Geographic Information Systems
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Lecture-45
Common Derivatives of DEMs - Slope and Aspect-02

Hello everyone! and welcome again for the next part of that discussion which we had in previous lecture that is common derivatives DEMs: slope and aspect and this is part 2. What we are going to discuss in this one is that planner and the other option which is available to us through the software's. So, basically 2D or 3D Cartesian or in that sense, we have to go for this because in various software's like for example in ArcGIS, different options are available when you go for Calculation.

I will show you little later about these options and what is the meaning of these options. Though for certain particular digital elevation model for if I am say working on a plane area like part of the Indo-Gangetic plain, I employ two different methods but when I see visually, I may not find much difference between two derivatives means both are slopes, calculated through two different options. However, if we go for cell values, again in some cases, you may not find much difference.

So, depending on our input data basically, these can be different. Depending on how much variations which are present within our cells. If cells elevation values are not very much then even if you choose to different options, still you may get almost same results but if there are large variations in the elevation values of cells of my input digital elevation model and I apply these different variants of calculating slopes, I need a different result.

So that is why I mentioned that depending on your input digital elevation model and values present which are having variations in their properties or values. So, if I go for first option is by default is Planner method which we have also briefly touched in the previous discussion.

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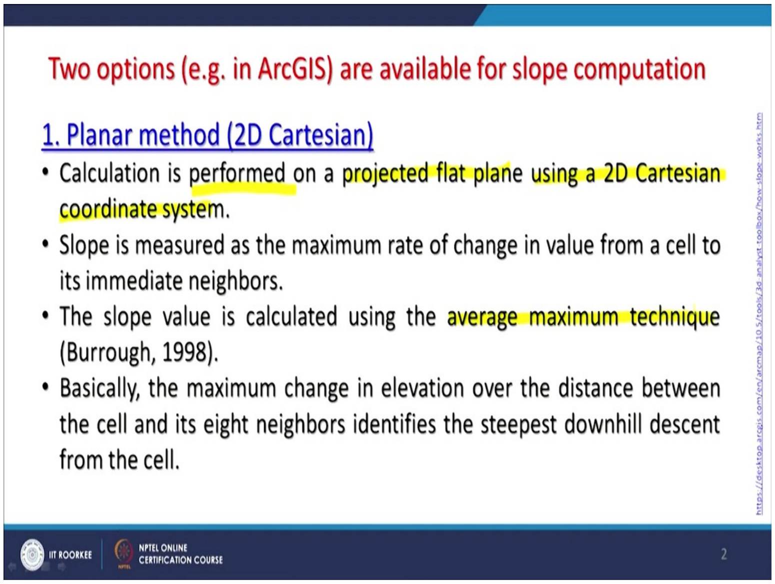

Basically, which calculates or perform on a project flat plain and using a 2D Cartesian co-ordinate system and that way, it calculates basically. So, slope is you know that it's a maximum rate of change in the terms of value. And slope value will be calculated for the average maximum technique. And this is what is there that average maximum technique which is calculated and basically the maximum change in elevation over the distance between the cell and its 8 neighbours.

So, that means we are talking about 3*3 matrix or kernel then identifies the steepest downhill descent from the cell compare to so, it's a neighbourhood analysis basically.

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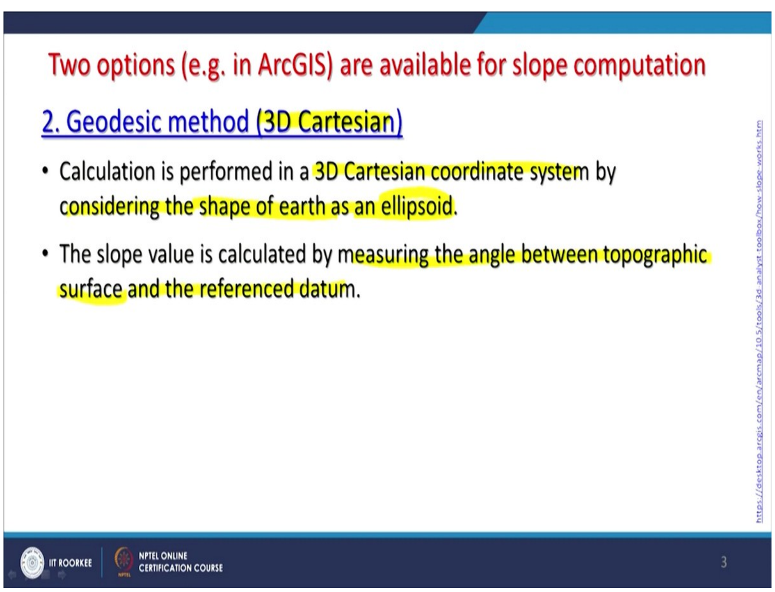

Now another method is Geodesic method or 3D Cartesian method. Here it is not assumed as a plane calculation but is a 3D so that means the calculation is performed in 3D Cartesian co-ordinate system by considering the shape of the earth as an ellipsoid. As I said that if lot of variations are not present in my input digital elevation model or in their values then no matter which option I go, I may get almost identical results. But if variations are there or I am covering a larger area then the shape of the earth will play very important role.

And there I have to take care about this ellipsoid problem or ellipsoid issue. Now the slope value in this geodesic method or 3D Cartesian method is calculated by measuring the angle between topographic surface and the reference datum.

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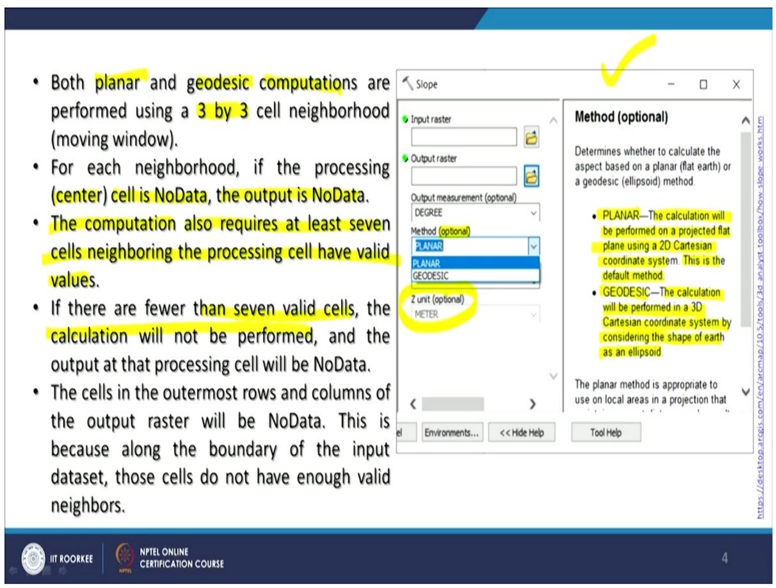

This reference data is very-2 important. So, if I take the example of ArcGIS which is here which you provide input raster and of course output raster. When you go for this method which is optional otherwise by default it is Planner then the two methods as we are discussing are introduced. But the planner calculation will be performed on a projected flat plane. So, your data which is of course representing the 3-dimensional data digital elevation model is projected on a 2D co-ordinate system and then it is calculated and of course, this is the default setting in most of the software's.

Now geodesic calculation will be performed in 3D Cartesian co-ordinate system by considering the shape of the earth as an ellipsoid. And if you want further details, of course those are available. And remember all the time this Z factor, though it is written optional but when our digital elevation models are in degree decimal and the cell values are in meters, we have to take care about this.

We have already discussed this and we will further discuss when I will have demonstration through a software. So, both these methods; planner and geodesic, computations are performed by using again the same 3*3 that moving window or kernel, for each neighbourhood that is remaining 8 cells. So, they will be 3*3, total number of cells are involved 9. But I exclude the centre cell then it is 8 neighbours.

For each neighbour if processing the centre cell because the calculation is done for Centre cell, is no data then output would be no data. This is true in all such calculations where neighbourhood calculations performed. If the center cell is having no data, output is also going by no data. That is why the concept on no data is very-2important. I also mentioned during early discussion that in older versions or early versions of GIS software, this concept of no data was not there, at all.

And that give lot of wrong outputs, wrong calculation because of this. But anyway, that issue has been solved. It is something like Y2K problem. And the computation also requires at least 7 cells neighbouring the processing cell which is having valid values. Even if single cell is having no data, nothing can be done so you can also imply. If there are fewer than 7 valid cells, that calculation will not be performed.

Only in case of 7, the calculations will be done. But if you are having more no data, of course the output would be that no data. Now this point I have already discussed but I will repeat again because this is very-2 important again, is that the cells in the outermost rows and columns, that means one cell thickness equivalent row and column and same with first row and first column, leftmost column and rightmost column, bottom row.

For that, calculation will not be done. So, that the cells in the outermost rows and columns of the output cell will be no data because in that surrounding, you do not have the side. This is because the boundary of the input dataset, for those you do not have enough valid neighbours. You would require at least 7. If one cell is bearing no data, it's fine. But in the corners or in the margins of your input digital elevation model, it would be more than 7 cells which will not have data and therefore the calculation is not done.

So, your output would be reduced by at least two cells by two cells, on the borders. This you have to remember one cell thick on all sides so it's a kind of border, it will create with no data.

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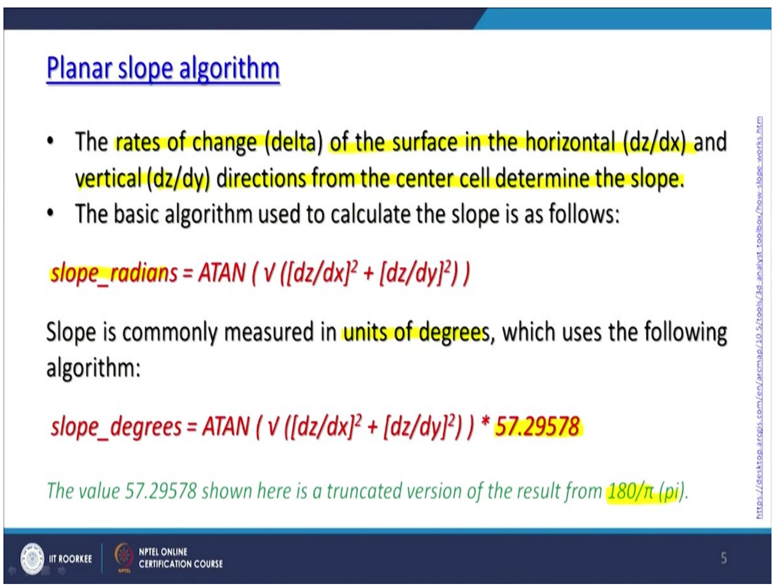

Now when we go for planers slope algorithm, the rates of change or delta of the surface in horizontal that is dz/dx and in vertical, dz/dy directions from the centre cell which will determine the slope. And the basic algorithms which is used for the slope calculation is like this. You can also have slopes in degrees or radians or whatever you want. And slope is commonly measured, generally most preferred one is units in degree.

And most of the software will also have by defaults in degrees. So, if we have to calculate in degree, we have already discussed this factor; 180/π. So, this will be required for slope calculation in degree. Now here is the truncated value of 180/π.

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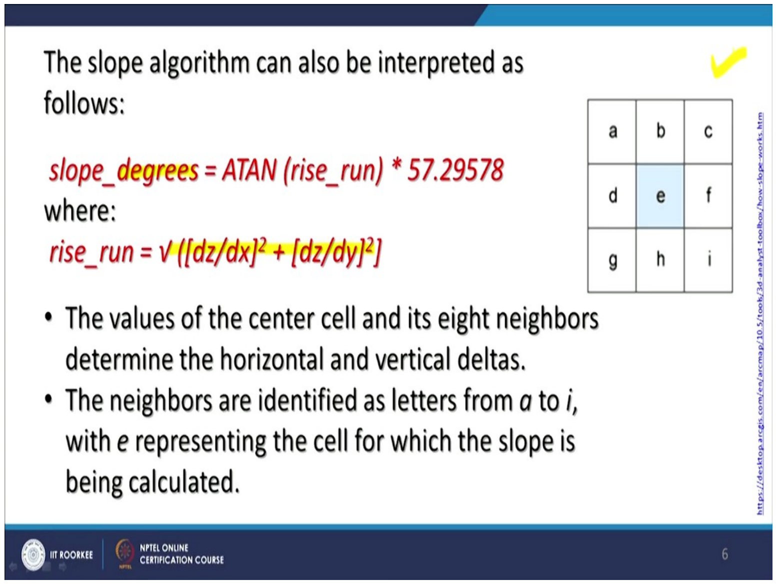

Now slope algorithm can also be interpreted as in this form like slope in degrees and where you know that rise/run will be decided based on this expression. And the value of the centre cell and its 8 neighbours like here in this example for each cell, the values are calculated and it will be in both directions. And neighbourhood are identified, here in this particular example between a to i which you are seeing; a to i and where e representing the cell for which the slope is being calculated.

Always in a 3*3 or 5*5 or 7*7 for the centre cell, the calculation is performed. Now rate of change in X direction for e cell calculated with the following formula.

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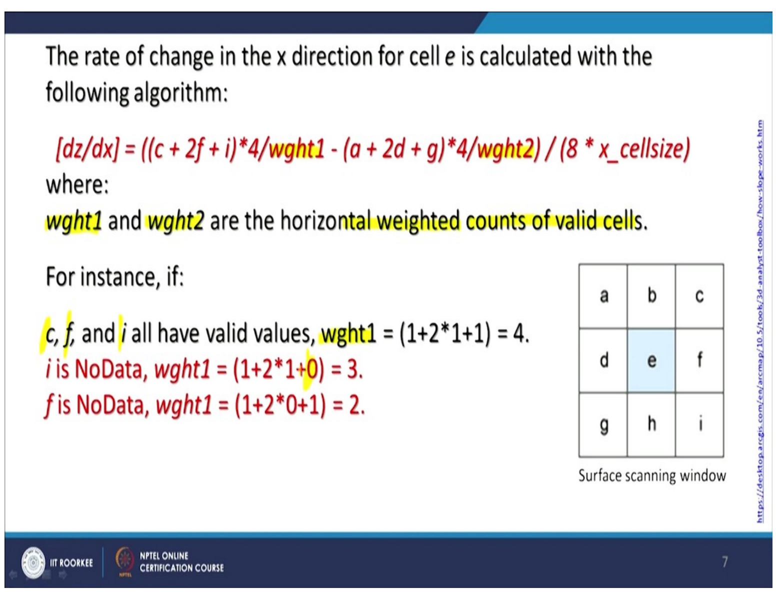

Again, the weights are there. Weights can be also given; this I have been also telling earlier. For each cell in the neighbourhood, the weight can be given and that becomes typical special filter, if every bring the analogy of digital image processing. So, weight 1 and weight 2 are horizontal weighted counts for valid cells, here weight 1 and weight 2 in the above equation. And for instance, I may assign some weights here. So, for values for my kernel against c, f and I, all have valid values and they will have weight 1; that is like this.

And i if no data then it will have something like this. This zero is also there. If f is no data, then the calculation will be like this.

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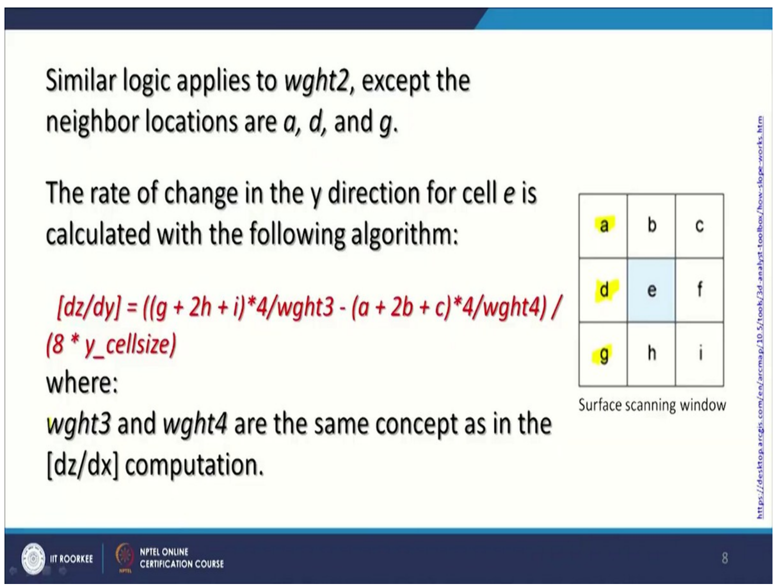

And similar logic will apply to weight 2 also except neighbourhood operators for a, d and g likewise. So, for each cell, the calculation is done. The rate of change in y direction for the cell is calculated with the following. So, this slope calculation become complicated. Once we go for assigning weight for different cells of my kernel and that will bring obviously different results. Again, it will depend on the input; the variations which are present in the input digital elevation model.

Here the weight 3 and weight 4 are the same concept as in computation for our in other x,y calculation or x calculation, now it is for the y scale calculation.

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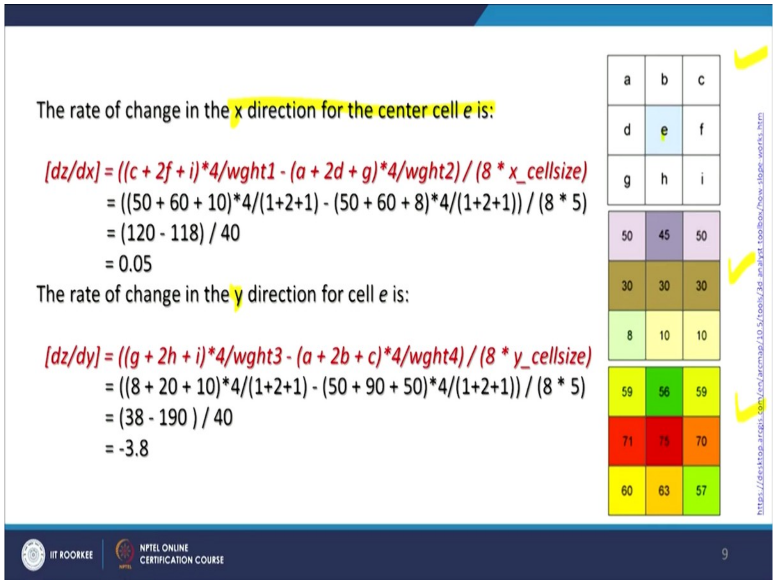

Now this rate of change in X direction for the centre cell e, which we can also observe in this example that this is my kernel; the top one. This is my input digital elevation model and this is my output. So, if these values are substituted in the above equation in the red colour then this is what the calculation will end and ultimately, we get 0.05 for that one; in one direction. Now, in the Y direction. That was for X direction, so for Y direction, similar calculations will be done and again if I substitute these values, I get -3.8.

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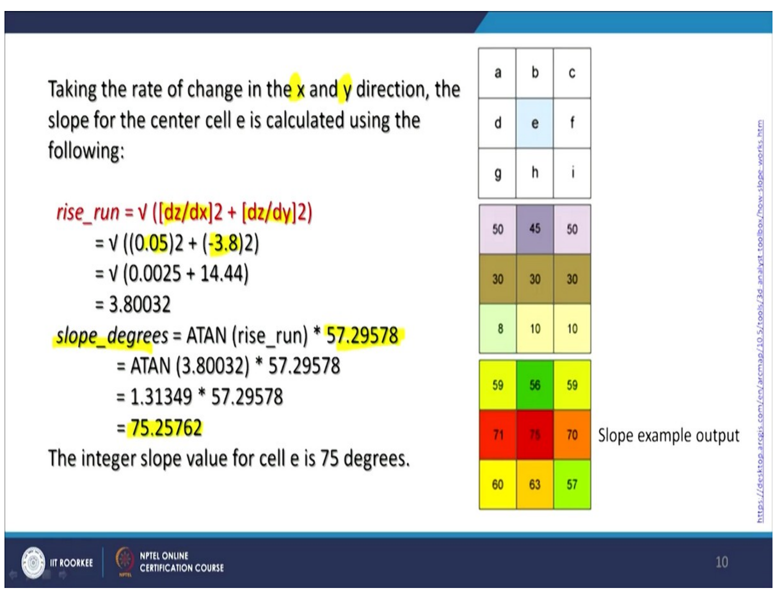

So, for taking all this together; X and Y direction, the slope of the centre cell is calculated using following formula, that now I am bringing X direction, I bring you by direction. Same concept; rise_run. dz/dx, we have calculated 0.05 and where dz/dy in the earlier slide, we have seen is -

3.8. Of course, it is a square so then negative values will disappear and it is 3.8, that is slope in degree which will be multiplied by this factor.

And then finally, we get this one, is 75 degree. Now, these are not represented in our results in fraction because for that kind of accuracy, topographic slope is not really required. So, rounded figure is represented for degrees. And if somebody is very keen, definitely some modifications can be done so that you can get even after decimal places so 75 degree. So, we started with this center cell e and in centre cell value, initially was 30 degree in input digital elevation model but through this neighbourhood analysis for planner x, y direction slope calculation.

Ultimately, we get for the centre cell value, the slope is 75 degree compared to what. So, it's a basically neighborhood analysis.

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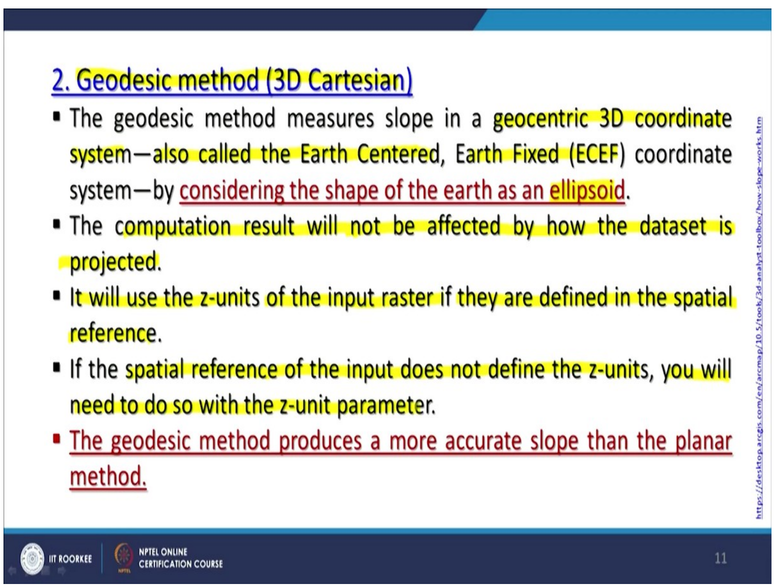

Now when we go for second option that is geodesic method 3D Cartesian system then this geodesic method as you know measures the slope in geocentric 3D co-ordinate system. Things may become little complicated in that sense because now we are bringing an ellipsoid which is earth centered, earth fixed (ECEF) coordinates system. This concept is also valid in case of these Navigation systems like in GPS, BeiDou or Navic of India or Galileo or Glonass. There also, different ellipsoids are used. Not all navigation systems are using the same ellipsoids.

They are using different concept. But GPS like for example uses this ECEF concept, that is earth centered, earth fixed co-ordinate system. Because there also, the 3D Cartesian co-ordinate system is required. So, for computation of results will not be affected by how the dataset is projected. So, from that point of view, there in case of 2D Cartesian or planner calculations, the data is first project in 2D plane and then calculation is done.

Here, no matter whatever the data is projected, it will do the calculation. It will use the z units of the input raster, that is the cell values if they are define by spatial reference. That means they are georeferenced and generally we are having. So, if the special reference of the input does not define the z units, one has to be provided that z unit parameters. In the geodesic method which will produce more accurate slope then planer method.

Again, I will tell you that important point is how variations exist in input digital elevation model. If values of elevations are not varying much then you may get almost the same results. Though on cell basis, if you do the comparison or created a subtracted slope having two different options, you may get some result. Otherwise visually there are not much difference. However, if elevation values variations are much higher within 5, you may get a completely different result, one.

And also, it will matter how big area is being represented, though calculation is done only by using a 3*3 matrix. So, this geodesic method produces a major accurate slope compared to planer method.

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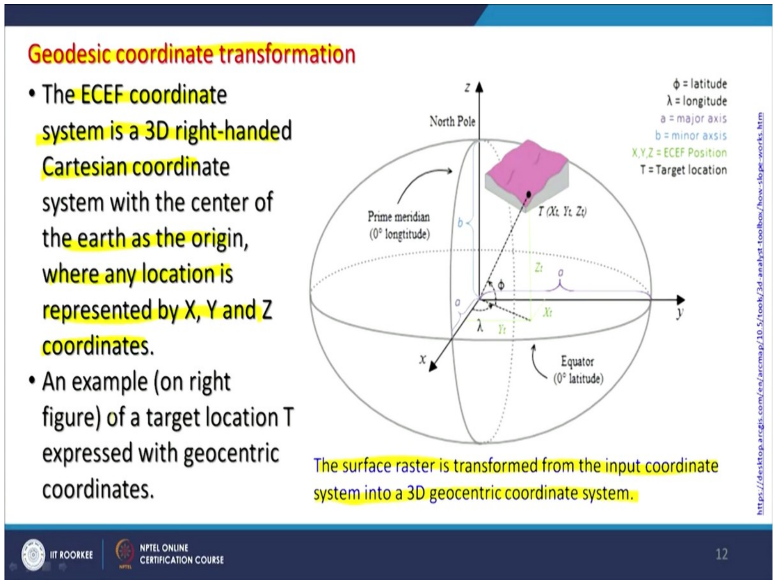

Now very briefly, I will touch about this geodesic co-ordinate transformation that is earth centre earth fixed one. Here because Earth is not perfectly spread so here it is little exaggerated thing for the Polar Regions. Just to explain that the surface is transform from input co-ordinate system into a 3D geocentric co-ordinate system, that is ECEF; earth centre earth fix co-ordinate system. So, whatever the input which is coming through the 2D, will go in the 3D.

So, ECEF co-ordinate system is 3D right-handed Cartesian co-ordinate system because you know coming from North, East and likewise so right-handed. Centre of the earth is origin where any location is represented by x, y and z co-ordinate system. So, in 2D, you will have only x,y but here also z co-ordinate system. In GPS or GNSS, you get the estimation of the height. That is why they are also following similar Geodesic Cartesian co-ordinate system or this 3D thing; ECEF.

And like here for a target location T which is shown here x1, y1 and z1 which is expressed in the geocentric co-ordinate system and then you get better results.

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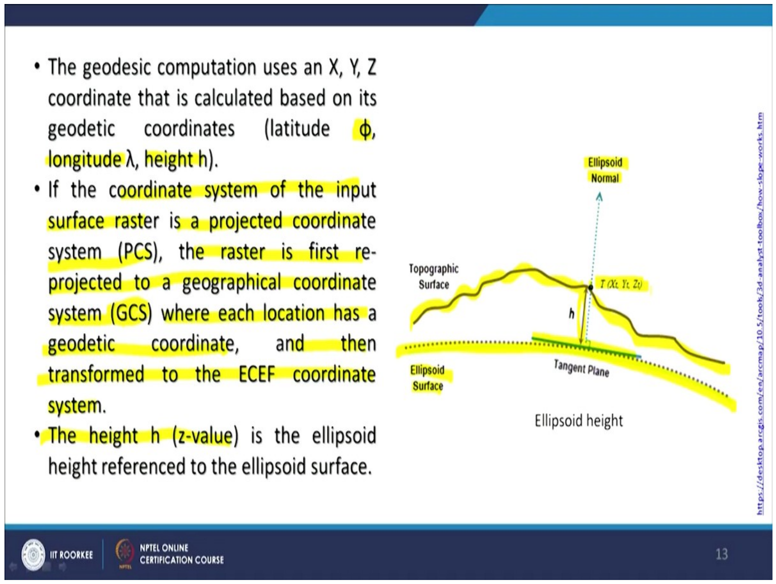

Now in this geodesic computation for x, y, z coordinates are calculated based on this geodesic coordinate that is latitude which is represented by Φ, longitude λ and height h. So, all three are there. Now as I mentioned that more complications will come once you go for more accurate and accurate estimation of slope. Still, I am using word estimation because after all digital elevation model is a model. It is not a completely true one to one scale thing.

The real one is on the ground only. So, in a model, you are assuming few things here and therefore the same thing also happens once we go for height calculations or altitude estimation by employing these GNSS systems; there we also encounter this problem. So, similar kind of problem will also come about the height above mean sea level or whatever.

So, like here, this is the ellipsoid surface which you are seeing like this and what basically you are calculating; the T which we have this just discussed in the previous figure. This t, x1, y1 and z 1 is basically what we are getting above the topographic surface but whether that height is perpendicular or this is our tangent plane which is just touching this ellipsoid surface. So, weather it is perpendicular to that or not.

So, again if it is coming from the Earth Centre then definitely it would be the ellipsoid normal otherwise not. So, the variations may come in our result. That is why in GNSS, you get the variations. You expect something but you may not get the accurate elevation value. Though for x and y that is 2D system, that issue is not there. So, generally the coordinates like GNSS systems, you get very accurate. Almost the same issue is here that we are representing a topographic surface which I want to know the slope for my input x, y and z, whether are having this kind of relationship or not.

So, if co-ordinate system of input surface raster is a projected co-ordinate system. The raster is firstly reprojected to a Geographic co-ordinate system that is GCS, where each location has a geodesic co-ordinate and then transformed to the ECEF coordinates. Lot of calculations will be required to get idea or to get a slope map through this geodesic computation. So, if you choose this obviously on an even small file, it may take some time and it may require some extra input from.

And as height, that is the z value here. The z value is the ellipsoid height above the surface whereas we say above mean sea level. So, this reference becomes our ellipsoid, though that can also be adjusted above mean sea level. But again, that is also depend on the model. Based on our gravity surveys of the earth, the model is there that is the geoid, not ellipsoid but the geoid and based on that then heights are estimated.

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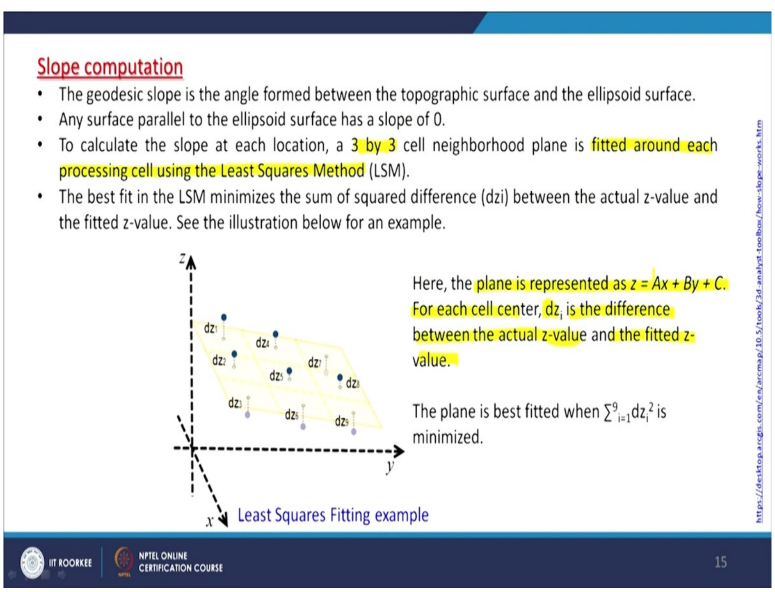

So, to transform our ECEF coordinates system to a geodetic coordinates system, that is in terms of latitude, longitude and height; the following formulae can be employed. For X, like this and for Y is like this and of course for Z is like this. So, whereas you know that further details are there for latitude, longitude and heights. You have also seen in case of ellipsoid that a represents the major axis of the ellipsoid and of course, the b than represent the minor axis of ellipsoid.

Because unfortunately, Earth is not perfect spheroid and therefore these complications come and we have to while going for more accurate and accurate slope calculation in current discussion then we have to take care about this. So, ellipsoidal height; h is in meters in above formula. Through these above formula, your input raster's z unit is specified. If any other unit, then internally transform to meters.

Because it has to be converted then only you can do. So, we go for this option, that is geodesic slope calculation, is this angle formed between the topographic surface and ellipsoid surface. And any surface parallel to the ellipsoid surface will have of course, slope equal to zero; that is the flat. And the calculate slope for each location again employing 3*3 cell kernel and also fitted around each processing cell using an LSM; that is Least Square Method.

And the best fit in this LSM, minimizes the sum of squared differences; that is dzi between the actual z value and fitted z value. And this is what we see here that the plane is represented here with the $z = Ax + By + C$. For each cell, the dz is the difference between the actual z value and the fitted z value because instead of in 2D, now we are talking in 3D. So that much care is required. And the best fitted plane is expressed like this which minimized the variance.

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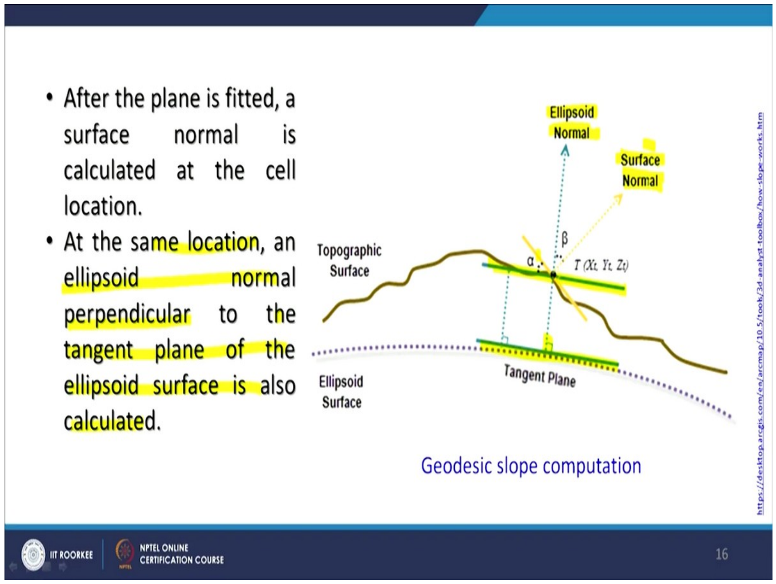

After this plane is fitted, a surface normal is calculated for each cell or at the cell location. Surface normal which is a tangent and going like this; this is represented here with blue lines. Now surface normal is this one when again, it is taken as above this level; topographic surface and then it is done. So, after plane is fitted, a surface normal is calculated at each cell location and in the same location, an ellipsoid normal perpendicular to the tangent plane of the ellipsoid surface is also calculated.

So here, one normal, ellipsoid normal. Here another normal which is surface normal, so, all these have to be calculated.

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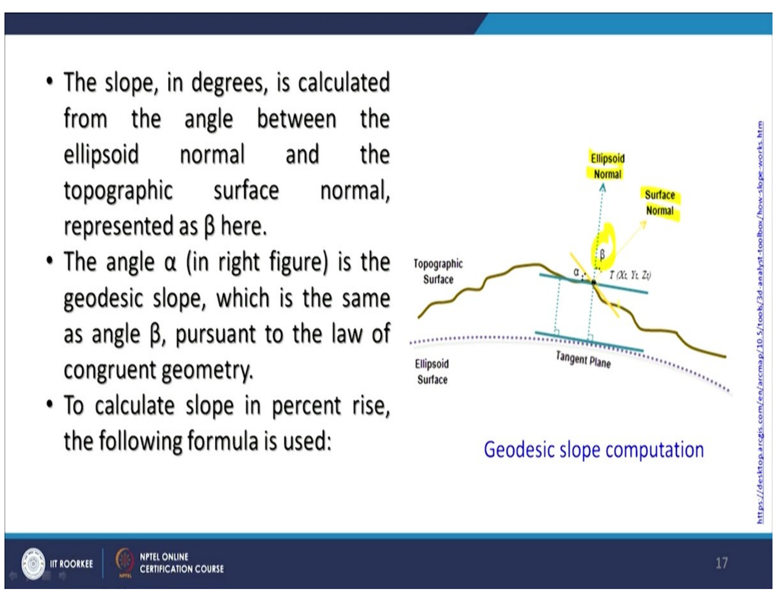

Now the slope in degrees is calculated from the angle between the ellipsoid normal and the topographic surface normal, is this one which is the β here. So, this angle α is the same angle as β pursuant to law of congruent geometry. And to calculate the slope in percentage wise of course, the formula would be little different. You have to multiply by hundred to get that one.

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Horn Method	Elevation (m)	Slope (max)	STD (relative value)
Everglades	28.1161 – (-2.20066)	8.63692	3.19169
San Francisco	399.963 – 2.29147	126.518	54.2024
Grand Canyon	2461.65 – 547.224	1375.69	596.265

Fleming & Hoffer	Elevation (m)	Slope (max)	STD (relative value)
Everglades	28.1161 – (-2.20066)	9.87327	3.31226
San Francisco	399.963 – 2.29147	159.937	69.4942
Grand Canyon	2461.65 – 547.224	1740.24	763.704

Now 3 examples are shown for different types of terrains of USA and with 3 examples and 2 different methods; that is Horn method and this Fleming and Hoffer method and what we see that elevation range, it is showing like this. Slope maximum is this and relative value is this. Similarly, for same different regions, the values are varying, slope of course will vary and STDs are varying.

Now if we compare this slope maximum with this slope maximum obviously earlier, I also discussed that in the previous lecture, the results are going to be a little different. Similarly, here that this deviation so relative value here is this. It is also having little different values. The purpose here to show this comparison because the terrain is different. Methods might be same but the results would be different because our algorithms which are being employed are completely different.

If we employ this diagonal ritter and simple method then again, these results are having huge difference here; 8.4, this is 13.36 and standard deviations, related values again 3.09 and 5.02. So, different for different terrain; this is the point which I have been mentioning. If lots of variations are present, results are going to be different. If less variation is present, results are going to be different. No matter what method you employ.

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Slope tool	Elevation (m)	Slope (max)	STD (relative value)
Everglades	28.1161 – (-2.20066)	84.0349	31.0543
San Francisco	399.963 – 2.29147	160.877	51.0929
Grand Canyon	2461.65 – 547.224	1422.6	545.238

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And last one; the slope tool which is available in ArcGIS in the spatial analysis extension or toolbox, there it may give different calculations. So, with this, I end this discussion by repeating only one important thing is no matter whichever the method you employee, you first take care about the z factor. If you do not take z factor care and you just go as default which is 1; that mean the system will assume that your x,y scale and z scale are same.

If in input digital elevation model, these are not same then you should convert one to another. And for India, already average value has been calculated and that is 0.0000089. If you provide that z factor, you may get better results than going for 1, which obviously will give bad result. Visually, you may not see much difference but when you see based on individual cells or zoom it and then compare it, definitely you would get different results.

Similarly, if you employ for different methods like Horn method, Javen Berger and Thorne method or simple slope tools or any other tool like diagonal ritter, you will get definitely different results because the based on the algorithm which they are using, are little different. So, with this, I end this discussion. Thank you very much.