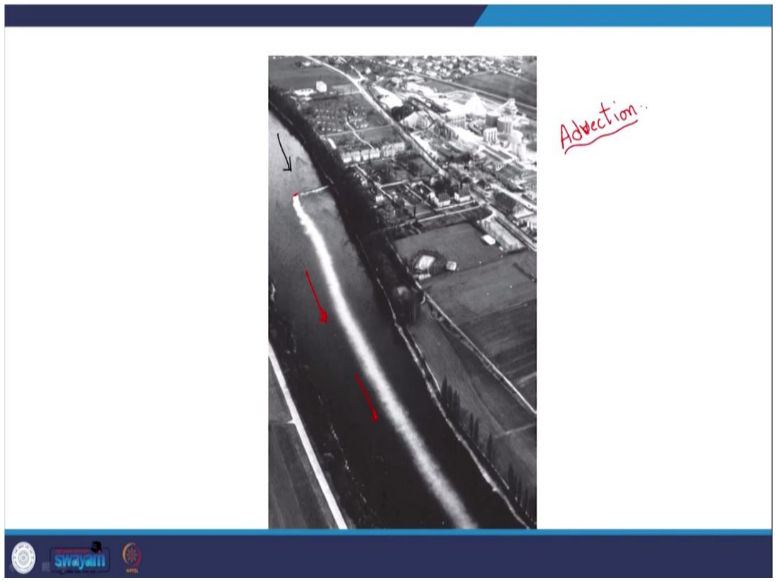

Here we have, the river or the canal flowing in this direction, and we have some particular plant out here, a sewage treatment plant. We see that, through a particular pipe or such, they are releasing the waste out, and then we see that the compound, or the waste is being transported in this direction along the net fluid flow of the liquid or the river. When we have a net fluid flow in one direction, and the compound is being transported along with that fluid. That is what we call as advection flow due to advection. That is the example that you see out here.

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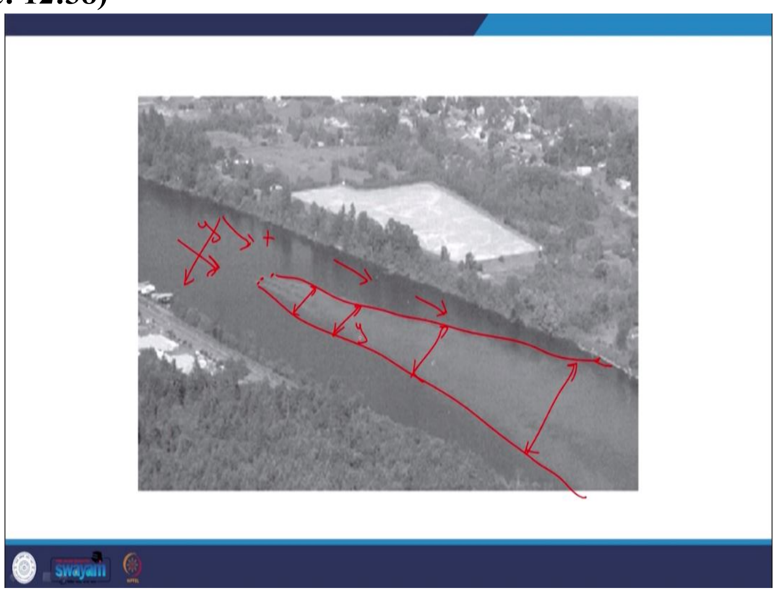

In the next example that we are going to look at, I am not sure if it is remarkably clear, but as you can see, through an underground diffuser, or the diffuser at the bottom of the canal, you see that the waste is being given out here. But unlike the previous slide that we looked at, we see that with time, this width of this waste is increasing? Here, it is X here, it is maybe 2X, and so forth.

We know that due to advection, compound is being transported along this direction. That is due to advection, but why is it being transported along this? Let us say if this is X direction, this is Y direction? Why is it being transported along Y direction? Why is it well that due to dispersion or turbulent diffusion that we talked about earlier, but we are not going to consider that?

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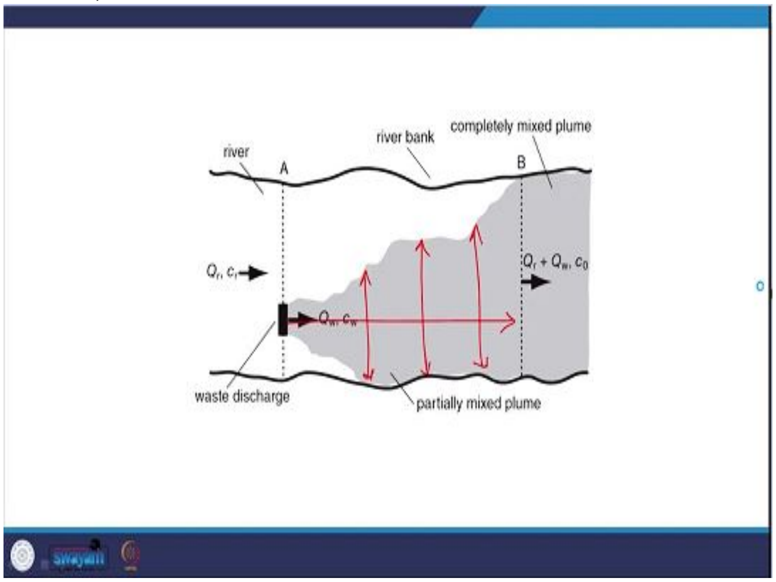

Again, advection and diffusion. Here advection is carrying it along with the fluid flow, and diffusion or dispersion, turbulent diffusion or dispersion is leading to this mixing, this mixing in this direction . That is something to keep in mind. As I mentioned, though, in this class, we are not going to look at diffusion and dispersion, we will only be looking at advection. We will look at advection u dot c dot product, assuming that this velocity what is this u, u is the velocity of the fluid flow.

And c is the concentration of the component. The units are mass per area time for per time, assuming that the flow is only in one direction i.e., only in X direction, and then this del product of the flux will turn out to be and assuming that the velocity is constant, u dc / dx? What did we assume velocity is constant? Flow is only in X direction. , this turns out to be u dc / dx. But if there was diffusion too , that would have been minus and diffusion, in 1 dimension , this is the diffusion coefficient, dc / dx.

And if I take the del product in the X direction, this is going to be the second derivative with respect to X. But as I mentioned, we are going to do away with diffusion and dispersion in this class. In this class, we are just going to look at transport due to advection. Advection as and when the there is a net fluid flow in one direction, and the compound is being transported along with that fluid that I am referring to as advection.

What does this equation transform into? Let me write that out here dc / dt plus, this is what I have out here, u dc / dx, u is velocity of flow of that fluid c is the concentration of that compound that is equal to plus or minus sources or sinks. As in if my particular point of not point control volume of interest is a lake . We looked at this example earlier, mass is coming in, and mass is also going out.

There are any reactions occurring within this particular control volume, which is the lake. We will have to take care of that in sources or sinks. This is the fundamental mass balance equation, and if there are reactions occurring and nothing else, what will that turn out to be equal to rate of formation of the compound minus rate of loss of the compound? Dc / dt plus u dc / dx, is equal to rate of formation minus rate of loss of that particular compound.

$$\frac{dC}{dt} + u\frac{dC}{dx} = \pm S$$

But this is the fundamental mass balance equation. There is another equation, which is typically applicable for macroscopic systems, which we will discuss later. But this is something that I wanted to mention, if this is the room, and this room is my control volume, and I open up the door on that end and this end, and we have air flowing in and while flowing in, if it is bringing a compound X, compound X is flowing in.

What that mass is coming in, and then the mass is also leaving through this door on my right hand side, mass coming in mass going out, and I have a reactor here that leads to decrease in concentration of this particular compound X that is being blown through the door. That I have to consider in my sources or sinks.

And that is when the system is non-conservative. That is what we have mass coming in, mass going out and mass being transformed within the reactor here, the control volume is my room out here. That is something to keep in mind, and we will look at this in great detail. Advection, we already looked at it advection and diffusion, and we already looked at this example, the plan or the top view.

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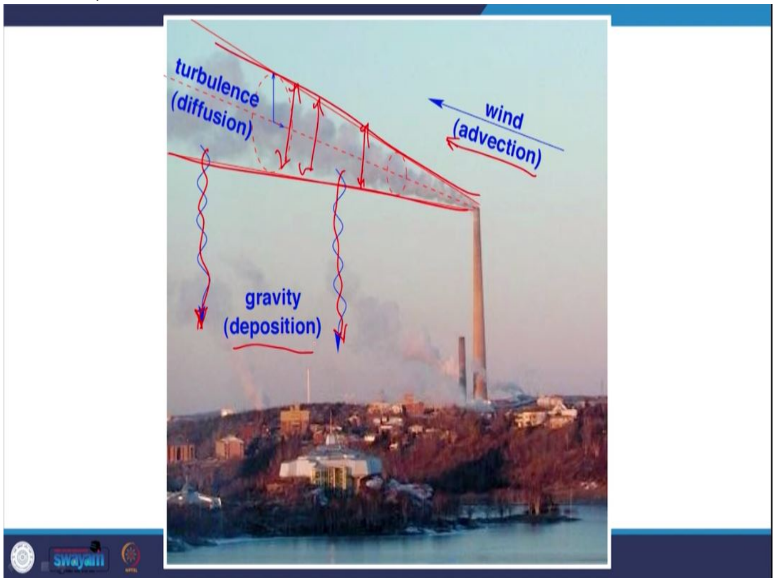

It has applications in every aspect, or in every field out there, for example, this chimney, and this is my control volume, and when which is blowing in one direction, so that is leading to transport of the compound of interest. That is due to advection, and due to turbulence, you see this lateral dispersion, this is due to turbulence, and then you have different sinks, which is if the particles are heavy, the particle is going to be deposited. That is a sink out here, that due to gravity which is a different example out here.

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Just to understand, if we have just advection how is it going to look like and if it is advection diffusion, and so forth, how is going to look like. Here we have concentration and distance, and if I put in a tracer at one particular point or such, and at time t_1, the concentration is out here. At time t_2 also the same plug will move and the concentration at time t_2, or the distance X_2 here we are talking about distance.

After it has traveled a certain distance X_1, which takes time t_1. This is the profile and after it has traveled X_2 distance, taking time t_2 it, the same profile. Let us say if it is only diffusion, there is no flow? This is the flow that is why the particular tracer or plug mode from this location X_1 to location X_2 with the same profile, but it fits due to diffusion. What is going to happen initially, you do concentration gradient, we are going to have something like this.

Then slowly but surely like this well it should not start from out here, or if , if this is a graph, and I put something out here at time t_0, after t_1, due to diffusion, it is going to be something like this. After t_2 time, it is going to be something like this, and t_3 something like this, diffusion, we will try to see to it that the concentration is uniform throughout your particular system or control volume.

If you have both advection, and diffusion, well, we look at this, and we also looked at the example or the picture previously. But in general, diffusion is pretty slow. When the flow is going in this direction, we do not consider the diffusion or the transport due to diffusion in this direction. That is something to keep in mind.

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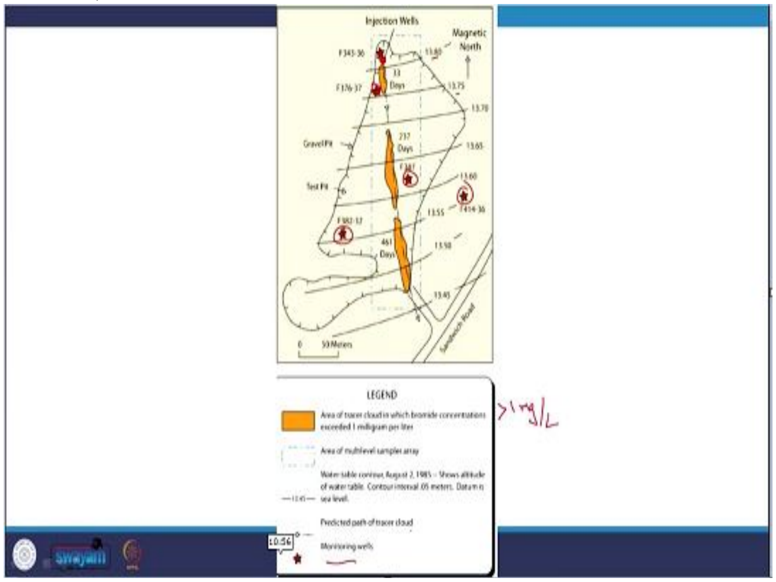

This is from , the USEPA document. Here we have an injection well, this is groundwater here. Keep in mind that we are talking about groundwater, and what do we have out here we have the LEGEND area of tracer cloud in which bromide concentrations exceeded 1 mg/L. The plume we are looking at is when the concentration is greater than 1 mg/L for bromide, and we have water table content levels, predicted path of tracer cloud and monitoring wells, we had the monitoring wells out here.

And we have the contour levels, as you see 13.875 and so. The flow should be in this direction? That is something that we understand from this figure, and they injected, this bromide, this particular location injection well, but did it stay? But did the shape, stay the same at different points in after different times or distances? No, as you see initially itself, this is the plume, and after many days 237 days.

You know, rather than retaining the shape, due to advection, this is groundwater due to groundwater flow, we have this particular shape. This particular plume transported from here to here, and it took around 200, and 4 days, to go from here to here. But why did it elongate it is due to dispersion or diffusion? Let us say and because groundwater flow is very slow. That is why you have what we say. Or you can see the effects of dispersion or diffusion and or diffusion in the same direction as the groundwater flow.

If it was surface water flow, we would not have seen this, and after 461 days to we see this profile, an example of understanding or an example that allows us to understand the effects of advection and dispersion, so that is something we see.

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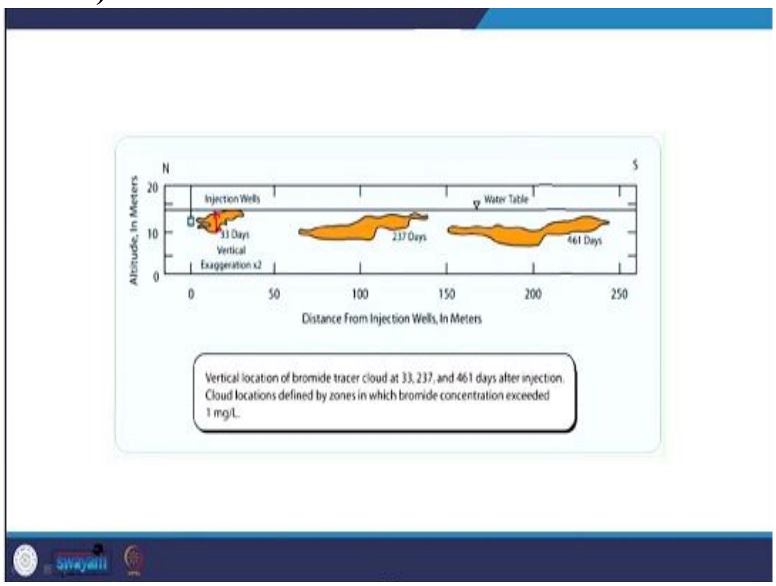

Let us see it from the other direction, but with respect to Z direction, we see that there is not much dispersion out here.

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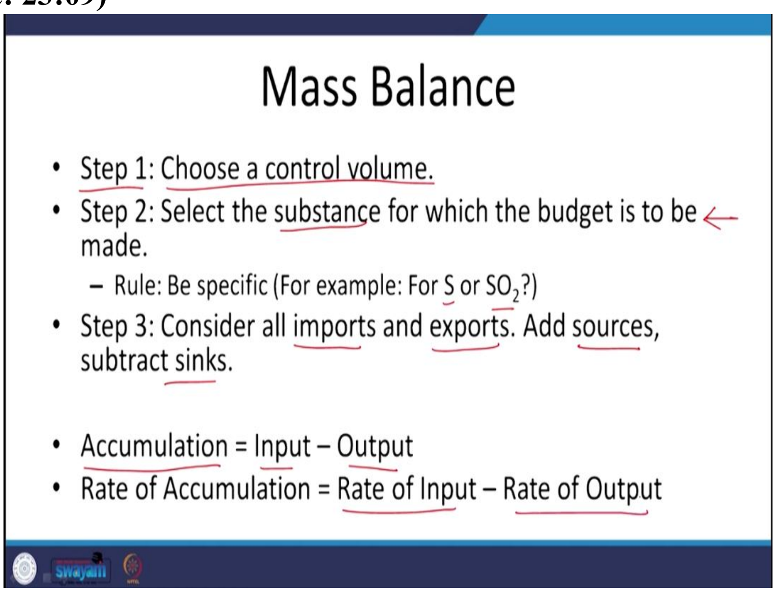

Mass balance, let us come back to what we are discussing or what we have been discussing. Step 1 is to choose the control volume. When we are talking about the example of this room, what is the control volume, the control volume is this room, this particular cuboid or such, that is this, but the room, that is my control volume, then select the substance on which you want to apply the mass balance, wind is blowing in multiple compounds.

You have to be clear about which component is that you are applying the mass balance one that is what we are trying to mention out here is it S or SO_2 or so on and so forth, and then try to understand what are all the sources and sinks all the imports and exports, sources and sinks. Here I am trying to understand the mass coming in mass going out and then sources are sinks is there any reactor or reaction here leading to the mass of that compound increasing or the mass of that compound decreasing by transformation into another compound?

I need to understand these sources or sinks and imports? Is the mass coming in through the door? Or is the mass going outside I am in through the door or such we are talking about the room as an example. We have accumulation dc / dt is equal to input minus output rate of accumulation equal to rate of input minus rate of output . We will also have to take into account import export and sources or sinks within this particular system. We will look at an example later, but let us go through and understand mass balance.

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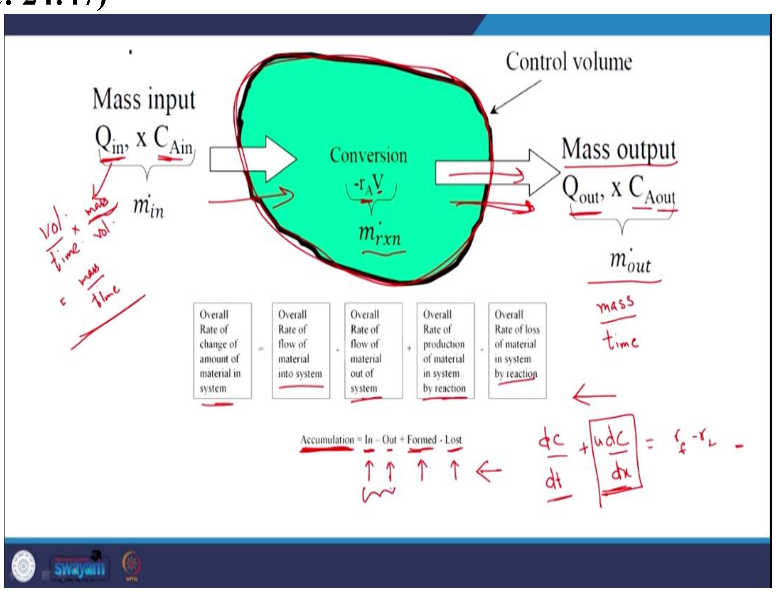

Here I have this particular control volume. This is my control volume. If mass input is Q x $C_{A\ in}$ Q_{in} is the rate flow of my liquid. The units will be volume per time. Here we are trying to understand mass balance in the way that we will typically use it in this class, fundamental mass balance equation, as discussed previously, in this class can certainly be applied. But we will also be looking at it from a different perspective.

Here, you have your particular reactor that you are going to use for your wastewater treatment, and the mass is coming in via this flow of water flow of Q_{in}, and the units are volume per time, and the concentration of the compound in that particular flow is C_{Ain} concentration of compound D. The units are going to be mass per volume, or moles per volume, depending on how you want to take it volume per time into mass per volume.

As you see the units will turn out to be mass per time here? And then looks like we have a reaction out here. Mass, due to the reaction, and we seem to be having a certain loss units of the rate of this particular reaction or any reaction are typically mass per volume per time. That is why we multiply with volume, and we get the same units as what we had earlier. That is mass per time.

And then mass is leaving my reactor, if not, it is going to over flow or such at some point, but we are going to have an output. Mass output, Q_{out}, flow rate of flow of the fluid going out, and concentration of A in that fluid going out. The units, as you can see, will turn out to be mass per time. Let us see how we can get this up here. We have different terms out here. Let us understand what we have.

Again, please note that even the fundamental mass balance equation that we had earlier is more or less similar to what we have out here. Overall rate of change of amount of material in the system. I am concerned with the compound in this control volume, and I am specifically concerned about rate of change of amount of material in the system overall rate. That is what I am referring to as accumulation.

And then it depends upon the overall rate of flow of material flow import of the material coming into the system, in an overall rate of flow of material going out of the system, input, and output, input minus output, and then what else overall rate of production of the compound in the system by reactions. Are any reactions leading to formation of the compound that is what we have out here formed?

If you are looking at formation of the compound, we also have to look at the overall rate of loss of the compound or material in the system by reaction that is lost, and , if there are any other sinks like settling out of the reactor or such, we are going to have to look at that. This is understanding mass balance. We will look at the applications in the next session, because I seem to be out of time.

I am trying to look at how it is that the compound or to look at the overall rate of change of the compound in my system that I am calling as accumulation that will depend upon mass coming into the system, mass going out of the system, mass being formed within the system, due to reaction and mass being lost within the system due to reaction. That is what we have, if you look at what we had earlier dc / dt + u dc / dx is equal to rate of formation minus rate of loss. If we take this term out here.

$$\frac{dC}{dt} + u\frac{dC}{dx} = r_f - r_l$$

And look at what we have, this is what we have accumulation term is dc / dt, and this will give you an idea about when it goes out to the hand side input minus output and formed minus lost. But we will try to take this to its natural conclusion but I will do that in the next session, and as usual, thank you for your patience. I will end today's session.