Remote Sensing Essentials
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Module No # 06
Lecture No # 28
Frequency Domain Fourier Transformation

Hello everyone and welcome to new discussion in remote sensing essential course. And today we are going to discuss frequency domain Fourier transformation. Early we have also discussed spatial domain filtering or spatial filtering. So instead of spatial domain there is another way of doing filtering by first transforming into Fourier domain and then doing whatever the filtering we want to do it.

And then again backward transformation to spatial domain. So by doing this thing we can also achieve very good results and there in a in this you know this discussion we will I will be also showing some examples from it.

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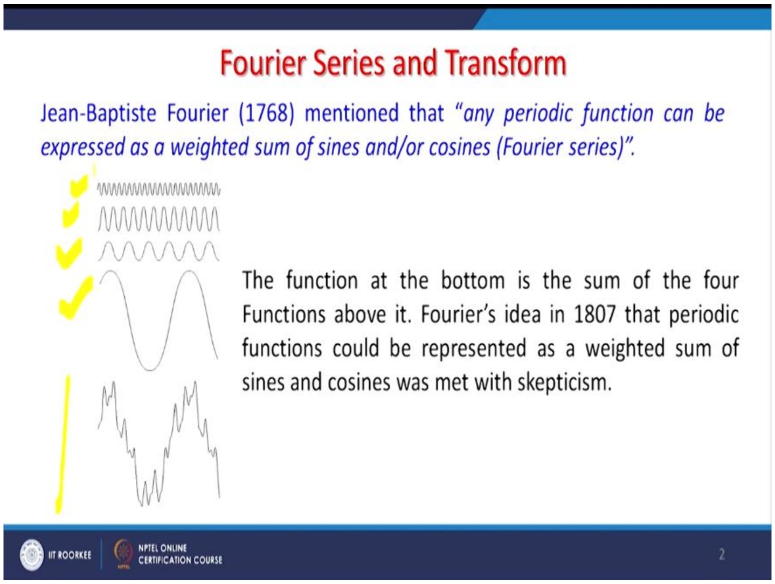

As you recall that when we have being discussing the spatial filtering at that time similar kind of curve came that bottom when which you are seeing here is the combined curve. But there by using filtering techniques single dimensional or linear filtering techniques. It is possible to you know take out all these frequency or waves separately and this is what it is shown here. That this is the input or the original wave and then this one and that one becomes a your output after you know filtering.

So this this Jean Baptiste Fourier build back in 1768 introduce this one that by identifying that any periodic function which we are seeing in form of waves can be expressed as a weighted sum which is the bottom one representing or sine’s and cosines or Fourier series. So having this concept in the background it is possible to you know filter out many things from original wave. And this is what the that the bottom image is the sum of the four functions which are shown above.

And this Fourier idea which was of course propagated in 1768 later on in 1807 the that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

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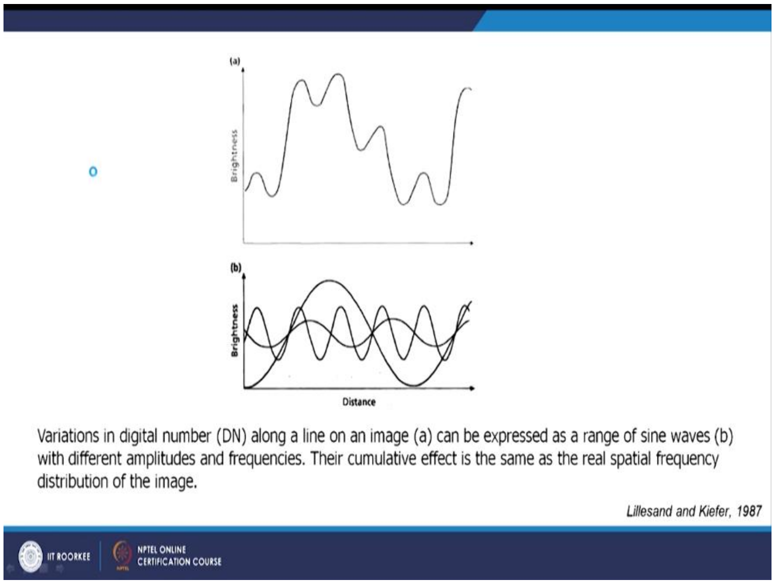

Means people did not believe initially they were not accepting but later on it was as accepted. This figure we have discussed why discussing the spatial filter in techniques or spatial domain filter in techniques. Where the top one filter is somehow the 3 waves which are shown in the bottom. So the bottom one is the cumulative effect the top one is cumulative effect of the 3 waves which are shown in the bottom image.

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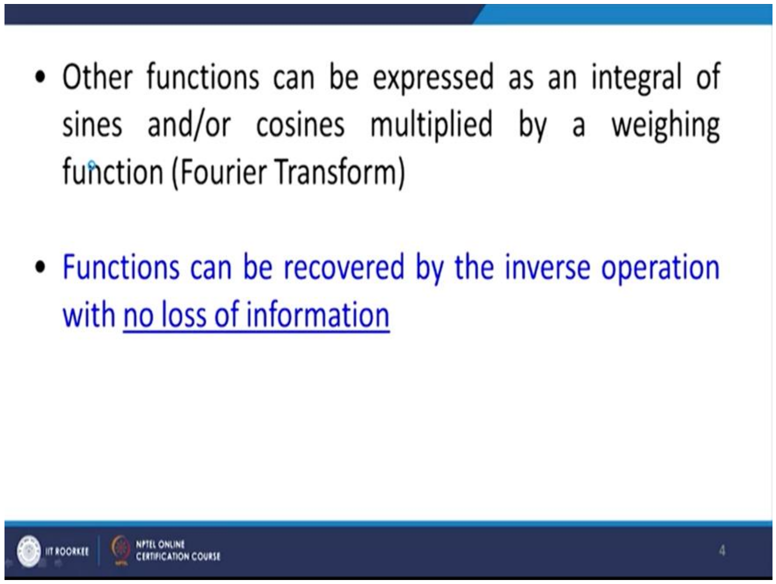

Other functions can be expressed as an integral of sine's and cosines multiplied by a weighing function or Fourier transform and the functions can also be recovered by the inverse operation with no loss of information. That means that If I put an image in the which is originally in the spatial domain and transform it through the a for in Fourier transform and whatever is the modification or filtering or anything which I want to perform I can do it Fourier transformation and again I can you know come back in the spatial domain.

And of course they will not be any loss and to the information of or the image but only thing the filtering can be performed very efficiently. And of course the main application of this Fourier transformation or frequency domain filtering technique is in to enhance our images like a spatial filtering techniques. And a image or natural scene can be regarded as a you know as a reconstructed from a spectrum of sin waves with different directions and wavelength and amplitudes.

After all in image is made from these things and such a reconstruction as known as Fourier synthesis. So an image can be converted into this and then again backward thing can be done that this as simply of an image into a family of sin waves in a Fourier analysis. So in sports lines bars that form the targets of the test of a cutie and form images to are themselves made of the sin waves.

So a small dots and a Conley space lines are drawn nominated by high amplitude at high frequency. And this is why they are difficult to recognize in the images or later on in the waves. So sin the size increases, a high frequency component will become lower in the amplitude and those in the frequency range to which the high is attenuated increase. And this way the perception and distinction becomes easier.

And beyond this however very low frequency components increase amplitude effectively degrading perception and distinction. So basically if you the call the original discussion which we started on image enhancement particularly the main purpose here ultimate purpose is to improve the image in such a way that our you know the interpretation or the perception or distinction between the different objects becomes much easier.

So this is of course a Fourier transformation is the mathematical technique which is separating an image into various frequency component is Fourier and that whole thing is called Fourier analysis.

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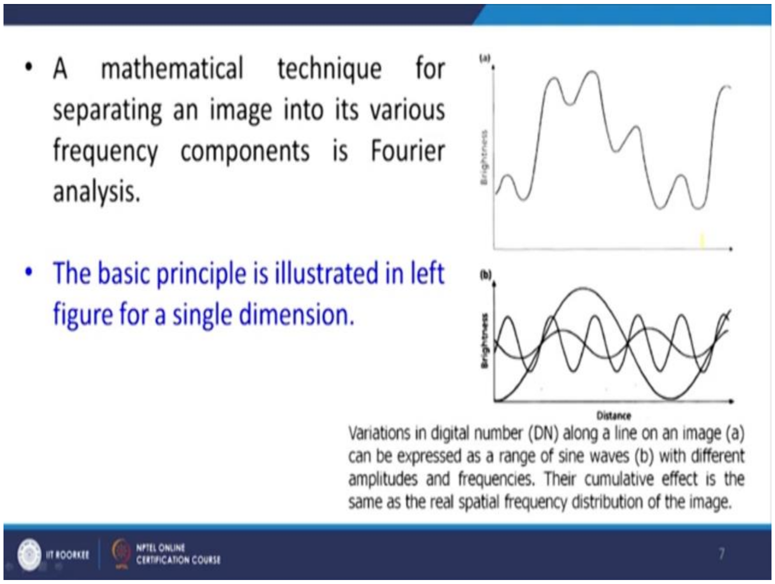

And the basic principles as we have already discussed that in a from a one accumulative image these component can be separated.

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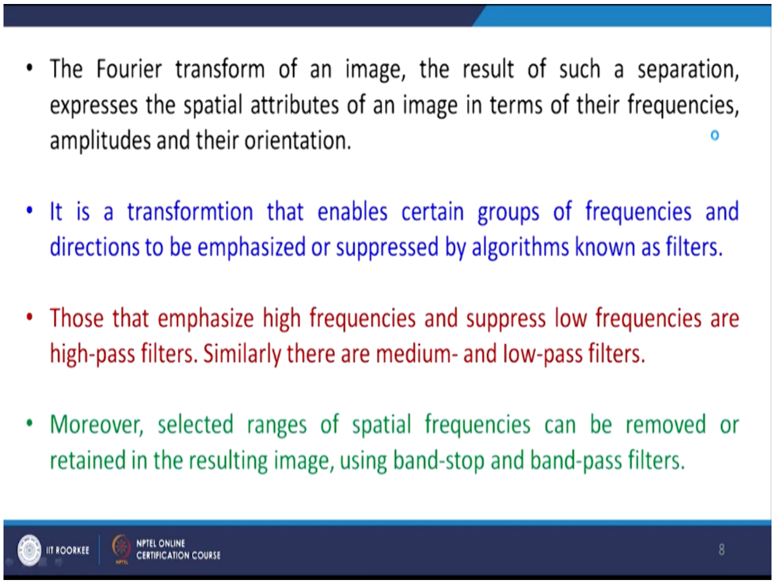

And whichever we want to get rid of may be noise and then we construct the original thing. So Fourier transform of an image when we do it so this wave scenario is a single dimensional whereas our images are in a spatial domain and they are 2 dimensional. So Fourier transformation image which is result of a separation expression expressionist the spatial attributes of an image of their frequency amplitude and orientation. Basically about a construction of a image using pixels.

And it is a transform that enable the Fourier transform basically enable certain groups of frequencies and direction to be emphasized or suppressed by algorithm know as filters. So when say a spatial domain image is transform into frequency domain image then it is possible using by some algorithms or some techniques we can remove certain things we can emphasize certain things like we have been doing in a spatial filtering and then those things can be suppressed or emphasize or later on we can come back again in spatial domain.

So those that are emphasize high frequencies and suppress low frequency or high pass filter. Similarly in spatial domain also when we would like to highlight the local variation present in the image and we want to suppress the regional variations then we use the high pass filter. And in reverse also when we want to highlight the regional features and we want to suppress the local feature then we say low pass filters.

So similar thing here with that which emphasize high frequency and suppress the low frequency or high pass filter and the lowest approach is the low pass filters. More over selected ranges of spatial frequencies can be removed completely may be noise or retain in the resulting image using band stop and band pass filters. So similar kind of filters are also there in frequency domain too.

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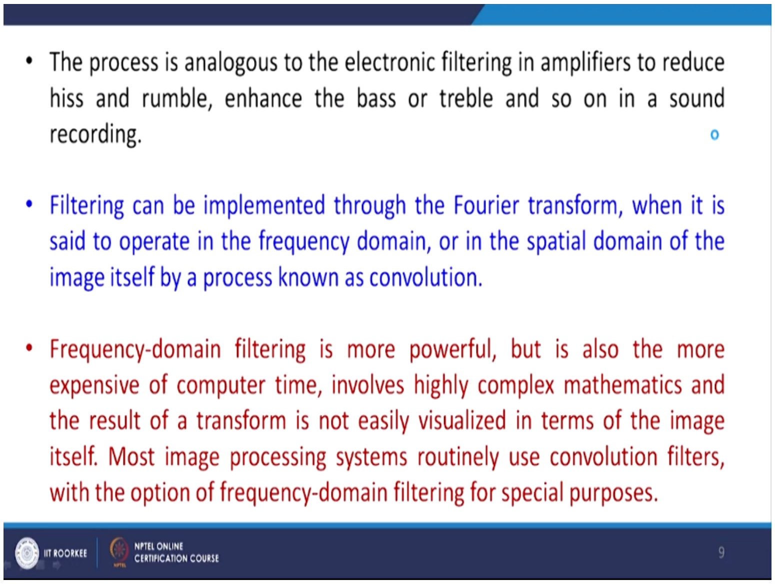

And this process is analogous to the electronic filtering in amplifiers in a electronic devices to reduce certain noises like hiss and rumble enhance the bass or treble and so on in a sound recording. So when we turn these nobs this is what because the information is already data is already in the frequency domain. But incase of images original images are satellite images which we are talking which we want to enhance or in spatial domain.

So first that is step that from spatial domain to frequency domain as to be performed and filtering can be implemented through the Fourier transform when it is said to operate the frequency domain or in spatial domain of image itself by process there is in spatial domain the process is we say is conversion convolution filtering or a spatial filtering. And a frequency domain filtering is more powerful but is also the more expensive of compute time it takes lot of time.

Because first 2 times the transformation has to happen because the first from spatial domain to frequency domain then you do the filtering and then come back again in reverse transformation backward transformation from frequency domain to its spatial domain. And therefore it consumes lot of computer resources nonetheless it is sometimes on certain images we find that it is better than the spatial domain filtering.

And of course the mathematical part has a complex as compared to spatial domain and the results of transformation is not easy easily visualize in terms of image itself. Unless it is transformed back from frequency domain to spatial domain. And most image processing systems this support convolution support easily but if you want you do this spatial frequency filtering or frequency based filtering on Fourier transformation maybe that may also be supported. But that is not a common thing because it requires lot of coding and due to the complex mathematics.

So instead of using a spatial domain that is a x and y coordinate space of image row number and column number and alternative coordinate space can also be used for image analysis that is in frequency domain. And this approach image is separated in two various spatial frequency components through application of the mathematical operation knows as Fourier transformation. And conceptually this amounts to fitting a continuous function through discrete pixel values if they were plotted along each row and column in the image.

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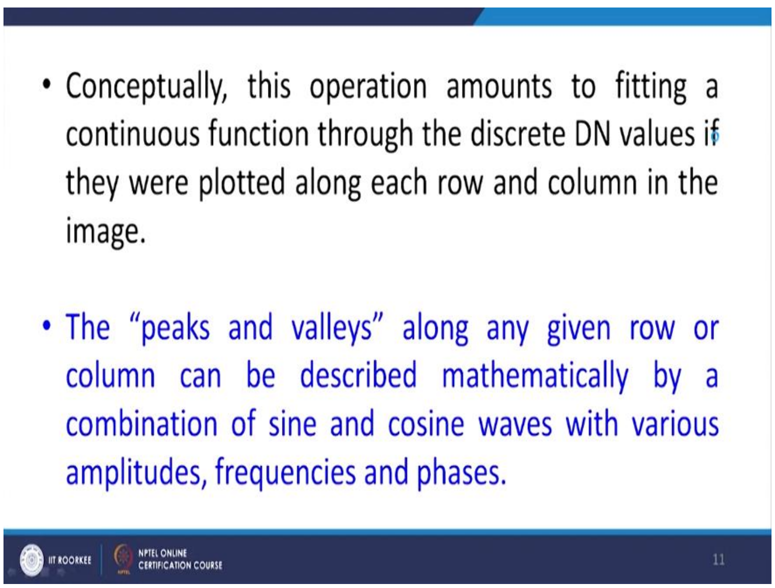

So when we find this peaks and valley along any give row and column can be described mathematically by a combination of sine and cosine waves with various amplitudes frequencies and phases because we will be getting a cumulative effect and that we can later on filter it. So frequency domain basically when we say frequency domain reverse to the plain of two dimensional discrete Fourier transform of an image and the basically the purpose of the Fourier transform is to represent a signal as a linear combination of sinusoidal signals of various frequencies.

They are you know different type some different types of you know functions or Fourier transforms and the frequency domains one we consider as one dimensional and the mathematics is given here.

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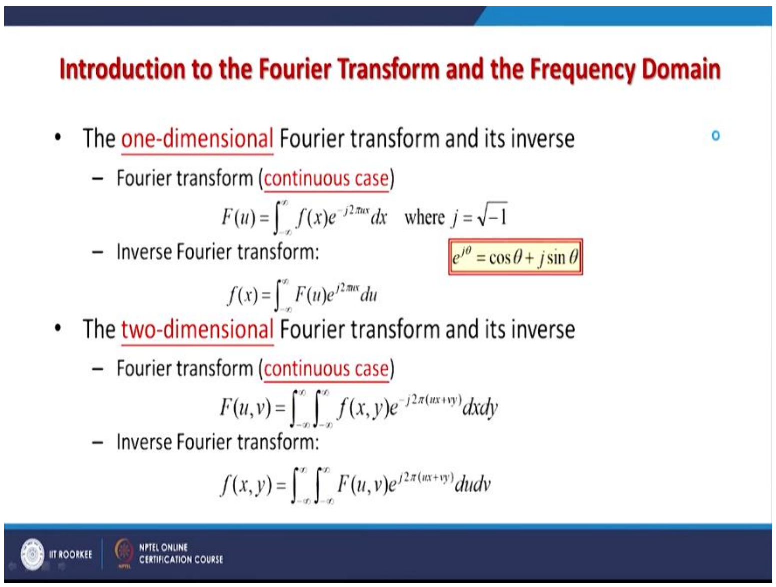

And the first step is for Fourier transform for continuous case and then inverse of course has to be done. So first forward and backward transformation and 2 dimensional Fourier transform and its inverse so the mathematics parts is here that the Fourier transform of continuous case again but in case of 2 dimensional.

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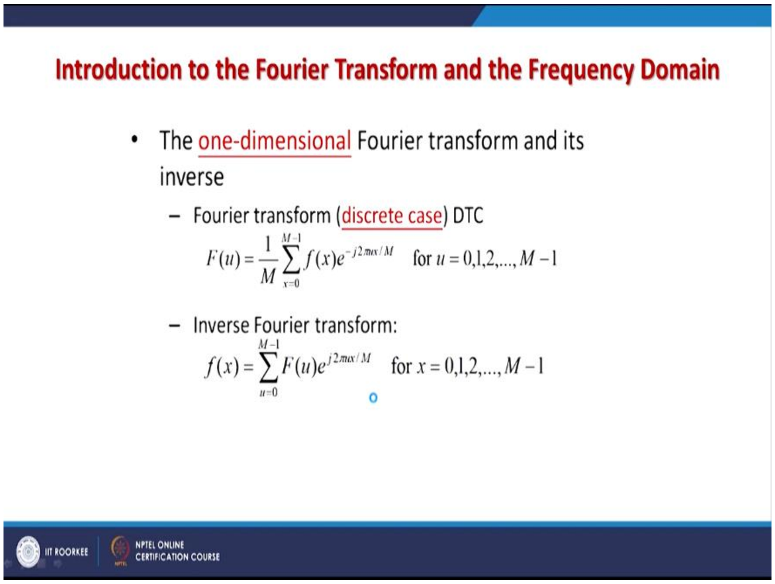

Further the 1 dimensional Fourier transform and its inverse is discrete rather than continuous case is also possible.

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- After an image is separated into its component spatial frequencies, it is possible to display these values in a two-dimensional scatter plot known as a Fourier spectrum.
- The lower frequencies in the scene are plotted at the centre of the spectrum and progressively higher frequencies are plotted outward.
- Features trending horizontally in the original image result in vertical components in the Fourier spectrum; features aligned vertically in the original image result in horizontal components in the Fourier spectrum.

And once image is separated that is from spatial domain into components to the frequency it is possible to display these values in a 2 dimensional scatter plot known as Fourier spectrum. As you can also recall in the two dimensional histogram or scatter plot almost similar thing here that you see different pixel values. And what we see in these scatter plot that a lower frequency in the scene or image are plotted at the center of the spectrum when you go away from the center progressively higher frequency or plotted outward and that way you get a Fourier spectrum.

This the features basically trending horizontally in the original image results in a vertical components is a sort of reverse plotting in Fourier spectrum and features which are align vertically in original image result in oriental components in the Fourier spectrum.

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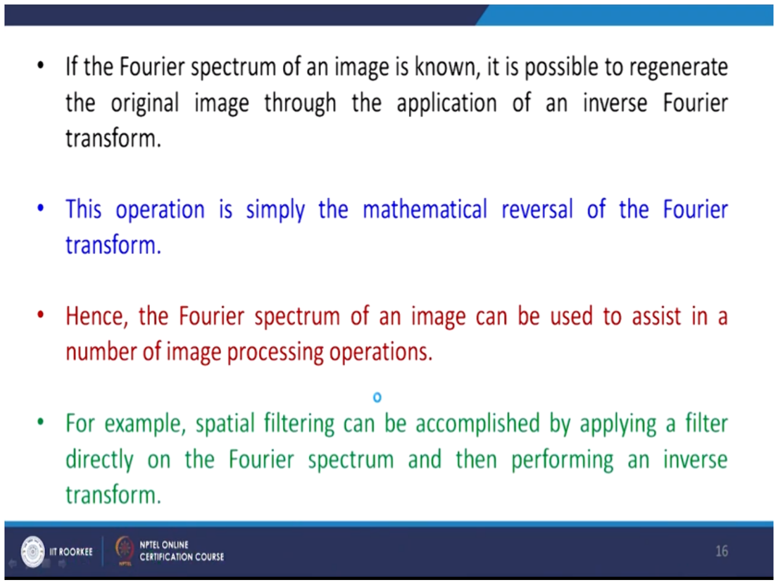

And if the Fourier spectrum of the image is known that has to be first created in frequency domain filtering or Fourier filtering. It is possible to regenerate the original image through backward transformation application of this inverse Fourier transform is simply the mathematical reversal of the Fourier transform. And therefore the Fourier spectrum of an image can be used to assist a number of image processing software.

The main application of Fourier transformation is basically for filtering though a spatial filtering can be accomplished by applying filtering directly by Fourier transform and then performing an inverse transform.

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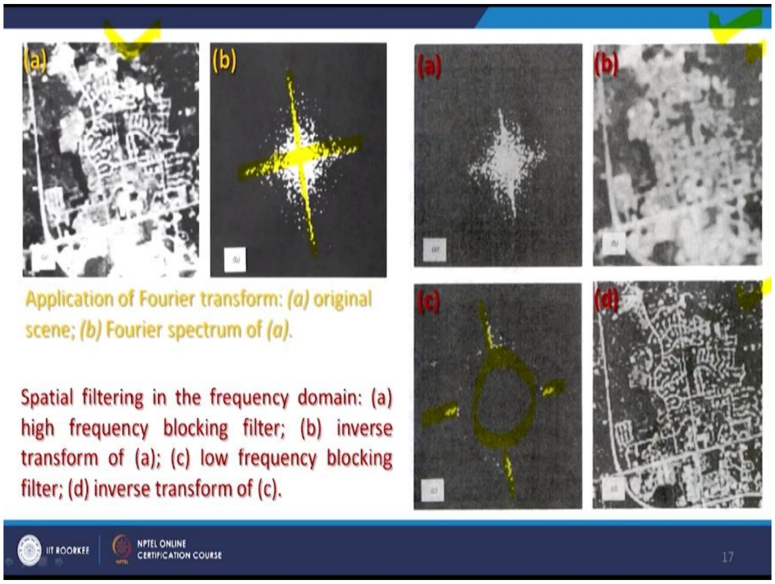

Now we will take some example here on the left me on the left side of figure A what you are seeing an image which is the original scene. And when we create a Fourier spectrum this is what we see. So you know the features which are north south direction are implemented here are represented here in the near horizontal direction and the features which are near horizontal in original image are presented in a vertical direction.

And then as a recall that low frequency feature will be coming in the center of the Fourier spectrum and high frequency feature will be will get plotted outside or away from the center. Now if I want to perform a filtering so the b one is the original Fourier spectrum. And the here the Fourier spectrum a in this example the same images basically here the inner part has been dropped or filtered out. And what we see that b represents here the image and d represents the output.

And therefore you know this directional things which were either oriented roughly east west or north south where highlighted in the image where the rest of the features have been suppressed. So in a spatial you know filtering high frequency blocking filter and b is of course a inverse transform of a in this one and a is the low frequency blocking as low pass filter blocking transform and then of course inverse transformation. So original image this is an no sorry the this a is original image here, b is the spatial filtered image and a is a basically a or this b almost are the same.

And then the c you are having a low pass frequency filtering and d is final output. So likewise, you can do filtering but it takes lot of time and care has to be taken while performing the filtering. So frequency filtering are there are these steps very quickly we will go as also shown to you the (()) (20:08) figures. So first originally images is transformed into its frequency representation using a Fourier transform and creating a Fourier spectrum.

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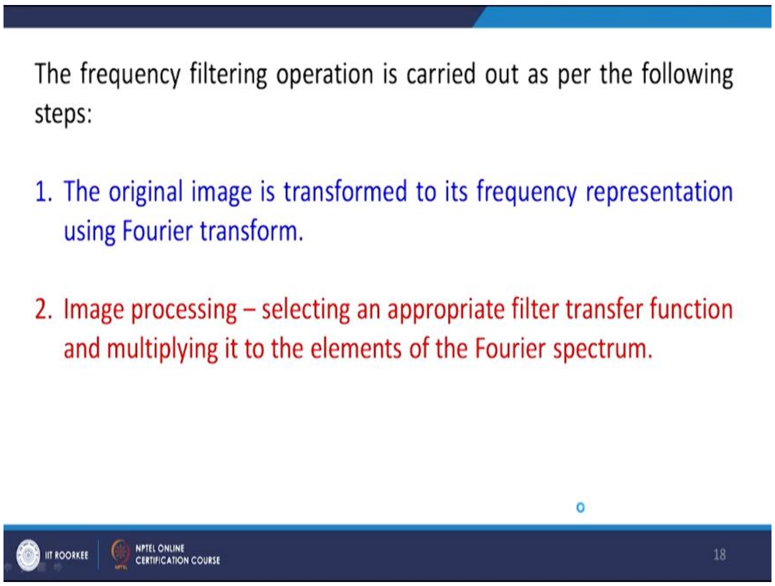

And by implying image processing selecting an appropriate filter in the example we selected this a low passing and filtered transform function and multiplying its elements of the Fourier spectrum. And of course finally once the filtering has been done then inverse transform function or Fourier transform is performed to return to the spatial domain for display purpose.

If you recall that there will not be any loss of information. If a if a simply suppose somebody transform its spatial domain to frequency domain and no change no filtering is done and later on if we do the backward transformation there should not be loss to the image quality. And this is what is done except that the middles stage when it is a we are having a Fourier spectrum at that time filtering either low pass or high pass filtering can be applied. And then inverse transformation and we can have a better results.

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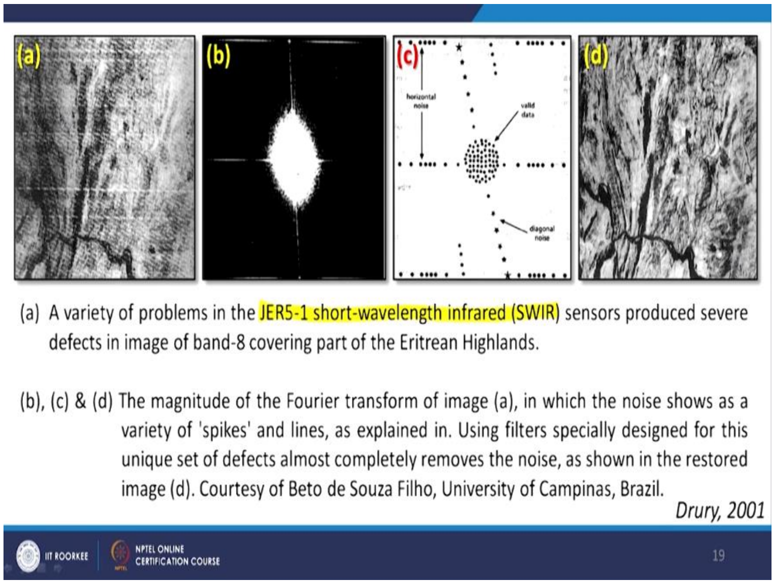

Few more examples are here and this is short wavelength infrared image of JER5 satellite and what you are seeing an image a there is lot of noises is there and how to remove that noise is a very quick way or very best possible way rather than implying its spatial filtering we can apply this spatial frequency domain filtering and can create a Fourier spectrum like B which we are seen.

As you can see that because there are lots of stripes are there and therefore vertical line in the Fourier transform spectrum you are seeing roughly not south lines or no south plotting and also some east west. And if we if we remove those parts and consider that this is our valid data rest is you know this is as mentioned here as horizontal noise. It is also in the top here also that is shown here so this is horizontal noise.

And diagonal noise also might be there so the central part is consider as a valid data in during the filtering and then when this is inverse transformation is done in frequency domain spatial domain we get this result d on the extreme right image. If we compare image a verses d then you can see the significant changes in the quality of an image. The noise is which was there because of you know this dropline maybe stripes problem and maybe some calibrations with the censor all have disappeared through this frequency domain filtering.

So that is the very big advantage this kind of error which we are seen in image a cannot be removed through spatial filtering. This I am telling from my own image processing experience but it is rather easy that a one can get rid of a such errors through Fourier transformation and frequency domain filtering. So this kind of advantage which we are seeing through this image is possible.

So one can also say here that these are in some way the spatial filtering and this frequency domain filtering they are complimented to each other when we does not get the desired results specially removing noise and other such features may be because of poor calibration in the sensor arrays. Then the rescue or the resort is for us is to go in the frequency domain do the filtering and do the backward transformation from frequency spatial and get the result like the right most image here.

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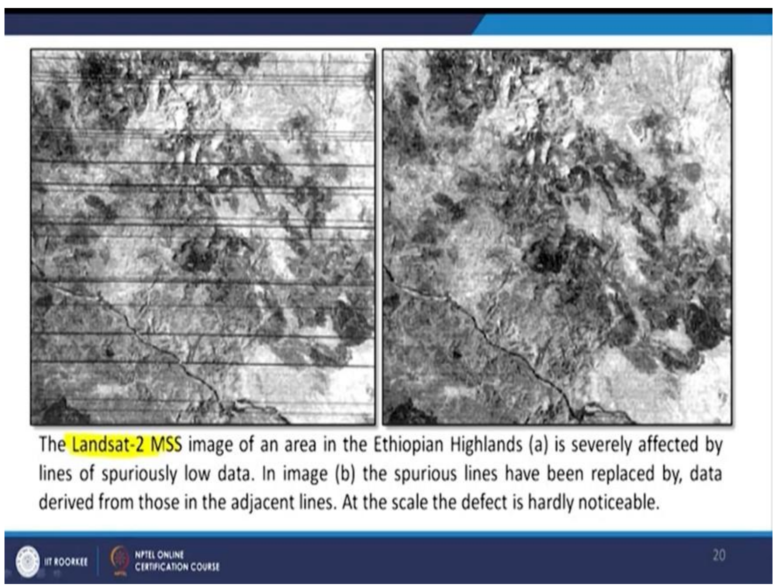

One more example shown here that was Landsat 2 MSS at that time when these satellites in 1975, 76 these census where not that reliable as todays census because electronics has improved significant in those 45 years. So these because of poor calibration or because of some cross track scanners some other problems there are lot of drop lines which we are seen in the left image the original image.

And when it is subjected to frequency domain filtering you can see that all that noise or those drop lines or a stripes effects which are seen here have completely removed that very high-quality image from the same data. So that is the advantage of a as a recall a you know that in one dimensional when you are getting a noises through your amplifier what you do? You filter certain noises and you get a clear noise. Similarly here in a spatial domain when there are noises like this on left image we can apply this frequency domain filter and can get rid of those noises.

So basically there is there cannot be for each image one has to be little innovative one has to find out a where noise has got plotted and you can get rid of that one so this one has to take care.

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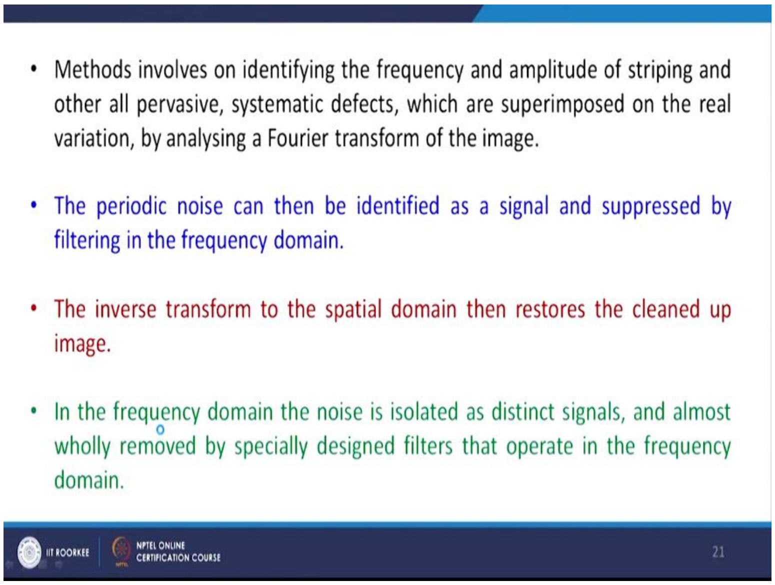

So methods which are which we involve on identifying the frequency and amplitude of striping. And all other pervasive, systematic defects, which we have seen in few of these examples are which superimposed on the real variation and why analysis a Fourier transform of an image. So this which part has to be removed that has to be carefully chosen. The filter out what should be the filter out the image. And this periodic noise like a stripping can then be identified as a signal and suppressed by filtering in the frequency domain.

And once it is done then inverse transformation from frequency domain to spatial domain and restores the original image without having the stripping effects or noise effects. So in the frequency domain the noise is isolated to distinct signal and almost wholly removed fully removed by spatial designed filter and that operates in the frequency domain.

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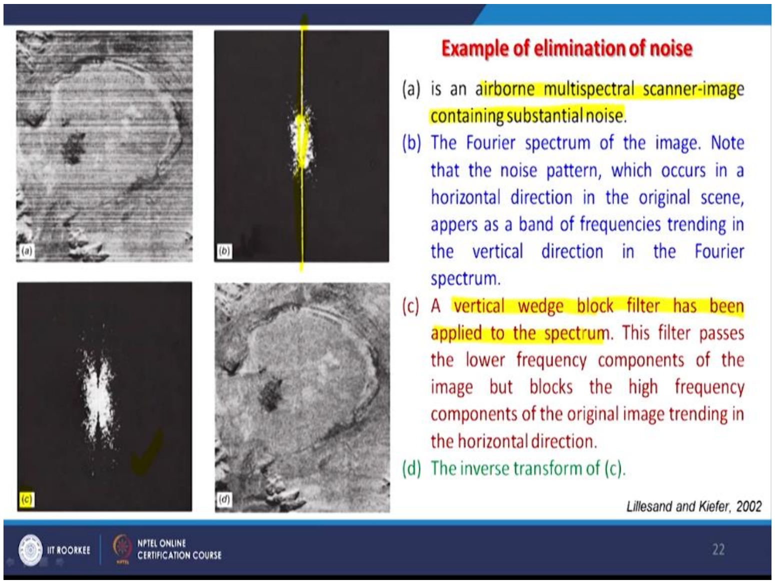

One more example here too many you know too many these lines are stripping example is there on the top left image and therefore in a Fourier spectrum you are seeing vertical line. These lines are plotted in Fourier spectrum opposite to what you see in the images this one has to remember. And a now we know that these vertical in the Fourier spectrum these vertical components has to be removed through filtering and this is what is done in c.

In figure c this is you see that this horizontal line in this image or vertical plots in the Fourier spectrum is removed and when the backward transformation is done and then the entire horizontal stripping effects in the image has completely disappeared. So this is an example of airborne multispectral scanner image which is containing substantial noise as you can also realize.

And this vertical badge which you are seeing has been blocked filtering and applied to the Fourier spectrum and this filter passes through a low lower frequency components of the image but blocks the high frequency components. Remember that low frequency components would be in the center. High frequency components would be outside. And while inverse transformation you will get basically the finally the d image.

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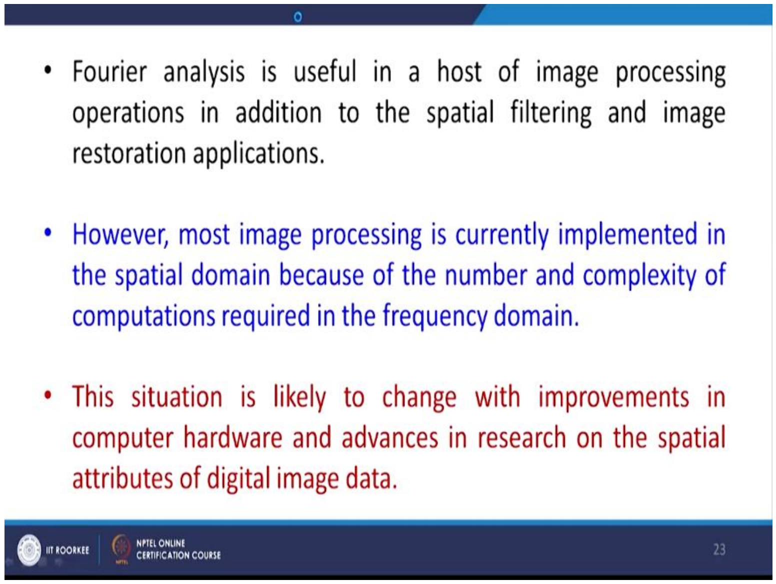

So Fourier analysis is useful as you must realize by now in many of image processing operation especially in filtering or image restoration also it is can be applied. However also one can realize that it is not a easy to implement because of complex mathematic and consequently complex computer programing. Nonetheless very popular and very powerful image processing software's have all implemented this and you may find those functions available with those software's.

And this is if a today some software's are not supporting frequency domain you know transformation filtering and backward transformation I am sure (()) (30:50) time in your future they will they two will implement this thing. Because this I said that in someday it is complement to spatial filtering it is not a competitive filtering technique but it is a complimentary filtering technique because the power it is having to remove these you know strives or knowledge in the image is much more useful much more powerful than doing in the spatial domain. So this bring to the end of this discussion. Thank you very much.