

**Water and Waste Water Treatment**  
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**Lecture-19**  
**Activated Sludge Process: Material Balance for Aeration Basin**

Hello everyone, welcome back to the latest session, let us try to pick up the thread of where we left off earlier. In the last session, we were looking at understanding the activated sludge process or the biological process by applying mass balance. In that context we applied the mass balance around the entire system which includes both the aeration tank and the secondary sedimentation tank or the secondary clarifier.

But we applied the mass balance twice once on the X which is the biomass or the microorganisms and the second time on the substrate are our waste are the food for the microorganisms and we looked at some variables.

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Material balance for completely mixed aeration basin

- SS on substrate around entire system

$$\blacksquare r_s = \frac{-(S_0 - S)}{\theta}$$

S

Note: similarities to balances for system without recycle. Substrate balance is the same, biomass balance differs only in substitution of solid retention time for hydraulic retention time

$$r_s = \frac{S_0 - S}{\theta}$$

Let us look at them. this is  $r_s$  net rate of microbial growth, net and we also looked at  $\theta_c$  which is a very important parameter cell residence time or solid retention time how much time will the relevant microorganisms spend in your system? we got this and from that we get that  $r$  net equal to  $x$  by  $\theta_c$ , that is also fine and by applying the mass balance on S or the substrate we got this equation S, we also have this  $r$  net, these are all  $r$  net. Why is this negative because waste water is being degraded that is why it is negative if I may say so.

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Material balance for completely mixed aeration basin

$$\bullet S = \frac{K_S(1+k_d\theta_c)}{\theta_c(\mu_m - k_d) - 1}$$

Note: same as no-recycle system, but SRT for HRT.

$$S = \frac{K_S (1+k_d\theta_c)}{\theta_c(\mu_m - k_d) - 1}$$

Let us move on and applying that and playing around with the relevant variables we come across this. And what do we see? We see that it is a function of theta c or the cell residence time or the solids retention time, it is not depend on the what is that flow of wastewater or such but the concentration of substrate in the effluent is going to be dependent mainly on the cell retention time or the solids retention time that is something to keep in mind.

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Material balance for completely mixed aeration basin

- Combine rate equation for biomass growth with material balance on substrate and material balance on biomass.
- $r_x = r_{x,D} - r_{x,G}$

$$r_{x,out} = r_{p,x} - r_{d,x} \quad Y_p = Y_r r_s$$

$$\frac{X}{\theta_c} = Y_{x,s} - k_d X$$

$$\frac{X}{\theta_c} = Y - \frac{(S_0 - S)}{\theta} - k_d X$$

$$r_x = r_{x,D} - r_{x,G}$$

And then let us move on what have we started looking at? We need to look at another application of mass balance that we are going to combine the rate equation for biomass growth, what is the rate equation for biomass growth?  $R_x$  net is equal to rate of production of  $x$  minus rate of loss of  $X$ .  $X_{net}$  what was that I think equal to  $x$  by  $\theta_c$ .

That is  $x$  by  $\theta_c$  rate of production of  $x$  you can write it in different terms but now we are going to write it in a different way minus rate of loss is  $K_d$ , the  $K$  constant into  $X$ , we know that the microbes how are they being formed how are they being produced? They are being produced when the substrate is being degraded, while the food is being degraded part of it is being used for cell synthesis and the microbes are being produced.

We know that rate of production of the microbes will be depend upon rate of loss of the substrate and we already calculated this earlier  $R_{net}$ . we know that a fraction of  $R_s$  will go towards cell synthesis. I will have this yield coefficient what is this giving me an idea about how much cells or biomass will be produced for how much or a unit what is it substrate or waste, that is what it will give me an idea about.

I can keep negative here because  $r_s$  is negative out here. let us play around with it or substitute that particular relevant aspect out here. But I think because we are taking  $r_s$  to be  $-S_0$  by  $S$ . let us not put that out here.  $X$  by  $\theta_c$  will be equal to  $Y$  into  $r_s$  net I think we calculated that here where is that equal to  $-S_0 - S$  by  $\theta_c - K_d$  into  $X$ . here you can play around all. And what do we get we can solve for  $X$

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Material balance for completely mixed aeration basin  $s: f(\theta_c)$

$$\bullet \underline{X} = \frac{Y}{1+k_d\theta_c} \frac{(S_0-S)\theta_c}{\theta} \quad x: f(\theta_c, \theta)$$

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$$X = \frac{Y}{1+k_d \theta_c} \frac{(S_0-S)\theta_c}{\theta}$$

We end up with this particular X. X is the concentration of microbes that need to be maintained in the aeration basin and that is depend upon various variables not just theta c, it is also going to depend upon the water how often or how much time is the water going to be present in the system. For example if theta is high as in if the water is going to be present in the system for longer time than X you can see the effect.

Unlike S which was depend only on theta c, here we see that X is depend upon both theta c and theta that is something to keep in mind and yield coefficient and decay coefficient but let us move on from here.

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Material balance for completely mixed aeration basin

$$S = \frac{K_s(1+k_d\theta_c)}{\theta_c(\mu_{max}-k_d)-1}$$

Note: same as no-recycle system, but SRT for HRT.

$$S = \frac{K_s(1+k_d\theta_c)}{\theta_c(\mu_{max}-k_d)-1}$$

I already mentioned this substrate function of theta c, biomass theta c and theta. one aspect to note is that I should look up the equation here as theta c. what do we see here as theta c goes to its minimum and if the minimum is 1 by mu max – K d as theta c goes towards this minimum value. What is going to happen out here? What do we see then the denominator turns out to be 1 - 1 0.

When  $\theta_c$  or the cell residence time decreases to its minimum cell residence time or solids retention time meaning how much time the microbes are going to spend in my system as they decrease what is going to happen to the substrate concentration or the wastewater BOD in the effluent it is going to go to infinity.

That is what it gives you an idea about, if the microbes are spending less time in the system then there is not enough time for the microbes to degrade the relevant wastewater. that is why the substrate or the concentration of your particular compound of interest which here is BOD, in your effluent is increasing. that is what we see out here.

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**Material balance for completely mixed aeration basin**

- Note :
  - $S = f(\theta_c)$  doesn't depend on  $\theta$ .
  - $X = f(\theta_c, \theta)$ .
  - $S \rightarrow \infty$  as  $\theta_c \rightarrow \frac{1}{\mu_m - k_d} = \theta_{c,min}$
  - $\frac{r_x}{-r_s} = Y_{obs} = \frac{Y}{(1 + k_d \theta_c)}$

$r_p = r_s$   
 $r_x,act$   
 $-r_s,net$   
 $r_p - r_d$

S goes to infinity as  $\theta_c$  goes to this value which is the  $\theta_c$  minimum and another aspect is if we play around with the relevant values we can come up with  $r_x$  net by  $r_s$  net, this  $r_x$  net will also take into account both the rate of production and rate of loss. Earlier we only looked at  $r_x$  and  $r_s$  and looked at Y here if we are looking at  $r_x$  net by  $r_s$  we get the Y observed.

And that is nothing but we can play around with the variables pardon me, it will be Y by  $1 + K_d$  into  $\theta_c$ , thus you see that the decay coefficient or the decay term also comes into picture. Here it is the observed one, yield I know that from this waste this from this much waste this much biomass will be produced, but I know that this biomass is also going to die. if we look at the observed yield so this is what it is going to look like slightly lesser if I may say so, that is something to keep in mind.

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Material balance

- Relationship of recycled ratio and recycled solids concentration
- SS balance around clarifier

$\frac{dC}{dt} = 0$

$$0 = (Q + Q_r)X - \{ (Q - Q_w)X_c + Q_w X_r + Q_r X_r \} + 0$$

One more mass balance relationship this is the last one. We are going to apply the material balance and we are going to try to look at the relationship of the recycle ratio and the recycled solids concentration, why is this important? We have our activated sludge process out here fine and that we know that there are 2 aspects. One is the aeration and one is this particular secondary settling tank or secondary clarifier where the sludge is settling down.

And thus comes out like this. Why is this that we are recycling some of it? Because here we need to maintain microbes and we know that the microbial concentration here is almost 0 the microbial concentration coming in. I need to know how much of this sludge do I need to recycle back into my aeration tank and how much I can waste here. this is critical out here.

That will depend upon how much or how well the sludge is settling. For example if the sludge is settling very well meaning the concentration of the microbes in that settled sludge is going to be very high, then my recycle rate need not be very high I can make do with relatively less recycled flow. But if my sludge is not settling well as in my plant is not being operated well.

When I say sludge I am referring to the suspended microbes and they are not settling very well. What is going to happen, then I have to increase the recycle flow and if I do not then the plant is going to be affected . Here we are going to look at steady state balance around the clarifier, keep in mind that until now we looked at the mass balance around the system entire system.

But now to be able to understand the relevant recycle ratio if I take this to be my entire control volume, if I take that to be my entire control volume I will lose out on the recycle flow, is the recycle flow coming into picture? No because it is within the system I will lose that

information. now what is it that we are trying to do? We are trying to apply the mass balance around this particular system around the clarifier.

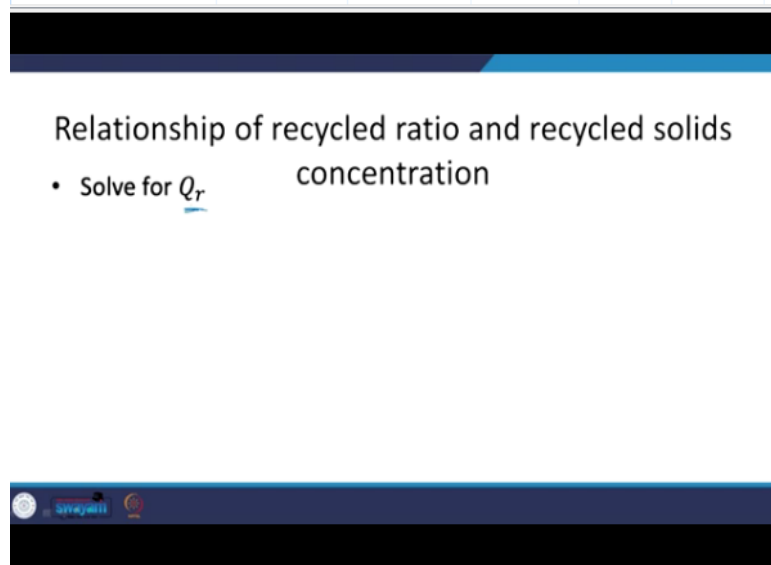
That I will get an idea about the recycle flow too. let us look at what we have, I know it is  $Q$  out here it is  $Q - Q_w$  out here, let me just erase this particular 1. That it is relatively clearer.  $t$  is  $Q - Q_w$ , because  $Q_w$  is going out here and this is  $Q_r$  how much is being recycled. this flow coming into the clarifier will be  $Q + Q_r$ . let us apply the mass balance, we are not concerned about substrate we are concerned about the sludge.

We are applying the mass balance on  $X$  around the clarifier. let us see that its steady state.  $V \frac{dc}{dt}$  will be 0. if it is 0 how are the microorganisms coming into the system only by  $Q + Q_r$  into  $X$  and how are they leaving the system via 3 means what is that one is  $Q - Q_w$  into  $X$  effluent, here note that I am assuming that some microbes are leaving from this treated wastewater.

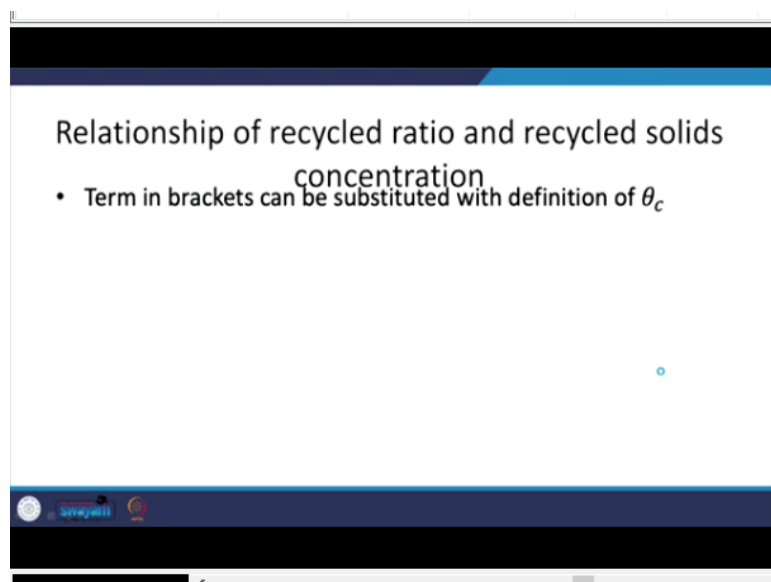
This treated waste water which is above after the suspended solids are removed at the bottom or the suspended microbes are removed at the bottom, the treated water is the supernatant and that is removed but we are assuming that there are still some microbes in the particular system, some people do not then expectants will be 0. How else, is the microorganisms are the microorganisms leaving the system  $Q_w$ .

$Q_w$  into  $X_R$  why  $R$  this settle sludge the concentration will be pretty high and what else we see that it is also being recycled  $Q_r$  into  $X_R$ ,  $Q_r$  into  $X_r$  that is how it is also leaving this system. And are there any reactions occurring that affect the microbial concentration? We know that in the clarifier we are assuming that no concentrations not, no concentrations pardon me no reaction circle. you know we can go ahead with what we have and assume that the reaction term is 0. we have this out here, let us see what else we need to do.

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It is simple to say we are trying to calculate how much we need to recycle  $Q_r$ .  
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The term in brackets can be substituted with the definition of  $\theta_c$  as in some of the terms in this brackets or after I play around with this rather can be substituted with  $\theta_c$ .

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• Solid retention time

$$\theta_c = \frac{\text{mass in system}}{\text{mass removal rate}} = \frac{VX}{Q_w X_r + (Q - Q_w)X}$$

Let us see what we have? Theta c is this particular term out here. I am not going to look at the algebra we can play around with that.

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Relationship of recycled ratio and recycled solids concentration

$$Q_r = Q(1 - \theta/\theta_c)X' / (X'_R - X')$$

$X \rightarrow \text{VSS}$       $X' = \text{TSS}$

$$\frac{VSS}{TSS} = \frac{X}{X'}$$

$$\downarrow \downarrow \quad Q_r = Q \left(1 - \frac{\theta}{\theta_c}\right) \frac{X}{X'_R - X}$$

And once I substitute with that I will come up here. One aspect I need to mention is here X is the concentration of the microbes VSS, MLVSS or VSS volatile suspended solids, but typically you have your X dash which is the total suspended solids, this is VSS and for a particular waste water based on experience or such you can know what is the ratio of this VSS by TSS or what is the ratio of X by X'.

0.6 or something like that, so based on that you can replace X with X' that is why we replace this. If not the actual what we say Qr will be equal to how much recycle will depend upon Q into 1 - theta by theta c into X into X recycle - X. this does not really look, I understand so it

has to be  $X$  dash by this or  $X'$  by this. minor correction there  $X$  by  $X$  recycle -  $X$ . this is how you will calculate how much  $Q_r$  is required.

what is it some of the aspects that you need to be concerned out here. one is  $X_r$ , if you look at the equation and also understand the system if the  $X_r$  is high, as you can see if  $X_r$  is high then you can make do with a lesser  $Q_r$ . If  $X_r$  is high you can make do with the lesser  $Q_r$ , why is that?  $X_r$  meaning how much sludge or how much is the concentration of microbes in the settled sludge.

If the sludge is settling well or it is good settling sludge, then the concentration of microbes that I am going to recycle within the unit or a particular flow rate is going to be pretty high. We are always concerned about the mass, but if this sludge is not settling well then I have to pump more of the relevant sludge. that is something to keep in mind. in that cases we are going to have issues with steady state but that is one aspect to keep in mind that  $X_r$  and  $Q_r$  we have close relationship out there.

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Relationship of recycled ratio and recycled solids concentration

- Biomass production ←
  - Define overall production rate in terms of volumetric rate.  $\theta = \frac{V}{Q}$

$$P = V r_x = \frac{V X}{\theta_c}$$

$$= \frac{V}{\theta_c} Y_{obs} \frac{(S_0 - S) \theta_c}{\theta} = \frac{V}{\theta} Y_{obs} (S_0 - S)$$

$$P = Q Y_{obs} (S_r - S)$$

Let us move on and look at what else we have. I also sometimes are such we need to look at how much biomass is being produced. let us look at this define overall production rate in terms of volumetric rate, overall production of microbes is what is this  $V$  times  $R_x$  and here  $R_x$  net  $R_x$  is rate of production of your biomass,  $R_{net}$  and I think we already solved that for that  $R_x = x$  by theta c.

And we have to substitute for  $x$  let me see if I have the equation written down out here somewhere  $X$  is this  $y$  by  $1 + kd$  into theta c or  $Y$  observed we know that that is equal to  $Y$

observed into  $S_0 - S$  into  $\theta c$  by  $\theta$  and  $R_x$  I know is  $x$  by  $\theta c$  and this is  $x$ . what do we have out here, let me try to substitute this  $x$  term out here  $Y$  observed into  $S_0 - S$ .

That is what I have  $\theta c$  by  $\theta$  apart me. what do I have cancel out  $\theta c$  I will be ending up with  $V$  by  $\theta$  into  $Y$  observed into  $S_0 - S$ , let me see if we can simplify this further  $\theta = V$  by  $q$ .  $V$  by  $\theta$  will be equal to  $Q$  into  $Y$  observed into  $S_0 - S$ . that will give me an idea about the overall production rate in terms of volumetric rate.

If you look at this, this much substrate is being consumed and this is the yield observed yield. this observed yield will tell me and give me an idea about how much substrate is being turned into biomass and if I multiply that by  $Q$  that will give me the relevant rate. that is what we have out here.

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**Biomass production**

- $P_x = Y_{obs} Q (S - S_0)$
- $P_r = Y_{obs} Q (S_0 - S)$

$X$   
 $S$   
 $Q_r$   
 $P_x$   
 $\mu_m$   
 $r_p = \mu X = \mu_{max} \frac{S}{K_s + S}$

Let us move on, we already did this and this is what we already ended up with. we just looked at the equation I think we got  $S_0 - S$ . probably the student made an error. it should be  $P_x = Y_{obs} Q (S - S_0)$ .  $S - S_0$  will be negative and  $P_{obs}$  or the biomass production cannot be negative we are having production of biomass. these are the terms.

What we have calculated over this course and the not course, session and the previous one we calculated equations for  $X$ , we calculated equations for  $S$ , we calculated equations for calculating  $Q_r$  or came up with the equation, we know the biomass production rate, we know  $\mu_{max}$  or we understood that we know that our rate of production of microbes or growth will be  $\mu$  into  $X$  and that is equal to  $\mu_{max}$  into  $S$  by  $K_s + S$  into  $X$ .

We looked at different variables whenever we are trying to design it what do I typically want, I want to what do you say achieve a particular S as initially I have 150 milligram per liter of BOD and I want to end up with 10 milligram per liter of BOD. How do I go about it, given that the expected yield as in for this particular waste this is the amount of biomass that will be produced. I get these aspects and we can solve them. let us go ahead and understand.

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### Biomass production

- Provides mass flow of VSS produced
- We need VSS/TSS for biological solids to obtain TSS mass flow
- 0.80 gm VSS/ gm TSS typical.

I have one other aspect. It gives you the mass flow of the biomass or volatile suspended solids produced, we need this VSS by TSS for biological solids to obtain the TSS mass flow, zero point grams of VSS per gram of TSS seems to be typical.

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### Comparison of FMT and AST

Parameter	FMT	AST (FMT with recycle)
$\theta, t_r$	V/Q	V/Q
$\theta_c$	$\rightarrow \text{L} \rightarrow$ $\frac{V}{Q} = \theta = t_r = \theta_c$	$\theta_c = \frac{VX}{1 + r - r(X_r/X)}$
$S$	$\left\{ \frac{K_s(1 + \theta K_d)}{t_r(\mu_{max} - K_d) - 1} \right\}$	$\left\{ \frac{K_s(1 + \theta_c K_d)}{\theta_c(\mu_{max} - K_d) - 1} \right\}$ $\leftarrow \theta_c < \theta$
$U$	$\frac{\mu_{max} S}{Y(S + K_s)}$	$\frac{\mu_{max} S}{Y(S + K_s)}$
$X$	$Y \left[ \frac{S_{in}}{1 + K_d t_r} - \frac{K_s}{\mu_{max} t_r - (1 + t_r K_d)} \right]$	$Y \frac{\theta_c}{t_r} \left[ \frac{S_{in}}{1 + K_d \theta_c} - \frac{K_s}{\mu_{max} \theta_c - (1 + K_d \theta_c)} \right]$

### Comparison of FMT and AST

Parameter	FMT	AST(FMT with recycle)
$t_r$	$\frac{V}{Q}$	$\frac{V}{Q}$
$\theta_c$	$\frac{V}{Q} = t_r = \theta$	$\frac{VX}{1 + r - r(X_r/X)}$
S	$\frac{K_s(1 + \theta K_e)}{\theta(\mu_{max} - K_e) - 1}$	$\frac{K_s(1 + \theta_c K_e)}{\theta_c(\mu_{max} - K_e) - 1}$
U	$\frac{\mu_{max}}{Y} \frac{S}{S + K_s}$	$\frac{\mu_{max}}{Y} \frac{S}{S + K_s}$
X	$Y \left[ \frac{S_{in}}{1 + K_e t_r} - \frac{K_s}{\mu_{max} t_r - (1 + t_r K_e)} \right]$	$Y \frac{\theta_c}{t_r} \left[ \frac{S_{in}}{1 + K_e \theta_c} - \frac{K_s}{\mu_{max} \theta_c - (1 + \theta_c K_e)} \right]$

Here from the MIT open course where we have a comparison between a fully mixed this is with recycle what we looked at. And this is another case without recycle but we will never or usually will not have or look at this practically. theta or retention time V by Q, V by Q theta c. If I do not have the recycle whatever is coming in is going out the microbes are not spending any additional time because there is no recycle.

Then theta c will be equal to V by Q or theta, but if I have recycle the theta will increase. that is what you see in system without recycle, theta c will be less as in cells well or the biomass or the microorganisms will spend lesser time than in the case with where we have recycle, you know some of the variables might be slightly off . But this is from you can play around and look at the equations that we developed.

Substrate equation here and here we already looked at this as we looked at it in our case. Here it is the only hydraulic retention time, here it will be cell retention time, typically hydraulic retention time in this case will be typically much lesser than theta c. here that is why forgiven similar characteristics the substrate or the effluent substrate or the BOD in the effluent will be much less for the case without recycle compared to the case with the recycle. And then you have the substrate utilization rate, substrate utilization rate different terms we will not go into this for now.

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## Oxygen transfer

- Oxygen required
  - Fundamental approach
    - $O_2$  required =  $O_2$  required to full oxidize organic removed –  $O_2$  equivalent of biomass solids.
    - $M_{O_2} = C_{o/s}P_s - C_{o/x}P_x + C_{o/n}P_n$

Note: Equation has S in units of bCOD in that case  $C_{o/s} = 1.0$

$$M_{O_2} = C_{o/s}P_s - C_{o/x}P_x + C_{o/n}P_n$$

let us move on. the last aspect is oxygen transfer but it is pretty important why is that in a particular wastewater treatment process 50 to 60% of your cost maintenance costs are from you know pumping air in BOD oxygen demand you do not want that oxygen demand to be exerted in the river, you want that to be taken care of in your wastewater treatment plant.

You need to provide the oxygen here. you know you have some aspects to consider but we will look at this in a new session and as usual thanking you for your patience I will end this session.